Problem Set 5

① R = draw a red card

A = draw a black Ace

E = draw an even number

F = draw a face card

a) $Pr(R \vee A) = Pr(R) + Pr(A)$

$= 26/52 + 2/52 = 28/52 = \boxed{7/13}$

b) $Pr(E \vee F) = Pr(E) + Pr(F)$

$= 20/52 + 12/52 = 32/52 = \boxed{8/13}$

② A_n = draw an Ace on the n^{th} draw

Ω_n = draw any suit on the n^{th} draw

S_2 = draw the same suit on the 2^{nd} draw as the suit drawn on the 1^{st} draw

a) $Pr(A_1 \& A_2) = Pr(A_1) \times Pr(A_2) = 4/52 \times 4/52$

$= 16/2704 = \boxed{1/169}$

b) $Pr(A_1 \& A_2) = Pr(A_1)Pr(A_2 | A_1) = 4/52 \times 3/51$

$= 12/2652$

$= \boxed{1/221}$

c) $Pr(\Omega_1 \& S_2) = Pr(\Omega_1) \times Pr(S_2) = 1 \times 1/4$

$= \boxed{1/4}$

d) $Pr(\Omega_1 \& S_2) = Pr(\Omega_1) \times Pr(S_2 | \Omega_1)$

$= 1 \times 12/51 = \boxed{4/17}$

3) G_n = pull a green ball on n^{th} trial

R_n = pull a red ball on n^{th} trial

A_n = pull from Urn A on n^{th} trial

B_n = pull from Urn B on n^{th} trial

$$Pr(G_1 \& R_2) = Pr(G_1) \times Pr(R_2)$$

Note that $Pr(R_2) = Pr(R_1) = 1 - Pr(G_1)$, so

$$Pr(G_1 \& R_2) = Pr(G_1) \times (1 - Pr(G_1)) \quad !$$

$$\Pr(G_1) = \Pr((G_1 \,\&\, A_1) \vee (G_1 \,\&\, B_1))$$
$$= \Pr(G_1 \,\&\, A_1) + \Pr(G_1 \,\&\, B_1)$$
$$= \Pr(A_1)\Pr(G_1 \mid A_1) + \Pr(B_1)\Pr(G_1 \mid B_1)$$
$$= (1/2)(3/7) + (1/2)(1/3)$$
$$= (1/2)(3/7 + 1/3) = (1/2)(9/21 + 7/21)$$
$$= (1/2)(16/21) = 8/21 \quad \text{(About 38\%)}$$

$$\Pr(G_1 \,\&\, R_2) = \Pr(G_1) \times (1 - \Pr(G_1))$$
$$= (8/21)(13/21) = \boxed{104/441} \quad \text{(About 24\%)}$$

(4) E = roll an even number

R_n = roll an n

$$\Pr(E \mid [(R_4 \vee R_5) \vee R_6]) = \frac{\Pr(E \,\&\, [(R_4 \vee R_5) \vee R_6])}{\Pr[((R_4 \vee R_5) \vee R_6)]}$$

$$= \frac{\Pr(R_4 \vee R_6)}{\Pr(R_4 \vee R_5 \vee R_6)}$$

$$= \frac{Pr(R_4) + Pr(R_6)}{Pr(R_4) + Pr(R_5) + Pr(R_6)}$$

$$= \frac{(1/6) + (1/6)}{(1/6) + (1/6) + (1/6)}$$

$$= \frac{(2/6)}{(3/6)} = \boxed{\frac{2}{3}}$$

5. W = the randomly selected marker is from Walmart

S = the randomly selected marker is from Staples

E = the randomly selected marker has the manufacturer error

Note the following:

$Pr(W) = .8$

$Pr(S) = .2$

$Pr(E \mid W) = .04$

$Pr(E \mid S) = .01$

a) $Pr(W \& E) = Pr(W)\,Pr(E \mid W)$

$$= .8 \times .04$$

$$= \boxed{.032}$$

b) $Pr(E) = Pr((W \& E) \vee (S \& E))$

$$= Pr(W \& E) + Pr(S \& E)$$

$$= .032 + Pr(S)\,Pr(E \mid S)$$

$$= .032 + (.2 \times .01)$$

$$= .032 + .002 = \boxed{.034}$$

c) $Pr(W \mid E) = \dfrac{Pr(W \& E)}{Pr(E)}$

$$= \frac{.032}{.034} = \boxed{16/17}$$ (About 94%)

6) $Pr(A \mid R_1 \& R_2) = \dfrac{Pr(A \& R_1 \& R_2)}{Pr(R_1 \& R_2)}$

$$Pr(A \,\&\, R_1 \,\&\, R_2) = Pr(A)\, Pr(R_1 \,\&\, R_2 \mid A)$$

$$= (1/2)\, Pr(R_1 \mid A)\, Pr(R_2 \mid A)$$

$$= .5 \times .6 \times .6 = \underline{.18}$$

$$Pr(R_1 \,\&\, R_2) = Pr((A \,\&\, R_1 \,\&\, R_2) \vee (B \,\&\, R_1 \,\&\, R_2))$$

$$= Pr(A \,\&\, R_1 \,\&\, R_2) + Pr(B \,\&\, R_1 \,\&\, R_2)$$

$$= .18 + Pr(B)\, Pr(R_1 \,\&\, R_2 \mid B)$$

$$= .18 + (1/2)\, Pr(R_1 \mid B)\, Pr(R_2 \mid B)$$

$$= .18 + (.5 \times .2 \times .2)$$

$$= .18 + .02 = \underline{.2}$$

So $Pr(A \mid R_1 \,\&\, R_2) = \dfrac{.18}{.2} = \boxed{.9}$ (90%)