MCV 4UI **PRACTICE EXAM**

VECTORS - LINES and PLANES

1. Given $\vec{u} = (4,-7,-2)$ and $\vec{v} = (3,-5,8)$, determine the following:
   a) the angle between $\vec{u}$ and $\vec{v}$ to the nearest degree.
   b) $4\vec{u} - 3\vec{v}$ in component form.
   c) the vector projection of $\vec{u}$ on $\vec{v}$ in component form.
   d) a vector perpendicular to both $\vec{u}$ and $\vec{v}$ in component form.
   e) a vector with a magnitude of 7, in the opposite direction of $\vec{u}$ in component form.

2. Given vectors $\vec{a}$ and $\vec{b}$, such that $\left|\vec{a}\right| = 7$, $\left|\vec{b}\right| = 15$ and the angle between them is 28°, determine $\left|2\vec{a} + 3\vec{b}\right|$ to one decimal place. Include a correctly labelled diagram in your solution.

3. A river is 400 metres wide and flows at 2 km/h. A canoeist who can paddle at a speed of 5 km/h in still water heads out 24° from the bank downstream. A marina lies directly across the river on the opposite bank.

a) How long will it take the canoeist to touch the other side to one tenth of a minute?
b) How far downstream from the marina will the canoeist touch the other bank to one decimal?

4. A dunk tank with a weight of 952 N is at rest on a ramp. The ramp has an incline of 17º to the horizontal ground. The force due to friction up the ramp is 53 N. What force must be applied to the dunk tank at an angle of 27º to the ramp to maintain equilibrium?

5. Determine the vector equation for the line perpendicular to $\frac{x-4}{5} = \frac{y+7}{-3}$ and passing through A$(-5,17)$

6. Determine the parametric equation of the line parallel to the line $\frac{x-5}{-2} = y-2, z=5$ and having the same *x*-intercept as the line $(x,y,z) = (-3,14,35) + t(18,-12,-30)$, $t \in \Re$.

7. a) Determine the scalar equation of the plane containing the line and the point below.
   b) Determine the perpendicular distance between the point (-1,5,7) and the plane in part a) to 2 decimal places.
   c) Determine the direction cosines for the line.

   line: $(x,y,z) = (7,-3,1) + k(4,-8,3),\ k \in \Re$ **point:** $B(-1,9,3)$

8. a) Verify the following lines are skew, using the **Test For Skew Lines**.
   b) Determine **the distance** between the lines **to 4 decimal places**.

$$\ell_1 : \begin{aligned} x &= -1+t \\ y &= 3+4t,\ t \in \Re \\ z &= 6+5t \end{aligned}$$

and

$$\ell_2 : (x,y,z) = (4,17,30) + k(-1,2,-5),\ k \in \Re$$

9. a) **Without actually solving the system of linear equations**, narrow down the geometric relationship(s) between the following planes. Give reasons for your conclusions**.**
   b) If necessary use **Gauss-Jordan** elimination to determine the true geometric relationship.
   c) If there is any intersection, state it. If there is no intersection, then say so.

$$\begin{aligned} \pi_1 &: \quad 2x + y + z = -4 \\ \pi_2 &: \quad 3y - 2z = 2 \\ \pi_3 &: \quad 3x + y + 2z = -7 \end{aligned}$$

10. a) **Without actually solving the system of linear equations**, state the geometric relationship between the following planes. Give reasons for your conclusions**.**
   b) Determine the intersection. If there is no intersection then say so.

$$\begin{aligned} \pi_1 &: \quad x + 3y - 2z = 8 \\ \pi_2 &: \quad 5x - 2y + z = -3 \end{aligned}$$

   c) **Classify** the system of linear equations.

<u>CALCULUS</u>

1. a) Define the derivative of $f(x)$ with respect to x in terms of a limit.
   b) Using your definition in a) find $f'(x)$ given $f(x) = \dfrac{4x}{x-4}$.

2. Determine the following limits

a) $\displaystyle \lim_{x \to \frac{\pi}{3}} \frac{\cos(x) + \csc\left(\frac{5}{2}x\right)}{2\ln\left(e^{3x}\right)}$

b) $\displaystyle \lim_{x \to 3} \frac{9 - x^2}{x^3 - 2x^2 - 2x - 3}$

c) $\displaystyle \lim_{x \to 4^-} \frac{2 - x}{x - 4}$

d) $\displaystyle \lim_{x \to 0} \frac{\sin(12x)}{8x}$

e) $\displaystyle \lim_{x \to \infty} \frac{2x^3 - 4x^2}{x + 2x^2 - 7x^3}$

f) $\displaystyle \lim_{x \to -2} \frac{\sqrt{x^2 - 5x} - \sqrt{10 - 2x}}{x + 2}$

g) $\displaystyle \lim_{t \to 0} (1 + 10t)^{\frac{3}{t}}$

(h) $\displaystyle \lim_{x \to 0} \frac{(4x+1)^{\frac{1}{3}} - 1}{x}$

3. Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify.

a) $y = 5\sqrt{x} + \dfrac{1}{4x^2} + \sec(7x) + 8\ln(3x+1) + \tan\left(\sin\left(e^x\right)\right) + e^{10}$

b) $y = e^{4x^2} \cos^4(3x)$

c) $y = \dfrac{\cot(10x)}{x^2 + 3}$ use the quotient rule

d) $y = -2\left[\csc(x) + \left(3^x - \log_7(x)\right)^{-4}\right]^{\sqrt{5}}$

e) $y = x^{\sin(x)}$

4. Determine the equation of one of the tangents to the curve $y = x^3 + 6x^2$ that is parallel to the line $y = 15x + 6$.

