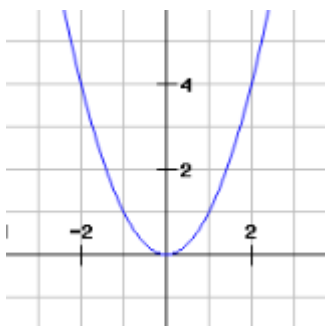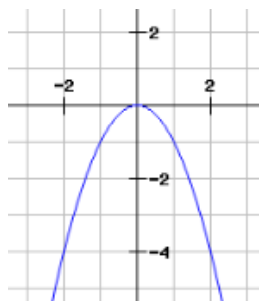## Quadratic Equations

Part 1. Factorization

Simply put, these are the equations that have an x^2 term in them.   They can also be called parabolas.
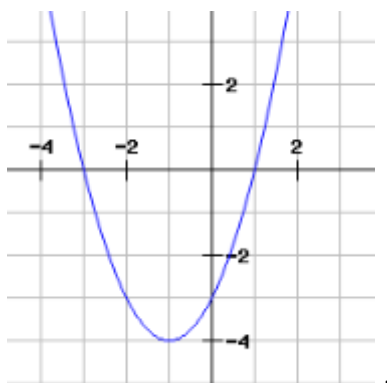
The simplest one is $y = x^2$:

The ones that have a negative, ex. $y = -x^2$ term are inverted:

They can be more complicated of course. They are often written in the form: $y = ax^2 + bx + c$. An example is: $y = x^2 + 2x - 3$:

For this example, $a = 1, b = 2 \text{ and } c = -3$.

Find the values for a, b and c for the following equations. Also note whether it would face upward or downward when graphed:

1. $y = x^2 - 5x + 6$

2. $y = 5x - 9x^2$

3. $y = x^2 - 4$

4. $y + x^2 = 3 - 6x$

There are many different types of exam questions that deal with quadratic equations.

To begin with, you are often asked to **FACTORIZE** them. This can be done in a few different ways. The simplest is **factorizing by inspection.**

Example 1:

$$y = x^2 + 7x + 12$$

Step 1.

Make two sets of brackets. Since the first term is $'x^2'$ put an 'x' into the first position in both sets of brackets. Why? Because x times x is the only way to get x^2!! Easy!

$$(x \quad)(x \quad)$$

Step 2.

Consider the ***last term***, **'c'** of the equation.   In this example, it is '**+12**' (yes, the sign matters). Find all the factors of '+12':

$$1 \times 12, \quad 2 \times 6, \quad 3 \times 4$$

Step 3.

Consider the middle term, b, '**+7**'. Which of the sets of factors above will sum (think ***combine***) to give '**+7**'?

$$+3 + 4 = +7$$

Step 4.

Put the values you have found, **+3** and **+4** into the brackets (order doesn't matter):

$$y = (x + 3)(x + 4)$$

YOU HAVE NOW FACTORIZED THE EQUATION!

Step 5.

Check your answer. The technique is called 'FOIL'.

F for FIRST: Multiply the **first** values in each bracket:

$$(x + 3)(x + 4) \; so \; x \; multiplied \; by \; x \; = \; x^2$$

O for OUTER: Multiply the **outer** values together

$$(x + 3)(x + 4) so \; x \; multiplied \; by \; '+4' \; = \; +4x$$

I for INNER: Multiply the **inner** values together

$$(x + 3)(x + 4) \; so +3 \; multiplied \; by \; x \; = \; +3x$$

L for LAST: Multiply the **last** values together

$$(x + 3)(x + 4) so +3 \; multiplied \; by +4 = \; +12$$

Now put them together : $x^2 + 4x + 3x + 12$, and then simplify: $y = x^2 + 7x + 12$.

This question has been correctly factorized because I got back to the correct equation when I checked my work.

Try to factorize the following equations (make sure they are written in the form: $y = ax^2 + bx + c$ first). Include FOIL to check your answer. With practice, you will get faster at quickly spotting the correct factors.

1. $y = x^2 + 6x + 8$

2. $y = x^2 + 2x + 1$

3. $f(x) = x^2 + 13x + 36$

4. $y = x^2 + 11x + 10$

5. $f(x) = x^2 + 10 + 7x$

They get more complicated, because they often include negative signs:

Example 2:

$$y = x^2 - 5x - 6$$

Step 1.

Start with brackets as before:

$$y = (x \quad)(x \quad)$$

Step 2.

Find factors of '-6' (BE CAREFUL WITH THE SIGNS!):

$$-1 \times 6 \ \ or \ 1 \times -6,$$

$$-2 \times 3 \ \ or \ 2 \times -3$$

Step 3:

Find the factors that sum (think *combine*) to give b, '-5'. BE CAREFUL! The negative sign makes a difference:

$$-1 + 6 = +5, \ \ (Incorrect, b = -5).$$

$$+1 - 6 = -5. \ \ (Correct)$$

Step 4:

Fill in the values from step 3: $(x + 1)(x - 6)$. This is now factorized.

Step 5.

CHECK YOUR WORK:

First : x^2, Outer: $-6x$, Inner: $+1x$ and Last: -6.

$$y = x^2 - 6x + 1x - 6$$

$y = x^2 - 5x - 6$, so, the factorization is correct.

Try to factorize the following. Again, use FOIL to check your work.

1. $y = x^2 - 3x - 10$

2. $y = x^2 - 11x - 12$

3. $f(x) = x^2 - 8 - 2x$

4. $y = x^2 + 4x - 12$

5. $y = x^2 - 4x + 4$

6. $f(x) = x^2 - 9x + 8$

7. $y = x^2 - 2 + 1x$