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Shape optimisation of manufacturable and usable cold-formed steel singlysymmetric and open columns



THIN-WALLED STRUCTURES

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ABSTRACT

This paper aims at incorporating manufacturing and assembly features into a shape optimisation algorithm for cold-formed steel (CFS) profiles. Genetic algorithm (GA) is used as the search algorithm and is combined with the augmented Lagrangian constraint-handling method to avoid ill-conditioning. Manufacturable cross-sections are arbitrarily drawn in the initial generation and subsequently treated as an integral part of the GA. The assembly features considered in the study reflect the ones commonly encountered in the construction industry. They include fastening elements (horizontal flange and vertical web) and allowances for utilities, and are treated as constraints. The algorithm is applied to simply-supported singly-symmetric, free-to-warp open section columns with various numbers of manufacturing bends. Three assembly cases for half sections are investigated: (a) a horizontal flange, (b) a horizontal flange and a vertical web, and (c) a horizontal flange and a vertical web with a utility clearance. A two-step optimisation process is used to optimise the columns: (i) the optimum positions of the fastening elements (horizontal flange and vertical web) are determined first and (ii) the cross-sectional shapes are then optimised. The optimised columns are discussed and compared to the unconstrained optimised columns and the conventional lipped Cee-sections. The results demonstrate the robustness and efficiency of the algorithm.

1. Introduction

Cold-formed steel (CFS) members, having a high strength-toweight ratio [1], have been widely used in engineered structures such as low-to-medium rise buildings and storage racks where hot-rolled steel profiles have been proven uneconomic. CFS members are manufactured by bending thin-walled steel coils at room temperature to the desired cross-sectional shapes. These light-gauge structural components provide flexible member profiles for architects and engineers, and facilitate onsite manufacturing and/or installation.

Finding new and optimised cross-sections can significantly enhance the member capacity and reduce costs compared to the commonly used "Cee", "Zed" and "Sigma" shapes. However, previous research efforts on shape optimisation of CFS members usually focused on unconstrained solutions [2–8], leading to cross-sections that cannot be manufactured by the current cold-forming processes and/or be used in construction.

Recently, manufacturing constraints have been introduced into shape optimisation algorithms for CFS members in [9-12]. This inclusion resulted in cross-sections with a marginal reduction in the member capacity relative to their unconstrained counter-parts. Franco et al. [12] developed a grammar-based model to shape optimise CFS open profiles in which the manufacturing features, with given stiffener sizes, were intrinsically considered in the algorithm. Leng et al. [9] and Wang et al. [10] incorporated manufacturing constraints into CFS shape optimisation algorithms by allowing limited numbers of rollforming bends per cross-section. Handling constraints, however, may add complexity to the algorithm and cause convergence issues. For this reason, Wang et al. [11] have improved their own algorithm and proposed a method of incorporating the manufacturing features as an integral part of the algorithm, addressing the challenges of dealing with manufacturing constraints.

For assembly purposes, Leng et al. [13,14] introduced end-use constraints into a stochastic shape optimisation algorithm, namely simulated annealing (SA). The constraints include singly-symmetry or point-symmetry, number of rollers (bends), vertical webs located at the mid-point (mid-height) of the cross-section, parallel flanges, rear lips and utility pass-through allowance. Both short (610 mm) to long (4880 mm) columns were investigated. The SA converged to singly-symmetric cross-sections for short and intermediate columns and to

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point-symmetric cross-sections with large number of rollers for long columns.

The present study focuses on incorporating manufacturing and assembly features into the published GA-based "self-shape" optimisation algorithm for CFS columns [4,5]. The algorithm is applied to simply-supported singly-symmetric, free-to-warp open section columns. Manufacturing features, defined in [10], are incorporated into the algorithm as an intrinsic part of the cross-sections. Assembly features, defined in this paper, are introduced as constraints into the fitness function. Two main types of assembly features, aimed at creating cross-sections that can be practically used on site, are taken into account by having (i) fastening elements (horizontal flange and/or vertical web elements) to connect the profile to adjacent structures/ elements and (ii) a clearance distance in the open part of the crosssection to allow utilities passage. The optimised cross-sections are discussed and compared to their unconstrained counter-parts and the conventional lipped Cee-sections.

2. Shape-optimisation algorithm

2.1. Overview

This paper employs the previously developed GA-based shape optimisation algorithm [4,5], referred to as the "self-shape optimisation algorithm". The algorithm uses the coordinates (x, y) of the floating points constituting the cross-sections to define the individuals in a given design space. The stochastic search algorithm yields superior off-springs that bear similarities to their parents in succeeding generations. An augmented Lagrangian constraint-handling method [15] is used to automatically control the penalty function coefficients to remain finite in order to avoid ill-conditioned solutions. Cross-over and mutation operators are performed on the floating points constituting a cross-section (see [4,5], Sections 2.4 and 2.5 for more details). As the problem is singly-symmetric, only half of the cross-section is considered by the GA operators. Nevertheless, the entire cross-section is used to calculate the member capacity presented in Section 3.1.

2.2. Constraints

2.2.1. Manufacturing constraints

The same manufacturing rules published in [10], emerged from the limitations of rolled-forming machines, are applied to the algorithm in this paper. The rules are:

- (1) The minimum length L_{min} of a single straight cross-sectional element (i.e. between roll-forming bends) is equal to 10 mm.
- (2) The maximum number of straight cross-sectional elements is limited to 20 (i.e. a maximum number of 19 roll-forming bends per cross-section).

