Likelihood of buckling mode interaction in shape optimisation of manufacturable cold-formed steel columns

Bin Wang[1], Benoit P. Gilbert[2], Hong Guan[3] and Lip H. Teh[4]

[1] Griffith University, QLD, Australia, b.wang@griffith.edu.au
[2] Griffith University, QLD, Australia, b.gilbert@griffith.edu.au
[3] Griffith University, QLD, Australia, h.guan@griffith.edu.au
[4] University of Wollongong, NSW, Australia, lteh@uow.edu.au

1. Abstract
This paper investigates the likelihood of buckling mode interaction in shape optimisation of manufacturable cold-formed steel columns. A literature review is carried on local, distortional and global buckling mode interactions. Optimised columns found in both the literature and the research performed by the authors are analysed. Their average elastic buckling stresses are reported herein and the need of incorporating buckling mode interaction into shape optimisation algorithms is quantified.

2. Keywords: Shape optimisation; Cold-formed steel structures; Buckling mode interactions.

3. Introduction
Cold-formed steel (CFS) structures are widely used due to their advantages over hot-rolled steel, which include a high capacity to weight ratio, lightweight profiles, reduced installation time and manufacturing processes at room temperature [1]. The manufacturing processes, namely "roll-forming" and "brake-pressing", allow the formation of any cross-sectional shape. Yet, CFS cross-sections used in practice are mainly limited to "Cee", "Z" and "Σ" cross-sectional shapes, with or without intermediate stiffeners [2]. As the cross-sectional shape of CFS profiles controls the fundamental buckling modes (local (L), distortional (D) (for open sections) and global (G)), discovering innovative and optimum cross-sectional shapes is a key element in saving material and enhancing the profitability of CFS members. The need is reinforced by the recent development of a new structural design method, the Direct Strength Method (DSM) [3], which allows designing any cross-sectional shape with the same degree of difficulty.

In the literature, shape optimisation of manufacturable CFS profiles has mainly been performed by Leng et. al. [4], Wang et. al. [5, 6] and Franco et. al. [7]. In these studies, different optimisation methods were used to achieve similar objectives. Nevertheless, the capacity of the optimised profiles was always calculated using the DSM as published in the North American [8] and Australian [9] design specifications. Therefore, only local-global buckling mode interaction was considered in the DSM equations [8, 9]. In all of the above studies, various column lengths and number of manufacturing folds per cross-section were investigated.

Results from the research performed by the authors [5, 6] show that, (i) the nominal member axial compressive capacity of the optimised manufacturable cross-sections is typically governed by global buckling and (ii) the nominal distortional and global axial compressive capacities are close This indicates that global-distortional buckling mode interaction may occur. However, as the different buckling modes have different post-buckling reserves [10], close local, distortional and global capacities does not necessarily involve interaction. The aim of this paper is to quantify if buckling mode interaction needs to be considered in the design equations in shape optimisation of CFS manufacturable columns.

In this paper, the literature on local-distortional (LD), distortional-global (DG) and local-distortional-global (LDG) buckling mode interactions is reviewed. The proposed corresponding DSM equations in compression are also reviewed. The available average elastic buckling stresses of the optimised cross-sections found in the literature [4] and the authors' previous study [6] are summarised herein and the likelihood of the buckling mode interaction occurring in shape optimisation of manufacturable CFS columns is quantified.

4. Literature on buckling mode interaction
Buckling mode interaction was shown to significantly affect the post-buckling behaviour and ultimate strength of CFS members [11-13]. Yet, only LG buckling mode interaction is currently considered in design specifications [8, 9]. Numerical and experimental investigations, such as in [11-14], are currently performed to better understand LD,

DG and LDG buckling mode interactions and new DSM buckling mode interaction equations are being developed. However, as the new equations are usually conservative when no buckling mode interaction occurs, the domain of validity of these equations is currently unclear. A summary of the DSM equations can be found in [15] and in Sections 4.1 to 4.3.

CFS columns usually experience buckling mode interaction due to close values of elastic buckling stresses [15], i.e. $f_{ol} \approx f_{od}$ (for LD buckling mode interaction), $f_{od} \approx f_{oc}$ (for DG buckling mode interaction) and $f_{ol} \approx f_{od} \approx f_{oc}$ (for LDG buckling mode interaction), where f_{ol}, f_{od} and f_{oc} are the local, distortional and global elastic buckling stresses, respectively.

