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Parameter based non linearity in a state variable model of a practical system: A case study

A.B.Chattopadhyay[1], Shazia Hasan[2], Sunil Thomas[3]

[1]Faculty, Electrical and Electronics Engineering Dept, BITS Pilani Dubai, Dubai

[2]Faculty, Electrical and Electronics Engineering Dept, BITS Pilani Dubai, Dubai

[3]Faculty, Electrical and Electronics Engineering Dept, BITS Pilani Dubai , Dubai

[1]chattopadyay@dubai.bits-pilani.ac.in, [2]dr.shaziahasan@gmail.com, [3]sunilthomas@dubai.bits-pilani.ac.in

Corresponding Author: Shazia Hasan

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Abstract

The small perturbation method is widely used in attempting to model non-linear systems. Many systems nowadays in different domains exist as adaptive-parameter type models, where the control effort is not applied as in input to the system (as is usually the case) but as a change to the parameter within the system itself. This paper attempts to analyze a non-linear adaptive-parameter type system, using the small perturbation method for linearization. The Ward-Leonard DC Motor with thyristor field control is used as a "test bench" here as it is suited for being an adaptive parameter system. The results and inferences from this study can easily be generalized to a wide variety of systems in applied mathematics, general control systems, power systems, robotics etc.

Keywords: Non-linear adaptive-parameter, Small perturbation approach, DC drive, Thyristorized W-L method, silicon-controlled rectifier.

I. Introduction

In many domains in engineering and science, attempts are made to develop mathematical model of different systems. This is done for a variety of reasons: in order to analyze the system, to make it efficient and/or profitable, to improve its performance and so on. The systematic way of thinking is a very popular way in academia and industry in order to understand the combination of different domains. An example of this is mechatronics, combining seemingly disparate domains such as mechanics, embedded systems, control theory, power electronics etc., into a system useful for society or business (such as an automobile or a humanoid robot), and itself consisting of a number of sub-systems.

A system is considered as a black-box, with certain inputs, states and outputs. Control theory attempts to control a system by monitoring its outputs or states, and controlling its inputs such that the system achieves a target state and provides a required output. In fact most of the control systems are of this type, where the input to the system is manipulated, and where the system parameters are fixed.

However it has been seen that there are many systems across the globe, relevant to the society, and it becomes very interesting to observe that these systems have certain parameters which are non constant. For example an airplane or rocket burns of fuel as it flies, therefore changing its mass continuously. The Airbus A-380 burns of fuel in order of hundreds of kg/sec. A robot with multiple arms in series changes its moment of inertia with respect to the base as it moves its arms in different directions. In this context, it may be mentioned that a linear differential equation with non constant co efficient becomes very tedious to solve as close as to the solution of a non linear differential equation.

The dynamics of the system in such case is different from normal control theory, which leads to different paradigm of thinking. It is further complicated by the addition of non-linearity in the system. To assist in the analysis of this class of non-linear adaptive-parameter systems. The present paper looks into linearization of such systems using the small perturbation approach, without sacrificing much to the accuracy of the concerned solution or system response. But this linearization technique is not a traditional linearization process of a variable about a quiescent point. The details of the proposed experimental circuit involving a power electronic device (SCR) will be discussed in section2, which deals with to a parameter – perturbation problem in contrast to a traditional variable perturbation problem.

To assist the analysis of such systems, a “test bench” system is generally selected. This will help in relating this to real world systems, and in attacking such problems from an application perspective. Nevertheless, the inferences and understanding obtained from this paper can be generalized to any system with such characteristics.

An interesting work has been reported in[1], where the authors has implemented the speed control of DC motor using Ward-Leonard system by remotely accessing the system using LabVIEW™10. The hardware is linked to the webserver by interface cards. Qing-Chang Zhong[2] has proposed a control algorithm for AC Ward Leonard drive systems without a speed senor. However electric drive control using fuzzy systems was by A Bara et.al [3]. In this work the authors have addressed on how to maintain the control performance for a wide range of speed variation based on the operating conditions. This method involved parameter variations to changing load/operating condition of the drive system. simulation results with adaptive fuzzy engine are also been presented in this work. Modeling of SCR controlled Ward-Leonard system has been reported in [4]. The mathematical models of the generator, motor, mechanical load, rectifiers, speed regulator, current regulator are detailed and the complete control system is implemented using SIMULINK.F P A Vaccaro et. al has reported Digital control of a Ward Leonard drive system [5] and in this research work it has been shown that iron losses and line losses have an effect on the sizing of the thyristor set and that the voltage forcing has to be limited to the dynamic optimum of the system.

