Variable-based Intensionality for Structured Propositions

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Abstract: Variable-based theories of linguistic intensionality introduce covert possible world variables at logical form in order to account for cases of transparency, such as the third reading of indefinites, that traditional scope-based theories cannot explain. These variable-based theories generally presuppose a Hintikkan, possible worlds semantics for propositional attitude verbs, leaving a gap in the literature on the proper treatment of third readings for advocates of more finely grained, structured propositions. Recently, Lederman (2022) has attempted to bridge this gap for Fregeans, but his theory faces a counterexample unrelated to Fregeanism. Independently, many theorists prefer a simpler, Russellian account of propositions. In this paper, I develop a variable-based theory of intensionality for structured, Russellian propositions that explains the third reading of indefinites and is not susceptible to the counterexample. The formal implementation involves an intensionalized version of the semantics for structured propositions found in Pickel (2019), supplemented with novel choice-function machinery. This result puts the theory of structured, Russellian propositions one step ahead of the only Fregean account, and brings it up to speed with Hintikkan approaches to linguistic intensionality.

1 Introduction

It's widely recognized that traditional scope-based theories of intensionality, such as Russell (1905), Montague (1973), Ladusaw (1977), Stowell (1993), and Ogihara (1996), struggle to account for the behavior of determiner phrases (DPs) in the context of certain attitude reports. The so-called "third reading of indefinites" first identified by Janet Fodor (1970), for example, shows that scopal relations alone fail to generate all of the accessible readings for certain reports in context. In response to these cases, variable-based theories of intensionality, such as Percus (2000), Schueler (2011), and Schwarz (2012), explain third readings by positing covert possible world variables at logical form (LF) under a possible worlds semantics in the style of Hintikka (1969). There remains a gap in the literature, however, on the proper treatment of third readings by advocates of more finely-grained semantics, such as those according to which propositions are structured entities individuated by the identity and arrangement of constituents.[1]

Recently, Lederman (2022) has attempted to bridge this gap by providing a variable-based theory of intensionality under broadly Fregean assumptions about the nature of

[1] I'll assume that propositions are the objects of psychological attitudes such as belief and desire, the designations of 'that'-clauses in context, and the compositional semantic values of declarative sentences in context. For ease of exposition, I'll sometimes omit references to context and, more generally, other relevant parameters of relativization once they have already been explicitly introduced and are clear from context.

structured propositions. For our purposes, Fregeanism is the view that senses or modes of presentation – as in Frege (1892) – of objects, properties, and relations are among the compositional semantic values of subsentential expressions and the constituents of structured propositions. Fregeanism contrasts with Russellianism, which takes not senses but objects, properties, and relations themselves – as in Russell (1903) – to play the roles of semantic values and propositional constituents. In this paper, I'll show that Lederman's theory is susceptible to a counterexample unrelated to any shortcomings of Fregeanism, and I'll propose a more successful theory in its place that assumes Russellianism for two reasons. First, it is simpler to understand how the core of Lederman's theory faces a counterexample once we abstract away from its semantic layer involving Fregean senses. Second, many structured propositionalists remain unconvinced that Fregeanism is correct, and Russellianism tends to be the default alternative. My goal in this work is to therefore propose a variable-based theory of intensionality for structured, Russellian propositions that explains the third reading of indefinites without being susceptible to the counterexample to Lederman's theory. If successful, the upshot will be to put Russellians one step ahead of the only Fregean proposal while bringing them up to speed with Hintikkan approaches to linguistic intensionality.

The paper proceeds as follows. In §2, I'll set the stage by introducing the third reading of indefinites and its standard, variable-based explanation under a Hintikkan semantics for attitude verbs. It's tempting to think that the main obstacle preventing structured propositionalists from adopting this style of explanation is precisely the rejection of possible worlds semantics, but this would be an oversimplification. In §3, I'll argue that there are deeper theoretical constraints involving the nature of compositionality that must be settled in order for a variable-based theory to be a live option for them. These constraints contextualize Lederman's theory, which I'll present in §4 in abstraction from Fregeanism and argue against by way of counterexample. The shortcoming of Lederman's theory segues in §5 into the development of a new variable-based theory of intensionality for structured, Russellian propositions. Its formal implementation involves an intensionalized version of the semantics for structured propositions found in Pickel (2019), supplemented with novel choice-function machinery. This presentation of what I'll call "the Property Swap theory" breaks down into three parts that respectively address semantic preliminaries, opacity, and transparency, including third readings. I'll show how to extend the framework to some basic linguistic embeddings in §6 in order to gesture at the need for more sustained work on the landings sites for existential quantification over choice functions. Since the semantic theories on offer have become quite complex in this space, I'll emphasize the importance of getting them right in §7 by briefly explaining the difficulties one will encounter if one instead tries to offer pragmatic explanations of intensional phenomena instead. I'll conclude in §7 by recapitulating a few key points and explaining the theoretical significance of the new theory.

2 Third readings and variable-based intensionality

Consider the following belief report:

(1) Betty believes that every senator spies.

As is well-known, this report admits of readings that are true in different contexts, such

as the following:

> Suspicious Senators
> Betty the reporter is suspicious of the government and thinks that maintaining a senatorial seat requires spying. Despite not knowing any senators, she pens a story with the headline, 'EVERY SENATOR SPIES.'
>
> Suspicious Suits
> Betty the reporter thinks that anyone rich enough to wear an Armani suit must engage in espionage. Walking home one night, she passes a convention center and sees all of its attendees wearing Armani suits. Unaware that these are the senators, she pens a story with the headline, 'EVERYONE AT CONVENTION SPIES.'

On the reading of (1) true in Suspicious Suits, the NP-complement 'senator' can be substituted with a co-extensional expression such as 'convention-goer' salve veritate, but this isn't the case on the reading of (1) true in Suspicious Senators. When a sentential operator O operates on a sentence S whose surface form contains a DP with NP-complement φ, we'll say that the DP is *transparent*, and otherwise *opaque*, with respect to O in context c just in case, for every expression φ^* co-extensional with φ in c, $\ulcorner OS \urcorner$ is true in c if and only if $\ulcorner OS[\varphi^*/\varphi] \urcorner$ is true in c.[2] Then the reading of (1) true in Suspicious Suits contains an occurrence of 'every senator' that is transparent with respect to the attitudinal operator 'Betty believes,' and the reading true in Suspicious Senators contains an occurrence that is opaque.

According to traditional scope-based theories of intensionality, such as Russell (1905), Montague (1973), Ladusaw (1977), Stowell (1993), and Ogihara (1996), whether a DP is transparent or opaque with respect to a sentential operator depends exclusively on the syntactic position of the DP relative to that of the operator at LF.[3] Such a view predicts the different readings of (1) by assigning it two possible logical forms, which can be represented at the relevant degree of abstraction as follows:

(1a) $\forall x(\mathit{senator}(x) \rightarrow \mathit{Betty\text{-}believe}\text{: } \mathit{spy}(x))$ ← wide scope
(1b) $\mathit{Betty\text{-}believe}\text{: } \forall x(\mathit{senator}(x) \rightarrow \mathit{spy}(x))$ ← narrow scope

The prediction is that a DP δ is transparent with respect to a sentential operator O just in case δ c-commands O at LF, as in the wide scope reading (1a). Otherwise, the DP is predicted to be opaque, as in the narrow scope reading (1b). Since May (1977), the

[2] $\ulcorner S[\varphi^*/\varphi] \urcorner$ is the sentence that results from substituting every occurrence of φ in S with φ^*. Characterizing the readings of (1) in terms of opacity and transparency is an intentional departure from their traditional characterization in terms of "de dicto" and "de re" readings as in Quine (1956). The present work brackets questions surrounding the proper derivation of ascriptions of singular (or "de re") attitudes, which might be more faithfully represented by reports such as the following (cf. §2.5 of Hawthorne and Manley (2012)):

(i) Betty believes of every senator that he spies.

For a similar point, see footnote 2 of Lederman (2022), and for a recent account of the syntax and semantics of reports in the form of (i), see Rausch (2021).

[3] Traditional scope-based theories of intensionality contrast with non-traditional scope-based theories such as Keshet (2011), the consideration of which falls outside the scope of this work.

predominate approach to syntactically deriving such logical forms is through Quantifier Raising (QR), a form of movement by which a DP adjoins a higher sentential node and leaves behind a trace at LF. While this view enjoys significant empirical coverage, a number of troubling counterexamples to it, including the third reading of indefinites, were identified in the latter half of the twentieth century.[4]

Consider the following context and belief report:

> GAME SHOW
> You're watching a live game show with Betty, who sadly thinks that you have no friends. She also holds the superstitious belief that anyone with green eyes is extremely lucky. As it so happens, your only two friends, Xavier and Yvette, are contestants on the show, and they are the only ones with green eyes. Seeing them, Betty turns to you and says, "I think one of those green-eyed contestants will win."

(2) Betty believes that a friend of yours will win.