Silvestre et. al. [13, 16] numerically investigated the influence of LD buckling mode interaction on CFS lipped channels. The ratio between the distortional and local elastic buckling stresses (f_{od} / f_{ol}) was chosen between 0.9 and 1.1. The studies conclude that for stocky columns against distortional buckling (distortional slenderness ratio $\lambda_d \leq 1.5$) and the f_{od} / f_{ol} ratios range considered, the LD interactive compressive strength is fairly accurately estimated by the DSM pure distortional nominal capacity in compression N_{cd}. For slender column against distortional buckling ($\lambda_d > 1.5$), the LD interactive compressive strength can be estimated by the NDL approach modified DSM equations presented in Section 4.1. Young et. al. [14] and Kwon et. al. [17] experimentally tested CFS lipped channels that experienced LD buckling mode interaction despite large f_{od} / f_{ol} ratios, between 1.1 and 2.7 in [14] and 1.4 and 3.2 in [17]. In these two studies, interaction was deemed to occur due to the high yield stress of the specimens and therefore the possibility of the emergence of secondary bifurcation phenomenon before yielding.

Dinis and Camotim [12] studied DG buckling mode interaction and selected the length of the columns so $f_{od} = f_{oc}$. To avoid LD buckling mode interaction, the columns were designed for the distortional elastic buckling stress to be 20% lower than the local elastic buckling stress. This suggests that a ratio f_{od} / f_{ol} less than 0.8 is enough to prevent LD buckling mode interaction.

Dinis et. al. [11, 18] experimentally and numerically investigated LDG buckling mode interaction of CFS lipped channels and designed the profiles to ensure strong interaction with the elastic buckling stresses, f_{ol}, f_{od} and f_{oc}, no more than 3-4% apart.

4.1 LD buckling mode interaction DSM equations

Schafer [10] proposed to estimate the nominal capacity of CFS columns against LD buckling mode interaction by replacing the nominal yield capacity N_y in the DSM equations for pure local buckling by the nominal distortional capacity N_{cd}. The nominal capacity in compression N_{cld} for LD buckling mode interaction is then given as,

$$\begin{cases} \text{For } \lambda_{ld} > 0.776: N_{cld} = \left[1 - 0.15\left(\dfrac{N_{ol}}{N_{cd}}\right)^{0.4}\right]\left(\dfrac{N_{ol}}{N_{cd}}\right)^{0.4} N_{cd} \\ \text{For } \lambda_{ld} \leq 0.776: N_{cld} = N_{cd} \end{cases} \tag{1}$$

where N_{ol} is the elastic local buckling load and λ_{ld} is the LD non-dimensional slenderness ratio expressed as,

$$\lambda_{ld} = \sqrt{\frac{N_{cd}}{N_{ol}}} \tag{2}$$

Eq. (1) is referred to as an NLD approach. Yang and Hancock [19] used a similar method to Schafer [10] but replaced the nominal yield capacity N_y in the DSM equation for pure distortional buckling by the nominal local capacity N_{cl}. The nominal capacity in compression N_{cdl} for LD buckling mode interaction (referred to as the NDL approach) is then given as,

$$\begin{cases} \text{For } \lambda_{dl} > 0.561: N_{cdl} = \left[1 - 0.25\left(\dfrac{N_{od}}{N_{cl}}\right)^{0.6}\right]\left(\dfrac{N_{od}}{N_{cl}}\right)^{0.6} N_{cl} \\ \text{For } \lambda_{dl} \leq 0.561: N_{cdl} = N_{cl} \end{cases} \tag{3}$$

where N_{od} is the elastic distortional buckling load and λ_{dl} is the DL non-dimensional slenderness ratio expressed as,

$$\lambda_{dl} = \sqrt{\frac{N_{cl}}{N_{od}}} \tag{4}$$

4.2 DG buckling mode interaction DSM equation

Silvestre et. al. [20] expressed the nominal capacity in compression N_{cde} for DG buckling mode interaction in a similar way to [19] and replaced the nominal local capacity N_{cl} in the DSM equation for pure distortional buckling by the nominal global capacity N_{ce}. N_{cde} is then expressed as,