Nonlinearity study in engineering phenomena is a wide area. Even though the research work in the present paper comes under the category of nonlinear state variable model for power electronic drive systems. One such work has been reported in 2016 [6], which is based on the model predictive control and optimal control of an Inverted pendulum. Basically for an inverted pendulum problem, a suitable non-linear model has been developed using Langrangian mechanism. Another research paper [7] has been found to be of mathematical interest, in which a non-linear state variable method has been presented and it is used to solve the pseudo-2D Li-ion cell model under high frequency input current and temperature signals.

The system as a case study being considered here is a slight variation of the popular Ward-Leonard (W-L) DC Drive. In this paper the W-L drive involves a thyristorized field control system, where the thyristor may be triggered using different firing circuits such as resistor-resistor triggering, RC triggering and so on.

In this paper resistor-resistor triggering method has been used and the small change in a particular resistance parameter of the concerned gate circuit of the thyristor becomes the ultimate input (in small perturbation form) to control the speed of the motor of the said modified W-L drive system.

Development of the mathematical model

This section deals with the derivation of the necessary equations for the said modified W-L method. But before deriving the necessary equations , at this stage it becomes very important to give an outline of physical system taken up as a case study. These aspects are presented in the following section.

I.i Ward Leonard DC Drive with Thyristor Field Control

Ward-Leonard or W-L drive is a type of DC drive which was very popular before the advent of commercial power electronics in the speed control of DC Drives in industry. It was widely used in mining and construction, elevators, steel mills, cement plants, ships, gun turrets and so on. In many large DC W-L drive sets (often in tens of MW) operating even today, the operators have modified it by adding thyristors to the field circuits, rather than replacing the whole unit with costly AC drives. This allows for more fine-tuned control, while needing only a small cost increase as thyristors are relatively cheap, and are only required in moderate rating (carrying only field currents, no armature currents).

This modified W-L (MWL) will be used as a test bench concept for attempting to perform our analyses. Therefore a short introduction to the thyristorized W-L method will be introduced, as the mathematical model will be developed based on this test bench arrangement.

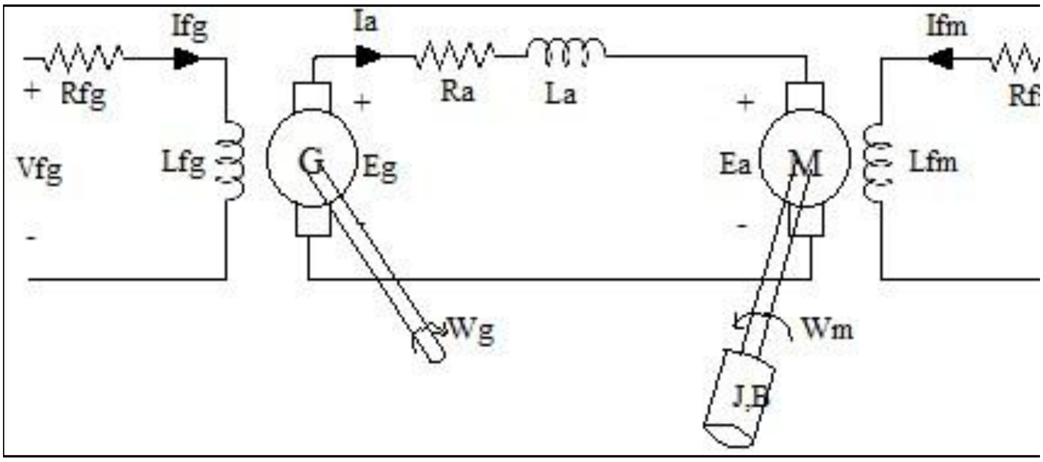

Fig.1: General topology of a Ward-Leonard system

Fig. 1 shows the topology of a general Ward-Leonard system. It consists of 2 sets of DC machines connected as in the figure. One set acts a generator, whose shaft is driven by a suitable prime mover. It feeds power to the second set of DC machine, which acts as a motor, and drives the load. The field windings are controlled for both machines. The DC generator field winding controls the voltage across the DC motor, while DC motor field winding allows for speed/torque control of the motor. One thyristor (SCR) is connected to the field winding of the DC Generator field winding, as shown in Fig.2. V_s in Fig.2 is a single constant voltage constant frequency source. There for Fig.2 van be considered as a modification of the experimental set up in Fig.1, such that the concept of parameter perturbation can be simulated after creating a suitable simulation environment. The motivation behind such modification is to develop an explicit type two column look up table involving variation in the ultimate output in one column due to the variation of ultimate input in the other column. The description of the associated circuits in the next section will make it clear that the input and output are the small changes in R_2 (ΔR_2 which is a particular variable resistor inserted in the gate circuit of the SCR) and, the small change in motor speed in radiations per second (Δw_m), respectively. The ultimate objective of the present research work is to develop a tool such that the change in motor speed can be predicted based on a specific value of perturbation of control input (ΔR_2) which is physically a parameter perturbation.

Another important point is that even though the necessity of developing a look-up table has been pointed out in the above paragraph, such table is more useful when the model is developed in discrete- time domain as a part of digital modeling. As this present model represents a state-space modeling in continuous time-domain, the graphical plot of Δw w.r.t ΔR_2 will work as efficiently as a look-up table. That is why, in section 4 the above said plot and time domain responces have been presented based on MATLAB simulation.

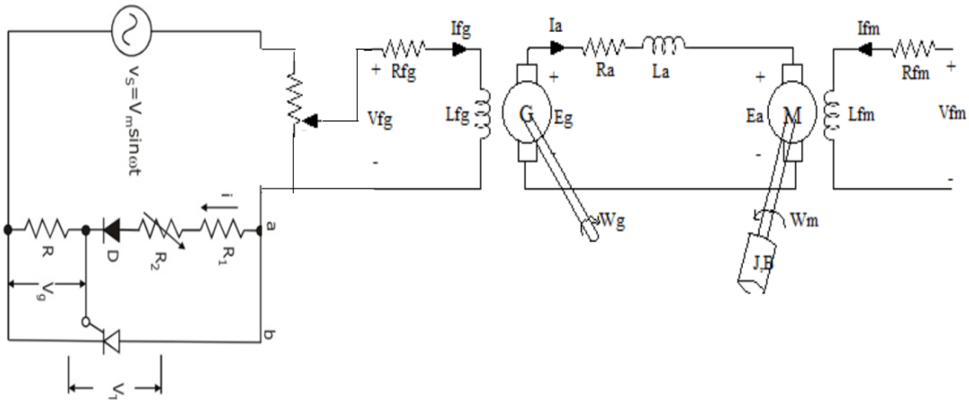

Fig.2 : Topology of the thyristorized rectifier circuit that feeds the DC generator field winding

Thyristor Rectifier Circuit

The thyristor, also known as a silicon-controlled rectifier or a SCR, is a widely-used power electronic switch. Here it is used to control the field winding of the generator part of the Ward Leonard system, in the form of a controlled rectifier. For a single phase sinusoidal voltage applied to a thyristor rectifier of voltage amplitude V_m, the r.m.s voltage at the output of the rectifier is:

$$V_{fg\,(rms)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\pi - \alpha + \frac{1}{2}\sin(2\alpha)} \qquad (2.2.1)$$

Where α is the firing angle of the thyristor.

The thyristor can fired using different kinds of firing circuits. The dynamics of the firing circuits also need to be considered as any change in the parameters of the firing circuit leads to a specific value of the change in the system output.

III. Development of the State Space Model of the modified W-L method

The resistor-resistor triggering is one of the simplest triggering system. It is relatively easy to analyze due to the lack of energy storage elements such as capacitor. Fig RR-Trig shows the schematic of a standard triggering circuit of a thyristor.

