EXAM REVIEW FOR MCV 4U1
VECTORS AND CALCULUS

PART A: CALCULUS

Limits, Continuity, and First Principles:

1. Evaluate each limit, if it exists.

a) $\lim_{x \to 3} \frac{x^2 - 9}{x^2 + 7x - 30}$ b) $\lim_{x \to 2} \frac{3\sqrt{x-1} - 3}{x - 2}$ c) $\lim_{x \to 2} \frac{\frac{1}{2} + \frac{1}{x-4}}{2x - 4}$

d) $\lim_{x \to 0} \frac{(x+64)^{\frac{1}{3}} - 4}{x}$ e) $\lim_{x \to \infty} \frac{x^2 - x^3 + 2x}{5x\left(x - 2x^2\right)}$

2. Determine all discontinuities and state which conditions of continuity fail for each discontinuity.

a) $f(x) = \frac{3x - 6}{x^2 + 8x - 20}$ b) $f(x) = \begin{cases} 3x^2 - 1, & x \geq 4 \\ 5x + 10, & 0 < x < 4 \\ 3x^2 + 10, & x < 0 \end{cases}$

3. Find the derivative of each function using **FIRST PRINCIPLES**.

a) $f(x) = 3x^2 + 2x - 1$ b) $f(x) = \frac{3}{2 - x}$ c) $y = 4 - 2\sqrt{x+3}$

Derivatives and Basic Applications

4. Find $\frac{dy}{dx}$ for each curve. (Do not simplify your final answer.)

a) $y = 5x^{-4} + \frac{2}{x^{-\frac{3}{2}}} - 5\pi + x$ b) $y = \frac{\sqrt{3x^2 - 2}}{5x}$ c) $y = \sqrt[4]{e^{4x} + 4x}$ d) $y = x^7 e^{x^7} + 8^{x-1} g4^{-x^2 - \sec x^3}$

e) $3y^2 - 8x^2 y + 2x = 5$ f) $7xy^3 - 2xy = 3y^4$ g) $y = (9\log_3 x)^5$ h) $y = 7x^2 \cos x^2 \tan\left(2x + 5\right)$

i) $y = \sin\left(\log\left(5x^2 + \pi^3\right)\right) + \cot^2\left(\cos\left(\ln\left(6x - 4\right)\right)\right)$

5. Find $\frac{dy}{dx}$ using Logarithmic Differentiation.

a) $y = \frac{(3x^2 - 2)^5 \sqrt{x+1}}{5x^3}$ b) $y = (14x^5 + 2x)^{3x}$

6. Find the equation of the tangent at the given value.

a) $f(x) = \frac{3x^2 - 2}{x^4 - 2}, at\ x = 1$ b) $f(x) = (6x^3 + 2)(5x - 1)^4, at\ x = 0$

c) $3x - y = 2\ln x$, at x = 1 d) $4x - e^{xy} = 2y$ at the y-intercept

7. If $f(x)=x^2+ax+b$ has a horizontal tangent line at (-3, -12), find the values of **a** and **b**.

Word Applications (Position, Velocity, Acceleration, Related Rates, and Optimization)

8. Determine the initial velocity and acceleration for the position function $s(t)=\dfrac{3t}{1+t^3}$ where s is in km and t is in minutes.

9. Given the horizontal position function, $s(t)=t^3-t^2-t+3$, where **s** is in meters and **t** is time in seconds, find:

   a) When is the object at rest? What is its position in terms of the fixed point? (Be specific)

   b) When is the object moving left?

   c) When is the velocity constant?

   d) Is the object speeding up or slowing down at 4 seconds?

10. A cylindrical bread role is growing at 3 $cm^3/\min$ and is increasing in length at 1 $cm/\min$. Determine the rate of change of the radius when the length of the role is 10 cm and the radius is 4cm.

11. A spherical balloon is being filled such that the circumference is increasing at $4\pi cm/\mathrm{s}$. Determine the rate at which the volume of the balloon is changing when the circumference is $24\pi cm$.

12. A water tank in the shape of an inverted cone is draining at $2m^3/\min$. If the tank has a height of 10 meters and a diameter of 6 meters, find the rate at which the water level is falling when the water is 1.3 meters deep. (Round your answer to 2 decimal places)

13. What is the maximum volume of a cylinder with a surface area of 142 cm^2? (Give your answer to the nearest cm^3)

14. Given that a square based box has a volume of 2.8 m^3. Find the dimensions of the box that will minimize the cost of the material to make the box knowing that the top and bottom of the box are FOUR times as expensive as the sides. (Round your answer to two decimal places.)

15. A piece of string 30 cm long is cut into 2 pieces to make a square and a circle. Determine the lengths of each piece of string in order to minimize the total area of both shapes. (Round to one decimal place.)

16. Find the largest volume of a cylinder that can be inscribed in a sphere with a radius of 9cm.

17. Find the absolute maximum and the minimum value on the interval $-3\le x\le 3$ for each function.

    a) $f(x)=x^3+3x^2+3x-7$ b) $f(x)=\dfrac{x-3}{x+5}$ c) $f(x)=\dfrac{6x}{1+x^2}$

18. Find the absolute maximum and the minimum value on the given interval:

a) $f(x)=\sin^2 x-\dfrac{x}{2}$ on $0\le x\le 2\pi$ b) $y=\dfrac{1}{4}x^2(2\ln x-3)$ on $1\le x\le 3$ c) $g(x)=x^2e^x$ on $-3\le x\le 1$

Curve Sketching:

19. Find all intervals of increase and decrease and local max and min values for each function:

a) $f(x)=x^3-9x^2+24x-15$ b) $f(x)=\dfrac{x^2-x-1}{x-1}$

20. Use the second derivative test on $f(x)=2x^3-9x^2+9$ to determine if the critical values are maximum or minimum points.