$$\begin{cases} \text{For } \lambda_{de} > 0.561: N_{cde} = \left[1 - 0.25\left(\dfrac{N_{od}}{N_{ce}}\right)^{0.6}\right]\left(\dfrac{N_{od}}{N_{ce}}\right)^{0.6} N_{ce} \\ \text{For } \lambda_{de} \leq 0.561: N_{cde} = N_{ce} \end{cases} \tag{5}$$

where λ_{de} is the DL non-dimensional slenderness ratio expressed as,

$$\lambda_{de} = \sqrt{\frac{N_{ce}}{N_{od}}} \tag{6}$$

4.3 LDG buckling mode interaction DSM equation

Dinis et. al. [11] proposed a new DSM equation for LDG buckling mode interaction and assessed its accuracy. The nominal capacity N_{clde} in compression against LDG buckling mode interaction is given as,

$$\begin{cases} \text{For } \lambda_{lde} > 0.776: N_{clde} = \left[1 - 0.15\left(\dfrac{N_{ol}}{N_{cde}}\right)^{0.4}\right]\left(\dfrac{N_{ol}}{N_{cde}}\right)^{0.4} N_{cde} \\ \text{For } \lambda_{lde} \leq 0.776: N_{clde} = N_{cde} \end{cases} \tag{7}$$

where N_{cde} is the nominal capacity for DG buckling mode interaction (Eq. (5)) and λ_{lde} is the LDG non-dimensional slenderness ratio expressed as,

$$\lambda_{lde} = \sqrt{\frac{N_{cde}}{N_{ol}}} \tag{8}$$

5. Results and discussion

The studies on shape optimisation of manufacturable CFS columns found in the literature [4] and the authors' previous research [6] are summarised in Table 1. The study in [7] is not considered herein as the elastic buckling stresses of the optimised sections are not reported. The study in [4] includes both construction (for end-use purposes) and manufacturing constraints, and considers singly and point-symmetric cross-sections. The algorithm converges to "Cee" and "Σ" cross-sectional shapes for the 610 mm and 1,220 mm long columns, respectively, and to "Cee" and squashed "S" cross-sectional shapes for the 4,880 long columns. The algorithm in [6] only considers singly-symmetric cross-sections and converges to closed or nearly closed "Cee" and "bean" cross-sectional shapes for all column lengths. The elastic buckling stresses in compression from the 4,880 mm long column in [4] are not presented and discussed herein as they are not reported in the paper.

Fig. 1 plots the elastic buckling stress ratios f_{od} / f_{ol}, f_{ol} / f_{oc} and f_{od} / f_{oc} for the cases reported in Table 1. The 610 mm and 1,220 mm long columns in [4] and the 500 mm long columns in [6] usually show close local and distortional elastic buckling stresses, with f_{ol} and f_{od} within 20% of each other. LD buckling mode interaction is therefore likely to occur. However, for these columns, the distortional slenderness ratio is less than 1.5 and the DSM equation for pure distortional buckling will accurately predict the strength of the columns (see Section 4 and [9]). No DSM interaction equations therefore need to be considered for these columns. For the 1,500 mm and 3,000 mm long columns in [6], f_{ol} is usually at least twice greater than f_{od} and no LD buckling mode interactions is likely to occur.

The global elastic buckling stress is always greater than the distortional elastic buckling stress. For 5 cases out of

22, the ratio f_{od} / f_{oc} is less than 0.5. Therefore DG buckling mode interaction is unlikely to occur for these cases. For the other cases, the f_{od} / f_{oc} ratio closer to unity (1.0) is 0.85 and the average f_{od} / f_{oc} ratio equal to 0.68, Therefore, there is usually no close proximity of the global and distortional elastic buckling stresses for the latter cases and DG buckling mode interaction is likely limited.

Additionally, the elastic local, distortional and global buckling stresses are never close to each other, let's say all within 15 % of each other, and LDG buckling mode interaction is therefore unlikely to be encountered.