It's widely agreed that there's a reading of (2) that is true in Game Show. But as Janet Fodor (1970, 226-232) first observed, it results from neither the wide nor narrow scope interpretation of the DP.[5] These interpretations result from logical forms such as the following:

(2a) $\exists x(friend(x)$ & *Betty-believe:* $win(x))$ ← wide scope
(2b) *Betty-believe:* $\exists x(friend(x)$ & $win(x))$ ← narrow scope

On the intended interpretation, '$friend$' is assigned to the property of being a friend of yours, and '*win*' to the property of winning the game.[6] The wide scope reading is false in this context because no individual friend of yours is such that Betty stands in the belief relation to the singular proposition that (s)he will win. Betty doesn't believe that Xavier will win, and she doesn't believe that Yvette will win, so there's no witness to the existential quantifier in (2a). The narrow scope reading is false in this context, too, because it's not the case that Betty believes you have any friends. Call the reading on which (2) is true in Game Show "the third reading." Since there don't appear to be any additional scopal interactions available at LF, traditional scope-based theories of intensionality prima facie predict incorrectly that the third reading is unavailable in this context.

The third reading seems to require a way of evaluating the NP-complement ('friend of yours') relative to the evaluation world of the matrix clause while simultaneously keeping the force of existential quantification within the subordinate clause. This poses a difficulty for traditional scope-based theories because the entire DP moves outside of the subordinate clause at LF through QR.[7] According to variable-based theories of

[4]For overviews of this literature, see Keshet and Schwarz (2019) and §4 of Grano (2021). For a canonical introduction to QR, see §7 of Heim and Kratzer (1998).

[5]For additional examples, see Ioup (1977) and Bonomi (1995).

[6]I'll abstract away from tense and aspect, which don't concern us here.

[7]Sophisticated forms of syntactic movement that bypass this difficulty are considered in §8.3.2 of von Fintel and Heim (2011) and Appendix B of Lederman (2022). If one is willing to countenance what are generally considered to be ad hoc movement operations, these accounts can correctly predict the third reading's truth conditions for Hintikkans and structured propositionalists alike. Following Lederman, I will prioritize developing a non-movement-based approach, so that structured propositionalists are not held hostage to controversial syntax.

intensionality, such as Percus (2000), Schueler (2011), and Schwarz (2012), the proper account of the third reading involves covert possible world variables at LF.[8] What von Fintel and Heim (2011) call *the standard solution*, for example, generates the third reading of (2) by assigning it a logical form such as the following:

(3) λw_1 *Betty-believe*$_{w_1}$: λw_2 $\exists x(friend_{w_1}(x)$ & $win_{w_2}(x))$

where variables $w_1, w_2, \ldots$ of semantic type s range over possible worlds.[9] According to this view, verbs select for possible world variables and project λ-operators co-indexed with them to the heads of their immediate clauses. Nouns also select for possible world variables, but their variables are bound by the λ-operators projected by verbs. In this case, '*win*' is evaluated with respect to the evaluation world of the subordinate clause ('w_2'), and '*friend*' with respect to the evaluation world of the matrix clause ('w_1').[10] Given the standard λ-calculus and intended meanings for predicates and logical vocabulary, the subordinate clause is therefore semantically interpreted as the intension that maps a world w to 1 just in case there exists an x such that x is your friend in the evaluation world of the matrix clause and x wins in w.

The attitudinal operator acts on this intension in accordance with a possible worlds semantics, assuming a semantic value in context c along the following lines:[11]

$$[\![\textbf{Betty-believe}]\!]^c = \lambda w_s \lambda T_{<s,t>}(\text{every } w^* \text{ compatible with Betty's beliefs in } w \text{ is such that } T(w^*) = 1)$$

As a result, the semantic interpretation of the matrix clause in c is the intension defined as follows, for any world w:

$[\![(3)]\!]^c(w) = 1$ if and only if the following condition is met:
for every world w^* compatible with Betty's beliefs in w,
there is an x such that x is your friend in w, and
x wins the game in w^*

When '*friend*' in (3) is substituted by a noun co-extensional with it in the evaluation world of the matrix clause, the same coarse-grained intension as above results. So, the DP is correctly predicted to be transparent. In general, for any DP δ, NP-complement φ, and sentential operator O, the standard view predicts δ to be transparent with respect to O just in case φ operates on a world variable w_i and the λ-operator $\ulcorner \lambda w_i \urcorner$ adjoins the main clause at LF. On this view, a sentence is true in a context just in case the semantic interpretation of its logical form in the context maps the world of the context to 1. In this way, (2) is correctly predicted to be true in Game Show.

[8] The original implementation in Percus (2000) appeals to situation variables, but I'll conflate situations and possible worlds.

[9] I'll display possible world variables as subscripts to verbs and nouns at LF, but these variables should be understood as semantically contributing arguments for intensions.

[10] Percus (2000) and Keshet (2008) note that the standard solution requires a variable binding theory that restricts the availability of certain co-indexations.

[11] I'll treat 'Betty believes' as a single lexical item for now because the details of the compositional derivation that this simplification obfuscates are not at issue. Semantic values in this work are relativized to both a context c and the variable assignment function determined by c, g_c, but I'll only make variable assignment functions explicit when treating the semantics of quantification.

3 Structured propositions

Possible worlds semantics for propositional attitude verbs play an important role in the standard solution. Since the subordinate clause is semantically interpreted as a coarse-grained intension, the NP-complement can be evaluated with respect to the actual world and remain within the scope of the attitudinal operator without placing any implausible requirements on the attributee's conceptual repertoire. But such coarseness of grain, which is a theoretical advantage in this case, comes with the well-known cost of seeming to incorrectly predict necessarily equivalent sentences to be substitutable salve veritate in the context of attitude reports. For example, since the worlds in which Biden is president are exactly those in which Biden is president if and only if arithmetic is undecidable, the following belief reports are prima facie predicted incorrectly to be true in the same contexts:

(4) Betty believes that Biden is president.
(5) Betty believes that Biden is president if and only if arithmetic is undecidable.

Moved by this consideration among others, many theorists have adopted the view that psychological attitude verbs semantically express relations between subjects and more finely-grained, structured propositions individuated by the identity and arrangement of constituents.[12] According to this view, the proposition reported as the object of Betty's belief in (4) is not identical with the proposition reported as the object of her belief in (5), because only the latter contains the semantic values of 'arithmetic' and 'undecidable' as constituents. Structured propositionalists therefore maintain that some distinct propositions have the same truth-value in every possible world.

Despite the differences between their views, structured propositionalists generally take there to be a correspondence between propositional constituents and subsentential semantic values. For our purposes, the following principle serves as a representative example of this commitment:

> **Terminal Node Constituency:** For any proposition P and thing x, x is a constituent of P if and only if for some sentence S and context c, S designates P in c and x is the semantic value of a terminal node in the logical form of S in c.

Suppose, for example, that the sentence 'Serena smiles' designates in c the proposition that Serena smiles, that 'Serena' and 'smiles' are terminal nodes in the logical form of 'Serena smiles' in c, and that the semantic values of 'Serena' and 'smiles' in c are, respectively, Serena and the property of smiling. Then Terminal Node Constituency entails that Serena and the property of smiling are constituents of the proposition that Serena smiles. A couple of caveats are worth mentioning. First, it might be desirable to restrict this principle to sentence-context pairs that are "canonical" – in some sense incumbent upon the structured propositionalist to explain – in order to bracket degenerate cases arising

[12] See, e.g. Frege (1892), Russell (1903), Carnap (1947), Lewis (1972), Cresswell and von Stechow (1982), Zalta (1983, 1988), Cresswell (1985), Soames (1985, 1987, 1989), Salmon (1986a, 1986b, 1989a,1989b), Menzel (1993), King (1995, 1996, 2007, 2009), Richard (2013), and Bacon (2023). Stalnaker (1984) provides the traditional defense of the possible worlds view. For a more general argument against propositions as sets of truth-supporting circumstances, including possible and impossible worlds, see Soames (1987).

from stipulated conventions.[13] Second, it might be desirable to restrict the attribution of propositional constituency to the semantic values (in context) of certain occurrences of lexical items.[14] Complications aside, a commitment of this nature is motivated by the thought that propositional constituency must be determined in a principled manner, since allowing for other sources of constituency threatens to trivialize semantic theorizing, generally.[15] For our purposes, Terminal Node Constituency is intended as an extensionally adequate characterization of propositional constituency, and not as a metaphysical analysis of what it is to be a propositional constituent.

It's not immediately obvious whether – and, if so, how – a structured view of propositions with such commitments can account for the third reading by incorporating possible world variables into logical forms. The standard solution is unavailable as it stands because structured propositionalists can't take propositions to be unstructured sets of possible worlds, but there are more foundational difficulties preventing adherents of Terminal Node Constituency from easily adopting a syntax inspired by the standard solution. Understanding these limitations will help to contextualize the semantic theories we'll consider in this work, and the best way to bring them out is to see what goes wrong when a structured propositionalist attempts to adopt the syntax of the standard solution in the simplest way.

For this exercise, let's grant the structured propositionalist a method of compositionally deriving structured propositions as the semantic values of 'that'-clauses in context; we'll surround a clause with angle brackets at LF to indicate the application of such a method. Let's also grant that possible world variables are not syntactically required but may nevertheless occur covertly as needed at LF. Now suppose a structured propositionalist tried to implement a variable-based theory of intensionality by assigning the third reading of (2) in context c a logical form along the following lines:

(6) $\mathit{Believe}[\mathit{Betty}, \langle \exists x(\mathit{friend\text{-}in}(w_c, x) \;\&\; \mathit{win}(x))\rangle]$

On the intended interpretation, '*Believe*' is assigned to the belief relation, '*Betty*' to Betty, '*friend-in*' to the relation of being a possible world w and individual x such that x is a friend of yours in w, and 'w_c' to the world of c.[16] As indicated by the angle brackets, the subordinate clause is interpreted as designating a structured proposition, which by Terminal Node Constituency has constituents corresponding to the machinery of existential quantification, '*friend-in*,' 'w_c,' '*wins*,' and the conjunction operator.[17] Our

[13]Suppose, for example, that a community of speakers stipulates that the sentence 'Serena smiles' designates in context the proposition that the present day is good, even though 'Serena' might still occur as a terminal node and have Serena herself as its semantic value (in context). Then Terminal Node Constituency will entail that Serena is a constituent of the proposition that the present day is good. However problematic the supposition might be, the restriction to canonical sentence-context pairs avoids the difficulty altogether.

[14]Collins (2007) argues, for example, that if propositional structure mirrors syntactic structure, then syntactic copying at LF results in too many, duplicative propositional constituents.

[15]For considerations against so-called "unarticulated constituents," see Stanley (2000).

[16]More generally, for any context named by expression α, $\ulcorner w_\alpha \urcorner$ names the world of that context.

[17]In a fully worked out version of the view, (6) as a whole would also designate a structured proposition. For ease of exposition, I'll speak in terms of the truth-conditions of (6) in context instead; the considerations of this section run orthogonal to the compositional derivation of structured propositions as the designations of clauses in context, and the details of such derivations are taken up fully in later sections.

structured propositionalist will then propose on the basis of this logical form the following truth-conditions for the third reading of (2) in context c:

(2) is true in c if and only if Betty stands in the belief relation in w_c to the structured proposition that for some x, x is a friend of yours in w_c and x wins.

Their idea is to maintain that the instantiation conditions of the belief relation are loose enough for these truth-conditions to be satisfied in Game Show, even if Betty insists that she holds no belief the truth of which depends on who your friends are in any possible world. The advantage of including the world of the context in the specification of the object of Betty's reported belief is that its own truth-conditions will depend on who your friends are only in that world. So, this structured proposition has the desired modal profile when evaluated for truth at different possible worlds. If such an account could be made to work – let's call it *the simple theory* – then structured propositionalists would have a syntactically and semantically straightforward variable-based theory of intensionality at the cost of a less traditional account of the belief relation.

There are two problems with the simple theory, however, that suggest a more sophisticated approach. The first problem comes in the form of a slingshot argument in the style of Church (1943). Let ϕ be any variable-free, declarative sentence in English, and consider the third reading of the following report:

(7) ⌜Betty believes that a friend of yours such that ϕ will win.⌝

According to the simple theory, the truth-conditions of this reading in context c can be expressed by the following:

> ⌜(7) is true in c if and only if Betty stands in the belief relation in w_c to the structured proposition that for some x, x is a friend of yours in w_c such that ϕ and x wins.⌝

The problem is that any principled theory of the belief relation will plausibly require the following entailment relations to obtain in c under the assumption that Betty is minimally rational:

> ⌜Betty stands in the belief relation in w_c to the proposition that for some x, x is a friend of yours in w_c such that ϕ and x wins.⌝
>
> $\Longrightarrow$
>
> ⌜Betty stands in the belief relation in w_c to the proposition that for some x, x is a friend of yours in w_c such that ϕ.⌝
>
> $\Longrightarrow$
>
> ⌜Betty stands in the belief relation in w_c to the proposition that for some x, x is such that ϕ.⌝
>
> $\Longrightarrow$

⌜Betty stands in the belief relation in w_c to the proposition that ϕ.⌝

If these entailments obtain, then the truth of the third reading of (7) in c requires Betty to stand in the belief relation in the world of c to every proposition designated (in context) by a variable-free, declarative sentence in English – an absurdity. In order to avoid this result, an advocate of the simple theory will need a principled reason to reject one of the entailments, despite their seeming simplicity and plausibility, while simultaneously maintaining that the instantiation conditions of the belief relation are looser than has been traditionally supposed. In light of this, objecting to one of these entailments and not the others seems objectionably ad hoc, and any independent motivation for doing so has not yet been produced.[18]

The second problem with the simple theory involves third readings embedded within the consequents of counterfactual conditionals. Consider the following sentence:

(8) Even if you were friendless, Betty would (still) believe that a friend of yours would win.

There's a reading of this sentence that is true in Game Show in which the indefinite DP 'a friend of yours' is evaluated with respect to the evaluation world of the main clause.[19] Let w_c be the evaluation world of the main clause, and let $w_\diamond$ be the nearest possible world to w_c in which you have no friends. Then according to the simple theory, supplemented with a standard semantics for counterfactuals in the style of Stalnaker (1968) and Lewis (1973), the intended reading of (8) is true in c if and only if Betty stands in the belief relation in $w_\diamond$ to the structured proposition that a friend of yours in w_c will win. According to Terminal Node Constituency, these truth-conditions require Betty to stand in the belief relation in a counterfactual scenario to a proposition that contains the evaluation world of the main clause, i.e. the actual world, as a constituent. But this is problematic because it's possible for Betty to have the belief reported by the intended reading without believing anything at all about the actual world.[20] Moreover, if a special relation of causal or epistemic acquaintance must obtain between believers and the constituents of their beliefs – as many structured propositionalists maintain – then it would be especially implausible for Betty to bear such a relation in a merely possible world to the actual world.[21] With or without such an acquaintance constraint, there are

[18]One might worry that the slingshot argument problematically proves too much, namely, that a transparent report of the form ⌜S believes that a friend of mine such that ϕ will win⌝ entails ⌜S believes that ϕ⌝ is true, for an arbitrary sentence ϕ. The slingshot argument appeals to entailment relations, however, that don't obtain between belief reports themselves but on claims involving the belief relation and its propositional relatum. The final entailment, in particular, becomes problematic when transformed into an inference on belief reports; the transparent reading of ⌜Betty believes that something such that ϕ exists⌝ does not entail ⌜Betty believes that ϕ.⌝

[19]There's also a reading not at issue that is trivially false in Game Show, assuming that the counterfactual construction syntactically introduces a possible world variable binder allowing the NP-complement to be evaluated relative to the relevant counterfactual scenario(s). I'll discuss modal embedding more fully later.

[20]Soames (1998) deploys an analogous argument against rigidifying forms of descriptivism about proper names, although see Pickel (2012) for a reply.

[21]A variety of authors who endorse a special relation of causal or epistemic acquaintance on singular belief are included in Jeshion (2010), although see Hawthorne and Manley (2012) for a dissenting view.

strong reasons for thinking that the simple theory incorrectly predicts that there isn't a true reading of (8) in Game Show.

The moral of the simple theory's failure is that the truth of the third reading can't be explained in terms of the attributee standing in the belief relation to a proposition with a constituent corresponding to either the embedded NP-complement or a covert possible world variable.[22] This suggests that structured propositionalists interested in a variable-based theory of intensionality must somehow take certain embedded NP-complements and world variables to make semantic contributions without contributing propositional constituents, and this amounts to a rejection of Terminal Node Constituency.

4 Against Rigidification

Recently, Lederman (2022) has proposed a semantic account of third readings that rejects Terminal Node Constituency and adopts broadly Fregean assumptions about the nature of structured propositions. The guiding thought behind his proposal, abstracting away from the particular Fregean implementation, is that the third reading of (2) is true in Game Show because the NP-complement 'friend of yours' could just as well have been replaced with 'person identical with Xavier or Yvette,' on the basis of the fact that exactly Xavier and Yvette satisfy the NP ('friend of yours') in the context. After presenting a sketch of this theory, I'll argue that it is susceptible to a counterexample, the proper handling of which should inform the construction of a new theory.

To see how Lederman's idea can be formally implemented, let's grant the structured propositionalist a number of useful tools, including an intensional implementation of QR, intensions of semantic type *set* for NPs, and a semantics for generalized quantifiers on which determiners of semantic type $<set, <set, st>>$ deliver finely-grained "functions in intension," in the sense of Church (1941), as the semantic values of subordinate clauses.[23] Attitude verbs will be treated as expressing binary relations to such structured intensions. Finally, we'll assume that unbound world variables are interpreted by default as the world of the context of utterance.[24]

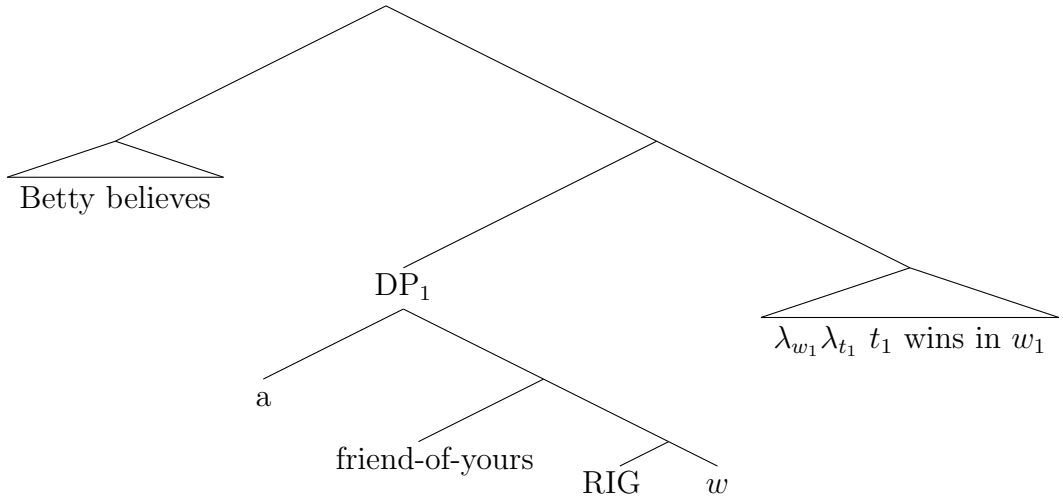
[22]The simple theory is a more complicated version of what I'll call *the flatfooted theory*, which isn't a variable-based theory of intensionality at all, according to which the true reading of (2) in Game Show is simply its narrow scope reading (2b), appearances to the contrary notwithstanding. While the simple theory requires covert possible world variables at LF, the flatfooted theory does not. I focus on the simple theory in the main text for the following reasons: (i) it's a variable-based theory of intensionality, (ii) it captures the correct modal profile of the structured proposition to which Betty reportedly stands in the belief relation, and (iii) it only requires loose instantiation conditions for the belief relation between subjects and a special class of propositions, namely, those with relations to possible worlds as constituents. Both views are susceptible to the slingshot argument, mutatis mutandis, although the flatfooted theory doesn't face the problem of third readings embedded in counterfactual conditionals.

[23]It doesn't matter for our purposes whether there is, upon consideration, a defensible notion of function in intension. We just need some notion of a finely-grained content that easily integrates within an intensional semantic framework in order to illustrate the central idea of Lederman's theory.

[24]This assumption is considered in §8.2.4 of von Fintel and Heim (2011). It's a divergence from the treatment in §3.2 of Lederman (2022), which binds the possible world variable by a λ-operator at the top of the clause in order to accommodate various embedded constructions. These details need not detain us here, however, since the relevant shortcomings of the theory under discussion run orthogonal to issues surrounding embedded constructions. I'll return to modal embeddings later.

Now consider the following abstract proposal for the logical form of the third reading of (2) in Game Show – hereafter, *GS* – after movement by intensional QR:[25]

(9)

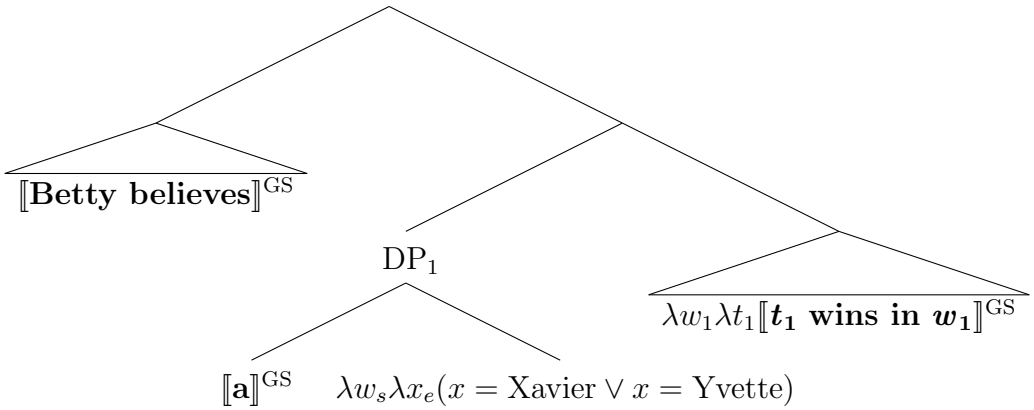


Call 'RIG' a *rigidifier*; it's purpose is to rigidify the interpretation of the noun with respect to the world of the context of utterance. To achieve this, its semantic value across contexts will be the function, κ, that maps any world and noun-meaning to the (intensional) characteristic function of the set of individuals satisfying the noun-meaning in the world of the context. That is, for any world w_s and noun-meaning P_{set},

$$\kappa(w)(P) = \lambda w'_s \lambda x_e (x = a_1 \vee x = a_2 \vee \ldots \vee x = a_n)$$

where each a_i is such that $P(w)(a_i) = 1$.[26] In the case of Game Show, successive functional application of κ to w_{GS} and $[\![\textbf{friend-of-yours}]\!]^{GS}$ yields the following intermediate stage:

(10)



[25]The syntax here is analogous to the first syntax tree in §3.2 of Lederman (2022), although note a slight typographic error there that displays [[**RIG** t_{4_w} **winners**] **lost**] as a sentential constituent, instead of the intended [**two** [**RIG** t_{4_w} **winners**]].

[26]This constant function is analogous to what Lederman (2022) calls a "mere list." For simplicity, I'll ignore cases where the noun-meaning's extension is infinite; accommodating these cases requires redefining κ in terms of set theory or infinitary logic.

The rejection of Terminal Node Constituency is apparent, as the semantic values of 'friend-of-yours,' 'RIG,' and 'w' (in context) have disappeared from the compositional derivation. Appealing to the rest of our simplifying semantic assumptions, we'll take the semantic value of the indefinite determiner to map the two meanings of type *set* to a structured intension which, after β-reduction, is defined as follows:[27]

11. $\lambda w \exists x((x = \text{Xavier} \vee x = \text{Yvette}) \ \& \ Wins(w)(x) = 1)$

Finishing the derivation with a relational semantics for the attitudinal operator, we arrive at the final prediction that the third reading of (2) is true (in GS) if and only if Betty stands in the belief relation (in w_{GS}) to (11), understood as a finely-grained function in intension. Informally, we can say that the third reading is true just in case Betty stands in the belief relation to the structured proposition that someone identical with Xavier or Yvette will win. And we'll take this proposition to involve Xavier and Yvette themselves as constituents. Since Betty does reasonably satisfy this condition in Game Show, Lederman's view correctly predicts the truth of the third reading in this context.

The theory just sketched entails the following principle:

> **Rigidification**: If δ is a transparent DP with NP-complement φ occurring in an attitude report true in context c, then the attributee of the report stands in the belief relation to a structured proposition that contains as constituents the individuals satisfying φ in c.

The system of Lederman (2022) adheres to Rigidification in its own terms.[28] As Lederman himself points out, however, it isn't immediately obvious that the attributee of any third reading whatsoever stands in the belief relation to a proposition containing the required constituents:

> [O]ne might wonder whether in every case where such reports are available, the ascribee can be reasonably said to stand in the belief-relation to a relevant thought *[proposition]*. In response to this concern, I note that a parallel assumption is also built into all competing treatments I am aware of. (1262, parenthetical mine)

This passage makes it clear that there is a gap in the literature surrounding a potential shortcoming of Rigidification.

Consider the following context:

> Blindfolds
> You're watching a live game show with Betty, who sadly thinks that you have

[27] In particular, we require $[\![\mathbf{a}]\!]^c = \lambda P_{set} \lambda Q_{set} \lambda w_s \exists x_e (P(w)(x) = 1 \ \& \ Q(w)(x) = 1)$.

[28] Lederman (2022) characterizes it as follows:

> the proposal requires that, when a person's beliefs can truly be reported using a transparent attitude ascription, the person must stand in the belief-relation to a thought composed in part of a sense of a rigid property. For instance, I assumed that John stood in the belief-relation to a thought composed in part of the sense of the list "Ann, Bill, Carol and Dan". (1262)

> no friends. She also holds the superstitious belief that anyone with green eyes is extremely lucky. The three contestants are: (i) Xavier, your friend with visible, green eyes, (ii) Yvette, your blindfolded friend with green eyes, and (iii) Zoe, a blindfolded stranger with brown eyes. Seeing them, Betty turns to you and says,"I don't know all of their eye colors, but I think a green-eyed contestant will win."

In this context, Betty knows that there's at least one green-eyed contestant, namely, Xavier, but she doesn't know who all the green-eyed contestants are due to the blindfolds. Still, she thinks that whoever the green-eyed contestants happen to be, one of them will win. The reading of (2) that's true in Blindfolds is a third reading for familiar reasons; no individual friend of yours is such that Betty stands in the belief relation to the singular proposition that (s)he will win, contra the wide scope interpretation, and Betty believes you have no friends, contra the narrow scope interpretation. As before, Lederman's theory predicts this reading to be true in Blindfolds only if Betty stands in the belief relation to the proposition that someone identical with Xavier or Yvette will win. The problem is that this is no longer plausible here, because Yvette and Zoe are indistinguishable contestants from Betty's point of view in virtue of their blindfolds. There does not seem to be a principled reason for taking Betty to stand in the belief relation to the required proposition instead of the proposition that someone identical with either Xavier or Zoe will win. So, Lederman's theory seems to incorrectly predict that there is no true reading of (2) in Blindfolds.

One might wonder whether Lederman could simply respond by insisting that, appearances notwithstanding, Betty really does stand in the belief relation in this case to the proposition that someone identical with Xavier or Yvette will win. While Betty might not put her belief in quite these terms, she might look back at the context after the blindfolds have been removed and say, "I didn't realize it at the time, but since I believed that a green-eyed contestant would win, and Xavier and Yvette were the green-eyed contestants, I did believe that one of them would win."

It is natural at this point to consider a further modification to the context, according to which Yvette and Zoe are placed behind curtains instead of being blindfolded. This prevents Betty from entering into any relation of causal or epistemic acquaintance with Yvette, and as such renders it increasingly implausible that Betty would stand in the belief relation to a proposition containing her as a constituent. The requirement, however, that believers stand in any such relation of acquaintance to the constituents of their singular beliefs has been challenged, for example by Hawthorne and Manley (2012). Anyone convinced by their arguments will need a different reason for resisting an insistence that Betty stands in the belief relation to the proposition that someone identical with Xavier or Yvette will win.

Here is another such reason upon which most will agree. The insistence, in the case at hand, that Betty stands in the belief relation to the proposition in question is on a par with the insistence that Betty stands in the belief relation to the proposition that a friend of yours will win. In neither case would Betty ascent to the relevant sentence, viz. 'Someone identical with Xavier or Yvette will win' and 'A friend of yours will win.' So, if someone is willing to loosen the instantiation conditions of the belief relation to permit the former instance but not the latter, then they are guilty of special pleading until they identify some relevant difference between the cases. And until such time, the insistence

that Betty stands in the belief relation to the proposition required for the truth of the third reading of (2) on Lederman's view is on a par with the insistence that there is no third reading at all, because the narrow-scope reading is true after all.

Having raised a counterexample to Lederman's model and provided reasons from both acquaintance relations and argumentative parity for taking it seriously, I'll now turn to developing a new, improved theory.

5 The Property Swap theory

Let's start fresh with what we know about the Game Show and Blindfolds contexts:

(i) Betty stands in the belief relation to the proposition that a green-eyed contestant will win.
(ii) The green-eyed contestants are your friends.
(iii) Facts (i) and (ii) suffice for the truth of the third reading of (2).

I want to suggest that our semantic theory should do justice to these simple observations, which point to the conclusion that the truth of the third reading should be explained in terms of the attributee standing in the belief relation to some proposition that contains as a constituent a property (e.g. *being a green-eyed contestant*) that is in some sense equivalent to the one expressed by the NP-complement (e.g. *being your friend*). Instead of Rigidification, more specifically, I propose the following principle:[29]

> **Property Swap**: If δ is a transparent DP with NP-complement φ occurring in an attitude report true in context c, then the attributee of the report stands in the belief relation to a structured proposition that contains a property instantiated in w_c by exactly the individuals satisfying φ in c.

Developing this view in earnest will require a preliminary semantic framework, an initial fragment for opaque DPs, and an extended fragment for transparent DPs. I will adopt a broadly Russellian semantic framework both for ease of exposition and because it is the most popular alternative to Fregean views about the nature of structured propositions.

5.1 Preliminaries

Structured propositionalists disagree about many aspects of their view, including the ontological category of propositions, their representational properties, and the means by which constituents are bound together into unified propositions. Abstracting away from these metaphysical distinctions, we can represent structured propositions in a Russellian semantic framework as ordered n-tuples consisting of a property or relation R followed by $(n-1)$-many things, as in the following:

$$\langle R, x_1, x_2, \ldots, x_{n-1} \rangle$$

The proposition that John loves Jane, for example, is represented by ⟨Love, John, Jane⟩. It contains the love relation, John, and Jane as constituents, arranged in such a way

[29] Similar principles have been proposed in the context of pragmatic explanations of third readings in Schwager (2011) and Sudo (2014). I aim to give a purely semantic implementation of the principle and discuss the limitations of pragmatic approaches in §7.

that the proposition is true when John and Jane instantiate (in that order) the loving relation. There's nothing deep about this system of representation; it's arbitrary, for instance, that the relevant property or relation appears as the first member of the n-tuple. The truth-conditions for any structured proposition are determined by its constituents and their arrangement. So, a world-relative theory of propositional truth can be given as follows:

> **Propositional Truth:** For any possible world w, property or relation R, and things $x_1, x_2, \ldots x_n$, $\langle R, x_1, x_2, \ldots, x_n \rangle$ is true relative to w if and only if $x_1, x_2, \ldots, x_n$ instantiate R in that order in w.

In our formal semantics, we will abbreviate truth relative to a world using the '$\vDash$' symbol, as in '$w \vDash \langle R, x_1, x_2, \ldots, x_n \rangle$'. Propositional Truth is a principle situated squarely within the metaphysics of propositions, which are commonly taken to be non-mental and non-linguistic entities. Nothing has yet been said about semantic values in context, composition, or linguistic meaning. On this view, the role of a semantic theory is to map sentence-context pairs to structured propositions, which have truth-conditions in accordance with Propositional Truth.[30] We'll then say that a sentence S is true in context c relative to a possible world w just in case the correct semantic theory maps S and c to a structured proposition that is true relative to w. Such a mapping between sentence-context pairs and structured propositions must be determined compositionally in order to explain linguistic competence and productivity, but the nature of semantic composition and the specific assignment of semantic values in context may vary by theory.

Consider, for example, a Russellian semantics along the lines of Salmon (1986a), Soames (1987), or King (2007), according to which the structured proposition corresponding to a sentence-context pair is determined by the order in which semantic values compose up the tree of syntax, rather than by any features of the semantic values themselves, such as whether one is in the functional domain of another. On these views, two sister nodes with semantic values x and y join at a mother node with $(x \oplus y)$ as its semantic value, where '$\oplus$' symbolizes a primitive operation mapping any two constituents to their unique fusion. Since this conception of semantic composition entails Terminal Node Constituency, it will not meet the demands of our new variable-based theory of intensionality.

A more flexible framework would be like the extensional system of Pickel (2019), which, building on Elbourne (2011), takes semantic composition to proceed by functional application. On this view, the semantic value of a predicate, such as 'loves,' is a function that maps the semantic values of proper names, such as 'Jane' and 'John,' to a structured proposition, such as ⟨Love, John, Jane⟩. Since the semantic value of 'loves' is a function that is not a constituent of the proposition that John loves Jane, this view is inconsistent with Terminal Node Constituency. The variable-based theory of intensionality I'll propose, consistent with Property Swap, exploits this kind of semantic machinery in order to prevent NP-complements and possible world variables from corresponding to propositional constituents in third readings. As a first step toward that end, I'll define an initial fragment that generates opaque DPs.

[30] I'll assume an answer to the question of what it is in virtue of which a given proposition has the truth-conditions it has. For discussion, see King et al. (2014).

5.2 Opacity

The Property Swap theory treats NPs, VPs, determiners, and the complementizer as purely extensional semantically; reference to possible worlds only appears for now in the derivations of truth-conditions in accordance with Propositional Truth. For this reason, we only require a standard type hierarchy with primitive types of e for individuals, which include properties and relations, and t for structured propositions. Here's a sampling of semantic values for each of these lexical category:

NPs
$[\![\textbf{Betty}]\!]^c$ = Betty
$[\![\textbf{friend-of-yours}]\!]^c = \lambda x \langle \textsc{friend}, x \rangle$

VPs
$[\![\textbf{wins}]\!]^c = \lambda x \langle \textsc{win}, x \rangle$
$[\![\textbf{believes}]\!]^c = \lambda p_t \lambda x_e \langle \textsc{bel}, x, p \rangle$

Determiner
$[\![\textbf{a}]\!]^c = \lambda P_{et} \lambda Q_{et} \langle \textsc{some}, \lambda x \langle \textsc{conj}, P(x), Q(x) \rangle \rangle$

Complementizer
$[\![\textbf{that}]\!]^c = \lambda p_t . p$

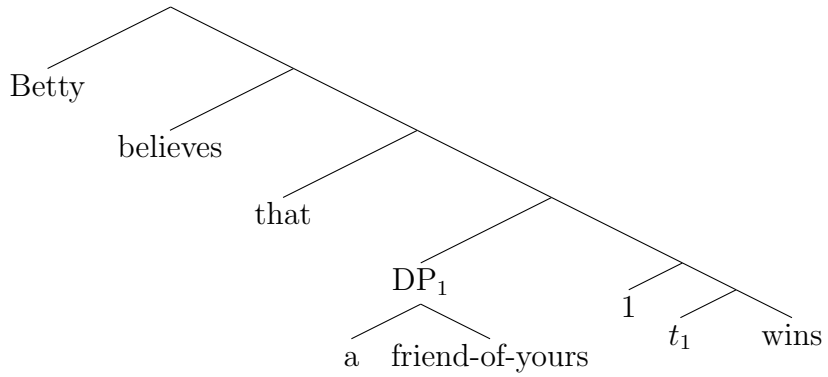
The properties and relations appealed to in these lexical entries require further specification in order to figure meaningfully in our statement of various truth-conditions. For any subject x, possible world w, propositional function f, and propositions p and q:

- x instantiates FRIEND (WIN) in w if and only if x is your friend (x wins the game) in w.
- x and p instantiate BEL in w if and only if x stands in the belief relation in w to p.
- f instantiates SOME in w if and only if for some y, $w \vDash f(y)$.
- p and q instantiate CONJ in w if and only if $w \vDash p$ and $w \vDash q$.

So far, this system is analogous to Pickel (2019), except that the instantiation relation has been relativized to possible worlds.

Now consider the opaque reading of (2) in context, which we'll suppose to have the following logical form after QR:

(12)



Over the course of syntactic movement, the numeral index is inserted in the customary way following Heim and Kratzer (1998). Composition is then assumed to proceed by functional application whenever possible:

> Functional Application
> If α is a branching node and $\{\beta, \gamma\}$ is the set of its daughters, then for any context c, α is in the domain of $[\![\cdot]\!]^c$ if both β and γ are, and $[\![\beta]\!]^c$ is a function whose domain contains $[\![\gamma]\!]^c$. In this case, $[\![\alpha]\!]^c = [\![\beta]\!]^c([\![\gamma]\!]^c)$.

At this point, I depart from Pickel (2019) by adopting a generalized version of the canonical account of quantification, which will be useful later:[31]

> Variable Predicate Abstraction
> Let α be a branching node with daughters β and γ, where β dominates only a numeral j co-indexed with a variable v of type ϕ. Then, for any context c, $[\![\alpha]\!]^c = \lambda v_\phi [\![\gamma]\!]^{c, g_c[v_j \mapsto v]}$.

The compositional derivation relative to a context c is then demonstrated in stages, starting in the interior of the subordinate clause with an application of Variable Predicate Abstraction:

$$[\![\ [\ \mathbf{1}\ [\ \boldsymbol{t_1}\ \mathbf{win}\ ]\ ]\ ]\!]^c = \lambda x [\![\ \boldsymbol{t_1}\ \mathbf{win}\ ]\!]^{c, g_c[t_1 \mapsto x]} = \lambda x \langle \textsc{win}, x \rangle$$

Through successive functional application, the semantic value of the determiner maps the value of the NP-complement and the propositional function above to the following structured proposition:

(13) $\langle \textsc{some}, \lambda x \langle \textsc{conj}, \lambda y \langle \textsc{friend}, y \rangle (x), \lambda y \langle \textsc{wins}, y \rangle (x) \rangle \rangle$

By β-reduction, this reduces to the following:

(14) $\langle \textsc{some}, \lambda x \langle \textsc{conj}, \langle \textsc{friend}, x \rangle, \langle \textsc{wins}, x \rangle \rangle \rangle$

[31] If this departure raises concerns about the compositionality of these semantics, see the recent iteration on the canonical account of quantification given by Glanzberg and King (2020), who refine and defend the appeal to semantic rules outside of functional application. The rule of Variable Predicate Abstraction can be seen as a version of their rule, Variable IFA, that is modified to interpret numeral indices in the syntax.

By Propositional Truth, (14) is true relative to possible world w just in case for some x, $\langle \textsc{friend}, x\rangle$ and $\langle \textsc{win}, x\rangle$ are true in w, that is, just in case for some x, x instantiates FRIEND and WIN in w. The complementizer is semantically vacuous, mapping this proposition to itself to yield the semantic value of the entire 'that'-clause. Finally, through successive functional application, the semantic value of the attitude verb maps this proposition and the attributee to the following structured proposition, which is the semantic value of (12) in c:

(15) $\langle \textsc{bel}, \text{Betty}, \langle \textsc{some}, \lambda x_i \langle \textsc{conj}, \langle \textsc{friend}, x_i\rangle, \langle \textsc{wins}, x_i\rangle\rangle\rangle\rangle$

By Propositional Truth, this proposition is true relative to possible world w just in case Betty stands in the belief relation in w to (14).

Substituting 'friend of yours' in (12) with a co-extensional expression in context can change the truth-value of the overall report by changing the object of Betty's reported belief to a proposition with distinct constituents. So, the DP is opaque with respect to the attitude verb, as desired. Now we can extend this initial fragment into a variable-based theory of intensionality that generates transparent DPs for third readings.

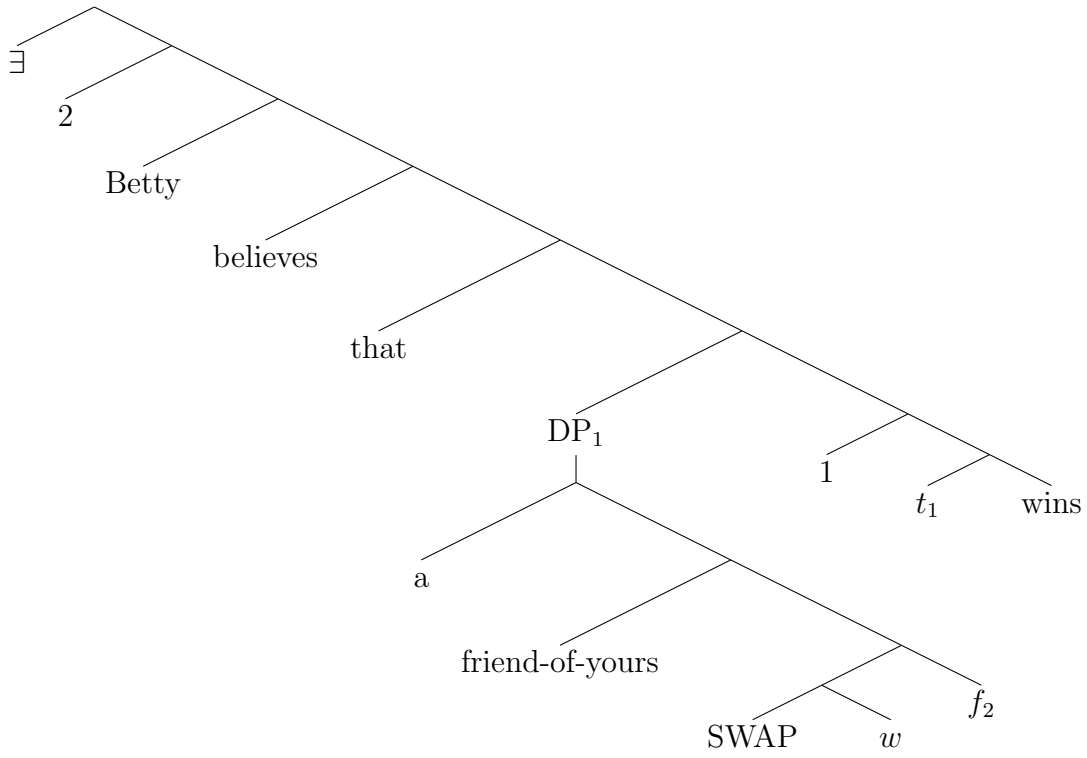
5.3 Transparency

According to the Property Swap theory, the intensional status of a DP is determined by the occurrence of a covert operator, 'SWAP', that selects for, in addition to a possible world variable, a choice function variable existentially bound at the top of the clause. The idea that lexical items could introduce variables over choice functions is not a new one, as it's pursued by Reinhart (1997), Winter (1997), Kratzer (1998), Matthewson (1999) and others in attempting to account for the exceptional scope-taking of indefinites. The Property Swap theory, however, uses choice functions defined on sets of NP-meanings rather than on NP-meanings themselves. So, it produces transparent DPs without analyzing them referentially.

A function f is a *choice function* on a non-empty collection of sets X if and only if f maps every member of X to an element of that member, i.e. for all $x \in X$, $f(x) \in x$. Since our choice functions operate on sets of NP-meanings, they require a new semantic type CH, where variables of that type $f_1, f_2, \ldots$ are mapped by variable assignment functions into $\{f \in D_{et}^{\mathcal{P}(D_{et})} \mid \forall x \in \mathcal{P}(D_{et}), f(x) \in x\}$, or D_{CH}. Variables over possible worlds are treated as before, requiring semantic type s, where variables of that type $w_1, w_2, \ldots$ are mapped by variable assignment functions into the set of all possible worlds, D_s. We'll continue to assume that unbound possible world variables are interpreted by default as the world of the context, and that each context c comes equipped with a world w_c and a single variable assignment function g_c, where g_c is accordingly defined on variables of type CH, s, and e.

According to the Property Swap theory, the third reading of (2) has a logical form such as the following:

(16)



The operator 'SWAP' selects for a possible world variable, choice function variable, and noun. The resulting DP has already undergone movement by QR but remains clause-bound, respecting established island constraints on movement outside of finite clauses.[32] Following Reinhart (1997) and Winter (1997), the choice function variable is bound at the top of the matrix clause by an existential quantifier, which in our case is a complex $\ulcorner[\ \exists\ [\ \boldsymbol{i}\ \ldots\ ]\ ]\urcorner$ for co-indexed numeral i.

For any $P \in D_{et}$, let $\downarrow_w P = \{x \in D_e | w \vDash P(x)\}$. Then the required lexical entries can be given as follows:

Choice Function Quantifier
$[\![\exists]\!]^c = \lambda P_{<CH,t>} \langle \textsc{some}, P \rangle$

Operator
$[\![\mathbf{SWAP}]\!]^c = \lambda w_s \lambda f_{CH} \lambda P_{et} (f\{R \in D_{et} | \downarrow_w R = \downarrow_w P\})$

The derivation can begin working from the outside-in with an application of Variable Predicate Abstraction on the numeral index '2' and its sister node, yielding a propositional function over choice functions as the semantic value of the penultimate root node. The semantic value of the choice function quantifier (in context) maps this propositional function to the structured proposition true in a world w just in case some proposition in its range is true in w, yielding the following as the semantic value of (16) in c:

(17) $\langle \textsc{some}, \lambda f_{CH} [\![\mathbf{[Betty\ [believes\ [[a\ [foy\ SWAP}\ \boldsymbol{w}\ \boldsymbol{f}_2\mathbf{]]\ [1}\ \boldsymbol{t}_1\ \mathbf{wins]]]]]}]\!]^{c, g_c[f_2 \mapsto f]} \rangle$

[32] See May (1977) and, more recently, Grano and Lasnik (2018).

We've ignored the complemetizer again since it's semantically vacuous, and 'foy' abbreviates 'friend-of-yours.' Through successive functional application, the semantic value of 'believes' applies to the value of the subordinate clause and the attributee, resulting in the following:

(18) $\langle \textsc{some}, \lambda f_{CH} \langle \textsc{bel}, \text{Betty}, [\![[\textbf{a [foy SWAP } \boldsymbol{w\ f_2}\textbf{]] [1 } \boldsymbol{t_1}\textbf{ wins}]]\!]^{c,g_c[f_2 \mapsto f]} \rangle \rangle$

At this point, the semantic value of the determiner successively applies to that of its complement and that of the predicate abstract, which is treated in the same way as in the derivation of the opaque DP.

The value of the determiner's complement is determined by functional application on the lexical entry for the swapping operator; through β-reduction on the semantic value of 'SWAP' provided above, 'w' is replaced by '$[\![\boldsymbol{w}]\!]^c$', 'f_{CH}' by '$[\![\boldsymbol{f_2}]\!]^{c,g_c[f_2 \mapsto f]}$', and '$P$' by '$[\![\textbf{foy}]\!]^c$'. The result is as follows:

(19) $\langle \textsc{some}, \lambda f_{CH} \langle \textsc{bel}, \text{Betty}, ([\![\textbf{a}]\!]^c([\![\boldsymbol{f_2}]\!]^{c,g_c[f_2 \mapsto f]}\{R \in D_{et} | \downarrow_{[\![\boldsymbol{w}]\!]^c} R = \downarrow_{[\![\boldsymbol{w}]\!]^c} [\![\textbf{foy}]\!]^c\}))(\lambda x \langle \textsc{wins}, x \rangle) \rangle \rangle$

We'll interpret the choice function variable, possible world variables, and noun in the usual way, allowing the determiner's value to operate successively on its two *et* arguments, yielding the following:

(20) $\langle \textsc{some}, \lambda f_{CH} \langle \textsc{bel}, \text{Betty}, \langle \textsc{some}, \lambda x_i \langle \textsc{conj}, f\{R \in D_{et} | \downarrow_{w_c} R = \downarrow_{w_c} \lambda x \langle \textsc{friend}, x \rangle\}(x_i), \langle \lambda x \langle \textsc{wins}, x \rangle (x_i) \rangle \rangle \rangle \rangle \rangle$

To further simplify, for any monadic property Π and possible world w, let $\downarrow_w \Pi = \{x \in D_e | x \text{ instantiates } \Pi \text{ in } w\}$; note that $\downarrow_w \lambda x \langle \Pi, x \rangle = \downarrow_w \Pi$.[33] This gives us the following equality:

$$\downarrow_{w_c} \lambda x \langle \textsc{friend}, x \rangle = \downarrow_{w_c} \textsc{friend}$$

Applying this result and β-reducing the last λ-abstract yields the final formulation of the semantic value of (16) in c:

(21) $\langle \textsc{some}, \lambda f_{CH} \langle \textsc{bel}, \text{Betty}, \langle \textsc{some}, \lambda x \langle \textsc{conj}, f\{R \in D_{et} | \downarrow_{w_c} R = \downarrow_{w_c} \textsc{friend}\}(x), \langle \textsc{win}, x \rangle \rangle \rangle \rangle \rangle$

By Propositional Truth, this proposition is true relative to a possible world w' just in case for some $R \in D_{et}$ such that $\downarrow_{w_c} R$ is $\downarrow_{w_c} \textsc{friend}$, Betty stands in the belief relation in w' to $\langle \textsc{some}, \lambda x \langle \textsc{conj}, R(x), \langle \textsc{win}, x \rangle \rangle \rangle$. The context and world of Game Show satisfy this condition, since R is witnessed by $\lambda x \langle \textsc{green-eyed contestant}, x \rangle$, where GREEN-EYED CONTESTANT is the property of being a contestant with green eyes. In this way, the Property Swap theory correctly predicts that (2) is true in Game Show.

With the theory now presented, I'd like to anticipate a possible reaction. Given that the formulation of (21) mentions both possible worlds ('w_c') and something approximating the embedded noun's meaning (FRIEND), one might reasonably wonder whether the supposed requirement to reject Terminal Node Constituency has been satisfied, and if not, whether it was needed in the first place. First, there is a trivial sense

[33] *Proof.* For any $w \in D_s$, $y \in D_e$ and property Π, $y \in \downarrow_w \lambda x \langle \Pi, x \rangle \iff w \vDash \lambda x \langle \Pi, x \rangle (y) \iff w \vDash \langle \Pi, y \rangle \iff y$ instantiates Π in $w \iff y \in \downarrow_w \Pi$

in which Terminal Node Constituency has been rejected, because (21) has exactly two constituents, namely, SOME and a complicated property of choice functions, neither of which are possible worlds or noun meanings. Second, and more importantly, Terminal Node Constituency as applied to the subordinate clause in (2) has been rejected because the 'that'-clause does not designate a proposition containing a possible world or friendship property as a constituent; while the proposition designated by (2) as a whole, i.e. ((21)) does involve possible worlds and a friendship property in some sense, albeit not as propositional constituents, technically, the proposition Betty reportedly believes which would explain the truth of the report does not. (21) is true relative to the world of Game Show, that is, partially because Betty stands in the belief relation in that world to $\langle \textsc{some}, \lambda x \langle \textsc{conj}, \langle \textsc{green-eyed contestant}, x \rangle, \langle \textsc{win}, x \rangle \rangle \rangle$. The fact that Betty's belief is constrained by the 'that'-clause without thereby containing a possible world or friendship property as a constituent is the relevant sense in which the Property Swap theory delivers on the requirement of rejecting Terminal Node Constituency.

6 Basic embeddings and landing sites

The current outline represents just the first steps toward a comprehensive articulation of the Property Swap theory. Numerous technical challenges remain, particularly around formal implementation, including constraints on permissible landing sites for the existential choice-function quantifier and extensions of the theory to handle more complex constructions, such as logical embeddings, doubly-embedded attitudes, and modal contexts. Before concluding, I aim to demonstrate the theory's robustness by applying it to some foundational embedding structures. Doing so will help clarify the syntactic parameters that govern acceptable landing sites for existential quantification over choice functions, which will, in turn, advance our understanding of the structural constraints involved in choice-functional interpretations.

First, consider the negation of the third reading of (2), i.e. the reading of the following sentence that is true (in context) if and only if the third reading of (2) is false (in context):

(22) It's not the case that Betty believes that a friend of yours will win.

The intended reading is accessible in a context in which (22) is uttered in disagreement over whether the third reading of (2) is true. It corresponds to a logical form in which the existential quantifier over choice functions lands below the negation operator 'NOT' at LF, as in the following:

(23) [NOT [∃ 2 [Betty believes that a foy SWAP w f_2 wins]]]

On the other hand, there is no accessible reading of (22) corresponding to the logical form in which the quantifier lands above the negation operation at LF, as in the following:

(24) [∃ 2 [NOT [Betty believes that a foy SWAP w f_2 wins]]]

Such a reading would be trivially true in most contexts, since the majority of choice functions would witness the existential quantification. This suggests that the only landing site available for the choice-function quantifier is immediately above a clause whose main connective is a propositional attitude verb, as opposed to a logical connective.

Now consider doubly-embedded attitude reports such as the following:

(25) Serena thinks Betty believes a friend of yours will win.

