Mathematics Analysis and Approaches

Diagnostic Exam 1

NO CALCULATOR PERMITTED

1. Consider the following function, $f(x) = \frac{2x-3}{x+2}$

   a) State the equation of the:

      a. Vertical asymptote

      b. Horizontal asymptote

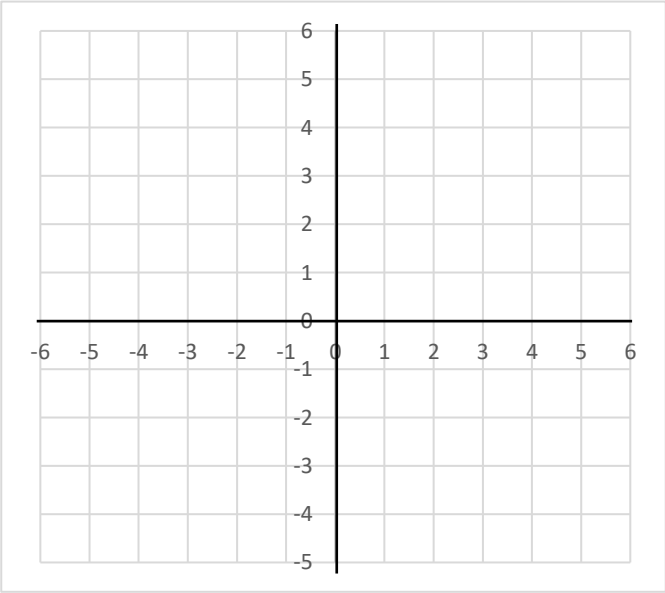
   b) Hence, state the domain and range for $f(x)$

   c) Find the value of the $y-$axis intercept

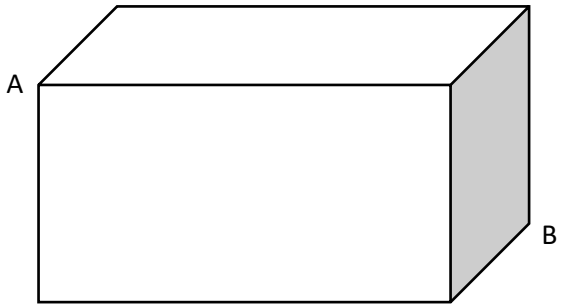
   d) Find the value of the $x-$axis intercept

Working and solutions:

   e) Sketch the graph of $f(x)$ ***without the use*** of a graphical calculator.



2. Determine the distance between A and B assuming the length of the box is 10 cm, the width is 2 cm and the height is 4 cm. Express your answer in the form $a\sqrt{b}$, where a and b are integers.



Working and solutions:

3. Private school tuition is 4000 EURO for the first year, 4300 EURO the second year, 4600 EURO the third year and so forth.

   a) Calculate the cost of tuition in the 12^{th} year of schooling.

   b) Calculate the total cost of a 12-year education.

Working and solutions:

4. Given that $f(x) = 4^x$ and $g(x) = 2x + 4$

Find

a) $f \cdot g(-1)$

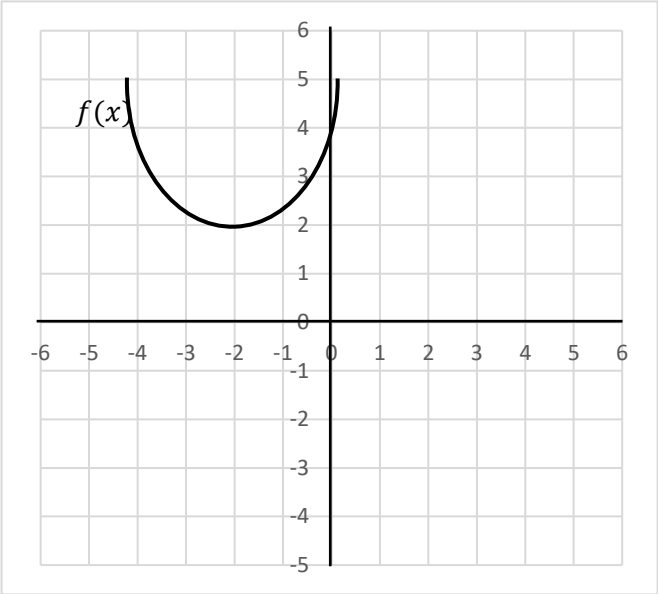
b) $f^{-1}(x)$

Solve:

