Problem Set 6 Solutions

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1. **Urn A has 30 red and 70 green balls. Urn B has 80 red and 20 green balls. An urn is chosen by flipping a fair coin.**

   A = drew from urn A (flipped heads)

   B = drew from earn B (flipped tails)

   R_1 = pulled red on first draw

   R_2 = pulled red on second draw

   G_1 = pulled green on first draw

   G_2 = pulled green on second draw

   $Pr(A) = Pr(B) = .5$

   (a) Two balls are drawn from this urn with replacement. Both are red. What is the probability that we have urn A?

   $Pr(R_1\&R_2|A) = (.3)^2 = .09$

   $Pr(R_1\&R_2|B) = (.8)^2 = .64$

   $$P(A|R_1\&R_2) = \frac{Pr(A\&R_1\&R_2)}{Pr(R_1\&R_2)} = \frac{Pr(A)Pr(R_1\&R_2|A)}{Pr(A\&R_1\&R_2)+Pr(B\&R_1\&R_2)}$$
   $$= \frac{Pr(A)Pr(R_1\&R_2|A)}{Pr(A)\times Pr(R_1\&R_2|A)+Pr(B)(R_1\&R_2|B)}$$
   $$= \frac{.5\times.09}{.5\times.09+.5\times.64} = \frac{.045}{.045+.32}$$
   $$= \boxed{\frac{.045}{.365} \approx .12}$$

   (b) Two balls are drawn from this urn without replacement. Both are green. **What is the probability that we have urn B?**

   $Pr(G_1\&G_2|A) = \frac{7}{10} \times \frac{69}{99} = \frac{483}{990}$

   $Pr(G_1\&G_2|B) = \frac{2}{10} \times \frac{19}{99} = \frac{38}{990}$

   $$P(B|G_1\&G_2) = \frac{Pr(B\&G_1\&G_2)}{Pr(G_1\&G_2)} = \frac{Pr(B)Pr(G_1\&G_2|B)}{Pr(A\&G_1\&G_2)+Pr(B\&G_1\&G_2)}$$
   $$= \frac{Pr(B)Pr(G_1\&G_2|B)}{Pr(A)\times Pr(G_1\&G_2|A)+Pr(B)(G_1\&G_2|B)}$$
   $$= \frac{.5\times\frac{38}{990}}{.5\times\frac{483}{990}+.5\times\frac{38}{990}} = \frac{38}{483+38}$$
   $$= \boxed{\frac{38}{521} \approx .07}$$

2. Suppose that 50% of our bananas come from Guatemala, 20% come from Honduras, and 30% come from Ecuador. 4% of bananas from Guatemala contain tarantulas, 2% of bananas from Honduras contain tarantulas, and 3% of bananas from Ecuador contain tarantulas. A banana is selected randomly and contains a tarantula. **What is the probability that it came from Ecuador?**

   G = the randomly selected banana came from Guatemala

   H = the randomly selected banana came from Honduras

   E = the randomly selected banana came from Ecuador

   T = the randomly selected banana contains a tarantula

   $Pr(G) = .5$

   $Pr(H) = .2$

   $Pr(E) = .3$

   $Pr(T|G) = .04$

   $Pr(T|H) = .02$

   $Pr(T|E) = .03$

   $$P(E|T) = \frac{Pr(E\&T)}{Pr(T)} = \frac{Pr(E)Pr(T|E)}{Pr(E\&T)+Pr(G\&T)+Pr(H\&T)} = \frac{Pr(E)Pr(T|E)}{Pr(E)Pr(T|E)+Pr(G)Pr(T|G)+Pr(H)Pr(T|H)}$$
   $$= \frac{.3\times.03}{.3\times.03+.5\times.04+.2\times.02}$$
   $$= \frac{.009}{.009+.02+.004}$$
   $$= \boxed{\frac{.009}{.033} \approx .27}$$

3. Suppose that in Logitropolis, 60% of taxis are green and 40% of taxis are blue. There was a hit-and-run accident involving a taxi, and our witness testifies that she saw the taxi responsible and that it was blue. We test her perceptual abilities and find that this witness is correct 90% of the time when presented with a series of green and blue cars under the same conditions of the accident. **What is the probability that the hit-and-run taxi is blue given that the witness testified that it's blue?**

   G = the hit-and-run taxi is green

   B = the hit-and-run taxi is blue

   W_b = the witness testifies that it's blue

   $Pr(B) = .4$

   $Pr(G) = .6$

   $Pr(W_b|B) = .9$

   $Pr(W_b|G) = .1$

   $$Pr(B|W_b) = \frac{Pr(B\&W_b)}{Pr(W_b)} = \frac{Pr(B)Pr(W_b|B)}{Pr(B\&W_b)+Pr(G\&W_b)} = \frac{Pr(B)Pr(W_b|B)}{Pr(B)Pr(W_b|B)+Pr(G)Pr(W_b|G)}$$
   $$= \frac{.4\times.9}{.4\times.9+.6\times.1} = \frac{.36}{.36+.06} = \boxed{\frac{.36}{.42} \approx .86}$$

1 Expected Value

4. A storm has 15% probability, and you have to decide between traveling via train or plane.

   The train will take at least 3 hours no matter what, plus EITHER (i) 3 additional hours, if there is no storm OR (ii) 5 additional hours, if there is a storm.

   The plane will take at least 2 hours no matter what, plus EITHER (i) 1 additional hour, if there is no storm OR (ii) 9 additional hours, if there is a storm.

   What's the expected travel time (expected value) of traveling by each?

   $Exp(train) = (3 \times 1) + (3 \times .85) + (5 \times .15) = 3 + 2.55 + .75 = \boxed{6.3 \text{ hours}}$

   $Exp(plane) = (2 \times 1) + (1 \times .85) + (9 \times .15) = 2 + .85 + 1.35 = \boxed{4.2 \text{ hours}}$

5. Suppose we run a lottery with 5,001 tickets of equal chance of being drawn. Tickets cost \$2.50 each. The prize is \$11,000. One quarter of the losers are randomly selected to receive a \$3.25 consolation prize. **What is the expected value of buying a ticket in this lottery?**

   $$Exp(ticket) = (-2.5 \times 1) + (11,000 \times \tfrac{1}{5001}) + (3.25 \times (\tfrac{5000}{5001} \times \tfrac{1}{4}))$$
   $$= -2.5 + \tfrac{11,000}{5001} + \tfrac{16,250}{20,004} \approx -2.5 + 2.2 + .81 \approx \boxed{\$0.51}$$

6. Suppose you are betting on whether a fair coin will lands heads or tails. If you bet correctly, you win back double your bet. (For example, if you bet \$10 and are correct, you win \$20 so are "up" \$10.) You decide to try out the *Martingale betting strategy* (see p. 90 of PIL, or my slide #13) and would like to guarantee that you eventually profit (are "up") \$5 total. (Assume you have infinite money, and that there is no maximum bet amount.) **How many times would you have to bet and lose in a row until the next bet is the first one greater than \$700?**

Spin	Bet Required
1	$2^{(1-1)} \times 5 = \$5$
2	$2^{(2-1)} \times 5 = \$10$
3	$2^{(3-1)} \times 5 = \$20$
4	$2^{(4-1)} \times 5 = \$40$
5	$2^{(5-1)} \times 5 = \$80$
6	$2^{(6-1)} \times 5 = \$160$
7	$2^{(7-1)} \times 5 = \$320$
$\boxed{8}$	$2^{(8-1)} \times 5 = \$640$
9	$2^{(9-1)} \times 5 = \$1,280$

   $\boxed{\text{You would have to bet and lose 8 times in a row.}}$