R and R_g form a resistor divider circuit. When the voltage across the lower resistor R_g is above the gate threshold voltage V_{gt}, the thyristor switches on, and remains on until the forward current drops to zero. Therefore the relation between R and R_g establishes the firing angle α. In this circuit, the variable is the resistor R_2.

Therefore the firing angle α is a function of the variable resistor R_2. However this function is non-linear due to the inverse sine function. Hence the non linearity is taken care by considering an operating point α_0, and then defining the problem to be a small perturbation problem around this.

Based on this philosophy the whole problem formulation is presented in the following section.

III.i. Problem Formulation

With reference to Fig:2, the bank voltage balance equations in the motor-generator armature circuit and generator field circuit and the standard torque equation of D.C motor and torq-balance equation of the same motor have been presented as follows:

$$km = Kv.I_{fm} \tag{3.1}$$

$$kg = Kv.w_g \tag{3.2}$$

$$Eg = kg.I_{fg} \tag{3.3}$$

$$Em = km.w_m \tag{3.4}$$

$$V_{fg} = I_{fg}.R_{fg} + L_{fg}.I_{fg} \tag{3.5}$$

$$Eg - Ea = R_a.Ia + L_a.\dot{Ia} \tag{3.6}$$

$$T_e = km.I_a \tag{3.7}$$

$$J.\dot{w_m} = B.w_m + T_e \tag{3.8}$$

State Space model for a Ward-Leonard system can be represented as

$$\begin{bmatrix} \dot{I_a} \\ \dot{I_{fg}} \\ \dot{w_m} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{kg}{L_a} & \frac{-km}{L_a} \\ 0 & -\frac{R_{fg}}{L_{fg}} & 0 \\ \frac{km}{J} & 0 & \frac{-B}{J} \end{bmatrix} . \begin{bmatrix} I_a \\ I_{fg} \\ w_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_{fg}} \\ 0 \end{bmatrix} . V_{fg} \qquad [W_m] =$$

$$[0 \quad 0 \quad 1] . \begin{bmatrix} I_a \\ I_{fg} \\ w_m \end{bmatrix} + [0] . V_{fg} \tag{3.9}$$

The basic state variable formulation of the W-L drive system has been presented above. But as per the proposed control scheme, V_{fg} is not the ultimate input rather a particular resistor parameter R_2 in the gate circuit of the concerned thyristor becomes the controlling parameter. While this modified mathematical model is being developed, the corresponding gate control circuit which is a part of Fig.2 should be available for ease of modeling. That part is presented in Fig.3.

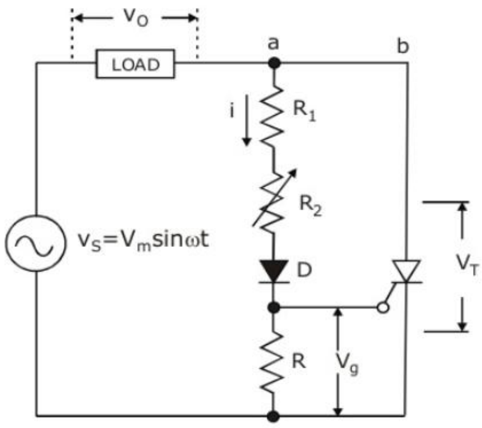

Fig.3. Resistor Triggering of the Thyristor

For firing angle α (rad), rms output voltage,

$$V_{fg(rms)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\pi - \alpha + \frac{1}{2}\sin(2\alpha)} \quad (3.10)$$

The state space model can be presented as,

$$\dot{x} = Ax + BV_{fg(rms)} \quad (3.11)$$

Applying Linearization technique, equation (3.11) can be modified to

$$\Delta\dot{x} = A\Delta x + B\Delta V_{fg(rms)} \quad (3.12)$$

The Thyristor will turn on when the gate voltage reaches a minimum threshold *Vgt*.

For the firing angle α, By KVL,

$$V_g = \frac{V_m.\sin(\alpha).R}{R+R_1+R_2} = V_{gt} \quad (3.13)$$

R_2 is varied in the control circuit in Fig.3

$$\sin(\alpha) = \frac{V_{gt}}{V_m}\left[\frac{R_1}{R} + \frac{R_2}{R} + 1\right] \quad (3.14)$$

$$\frac{d\sin(\alpha)}{dR_2} = \frac{V_{gt}}{V_m R} \quad (3.15)$$

$$\frac{d\sin(\alpha)}{dR_2} = \frac{d}{d\alpha}\sin(\alpha)|_{\alpha=\alpha 0}\left(\frac{d\alpha}{dR_2}\right) = \cos(\alpha_0)\left(\frac{d\alpha}{dR_2}\right) \quad (3.16)$$

Hence

$$\left(\frac{d\alpha}{dR_2}\right) = \frac{V_{gt}}{V_m R\cos(\alpha_0)} \quad (3.17)$$

Where $\alpha 0$ isoperating or steady state value of α.

Hence $\frac{\Delta\alpha}{\Delta R_2} \cong \frac{d\alpha}{dR_2} = \left(\frac{V_{gt}}{V_m R}\right)\left(\frac{1}{\cos(\alpha_0)}\right)$ (3.18)

From equation(2.2.1), it yields

$$\frac{dV_{fg(rms)}}{d\alpha} = \frac{V_m}{2\pi}\frac{d}{d\alpha}\sqrt{\pi - \alpha + \frac{1}{2}\sin(2\alpha)} \quad (3.19)$$

Equation (3.19) can be further simplified as,

$$\frac{dV_{fg(rms)}}{d\alpha} = \left(\frac{V_m}{2\pi}\right)\left(\frac{1}{2\sqrt{\pi-\alpha+\frac{1}{2}\sin(2\alpha)}}\right)\Bigg|_{\alpha=\alpha 0}\left\{\frac{d}{d\alpha}\left(\pi - \alpha + \frac{1}{2}\sin(2\alpha)\right)\Big|_{\alpha=\alpha 0}\right\} \quad (3.20)$$

Where $\alpha 0$ is the quiescent value of α.

Equation (3.20) can be expressed as,

$$\frac{\Delta V_{fg(rms)}}{\Delta\alpha} \cong \frac{dV_{fg(rms)}}{d\alpha} = \left(\frac{V_m}{4\pi}\right)\left(\frac{1}{\sqrt{\pi-\alpha+\frac{1}{2}\sin(2\alpha)}}\right)(\cos(2\alpha_0) - 1) \quad (3.21)$$

Or,

$$\frac{\Delta V_{fg(rms)}}{\Delta\alpha} = \left(\frac{V_m}{4\pi}\right)\left(\frac{1}{\sqrt{\pi-\alpha+\frac{1}{2}\sin(2\alpha)}}\right)(\cos(2\alpha_0) - 1) \quad (3.22)$$

Which reduces to,

$$\frac{\Delta V_{fg(rms)}}{\Delta R_2} = \left[\frac{\Delta V_{fg(rms)}}{\Delta\alpha}\right]\left[\frac{\Delta\alpha}{\Delta R_2}\right] = \left(\frac{V_m}{4\pi}\right)\left(\frac{1}{\sqrt{\pi-\alpha+\frac{1}{2}\sin(2\alpha)}}\right)\Bigg|_{\alpha=\alpha 0}(\cos(2\alpha_0) - 1)) \times \left(\frac{V_{gt}}{V_m R}\right)\frac{1}{\cos(\alpha_0)} \quad (3.23)$$

$$\Delta V_{fg(rms)} = M\Delta R_2 \quad (3.24)$$

Where, M is the multiplier obtained due to small perturbation model application.

$$M = \left(\frac{V_m}{4\pi}\right)\left(\frac{1}{\sqrt{\pi-\alpha_0+\frac{1}{2}\sin(2\alpha_0)}}\right)\left(\frac{V_{gt}}{V_m R}\right)\left(\frac{\cos(2\alpha_0)-1}{\cos(\alpha_0)}\right) \quad (3.25)$$

Equation (3.25) can be simplified as,

$$M = \left(\frac{1}{\sqrt{\pi-\alpha_0+\frac{1}{2}\sin(2\alpha_0)}}\right)\left(\frac{V_{gt}}{4\pi R}\right)\left(\frac{\cos(2\alpha_0)-1}{\cos(\alpha_0)}\right) \quad (3.26)$$

The initial state space model as in equation (3.9) can be modified to

$$\Delta\dot{x} = A\Delta x + B_1\Delta R_2 \tag{3.27}$$

Where

$$B_1 = BM \tag{3.28}$$

ΔR_2 is the small change in the gate control circuit resistance parameter.
Therefore finally the modified state space model can be written as

$$\begin{bmatrix} \Delta\dot{I}_a \\ \Delta\dot{I}_{fg} \\ \Delta\dot{W}_m \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{kg}{L_a} & \frac{-km}{L_a} \\ 0 & -\frac{R_{fg}}{L_{fg}} & 0 \\ \frac{km}{J} & 0 & \frac{-B}{J} \end{bmatrix} . \begin{bmatrix} \Delta I_a \\ \Delta I_{fg} \\ \Delta W_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_{fg}} \\ 0 \end{bmatrix} . \left(\frac{1}{\sqrt{\pi - \alpha_0 + \frac{1}{2}\sin(2\alpha_0)}} \right) \left(\frac{V_{gt}}{4\pi R} \right) \left(\frac{\cos(2\alpha_0) - 1}{\cos(\alpha_0)} \right) . \Delta R_2 \tag{3.29}$$

From state space model using small perturbation,

$$\Delta\dot{x} = A.\Delta x + B_1\Delta R_2 \tag{3.30}$$

Applying Laplace Transform with initial conditions relaxed, we get

$$=> s.\Delta X(s) - A.\Delta X(s) = B_1.\Delta R_2(s) \tag{3.31}$$

$$=> \Delta X(s).[sI - A] = B_1.\Delta R_2(s) \tag{3.32}$$

Where [I] is the identity matrix,

$$=> \Delta X(s) = [sI - A]^{-1}.B_1.\Delta R_2(s) \tag{3.33}$$

$$=> \Delta x(t) = [B_1][\varphi(t) * \Delta R_2(t)] \tag{3.34}$$

Where $\varphi(t)$ is state transition matrix

$$\varphi(t) = L^{-1}([sI - A]^{-1}) \tag{3.35}$$

And $\Delta R_2(t) = L^{-1}[\Delta R_2(s)]$
(3.36)

Applying the expression for convolution Integral, it yields

$$=> \Delta x(t) = B_1.\int_0^t \varphi(t-\tau).\Delta R_2(\tau)\, d\tau \tag{3.37}$$

Considering $\Delta R_2(s) = \frac{1}{s}$, where $\Delta R_2(t = 1)$,(i,e step change in that particulate gate circuit resistance has been considered) $\Delta x(t)$ has been numerically computed.
The response is simulated with the following design aspect

(a) $R_1 \geq \frac{V_m}{I_{gm}}$

where I_{gm} is maximum permissible gate current. and

(b) $R \quad \leq \frac{V_{gm}}{V_m - V_{gm}} R_1$

With the above said formulation, the MATLAB simulation has been performed. For unit step response of $\Delta I_a(t)$, $\Delta I_{fg}(t)$ and $\Delta W_m(t)$ the interpretations of the results are presented in the next section.

IV. Results and Discussion:

From the matrix equations (3.29) and (3.37), it becomes clear that the time responses of the small change in state variables like $\Delta I_a(t)$, $\Delta I_{fg}(t)$ and $\Delta W_m(t)$ are dependant on the selection of initial value of the thyristor firing angle α_0. Based on this observation, three set of values of α_0 have been chosen as $\alpha_0 = 9.6°, 19.2°, 28.8°$.
The time response of $\Delta I_a(t)$, $\Delta I_{fg}(t)$ and $\Delta W_m(t)$ have been plotted in Figs. 4(a),4(b) and 4(c) respectively for $\alpha_0 = 9.6°$,. Similarly, these three plots for $\alpha_0 = 19.2°$,and for $\alpha_0 = 28.8°$, have been shown in Figs. 5(a),5(b), 5(c) and Figs. 6(a),6(b) and6(c) respectively. All the plots indicates the stability of the system as the transients die out as time tends to infinity. However the nature of the transient in $\Delta I_a(t)$ and $\Delta W_m(t)$ are different from those in $\Delta I_{fg}(t)$. It is clearly seen from the state variable formulation that $\Delta I_{fg}(t)$ involves a first order transient only where as each of $\Delta I_a(t)$ and $\Delta W_m(t)$ involves transients of higher order.