5. a) Using the chain rule, determine $\frac{dy}{dx}$ at $x = -1$, given

$$y = 2u^3 - u, \qquad u = \frac{6}{v} + v, \qquad v = x^2 - x$$

   b) Determine $\frac{dy}{dx}$ at $x = 3$, given $f'(25) = 6$, $g(9) = -2$ and $g'(9) = -1$ for

$$y = f\left(x^3 + g(3x)\right)$$

6. For the following function, determine the x-coordinates of all relative extrema using the second derivative test, if possible. If the second derivative test fails, use the first derivative test.

$$f(x) = \frac{1}{5}x^5 - \frac{4}{3}x^3 - 6$$

7. A function, f, has all the properties listed below. Carefully sketch the function showing all points of extrema and all points of inflection if any exist.

   a) $f(0) = 0$
   b) $\lim_{x \to +\infty}\left[f(x) - \left(\frac{-3}{2}x + 3\right)\right] = 0$
   c) $\lim_{x \to -4} f(x) = +\infty$, $\lim_{x \to 2} f(x) = -\infty$
   d) $\lim_{x \to -\infty} f(x) = 3$
   e) $f(x) = -5$, $f'(x) = 0$ and $f''(x) < 0$, when $x = 4$
   f) $f''(x) < 0$ when $0 < x < 2$
   g) $f''(x) > 0$ when $x < 0$, $x \neq -4$

8. Given the function, $f(x) = \frac{1}{10}x^5 - 3x^3 + 5x - 1$, determine the intervals on which the function is concave up/concave down.

9. Determine the absolute minimum and maximum values of the following function on the given intervals. State where the absolute max and min occur.

$$f(x) = \cos^2(x) - \cos(x), \qquad 0 \leq x \leq \pi$$

10. A rectangle is inscribed under the curve $y = e^{-2x}$, with its base along the *x*-axis. Determine the dimensions of the rectangle of greatest area, subject to the restriction that the base does not exceed 2 units.

11. A spherical hot air balloon is being blown up at such a rate that its surface area is increasing at $8\pi \ m^2/\min$. Determine how the volume is changing at 50 minutes.

Answers:

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1. a) $\theta = 68^\circ$ b) $(7,-13,-32)$ c) $\left(\frac{93}{98}, \frac{-155}{98}, \frac{124}{49}\right)$ d) $(-66,-38,1)$ e) $\left(\frac{-28}{\sqrt{69}}, \frac{49}{\sqrt{69}}, \frac{14}{\sqrt{69}}\right)$

2. $\left|2\vec{a}+3\vec{b}\right| = 57.7$ 3. a) t = 12 minutes b) d = 1.32 km

4. $\left|\vec{F}_{rope}\right| = 252.9N$ 5. $(x,y) = (-5,17)+k(3,5)$ 6. $x = 18-2k$, $y = k$, $z = 0$

7. a) $13x+8y+4z-71=0$ b) 1.01 c) $\cos(\alpha) = \frac{4}{\sqrt{89}}$, $\cos(\beta) = \frac{-8}{\sqrt{89}}$, $\cos(\delta) = \frac{3}{\sqrt{89}}$

8. a) since $\vec{m}_1 \neq k\vec{m}_2$ and $\vec{P_1P_2} \bullet \vec{m}_1 \times \vec{m}_2 \neq 0$, the lines are skew b) distance is 0.1961

9. a) The planes intersect at a point. c) The point of intersection is $(1,-2,-4)$

10. a) The planes intersect at a line b) The line of intersection is $x = \frac{7}{17}+\frac{1}{17}t$, $y = \frac{43}{17}+\frac{11}{17}t$, $z = t$ c) dependent

CALCULUS

1. a) $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ b) $f'(x) = \frac{-16}{(x-4)^2}$

2. a) $\frac{5}{4\pi}$ b) $\frac{-6}{13}$ c) $+\infty$ d) $\frac{3}{2}$ e) $\frac{-2}{7}$ f) $\frac{-7}{2\sqrt{14}}$ g) e^{30} h) $\frac{4}{3}$

3. a) $\frac{dy}{dx} = \frac{5}{2}x^{\frac{-1}{2}} - \frac{2}{4}x^{-3} + 7\sec(7x)\tan(7x) + \frac{8}{3x+1}(3) + \sec^2(\sin(e^x))\cos(e^x)e^x$

   b) $\frac{dy}{dx} = e^{4x^2}(8x)\cos^4(3x) + e^{4x^2}4\cos^3(3x)(-3\sin(3x))$

   c) $\frac{dy}{dx} = \frac{-10\csc^2(10x)[x^2+3]-\cot(10x)[2x]}{(x^2+3)^2}$

   d) $\frac{dy}{dx} = -2\sqrt{5}\left[\csc(x)+(3^x-\log_7(x))^{-4}\right]^{\sqrt{5}-1}\left[-\csc(x)\cot(x)-4(3^x-\log_7(x))^{-5}\left(3^x\ln(3)-\left(\frac{1}{x}\right)\left(\frac{1}{\ln(7)}\right)\right)\right]$

   e) $\frac{dy}{dx} = x^{\sin(x)}\left[\cos(x)\ln(x)+\frac{\sin(x)}{x}\right]$

4. $y = 15x+100$ 5. a) $\frac{447}{2}$ b) 144 6. max at x = -2, min at x = 2

7. See solutions. 8. concave up on [-3,0] and [3,∞], concave down on [-∞,-3] and [0,3]

9. max of 2 when x = 0, min of $\frac{-1}{4}$ when $x = \frac{2\pi}{3}$

10. max area is $\frac{1}{2e}$ with dimensions $\frac{1}{2}$ by $\frac{1}{e}$

11. The volume is changing at $400\pi \ m^3/\min$