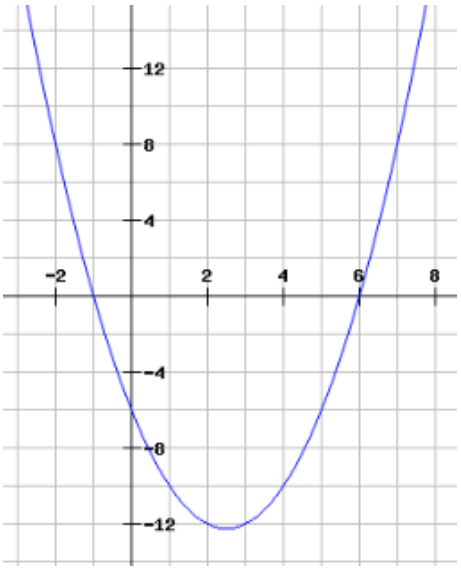
Quadratic Equations

Part 2. Finding the Roots, the Axis of Symmetry, and the Vertex

The reason ***why*** we factorize is to easily find the ***roots*** of the equation.

The ***roots*** are the ***values for x*** where the function ***crosses the x-axis (the x-axis intercepts)***. This is when $\boldsymbol{y = 0}$

This is called **SOLVING** the equation:

ME	**YOU**
We factorized $y = x^2 - 5x - 6$ and got $y = (x + 1)(x - 6)$.	Now factorize $y = x^2 - x - 12$ and you get: $y =$
Now to solve to find the roots we substitute 0 for y. I am also turning the equation around, because I prefer to have the x's on the left-hand side: $$(x + 1)(x - 6) = 0$$	Substitute 0 for y to SOLVE the equation:
Now, each set of brackets on its own is equal to 0. (If you want to know why, ask me during our session!). So, to find the first root: $$(x + 1) = 0$$ $$\boldsymbol{x = -1}$$ **So, the first set of coordinates is** $(\mathbf{-1, 0})$	Find the first root: **So, the first set of coordinates is**
Now the second root: $$(x - 6) = 0$$ $$\boldsymbol{x = 6}$$ **So, the second set of coordinates is** $(\mathbf{6, 0})$	Now the second root: **So, the second set of coordinates is**
I can also find where the equation will cross the y-axis. This occurs when $x = 0$. $$y = x^2 - 5x - 6$$ $$y = (0)^2 - 5(0) - 6$$ $$\boldsymbol{y = -6}$$ **(**Basically, the constant term at the end is always the y-intercept). **So, the third set of coordinates is** $(\mathbf{0, -6})$	Find the y-axis intercept: **So, the third set of coordinates is**
Now that I know the x and y intercepts it is very easy to sketch the graph. Note that I know it faces upwards since **x** is positive ($x^2 = 1x^2$ = positive number). 	Now sketch the graph, labeling each point:

The Axis of Symmetry

At this point, you can also find the ***equation of the axis of symmetry*** (the line that cuts the graph exactly down the middle). You can do this by simply looking at your graph, or by using a formula. Including it on your graphs helps you sketch it nicely.

For my example, $y = x^2 - 5x - 6$, the roots were -1 and 6.

If I look at my graph, then I can see that the midline of the graph must be halfway between -1 and 6:

$$x = \frac{(-1 + 6)}{2}$$

$$\boldsymbol{x = 2.5}$$

It is very important to write this as an equation ($x = \ the\ value\ for\ the\ axis\ of\ symmery$), since it *is* a line, after all.

If you don't have a graph drawn, or are trying to find it without having to spend time on a sketch, you can use the formula for the axis of symmetry:

$$x = \frac{-b}{2a}$$

Recall: $y = x^2 - 5x - 6$, so therefore $a = 1,\ b = -5$ (very important to include the negative sign!)

$$x = \frac{-(-5)}{2(1)}$$

$$x = \frac{5}{2}$$

$$\boldsymbol{x = 2.5}$$

As you can see you get the same answer.

The Vertex:

The **vertex** is the **maximum** or **minimum** point on the graph of the quadratic equation. Therefore, it has a set of coordinates (x, y).

The axis of symmetry crosses through the vertex, so the x value you found or calculated for the axis of symmetry IS the $x-$ coordinate of the vertex.

From our example above, the axis of symmetry was $x = 2.5$. If $x = 2.5$, this value can be substituted into the equation to find y:

$y = x^2 - 5x - 6$

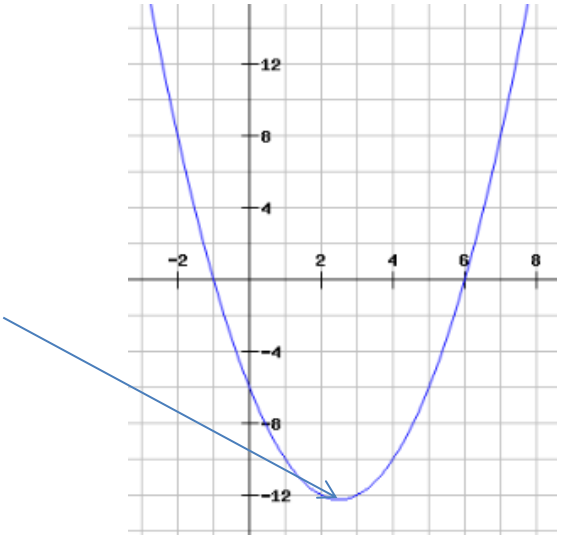
$y = (2.5)^2 - 5(2.5) - 6$

$y = 6.25 - 12.5 - 6$

$y = -12.25$

Now I know the co-ordinates of the vertex are (2.5, -12.25).

This allows me to sketch more accurately.



Try the following questions. Make a detailed sketch by:

a) Factorizing
b) Solving for x and y-intercepts (label coordinates),
c) Finding the **equation** of the axis of symmetry and
d) Finding the coordinates of the vertex (label):

1. $y = x^2 - 4x - 12$

2. $y = x^2 - 10x + 25$

3. $y = x^2 + 2x - 15$

4. $y = x^2 - 4x + 3$