

§1. Well-formed Formulas

1. Not well-formed
2. $(\sim)(\sim \sim A \vee (C \supset B))$
3. Not well-formed
4. Not well-formed
5. Not well-formed
6. $((A \equiv (B \& (\sim C \equiv A))) \supset ((D \& A) \& \sim Z))$

§2. Translations into SL

1. $P =$ Bob went to the park
 $S =$ It was sunny outside
 $C =$ It was partly cloudy outside
 $(P \supset (S \vee C))$
2. $S =$ Sue loves going to the movies
 $J =$ Jim loves going to the movies
 $X =$ Sue has gone to the movies recently
 $Y =$ Jim has gone to the movies recently

$$((S \& J) \& \sim (X \vee Y))$$

3. $A =$ Adam eats an apple
 $E =$ Eve gives Adam an apple.

$$(A \supset E)$$

4. $Y =$ You'll get an A in this course

$R =$ You do all the readings

$S =$ You skip at least one lecture

$$((R \& \sim S) \supset Y)$$

5. $S =$ That kid will get a stomach ache

$M =$ That kid takes some medicine

$$((S \vee M) \& \sim S)$$

6. $B =$ Bob loves Sue

$I =$ Sue is indifferent

$M =$ Bob is in a good mood.

$$((B \equiv I) \vee \sim M)$$

7. $O =$ John orders orange juice at the diner

$M =$ John orders milk at the diner

$C =$ John just needs one cup

$$(((O \vee M) \& \sim (O \& M)) \supset C)$$

8. $H =$ I hate it when it rains

$R =$ It rains sometimes

$$(H \& \sim R)$$

9. Y = The Yankees will win the big game tonight
 R = The Red Sox will win the big game tonight
 P = The Yankees' plane crashes on the way
 C = The Red Sox's plane crashes on the way

$$((Y \vee R) \vee (P \& C))$$

Also OK: $((Y \vee R) \& \sim(Y \& R)) \vee (P \& C)$

10. W = The street is wet because it rained last night.

W

§3. Truth-tables

1.

A	$A \equiv (\sim A \supset A)$
T	T
F	T

} truth-functionally true

2.

A	B	$\sim(A \& \sim B)$	\vee	$(A \equiv B)$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

} truth-functionally indeterminate

3.

A	B	$(B \supset \sim \sim B)$	\vee	$(\sim A \& \sim \sim A)$
T	T	T T T F T	T	F T F + F T
T	F	F T F T F	T	F T F + F T
F	T	T T T F T	T	T F F F + F
F	F	F T F T F	T	T F F F T F

truth-functionally true

4.

A	B	C	$((A \supset (B \equiv (\sim C \equiv A))))$	\vee	$((C \& A) \& \sim B)$
T	T	T	T F T F F + F T	F	T T T F F T
T	T	F	T T T T + F T T	T	F F T F F T
T	F	T	T T F T F T F T	T	T T T T T F
T	F	F	T F F F T F T T	F	F F T F T F
F	T	T	F T + T F T T F	T	T F F F F T
F	T	F	F + T F T F F F	T	F F F F F T
F	F	T	F T F F F T T F	T	T F F F T F
F	F	F	F T F T T F F F	T	F F F F T F

truth-functionally indeterminate

5.

A	B	$((A \supset B) \supset (B \supset A))$	\supset	$(\sim A \equiv \sim B)$
T	T	T T T T T T T	T	F T T F T
T	F	T F F T F T T	F	F T F T F
F	T	F T T F T F F	T	T F F F T
F	F	F T F T F T F	T	T F T T F

truth-functionally indeterminate

§ 4. Putting it All together

1. $C =$ MARY will buy a nice CAR
 $B =$ MARY gets a big bonus

P1. $B \supset C$

P2. $\sim B$

C. $\sim C$

B	C	$B \supset C$	$\sim B$	$\sim C$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

truth-functionally invalid

2. $T =$ The grocery store is open on Tuesday
 $W =$ The grocery store is open on Wednesday
 $A =$ Sue will buy Apples

P1. $(T \vee W) \& \sim (T \& W)$

P2. $T \supset A$

P3. $W \supset \sim A$

C. $A \vee W$

T	W	A	$(T \vee W) \& \sim (T \& W)$	$T \supset A$	$W \supset \sim A$	$A \vee W$
T	T	T	F	T	F	T
T	T	F	F	F	T	T
T	F	T	T	T	F	T
T	F	F	T	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	T	T
F	F	T	F	T	F	T
F	F	F	F	T	T	F

No counterexample, so truth-functionally valid

3. $Y =$ The Yankees win the big game

$R =$ It's raining

$U =$ The temperature is over 90 degrees

P1. $Y \equiv \sim R$

P2. $R \supset U$

C. $Y \supset \sim U$

\forall	R	U	$\forall \equiv \sim R$	$R \supset U$	$\forall \supset \sim U$
T	T	T	T F F	T	T F F
T	T	F	T F F	F	T T T
T	F	T	T T T	T	T F F
T	F	F	T T T	T	T T T
F	T	T	F T F	T	F T F
F	T	F	F T F	F	F T T
F	F	T	F F T	T	F T F
F	F	F	F F T	T	F T T

truth-functionally invalid