Sharp roll-forming bending corners are conservatively considered herein and actual internal bending radii can be determined prior to manufacture.

The new proposed method of allowing optimised cross-sections to be manufactured and initially published in Wang et. al [11] is used herein. Manufacturing rules are now introduced as an intrinsic part of the cross-sectional shapes. Specifically, instead of drawing the crosssections in the first generation with short elements (2 mm long) and aligning them using the Hough transform to ultimately create manufacturable cross-sections as in [10], the cross-sections are initially drawn herein with a fixed number of longer elements of random sizes, directly resulting in manufacturable cross-sections. The cross-over and mutation operators keep the cross-sections manufacturable. More details on this intrinsic feature of the algorithm are given Sections 2.3–2.5. The number of elements are set by the manufacturer, and is less than or equal to the maximum possible number of elements defined in the Rule (2). The self-avoiding random walks method defined by Gilbert et al. [4] is used to generate the initial cross-sections (see Section 2.3). The cross-over and mutation operators are detailed in Sections 2.4 and 2.5, respectively.

2.2.2. Assembly constraints

Adjacent members are connected to columns at the horizontal flange, the vertical web or both. Additionally, columns are often designed with an "utility clearance", as per [16], allowing bridging system and/or conduits to pass through the web. While specific applications of CFS columns are not considered in this paper, three combinations of the assembly features introduced above are considered herein for undertaking a case study:

- Case I: One horizontal flange per half cross-section, with the flange at least L_{min.fas}=25 mm long to connect a M12 bolt, as per [16].
- Case II: Case I with a vertical web per half cross-section, with the overall web at least *L_{min,fas}=25* mm long also to connect a M12 bolt.
- Case III: Case II with utility allowance of a minimum overall clearance of 2×*D_{min}*=30 mm.

These cases have similar end-use purposes to those described in [14].

Fig. 1 illustrates the assembly constraints per half cross-section for Case III, with the *x*-axis being the axis of symmetry. The fastening elements are formed in the initial population (see Section 2.3 for more details). To satisfy the utility allowance constraint, the *y*-coordinate of the last cross-sectional point $P_{\mathcal{A}}$ on the horizontal flange and all the succeeding points shall be greater than the minimum utility distance D_{min} (also see Section 3.1) per half cross-section. The cross-over and mutation operators allow the fastening elements to remain horizontal and/or vertical, as explained in Sections 2.4 and 2.5, respectively.

2.3. Initial population

Manufacturable half cross-sections are arbitrarily drawn in the first generation within the design space of x_{max} =100 mm× y_{max} =100 mm. The steps to create the manufacturable half cross-sections in the initial population are revised from [4,5], and are given below when fastening elements are not considered:

• Step 1: Create the first element, as shown in Fig. 2(a):

1.1 Set the starting point P_0 of the element on the symmetric xaxis of the design space at the coordinate $(x_{max}/4, 0)$, i.e. at (25 mm, 0 mm) herein.

1.2 Randomly select the length L_{ele} of the element in the interval $[L_{min}, L_{max}]$, where L_{min} is set in Rule (1) in Section 2.2.1 and



Fig. 1. Assembly constraints per half cross-section with the vertical web shown for the second element and the horizontal flange for the fourth element.



Fig. 3. Cross-over operator, illustrated with cross-sections (five segments per half section).

 L_{max} is an arbitrary maximum length of the element, selected by a trial and error method to achieve an uniform distribution of the initial population (see Section 3.1 and [17] for more details).

1.3 Randomly select the orientation angle θ_{ele} of the element in the interval [-45°, 45°] relative to the vertical direction.

1.4 Create the element.

- Step 2: Create a new element relative to the last built element, of the length randomly selected in the interval $[L_{min}, L_{max}]$ and of the direction randomly chosen in the interval $[-45^{\circ}, 45^{\circ}]$, as illustrated in Fig. 2(b).
- Step 3: Perform the following checks:

3.1 If the last built element intersects the cross-section or the boundary of the design space (x=0, $x=x_{max}=100$ mm, y=0 or $y=y_{max}=100$ mm), delete this element and go to Step 2. If this step is repeated more than 10 times per element, then the cross-section is considered to be self-trapped (see [1] for more details) and is subsequently removed.

3.2 Else, keep the element.

3.2.1 If the number of elements is less than the pre-defined number per half cross-section Nele determined by the manufacturer, go to Step 2,

3.2.2 Else, a new cross-section is generated. Go to Step 4.

Step 4: Check the number of individuals in the population.

4.1 If the number of individuals equals the target number of individuals per generation, then the initial population is created.

4.2 Else, go to Step 1.

When fastening elements are taken into account in the algorithm and an element is selected to be a flange or a web, the above steps are modified as follows:

- *L_{min}* in Step 2 is replaced by *L_{min,fas}* that is set in Cases I and II of Section 2.2.2.
- The orientation of the element is set to be horizontal (flange) or vertical (web).

The elements are labelled sequentially, as shown in Fig. 1, from the first element ('1') (with its first node on the symmetry axis) to the final element (' N_{ele} '). Similarly, the floating points representing the elements are sequentially ordered from point P_0 on the axis of symmetry to the last point P_{Nele} . In the algorithm, the horizontal flange is allowed to be located at any element except for the first element ('1'), and the vertical web to be at any element from the first element to the element before the horizontal flange.

2.4. Cross-over operator

The one-point cross-over operator used in this study is revised from Gilbert et al. [5] to keep the sections manufacturable and the fastening elements horizontal and vertical. Two parents, having the same number of elements and the same location of fastening elements, are selected for cross-over. This guarantees that the two offsprings have the same number of elements and the same location of fastening elements as their parents, therefore keeping the set number of elements and location of the fastening elements by the manufacturer. The concept is illustrated in Fig. 3.

A cross-over point, having the same sequential point number on each of the two cross-sections, is randomly selected between the second and the second last points constituting the cross-sections. These two cross-over points are referred to as P_{co1} and P_{co2} for Parent 1 and Parent 2, respectively, and are shown in Fig. 3(a). Two points P_{t1} and P_{t2} are then defined using a linear interpolation between P_{co1} and P_{co2} as [15],

$$P_{t1} = \delta P_{co1} + (1 - \delta) P_{co2} \tag{1}$$

$$P_{t2} = \delta P_{co2} + (1 - \delta)P_{co1} \tag{2}$$

where δ is a random number in the interval [0,1].

Two offsprings are created by stretching portions of the parents to the new points P_{t1} or P_{t2} as illustrated in Fig. 3(b) and (c) and detailed in the steps below. The first offspring is formed using the left-hand part of Parent 1 (from its first point to P_{co1}) and the right-hand part of Parent 2 (from P_{co2} to its last point), and the second offspring using the right hand part of Parent 1 (from P_{co1} to its last point) and the left hand part of Parent 2 (from its first point to P_{co2}). The procedure of creating the first offspring is detailed below; a similar procedure applies to the second offspring:

- Step 1: The left-hand part of Parent 1 is linearly stretched vertically and horizontally, using its starting point P_0 as the fixed (reference) point, so its cross-over point P_{co1} matches point P_{t1} in Eq. (1) (see Fig. 3(b)).
- Step 2: The right-hand part of Parent 2 is linearly stretched vertically and horizontally, using its end point P_{Nele} as the fixed (reference) point, so its cross-over point P_{co2} matches point P_{t1} in Eq. (1) (see Fig. 3(b)).
- Step 3: The two parts created in Steps 1 and 2 are joined together and form the first offspring, as illustrated in Fig. 3(b).
- Step 4: If the offspring intersects itself or the boundary of the design space, the offspring is deleted.

The above cross-over operator allows the length of all elements, including the fastening ones, to vary and therefore their length to be optimised. The starting point of the cross-section P_0 is a reference point common to all cross-sections and avoids having duplicate cross-sections with the exact same cross-sectional shape but different node coordinates. The end point of the cross-sections P_{Nele} is fixed during the cross-over operation but can mutate in Section 2.5, allowing new cross-sectional shape to be created.

Offsprings are created until the overall population is replaced. As the cross-over points P_{co1} and P_{co2} have the same sequential point number for the two parents, the offsprings thus have the same number of elements as their parents. Stretching the cross-sections vertically and horizontally, and not rotating and scaling them as in [5], allows the fastening element to remain horizontal or vertical in the offsprings. Similar to [5], a cross-over probability of 0.8 is used in this study.

2.5. Mutation operator

The mutation operator allows a new cross-sectional shape to be created by changing the coordinates of one or more cross-sectional points. All points can mutate except for the starting point P_0 . Similar to [5], each point has a probability of mutation of 0.01. If a point mutates, the following steps, illustrated in Fig. 4, apply:

Step 1: The point is randomly moved within a 20 mm radius circle from its original position. The radius is chosen as twice the minimum length of the manufacturable elements (see Section 2.2.1) and was found to adequately create new cross-sectional shapes for the overall dimensions of cross-sections optimised in this study. The new coordinates (x₀, y₀) of the mutated point is calculated as a function of the coordinates (x, y) of the original point as,

$$x_0 = r \times \cos \theta + x \tag{3}$$

$$y_0 = r \times \sin \theta + y \tag{4}$$

where *r* is the mutation radius randomly chosen in the interval [0 mm, 20 mm] and θ is the mutation angle randomly chosen in the interval [0°, 360°].

• Step 2: Perform the following checks.

2.1 If the cross-section intersects itself or the boundary of the design space, then move the point back to its original position (x, y) and go to Step 1. If Step 2.1 is repeated more than 10 times, the



Fig. 4. Mutation operator.

Table 1

Maximum length of manufacturable elements per Nele.

Nele	4	6	8
L_{max} (mm)	50	40	30

point is not mutated.

2.2 If one of the two points of a fastening element is mutated, the second point of the element is also mutated and moved by the same vector of the mutated point so as to keep the fastening element horizontal or vertical.

3. Optimisation problem

3.1. General

The algorithm is used to minimise the cross-sectional area A_s of simply-supported singly-symmetric, free-to-warp open CFS columns subject to an axial compressive load N^* of 75 kN. The column length is set to 2 m and the wall-thickness to 1.2 mm.

Numbers of cross-sectional elements per half-section N_{ele} =4, 6 and 8 are investigated for each of the three assembly cases (see Section 2.2.2). A uniformly distributed initial population in terms of cross-sectional areas (see [17] for more details) is created. If the maximum element length L_{max} is too large relative to N_{ele} , the algorithm tends to generate large cross-sections that do not fit in the design space and therefore have difficulties creating the initial population. To address this issue, the maximum element length L_{max} was adjusted using a trial-and-error method based on the value of N_{ele} . The different values of L_{max} chosen are reported in Table 1.

Ten runs are performed for each scenario being investigated to verify the robustness of the algorithm. The number of generations and individuals per generation is reported in Section 3.2. The yield stress f_y is 450 MPa, the Young's modulus *E* is 200 GPa and the shear modulus *G* is 80 GPa.

The general fitness functions, suitable for GA, are expressed for the

Table 2			
Analysed	position	of fastening	elements

three assembly cases as,

For Cases I and II:

$$f = \frac{A_s}{A_{squash}} + \alpha_N \max\left(0, \frac{N^*}{N_c} - 1\right) + \sum_{i=1}^{N_{ele}} \alpha_{L,i} \max\left(0, \frac{L_{m,i}}{L_{e,i}} - 1\right)$$
(5)

For Case III:

$$f = \frac{A_s}{A_{squash}} + \alpha_N \max\left(0, \frac{N^*}{N_c} - 1\right) + \sum_{i=1}^{N_{ele}} \alpha_{L,i} \max\left(0, \frac{L_{m,i}}{L_{e,i}} - 1\right) + \sum_{j=P_D}^{P_{Nele}} \alpha_{D,j} \max\left(0, \frac{D_{\min}}{y_j} - 1\right)$$
(6)

where A_{squash} is the squash area, defined as the lower bound crosssectional area of the profile,

$$A_{squash} = \frac{N^*}{f_y} \tag{7}$$

 α_{N} , $\alpha_{L,i}$ and $\alpha_{D,j}$ are the penalty factors associated with the capacity, the length of element *i* and the utility clearance violations of point *j*, respectively. $L_{e,i}$ is the actual length of element *i* and $L_{m,i}$ is the minimum allowable length of element *i*, either equals $L_{minvfas}$ for the fastening elements (see Cases I and II in Section 2.2.2) or L_{min} for the other elements (see the manufacturing Rule (1) in Section 2.2.1). If a vertical web is located at the first element of the half cross-section, $L_{min,fas}$ is then divided by two making the total length of the web as $L_{min,fas}$.

In Eq. (6), P_D is the last point number of the horizontal flange, P_{Nele} is the total number of cross-sectional points, and y_j is the ycoordinate of point number j. N_c is the nominal member capacity in compression evaluated by the Direct Strength Method (DSM) [18], as specified in Clause 7.2.1 of AS/NZS 4600 [19]. Therefore, the crosssections are optimised for global, local and distortional buckling. The rules to automatically calculate N_c for shape optimisation purpose as fully detailed in [5] are applied in the study. The open source software CUFSM [20] is used to perform elastic buckling analysis on the full cross-sectional elements are subdivided into two or three segments of equal length, as defined in [10].

To avoid ill-conditioning, the fitness functions in Eqs. (5) and (6) are re-arranged in Eqs. (8) and (9), respectively, to be used in the algorithm with the augmented Lagrangian method [21], as,

For Cases I and II:

$$f = \frac{A_s}{A_{squash}} + \frac{1}{2} \left\{ \gamma_N \left[\max\left(0, \frac{N^*}{N_c} - 1 + \mu_N\right) \right]^2 + \sum_{i=1}^{N_{ele}} \gamma_{L,i} \left[\max\left(0, \frac{L_{m,i}}{L_{e,i}} - 1 + \mu_{L,i}\right) \right]^2 \right\}$$
(8)

For Case III:

Nele	4		6		8					
Fastening elements	Vertical web	Horizontal flange	Vertical web	Horizontal flange	Vertical web	Horizontal flange				
Sequential positions	'1' to '3'	'2' to '4'	'1' to '5'	'2' to '6'	'1' to '7'	'2' to '8'				

Table 3

Parameters and results of fastening element/s position for all Nele and assembly cases.

Assembly case	N _{ele}	Initial number of individuals per position	Number of positions analysed	Total number of individuals per generation	Number of analysed generations per run	Number of runs converging to the same optimum position	Optimum position
Case I	4	400	3	1200	10	10	Horizontal '2'
	6	250	5	1300	25	10	Horizontal '2'
	8	200	7	1400	25	10	Horizontal '3'
Case II	4	300	6	1800	25	10	Horizontal '3'
							– Vertical '2'
	6	240	15	3600	35	9	Horizontal '3'
							– Vertical '2'
	8	200	28	5600	35	9	Horizontal '3'
							– Vertical '1'
Case III ^a	6	240	15	3600	35	9	Horizontal '3'
cuse III	Ū	210	10	0000	00	,	- Vertical '1'
	0	200	20	5600	95	Q	Homizontal 'A'
	0	200	28	3600	35	8	Horizoniai 4
							– Vertical '1'

^a N_{ele} =4 is not shown for Case III (see Section 4.4.2 for more details).



Fig. 5. Number of individuals per possible locations of the fastening element, shown for $N_{ele}{=}4$ and Case I.



Fig. 6. Number of individuals per possible locations of the fastening elements, shown for $N_{ele}{=}4$ and Case II.

$$f = \frac{A_{x}}{A_{squash}} + \frac{1}{2} \left\{ \gamma_{N} \left[\max \left(0, \frac{N^{*}}{N_{c}} - 1 + \mu_{N} \right) \right]^{2} + \sum_{i=1}^{N_{cle}} \gamma_{L,i} \left[\max \left(0, \frac{L_{m,i}}{L_{e,i}} - 1 + \mu_{L,i} \right) \right]^{2} + \sum_{j=P_{D}}^{P_{Nele}} \gamma_{D,j} \left[\max \left(0, \frac{D_{\min}}{y_{j}} - 1 + \mu_{D,j} \right) \right]^{2} \right\}$$
(9)

where γ_N , $\gamma_{L,i}$ and $\gamma_{D,j}$ are the penalty function coefficients associated with the real parameters μ_N , $\mu_{L,i}$ and $\mu_{D,j}$ for the capacity, the length of element *i* and the utility distance of point *j*, respectively. Adeli and Cheng [21] recommended real parameters to be zero. Gilbert et al. [4] investigated the initial penalty function coefficients from 0.35 to 5 and eventually recommended the coefficient of 2. The initial values of $\gamma_N=\gamma_{L,i}=\gamma_{D,j}=2$ and $\mu_N=\mu_{L,i}=\mu_{D,j}=0$ are therefore used in this paper. The penalty increasing constant is taken as $\beta=1.05$ and the convergence rate as $\rho=1.5$ [4].

3.2. Two-step optimisation process

The fastening elements, i.e. horizontal flange and vertical web, can be at various locations within a cross-section, and all possible locations need to be investigated to fully optimise the cross-section. Therefore, a two-step optimisation process is employed as below:

• Step 1: The location optimisation, solely aimed at finding the optimum locations of the fastening elements using a large population on a small number of iterations (generations), is performed first. All possible locations of the fastening elements are equally represented in the initial generation and compete to dominate the overall population. The optimum location of the fastening elements is found when all the individuals in the population have the fastening elements at the same location. This step is further articulated as follows:

1.1 Identify all possible locations of the fastening elements (vertical web and/or horizontal flange), as tabulated in Table 2, for all numbers of cross-sectional elements (N_{ele}). Then perform the GA analysis in the following manner.

1.2 Create an initial population (Section 2.3) composed of an equal number of individuals from each identified fastening element locations in Step 1.1. The number of individuals per fastening element location is given in Table 3, aiming to keep the overall population to a reasonable size, but with no less than 200 individuals for diversity per fastening element location.

1.3 Rank all individuals based on the AL fitness function in Eq. (5) (Cases I and II) or (8) (Case III).



Fig. 7. Convergence for assembly (a) Case I, (b) Case II and (c) Case III (see [5] for reference of no manufacturing and assembly features).

Table 4	ŀ					
Average	results	over	10	runs	for	assembly Case I.

Nele	Cross-sectional area		Nominal mer	nber capacity		Ultimate compressive stress	Element length constraint
	$A_s (\mathrm{mm}^2)$	CoV (%)	N_c (kN)	Error ^a (%)	CoV (%)	<i>N_c/A_s</i> (MPa)	$g \left(L_{ele} \right)^{b}$
4 6 8 ∞ ^c	343.9 339.4 339.6 337.4	0.30 0.16 0.49 0.25	75.1 75.0 75.1 75.1	0.24 0.18 0.22 0.21	0.28 0.25 0.38 0.24	218.4 221.0 221.1 222.6	3.3×10^{-4} 3.5×10^{-4} 8.7×10^{-4}

 $^{\rm a}$ Absolute error when compared to 75 kN.

^b Element length constraint per half cross-section expressed as the 3rd term in Eq. (5) without a_L .

^c Algorithm ran without manufacturing and assembly constraints (see [5] for reference).

1.4 Select the two parents for the cross-over operator amongst all individuals using the roulette wheel.

1.5 Reproduce the offsprings by performing the cross-over operator (outlined in Section 2.4) and the mutation operator (outlined in Section 2.5), and go to Step 1.4 until the entire new population is created.

1.6 Update the AL coefficients and go to Step 1.3 until the number of generations analysed, as given in Table 3, is met.

• Step 2: The shape optimisation aiming to find the optimum crosssection using a small population and a large number of iterations is then performed. This step is articulated as follows: 2.1 Create the initial population (Section 2.3) with 500 individuals, all having the fastening elements located at the optimum location found in Step 1.

2.2 Perform the GA analysis detailed in [4,5] using the cross-over and mutation operators described in Sections 2.4 and 2.5 over 80 generations.

2.3 Repeat Steps 2.1 and 2.2 over 10 runs.

Table 5

Average results over 10 runs for assembly Case II.

Nele	Cross-sectional	area	Nominal mer	nber capacity		Ultimate compressive stress	Element length constraint
	$A_s (\mathrm{mm}^2)$	CoV (%)	N_c (kN)	Error ^a (%)	CoV (%)	<i>N_c/A_s</i> (MPa)	$g (L_{ele})^{b}$
4	361.6	0.41	75.2	0.34	0.54	208.0	2.0×10^{-3}
6	340.3	0.20	75.0	0.17	0.30	220.4	9.1×10^{-4}
8	338.9	0.20	75.0	0.08	0.12	221.3	0.0
∞°	337.4	0.25	75.1	0.21	0.24	222.6	-

^a Absolute error when compared to 75 kN.

^b Element length constraint per half cross-section expressed as the 3rd term in Eq. (5) without α_L .

^c Algorithm ran without manufacturing and assembly constraints (see [5] for reference).

ble	6
	ble

Average results over 10 runs for assembly Case III.

N_{ele}	Cross-sectional area Nominal member capacity Ultimate compr		Nominal member capacity		Ultimate compressive stress	Element length constraint	Utility clearance constraint	
	$A_s (\mathrm{mm}^2)$	CoV (%)	N_c (kN)	Error ^a (%)	CoV (%)	N _c /A _s (MPa)	$g (L_{ele})^{b}$	$g(y)^{c}$
4 ^d	361.6	0.41	75.2	0.34	0.54	208.0	2.0×10^{-3}	0.0
6	348.8	0.41	75.0	0.07	0.08	215.0	0.8×10^{-4}	0.0
8	346.0	0.37	75.1	0.22	0.32	217.1	2.4×10^{-4}	2.7×10^{-3}
∞ ^e	337.4	0.25	75.1	0.21	0.24	222.6	-	-

^a Absolute error when compared to 75 kN.

^b Element length constraint per half cross-section expressed as the 3rd term in Eq. (6) without α_L .

^c Utility constraint per half cross-section expressed as the 4th term in Eq. (6) without α_D .

^d The results are replicated from N_{ele} =4 for Case II (Table 5).

^e Algorithm ran without manufacturing and assembly constraints (see [5] for reference).



Fig. 8. Average ultimate compressive stress for all the cases investigated.

4. Results and discussions

4.1. Optimum location of fastening elements (Step 1)

Figs. 5 and 6 plot the evolution of the average number of individuals per fastening element position over 10 runs for N_{ele} =4 for Cases I and II, respectively. Other configurations investigated show a similar trend to Figs. 5 and 6, with the individuals of one fastening element position dominating the entire population in a relatively small number of generations. In Figs. 5 and 6, typically, two or three fastening element position starts dominating afterwards.

The optimum locations of the fastening elements for all configurations investigated are given in Table 3. For Case I, all 10 runs converge to the same location of the fastening elements. For Cases II and III, at least 8 out of 10 runs converge to the same location. These results demonstrate the consistency of the algorithm in finding the optimum position of the fastening elements. The optimum position of the horizontal flange is usually found at element 2, 3 or 4, while the optimum location of the vertical web is usually observed at element 1 or 2.

4.2. Convergence (Step 2)

Fig. 7 illustrates the average fitness function f in Eq. (5) for Case I (Fig. 7(a)) and Case II (Fig. 7(b)), and f in Eq. (6) for Case III (Fig. 7(c)), with α_{N} , $\alpha_{L, i}$ and $\alpha_{D,j}$ =10, over 10 runs. The average fitness f in Fig. 7 is multiplied by A_{squash} for the equations to converge to the optimum cross-sectional area. For comparison purposes, the constrained (N_{ele} =4, 6 and 8 in this paper) and unconstrained (no assembly and manufacturing constraints published in [5]) cases are plotted. The comparison shows that the algorithm always converges to an optimised solution. The unconstrained case converges slightly faster than the constrained ones. The convergence rate of all constrained cases is similar to each other, in about 50–60 generations.

Based on the runs performed on a cluster consisting of a mixture of SGI Altix XE and SGI[®] RackableTM C2114-4TY14 servers, with the set no more than eight computer cores used per run, the average computation time of the constrained cases is 30 min per generation. This compares with an average computational time of 45 min per generation when the Hough transform was used to formulate manufacturable cross-sections in the authors' previous work [10]. In Fig. 7, to obtain convergence, 30,000 solutions (up to 60 generations) were investigated, compared to 70,000 solutions (up to 140 generations) in [10].

4.3. Average results

Tables 4–6 summarise the average results of the optimised solutions over 10 runs for all the cases investigated. The algorithm reasonably satisfies all the constraints, with the average violation on the element length and utility clearance constraints in Eqs. (5) and (6) being close or equal to zero. The maximum constraint violation is found for Case III and N_{ele} =8 (Table 6) on the utility clearance in Eq. (6), and is equal to 2.7×10^{-3} . The algorithm always converges to a





Fig. 10. Optimised cross-sections for assembly Case I and Nele=4, (a, b) fittest cross-sections and (c, d) least fit cross-sections.

consistent optimised solution, with a maximum CoV of 0.49% on the cross-sectional area for Case I and N_{ele} =8 (Table 4). This outcome demonstrates the robustness of the algorithm. The most optimised unconstrained cross-section (N_{ele} = ∞ in Table 4–6) has a higher average ultimate compressive stress, defined as the ratio between the capacity N_c and the cross-section area A_s , than all assembly cases by up to 7%. For each assembly case, the more the cross-sectional elements, the greater (more efficient) the average ultimate compressive stress is. As the assembly cases become more complex, i.e. from Case I to Case III, the average ultimate compressive stress of the optimised solutions usually decreases to satisfy the constraints, as illustrated in Fig. 8. Case I and Case II yield similar (within 0.3%) average ultimate compressive stresses for N_{ele} =6 and 8. For N_{ele} =4, the difference in average ultimate compressive stresses reaches 4.8%.

4.4. Cross-sectional shapes

4.4.1. Unconstrained shapes

Fig. 9 illustrates the two fittest (Fig. 9(a) and (b)) and two least fit (Fig. 9(c) and (d)) unconstrained optimised cross-sections (see [5] for reference). The ultimate compressive stress N_c/A_s is used to rank the cross-section from the fittest to the least fit. Closed or nearly closed rounded "bean" shapes are observed. The fittest solution in Fig. 9(a) has an ultimate compressive stress of 223.2 MPa, a cross-sectional depth of 108.3 mm, a width of 61.1 mm, and therefore a depth-to-width ratio of 1.8.

4.4.2. Case I shapes

Figs. 10–12 show the optimised cross-sections for assembly Case I (only horizontal flange) and all investigated number of elements N_{ele} per half cross-section. The two fittest cross-sections (subscript (a, b)) and two least fit ones (subscript (c, d)) for each number of elements

-40

-60

10 20 30 40 50 60 70 80

x (mm)

N_c/A_s = 222.1 MPa

(a) $A_s = 338.2 \text{ mm}^2$

 $N_c = 75.1 \text{ kN}$



Fig. 12. Optimised cross-sections for assembly Case I and N_{ele}=8, (a, b) fittest cross-sections and (c, d) least fit cross-sections.

-40

-60

 N_{ele} are presented in these figures. "Sigma" or "Cee" type shapes are found, with the cross-section being open for N_{ele} =4 and closed up when N_{ele} increases to 8. As N_{ele} increases from 4 (Fig. 10) to 8 (Fig. 12), the shape of the two fittest cross-sections tends to approach the unconstrained ones (Fig. 9). The fittest cross-section when N_{ele} =8 in Fig. 12(a) has a depth of 100.8 mm, a width of 60.6 mm and therefore a depth-to-width ratio of 1.7. It is therefore slightly shallower (6.9%) than the unconstrained solution in Fig. 9(a), while being of a similar width (within 0.7%).

-40

-60

10 20 30 40 50 60 70 80

x (mm)

(b) $A_s = 338.1 \text{ mm}^2$

 $N_{c} = 74.9 \text{ kN}$

 $N_c/A_s = 221.5 \text{ MPa}$

For all values of N_{ele} , the algorithm tends to have a horizontal flange about 25–35 mm long and to curve the cross-section next to the horizontal flange with a nearly uniform curvature radius (subscript (a, b) in Figs. 10–12) to resist flexural-torsional and distortional buckling. The web is typically composed of two (subscript (a, b) in Figs. 10 and 11) or three (Fig. 12(a) and (b)) segments (per entire section) forming a long web stiffener about 35–55 mm long located at the first or second element (per half section), to enhance the local buckling resistance of

the overall web.

4.4.3. Case II shapes

10 20 30 40 50 60 70 80

x (mm)

(c) $A_s = 340.0 \text{ mm}^2$

 $N_c = 74.9 \text{ kN}$

 $N_c/A_s = 220.3 \text{ MPa}$

Figs. 13–15 illustrate the optimised cross-sections for assembly Case II (horizontal flange and vertical web). Similar to Case I, open "Sigma" type shapes for N_{ele} =4 (Fig. 13) and closed or nearly closed "Sigma" type shapes are observed for N_{ele} =6 (Fig. 14) and N_{ele} =8 (Fig. 15). The vertical web and horizontal flange for N_{ele} =4 (Fig. 13) and 6 (Fig. 14) are consecutive and forms a "right-angle" bend. As the vertical web is positioned at the second element number, an odd number of manufacturing bends is observed for these cross-sections. For N_{ele} =8 in Fig. 15, as the vertical web is located at the first element and an even number of manufacturing bends is required for these cross-sections.

-40

10 20 30 40 50 60 70 80

x (mm)

(d) A_s = 342.4 mm²

 $N_c = 75.0 \text{ kN}$

 $N_c/A_s = 219.0 \text{ MPa}$

The cross-sectional shape of the fittest solution for $N_{ele}=8$ in Fig. 15(a) is similar to the one for assembly Case I (Fig. 12(a)), for which a vertical web was generated by the algorithm, despite not being



Fig. 13. Optimised cross-sections for assembly Case II and Nele=4, (a, b) fittest cross-sections and (c, d) least fit cross-sections.



Fig. 14. Optimised cross-sections for assembly Case II and Nele=6, (a, b) fittest cross-sections and (c, d) least fit cross-sections.

part of the constraints. Similar to assembly Case I, for the two fittest cross-sections when N_{ele} =6 (Fig. 14(a) and (b)) and 8 (Fig. 15(a) and (b)), the algorithm tends to curve the elements next to the flange and to create a large web stiffener.

For N_{ele} =4 in Fig. 13, the algorithm generates cross-sections that are on average about 38% deeper and 16% wider than those for N_{ele} =6 (Fig. 14) and 8 (Fig. 15). The depth and width of the fittest crosssections in Fig. 13(a) is 140.8 mm and 69.6 mm, respectively, therefore leading to a depth-to-width ratio of 2.0. Its cross-sectional area is 363.9 mm², which is 7.6% and 6.2% larger than the one of the fittest cross-section for N_{ele} =8 in Fig. 15(a) (assembly Case II) and N_{ele} =4 in Fig. 10(a) (assembly Case I), respectively. To satisfy the strength criteria with the small number of elements (N_{ele} =4) and the two fastening elements, the algorithm tends to enlarge and open the cross-section when compared to N_{ele} =8 for Case II and N_{ele} =4 for Case I.

Again for N_{ele} =4 in Fig. 13, the sections have a relatively long horizontal flange (approximately 50–55 mm long), when compared to N_{ele} =6 (Fig. 14) and 8 (Fig. 15), to resist lateral and flexural-torsional buckling by increasing the second moment of area about the axis perpendicular to the axis of symmetry. The sections also have a short



Fig. 15. Optimised cross-sections for assembly Case II and Nele=8, (a, b) fittest cross-sections and (c, d) least fit cross-sections.