c) $f(x - 3) = 64$

Working and solutions

5. Consider the following function, $f(x)$



a) The function undergoes transformations so that $g(x) = f(-x) - 5$

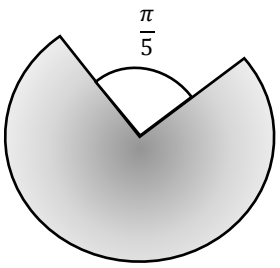
   i) Draw $g(x)$ on the graph above

   ii) Fully describe this transformation

b) The function $g(x)$ can be expressed in the form $g(x) = a(x-k)^2 + h$. Find the values of a, k and h.

Working and solutions:

6. This diagram, not to scale, shows a portion of a circle. The area of the shaded portion is $10\pi \text{cm}^2$. The value of the angle is as shown:



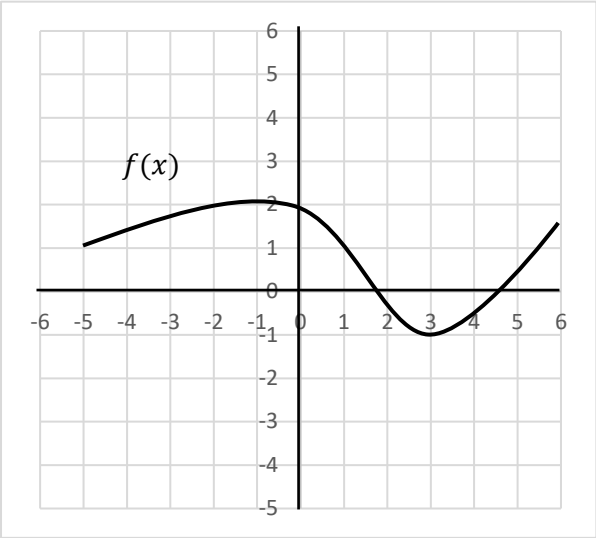
Find the perimeter of the shaded portion. Give your answer in terms of π.

Working and solutions:

7. An equation is such that $f(x) = 2x^2 - 2kx + 2$. Determine the values of k for which this function will have two equal roots.

Working and solutions:

8. Consider the following function, $f(x)$



c) Sketch each of the following transformations on the figure above. Label them clearly. Also, **describe the transformation** for each.

i. $g(x) = 2f(x)$

ii. $h(x) = f(2x)$

Working and solutions:

9. Make a detailed sketch, labelling all key points and stating the equation of the asymptotes, for

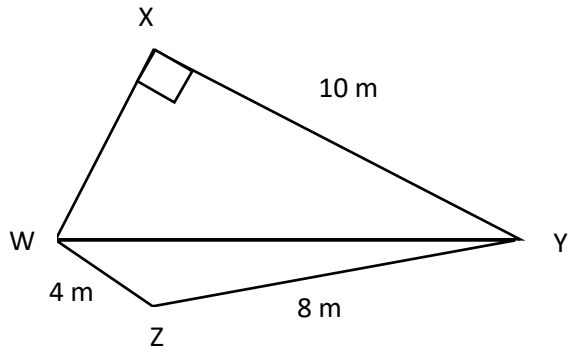
$f(x) = \frac{1}{x-3} + 2$, where $-5 \leq x \leq 8$

Working and solutions:

10. Find the value of the constant term in the expansion of: $(\frac{2}{x} - 3x^2)^6$

Working and solutions:

11. Two adjoining gardens have the following dimensions:



Given that $\mathrm{W\hat{Z}Y}$ is $120°$, determine the length of WX

Working and solutions:

12. A linear equation, L_1 is expressed as $2x - 4y + 10 = 0$
Determine the equation of the line, L_2 which runs perpendicular to L_1, and passes through (2, 8)

Working and solutions: