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PHIL 313Q: Inductive Logic

with “Probability and Inductive Logic” by Ian Hacking

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- **Deductive logic** dealt with arguments assessed for **validity.**
  - given the truth of the premises, the conclusion follows *by logical necessity*

    P1. Either Plum or Mustard killed Alex

    P2. If Plum killed Alex, then Alex died quickly.

    P3. Alex didn't die quickly.

    C. So, Mustard killed Alex.
- Deductively valid arguments are "risk-free", or "truth-preserving" , i.e. the premises "logically entail" the conclusion

    P1. (Kpa v Kma)

    P2. Kpa > Qa

    P3. ~Qa

    C. Kma

- There are also argument **whose premises merely support**, *without logically entailing*, their conclusion

  P1. A knife was found in Plum's car.

  P2. Alex was killed by stabbing.

  P3. All other culprits but Plum have alibis.

  C. So, Plum killed Alex.

- This argument is "risky" – *possibly*, the premises are true and conclusion false (even if this possibility is highly unlikely)

Kinds of Risky Arguments

1. **Inferences to the Best Explanation** (IBE)

    P1. If James wants a job, then he gets a haircut.

    P2. James gets a haircut.

    C. James wants a job.

- This argument is logically invalid:

    P1. $(J > H)$

    P2. H

    C. J

- But the conclusion might still be the best explanation for (one of) the premises.

Kinds of Risky Arguments

2. Inferences based on Testimony

P1. Alex is trustworthy.

P2. Alex said that the last argument is invalid.

C. The last argument is invalid.

- (But isn't this just an Inference to the Best Explanation?)
- Regardless: we won't study Inferences to the Best Explanation or Inferences based on Testimony.

Kinds of Risky Arguments

3. Sample & Populations

P1. This orange from the box is good.
C. All oranges from the box are good.

P1. This randomly selected orange from the box is good.
C. All oranges from the box are good.

P1. These 5 randomly selected oranges are good.
C. All oranges from the box are good.

Sample/Population Arguments

- **Sample to population**

  P1. These 5 randomly selected oranges are good.

  C. All oranges in the box are good.

- **Sample to sample**

  P1. These 5 randomly selected oranges are good.

  C. The next 5 randomly selected oranges will be good.

- **Population to sample**

  P1. Most oranges in the box are good.

  C. The next 5 randomly selected oranges will be good.

- **Population to population**

  P1. Most of the oranges are good.

  C. Almost all of the oranges are good.

Tangent/Hate

- Suppose we had 20 oranges in the box.

  P1. 12 randomly selected oranges are good.

  C. Most of the oranges are good.
- “Sample to population”? “Population to population”?
- Notice that if the premise is true, then the conclusion is true (*although not by logical necessity*)

Tangent/Hate

- Sample to population **Specific to general**

  P1. These 5 randomly selected oranges are good.

  C. All oranges in the box are good.

- Sample to sample **Specific to specific**

  P1. These 5 randomly selected oranges are good.

  C. The next 5 randomly selected oranges will be good.

- Population to sample **General to specific**

  P1. Most oranges in the box are good.

  C. The next 5 randomly selected oranges will be good.

- Population to population **General to general**

  P1. Most of the oranges are good.

  C. Almost all of the oranges are good.

"Probably"

- These arguments are related to what "probably" follows from what
- P1. These 5 randomly selected oranges are good.

  C. So, *probably*, almost all of the oranges are good.
- Does "probably" mean "at least 50%"?
- Probably just depends on the context.

Probability Basics

- A die has 6 sides (1–6). Suppose each side has equal chance of being rolled. Each side has ⅙ chance of being rolled. The probability of rolling a 1 is ⅙ = 0.166666…
- The probability of any event is between 0 and 1.
- The sum of the probabilities of all possible outcomes is 1. The probability of rolling 1–6 is ⅙+⅙+⅙+⅙+⅙+⅙ = 1
- The probability of two independent outcomes both occurring is the product of their probabilities. The probability of rolling a 1 and a 6 in 2 independent rolls is ⅙ x ⅙ = 1/36

Tangent/Hate

- Hacking says not all arguments dealing with probability are inductive; some are *valid*!
- P1. The die has 6 sides.
  <u>P2. Each side is equally probable.</u>
  C. The probability of rolling a 1 is 1.6
- **This is not (obviously) logically valid though!** Try to translate this into an argument of PL.
- This is only logically valid if probability- and number-vocabulary are logical vocabulary.

Gambler’s Fallacy

- Roulette wheel: bet on a color, win 2x if you’re right.

- Gambler sees 9 BLACK in a row and bets on RED!

Basics

- *Chance setups* have *trials*. *Trials* have *outcomes*.
    - Roulette, a coin are setups. Spins, flips are trials. Colors/numbers, heads/tails are outcomes.
- Some chance setups are **fair**, intuitively.
- A chance setup is **fair** iff it is **unbiased** and **independent**.

Unbiased

- A chance setup is **unbiased** iff the relative frequency of each outcome "in the long run" is equal to that of any other. (Hacking)
- A chance setup is **unbiased** iff there is no reason (other than chance) why one outcome would occur more often than any other given a sufficiently large number of trials.
- A coin flipped like this *might* be biased:

  HHHTHHHHTHHHHHHTHHHHHHTTHHHHHHTTHHHH

Independent

- But note that a coin flipped like this is unbiased:
  HTHTHTHTHTHTHTHTHTHTH
- So fairness also requires **independence**.
- Intuitively, a chance setup is **independent** if:
  - there are no predictable patterns/regularities
  - trials have no "memory" of previous trials
  - a successful gambling strategy is not possible
- A chance setup is independent iff previous trials don't affect future trials.

	Independent	Dependent
Unbiased	Fair! • An urn with 50 indistinguishable balls numbered 1-50 • A trial is to shake, pick, and return • "Sample with replacement"	Unfair! • An urn with 50 indistinguishable balls numbered 1-50 • A trial is to shake, pick, and NOT return • "Sample without replacement"
Biased	Unfair! • Sample with replacement on the 50-ball urn, but the even-numbered balls are weighted. • No regularity or pattern beyond the bias.	Unfair! • Sample without replacement on the 50-ball urn, and the even-numbered balls are weighted.

Gambler's Fallacy

- The gambler reasons:
  - "The wheel is fair."
  - "There have been 9 REDs, so the next will be BLACK!"
- But then the wheel would not be independent, hence unfair.
- If 9 REDs increased the chance of a 10th BLACK, then we could construct a successful gambling strategy.

Gambler's Fallacy

- In unbiased setups, the chance of each outcome is equal on every trial.
- Imagine a trial is 2 spins
  - RB, RR, BR, BB
  - ¼, ¼, ¼, ¼
- Imagine a trial is 3 spins
  - RRR, RRB, RBR, BRR, BBB, BBR, RRB, RBB
  - ⅛, ⅛, ⅛, ⅛, ⅛, ⅛, ⅛, ⅛
- Imagine a trial is 10 spins
  - **RRRRRRRRRR**, BBBBBBBBBB, … , **RRRRRRRRRB**, …
  - $1/(2^{10})$, $1/(2^{10})$, …, $1/(2^{10})$

ODD QUESTION #7

- Imitate the results of a 100-trial coin flip.
- How many of you had a sequence of 7 heads (or tails) in a row?
- What's the probability of flipping 7 heads in a row?
- 7 heads in a row would be ½ x ½ x ½ x ½ x ½ x ½ x ½ = 1/128
  - But you technically have 93 chances to flip 7 in a row.
- So, to NOT flip 7 in a row, you'd have to hit a (127/128) event 93 times.
  - $(127/128)^{93}$ = .48… = 48% chance of NOT flipping 7 in a row
- So, 52% chance of flipping 7 in a row

Thanks