Fig. 16. Optimised cross-sections for assembly Case III and Nele=6, (a, b) fittest cross-sections and (c, d) least fit cross-sections.

lip stiffener (approximately 20–25 mm long), oriented at approximately 45° to the horizontal flange, to resist distortional buckling. On the other hand, similar to assembly Case I, a long web stiffener, oriented at approximately 20° to the vertical web, allows deep crosssections by improving the local buckling resistance of the web. Due to the large open cross-section for N_{ele} =4, the utility constraint (set in assembly Case III) is satisfied. The *y*-coordinate of the last crosssectional point of all cross-sections in Fig. 13 is greater than the minimum utility clearance distance D_{min} =30 mm. N_{ele} =4 for assembly Case III is therefore not run and results from N_{ele} =4 for assembly Case II are used in the following Sections.

4.4.4. Case III shapes

Figs. 16 and 17 represent the two fittest (a, b) and two least fit (c, d) optimised cross-sections for N_{ele} =6 and 8, respectively, for assembly Case III. Open "Sigma" type cross-sectional shapes are typically observed. The fittest solutions for N_{ele} =6 and 8 have utility distances of 30.6 mm (Fig. 16(a)) and 29.4 mm (Fig. 17(a)), respectively. These distances are close (within 2%) to the minimum utility clearance D_{min} of 30 mm set per half cross-section (see Section 2.2.2). Similar to all



Fig. 17. Optimised cross-sections for assembly Case III and N_{ele}=8, (a, b) fittest cross-sections and (c, d) least fit cross-sections.

Table 7		
Comparison to	conventional	lipped Cee-sections.

Assembly category by cross-	Conventional	Difference in capacity					
sectional area	Depth (mm)	Width (mm)	Lip (mm)	Thick-ness (mm)	Cross-section area (mm ²)	Nominal member capacity (kN)	(%)
Case I, N _{ele} =4 Case II and III, N _{ele} =4	146.9 156.2	62.7 65.1	14.3 14.8	1.2 1.2	343.9^{a} 361.6^{b}	34.7 37.4	116 101

^a Same cross-sectional area as the optimised cross-section when N_{ele} =4 in Table 4. ^b Same cross-sectional area as the optimised cross-section when N_{ele} =4 in Table 6.

previous assembly cases, except for N_{ele} =4 in Case II, the algorithm tends to have a short horizontal flange, curve elements next to the flange and large web stiffener. The fittest cross-section for N_{ele} =8 in Fig. 17(a) has an ultimate compressive stress of 218.3 MPa. It is 118.8 mm deep and 67.7 mm wide, which is 8.8% deeper and 9.8% wider than the fittest unconstrained cross-section in Fig. 9(a). Its depth-to-width ratio is 1.8. It is worth mentioning that while the vertical web is located at the first element in Fig. 17(a), the third element is also vertical and forms a "right-angle" bend with the horizontal flange, creating the overall web stiffener.

4.5. Capacity improvement

The average nominal member capacity in compression of the constrained optimised solutions in Table 4–6 is compared to the capacity of conventional lipped channels that have similar aspect ratios to the cross-sections manufactured by Bluescope Lysaght [16] in Australia. These conventional lipped channels satisfy the geometric limitations for compression members for design using the Direct Strength Method given in Table 7.1.1 of the Australian Standard AS4600 [19]. The conventional cross-sections have a wall thickness of 1.2 mm (same as the optimised cross-sections) and are designed with a cross-sectional area equal to the ones of the optimised solutions for N_{ele} =4 in Tables 4 and 6. Table 7 summarises the comparison between the conventional cross-sections and the optimised ones for each assembly case. The nominal member capacity of the optimised

solutions is significantly higher (up to 116% and no less than 101%) than the conventional cross-sections.

5. Conclusions

This paper has introduced manufacturing and assembly features into a shape optimisation algorithm for CFS columns. The principles of the algorithm have been explained and applied to simply-supported singly-symmetry, free to warp open section columns. The major findings are:

- The robustness of the algorithm was verified by having consistent optimised solutions with a maximum CoV of 0.49% (*N_{ele}=8* in Case I) and 0.54% (*N_{ele}=4* in Cases II and III) for the cross-sectional area and the member capacity, respectively, over 10 runs.
- The algorithm converged rapidly in 50–60 generations to an optimised solution, and 500 individuals and 80 generations (40,000 solutions) are found adequate for convergence.
- The more the cross-sectional elements, the less the cross-section area is. Moreover, when the number of cross-sectional elements increases, the cross-section tends to close. Specifically, a small number of elements (*N_{ele}=4*) with a horizontal flange and a vertical web resulted in (i) largely open "Sigma" type cross-sectional shapes, (ii) long horizontal flange and web stiffener and (iii) lip stiffener oriented at 45° to the horizontal flange. A large number of elements (*N_{ele}=8*), on the other hand, resulted in (i) closed "Sigma" type

cross-sectional shapes, (ii) short horizontal flange and long web stiffener and (iii) curved elements next to the flange.

- When the utility constraint was considered (assembly Case III) with a large number of cross-sectional elements (N_{ele} =8), the algorithm tended to close the cross-section to its maximum while satisfying the minimum utility clearance. The curved cross-sectional shape next to the flange was also present.
- The optimised manufacturable and usable cross-sections are only up to 7% less efficient than the optimised unconstrained ones. The optimised solutions exhibit a member capacity in compression more than twice that of the conventional lipped Cee-sections of a similar cross-section area.

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