There's an accessible reading of this report that attributes to Serena a belief expressed by the third reading of (2) in Game Show, i.e. a belief the object of which is the structured proposition (21). This reading results from a logical form in which the existential quantifier lands above the first available matrix clause dominating the choice function variable, as in the following:

(26) [Serena thinks [$\exists$ 2 [Betty believes [a foy SWAP w f_2 will win]]]]

At this point, one might object to the plausibility of a subject's thought involving quantification over choice functions, especially when the subject doesn't know what choice function are. In response, we can note that the core semantics of the Property Swap theory require subjects to stand in the belief relation to structured propositions with constituents that include propositional functions, properties of propositional functions, and logical relations between propositions. So, if the intelligibility of constituents is objectionable, it's a problem well before the introduction of choice functions. Moreover, most if not all semantic theories involve pre-theoretically unrecognizable formal machinery, but it's often possible to colloquially paraphrase the function of this machinery. A linguistically competent subject might not be familiar with the mathematical representation of choice functions, but he plausibly understands various ways of replacing one meaning for another in sentences or thoughts. It's arguable that one's ability to do so is partly constitutive of one's linguistic competence.

If the choice-function quantifier can land immediately above any clause whose main connective is a propositional attitude verb, then there will also be an accessible reading of (25) corresponding to the following logical form:

(27) [$\exists$ 2 [Serena thinks [Betty believes [a foy SWAP w f_2 will win]]]]

This reading would be true in the Game Show context, even assuming that Serena believed you to be friendless, provided that she stands in the thinking relation to the proposition that Betty believes that a green-eyed contestant will win. My own intuitions are those of many of my respondents are mixed. It's clear that if there is such a reading, it's more difficult to access than (26). So, the choice-function quantifier has an easier time landing closer to 'SWAP,' and it may even be mandatory that it does so in some idiolects. In either case, the Property Swap theory can appeal to a straightforward syntactic rule, viz. that the choice-function quantifier either must land immediately above the nearest c-commanding clause with an attitude verb, or else can optionally land above any such c-commanding clause, as long as it does not pass over a logical connective.

Finally, consider counterfactual embeddings such as the following:

(28) If you had no friends, then Betty would believe that a friend of yours would win.

There are at least two readings of this sentence. On the first, (28) is true in Game Show, since the NP-complement is evaluated relative to the world of the context of utterance; in a counterfactual scenario, Yvette and Xavier are still friends fo yours in the actual world. On the other reading, (28) is trivially false in every context, since the NP-complement is evaluated relative to the counterfactual scenario in which you have no friends. The Property Swap theory can generate both readings under the assumption

that counterfactual constructions are syntactically analyzed in terms of quantifiers over possible world variables at logical form. At a high level of abstraction, the true reading of (8) can be assigned a logical form or intermediary stage of semantic composition along the following lines in context c:

(29) $[\forall w_1 : w_1 R_c w_c](w_1 \vDash [\![\textbf{you have no friends}]\!]^c \rightarrow$
$w_1 \vDash [\![\exists$ **2 Betty believes that a [foy [[SWAP** $\boldsymbol{w}$**]** $f_{\mathbf{2}}$**]] wins**$]\!]^c)$

where 'R_c' designates a contextually determined accessibility relation between worlds. The unbound w picks up the world of the context of utterance as its value. By contrast, the trivially false reading of (28) results from replacing the unbound occurrence of 'w'above with 'w_1,' thereby binding it to universal quantifier introduced by the counterfactual construction.

The Property Swap theory provides a promising framework for understanding basic cases of embedding, including negation, doubly-embedded attitudes, and counterfactual conditionals. These examples illustrate how the theory accounts for nuanced readings and respects syntactic constraints on landing sites for choice-function quantifiers. While this exploration offers a solid foundation, much remains to be addressed. Future work should extend this analysis to more complex embedding structures, as well as refine the formal implementation to address potential variations in acceptability and accessibility across different contexts.[34]

7 Pragmatics to the rescue?

On the basis of the syntactic and semantic complexity posited by variable-based theories more generally, it is tempting to wonder whether intensional phenomena such as the third reading of indefinites are not better understood as pragmatic in nature. I'd like to discuss two potential pragmatic approaches to third readings in order to highlight the difficulties they face.

First, suppose we took inspiration from Blumberg and Lederman (2021) and proposed the following principle:

> **Revisionist Reporting:** A belief report of the form ⌜S believes that p⌝ can be felicitously uttered in context c just in case the referent of S in c stands in the belief relation to some proposition Q, such that Q entails the proposition designated by ⌜that p⌝ under shared conversational background assumptions.

Although the authors do not directly apply this principle to third readings, it would seem to explain the felicity of uttering (2) in Game Show; Betty stands in the belief relation to the proposition that a green-eyed contestant will win, and under shared background assumptions, this proposition entails that a friend of yours will win. If this explanation were correct, then the third reading of indefinites would really be the result of a pragmatic process operating on de dicto readings of the reports in question.

[34] A variety of interesting complications not considered here can be found in Schwager (2011), including cases involving empty predicates, impossible buildings, and different containment relations between predicate extensions.

Unfortunately, any strategy – pragmatic or otherwise – that relies on this kind of entailment relation will not sufficiently generalize, because third readings are accessible for reports that would, if interpreted de dicto, ascribe beliefs in necessarily false propositions. Consider, for example, the following belief report under the assumption that it is impossible to be both your friend and sworn enemy:

30. Betty believes that a friend of yours is your sworn enemy.

We could easily modify the context of Game Show so that a third reading of this report is accessible; suppose Betty takes your sworn enemy, rather than the lucky winner, to be among the green-eyed contestants. Then the pragmatic approach inspired by Blumberg and Lederman (2021) predicts that this report is felicitous in context just in case Betty stands in the belief relation to some proposition that entails the proposition that a friend of yours is your sworn enemy. Unless Betty already believes something necessarily false, none of her beliefs entail this necessarily false proposition. So, the pragmatic approach incorrectly predicts that there is no accessible reading of the report in a suitably modified context.

A second kind of pragmatic approach can be found in Schwager (2011) and Sudo (2014), according to which there is a pragmatic rule of replacement that allows speakers to exchange co-extensional predicates in the context of attitude reports. We could formulate the rule as follows:

> **Replacement Rule:** In a context c in which a belief report of the form $\ulcorner S$ believes that A is $\phi\urcorner$ would be felicitous, speakers can felicitously utter $\ulcorner S$ believes that A is $\psi\urcorner$, for any predicate ψ co-extensional with ϕ in c.

While it is possible to understand the Property Swap theory I offer as a semantic implementation achieving much of the same result, there is an important difference when it comes to embedded constructions. Consider the case of embedding under counterfactual conditionals, in which the Property Swap theory accounted for one of the accessible readings by positing that the covert world variable operated on by 'SWAP' is co-indexed with the world variable introduced by the 'if'-clause. In this case, the predicates being swapped are co-extensional relative to the counterfactual scenario, not the actual world. It is unclear how a pragmatic rule of replacement can systematically track to which possible world the notion of co-extensionality should be applied. In the crude formulation above, the would only permits speakers to replace predicates that are actually co-extensional, leaving no principled explanation for third readings embedded under counterfactual conditionals.

Given the complexity arising out of semantic theories attempting to capture linguistic intensionality, it would be nice if there were a simpler pragmatic explanation with the same explanatory power. As semantic theories become more complex, our interest in pragmatic approaches should increase. But such a pragmatic approach for third readings has yet to be developed.

8 Conclusion

I have spent this work developing a variable-based theory of intensionality for Russellian structured propositions, specifically to address the third reading of indefinites in belief

reports. By introducing the Property Swap theory, I've shown how transparent readings can emerge through covert syntactic mechanisms, allowing structured propositionalists to account for intensional phenomena without needing to include entities corresponding to possible world variables or NP-complements as constituents. This approach offers a novel alternative to traditional scope-based theories, Hintikkan variable-based theories, and Lederman's recent Fregean-inspired model, filling a gap in the literature by providing a solution compatible with structured, Russellian propositions.

The Property Swap theory addresses cases like the third reading of indefinites by allowing DPs to adopt transparent interpretations through novel choice-function machinery and covert syntactic operators. This variable-based mechanism generates transparent readings without relying on rigidified propositional constituents or traditional Hintikkan possible worlds semantics for attitude verbs. By removing the need for rigidified interpretations, this model circumvents some of the limitations faced by Lederman's approach, while capturing transparency more flexibly across contexts. However, further work is needed to refine the model for more complex embeddings, such as counterfactuals and doubly-embedded attitudes, which may require a more precise approach to quantifier landing sites and intensional shifts in nested structures.

The Hintikkan orthodoxy of possible worlds semantics remains the predominant framework in linguistics for analyzing propositional attitudes, and works like Lederman (2022) are important contributions to the tradition that views propositions as more finely grained – a tradition rooted in both Frege and Russell. For those who see structured propositions as preferable to unstructured sets of worlds, establishing a theory that handles intensional phenomena as effectively as Hintikkan approaches is essential. Structured propositions enjoy broad support among philosophers but have yet to gain comparable traction in linguistic semantics. Lederman's work has laid valuable groundwork for exploring how structured propositions might handle transparency, and the Property Swap theory builds on this foundation, taking the analysis in a new, Russellian direction. If successful, this approach strengthens the case for structured propositions, bringing them closer to earning their keep within formal semantic theory.

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