Table 1: Summary of available studies found in the literature

Study	Yield stress f_y (MPa)	Objective of algorithm	Number of folds N_f	Column length (mm)		
Leng et. al. [4]	228	Maximise the column capacity for a 279.4 mm wide and 1 mm thick steel sheet	4, 6, 8, 10 and 12	610	1220	4880
Wang et. al. [6]	450	Minimise the cross-section area of a 1.2 mm thick columns subjected to axial compressive load of 75 kN	5, 7, 9, 11, 13 and 15	500	1500	3000

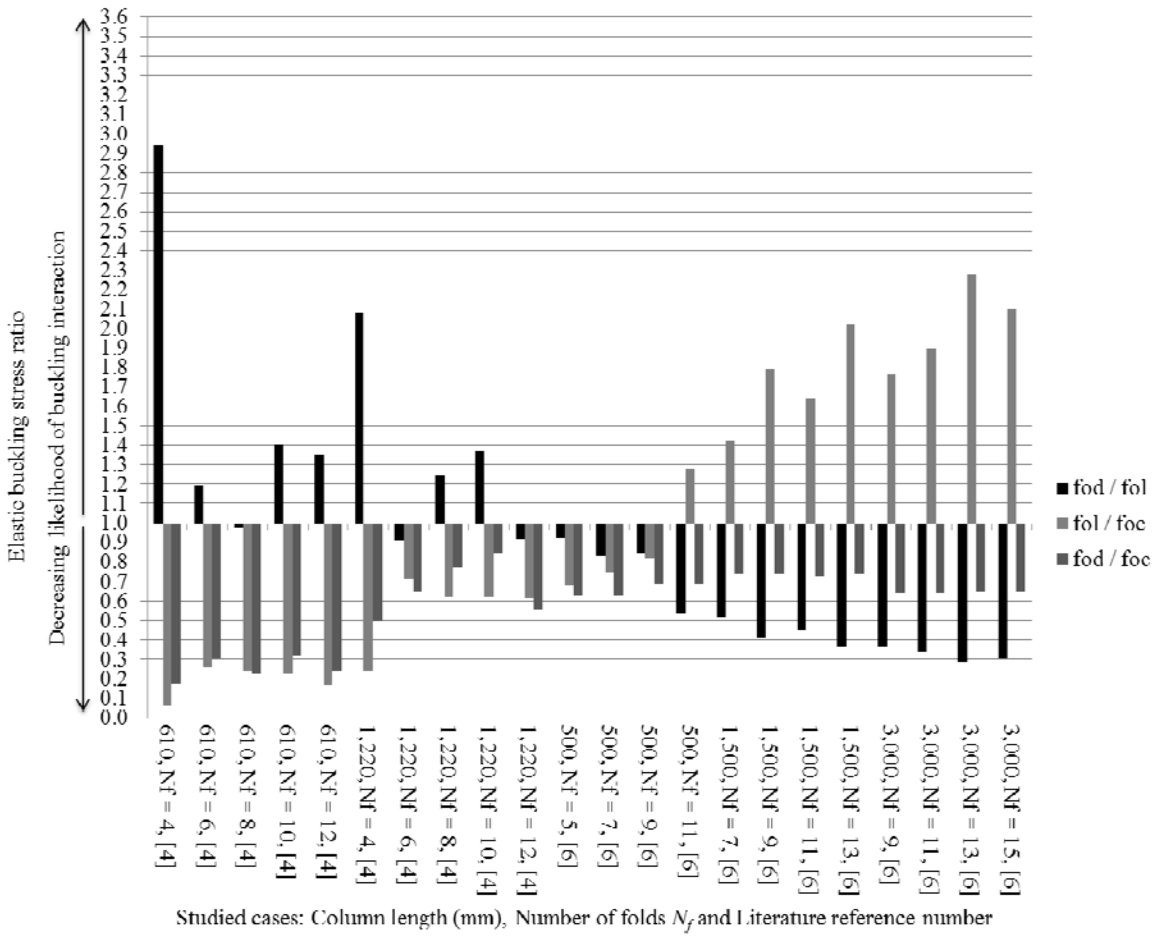

Fig. 1 Elastic buckling stress ratios for studied optimised columns in [4, 6]

7. Conclusions

The paper investigated the likelihood of buckling mode interaction in published shape-optimised manufacturable CFS columns. A quick literature review on buckling mode interaction was performed. Proposed DSM equations to determine the nominal capacity of a column experiencing LD, DG and LDG buckling mode interaction were reviewed. Despite the shape-optimisation algorithm in the authors' previous work optimising for all buckling modes, with nominal global and distortional capacities close to each other, this paper showed that strong buckling mode interactions are unlikely to be encountered for the studied optimised columns. The current DSM equations ignoring LD, DG and LDG buckling mode interactions are likely to accurately predict the capacity of the optimised cross-sections. However, buckling mode interactions still need to be further assessed in future studies on shape-optimisation of CFS members.

8. References

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