21. Determine any vertical, horizontal or oblique asymptotes for each function:

a) $f(x)=\dfrac{x^3-4x^5+2x}{x^2(x^3-8)}$ b) $f(x)=\dfrac{5x^2-11x+1}{x-3}$

22. Determine intercepts (where possible), asymptotes, critical points, intervals of increase/decrease, local max/min points, intervals of concavity, and points of inflection, and then sketch each function. (Show all work)

a) $f(x)=\dfrac{1+x^2}{1-x^2}$, given that $f''(x)=\dfrac{12x^2+4}{(1-x^2)^3}$ b) $f(x)=\dfrac{x^2-x+2}{x-2}$, given that $f''(x)=\dfrac{8}{(x-2)^3}$

23. Find the constants Q, R and S so that the graph of $f(x)=Qx^2+Rx+S$ has a relative maximum at (5, 12) and y-intercept of 3.

PART B: VECTORS

Geometric Representation

24. Determine the magnitude of the resultant vector of two forces of 54 N and 34 N acting at an angle of 55° to each other.

25. Two forces at an angle of 130° to each other act on an object. Determine their magnitudes if the resultant has a magnitude of 480 N and makes an angle of 55° with on of the forces.

26. The pilot of an airplane that flies at 800 km/h wishes to travel to a city 800 km due east. There is a 80 km/h wind from the north east.

   a) What should the plane's heading be?
   b) How long will the trip take?

27. A 10 kg weight is supported by two strings of length 5 m and 7 m attached to two points in the ceiling 10 m apart. Find the tension in each string.

Algebraic Representations

28. Which of the following vectors are unit vectors? Explain.

$\vec{a}=\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$ $\vec{b}=\left(\frac{1}{\sqrt{26}},\frac{4}{\sqrt{26}},\frac{-3}{\sqrt{26}}\right)$ $\vec{c}=\left(\frac{3}{4},\frac{-\sqrt{3}}{4},\frac{1}{2}\right)$ $\vec{d}=\left(\frac{-11}{2},4,\frac{7}{2}\right)$

29. Calculate the scalar and vector projection of $\vec{a}$ onto $\vec{b}$ given that:

a) $\vec{a}=(2,3)$ and $\vec{b}=(-1,1)$ b) $\vec{a}=(0,2,1)$ and $\vec{b}=(4,6,-1)$

30. Points A(2,4), B(0,0) and C(-2,1) define a triangle in a plane. Find angle ABC to the nearest degree.

31. Consider two lines with equations $\dfrac{x+8}{1}=\dfrac{y+4}{3}=\dfrac{z-2}{1}$ and $(x,y,z)=(3,3,3)+t(4,-1,-1)$.

   a) Show that the two line are perpendicular.

b) Find the point of intersection between the lines.

32. Find the vector, parametric, and symmetric equations of the line (if possible):

a) that passes through the point (0, -1, 2) and in parallel to the line x = 3 + 2t, y = 5t, z = -1 – t, $t \in R$.

b) That passes through points (0, 0, 1) and (1, 0, 0).

33. Find the Cartesian equation of the plane that:

a) passes through the points (3, 2, 3), (-4, 1 ,2) and (-1, 3, 2)

b) passes through the point and is parallel to the plane y + z = 5

c) contains the lines $\vec{r} = (2,1,7) + t(0,1,0)$ and $\vec{r} = (3,0,4) + t(2,-1,0)$

34. For what values of k, if any, will the planes $3x + ky + z - 6 = 0$ and $6x + (1-k)y + 2z - 9 = 0$ be:

a) Parallel b) Perpendicular

35. Find the shortest distance between the:

a) point (1, 3, 2) and the line $\frac{x-1}{-1} = \frac{y-3}{1} = \frac{z-7}{2}$

b) point (3,2,1) and the plane $3x + 2y + z = 10$

c) the planes $x + 2y - 5z - 10 = 0$ and $2x + 4y - 10z - 17 = 0$

36. Determine the intersection (if any) between the following sets of planes. Give a geometrical interpretation for each situation. Explain how many solutions each situation has (one unique solution, infinite solutions or no solution).

a) $\begin{aligned} 3x - y + 4z - 7 &= 0 \\ x + y - 2z + 5 &= 0 \end{aligned}$

b) $\begin{aligned} 2x + 4y + z &= 2 \\ 5x + 5y + 3z &= 17 \\ 4x - y + 3z &= 26 \end{aligned}$

c) $\begin{aligned} x + 2y + 3z &= 4 \\ 3x + 6y + 9z &= 12 \\ 3x + 5y - 2z &= 1 \end{aligned}$

d) $\begin{aligned} x - 3y + 2z &= -3 \\ 3x - 4y + z &= 11 \\ 8x - 9y + z &= 5 \end{aligned}$