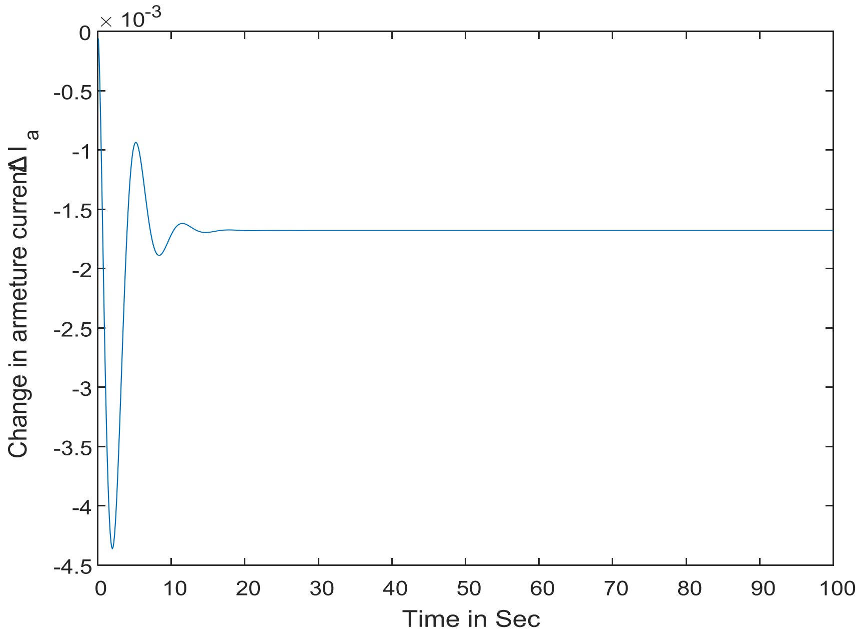

Fig.4(a). Unit step response of $\Delta I_a(t)$ for $\alpha_0 = 9.6°$

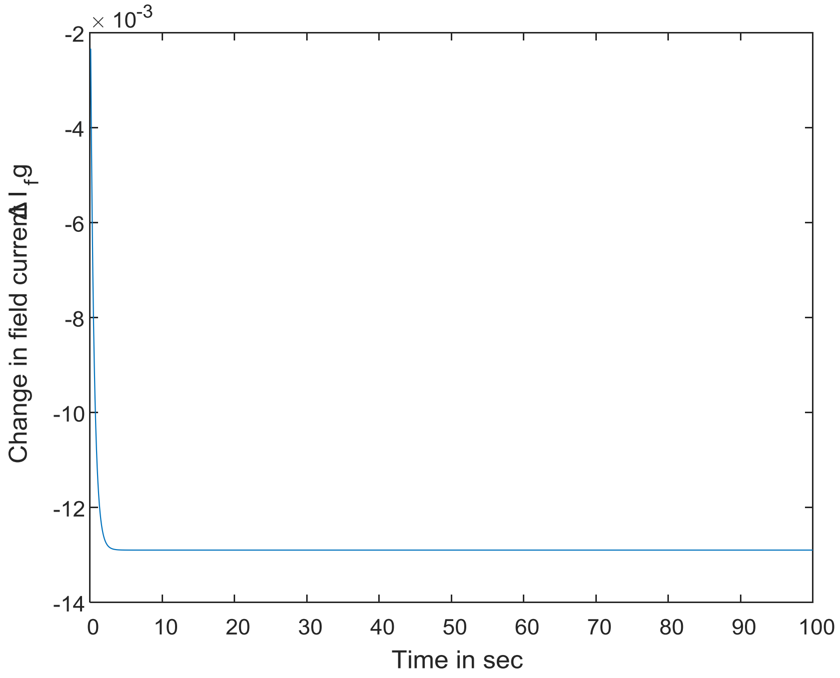

Fig.4(b). Unit step response of $\Delta I_{fg}(t)$ for $\alpha_0 = 9.6°$

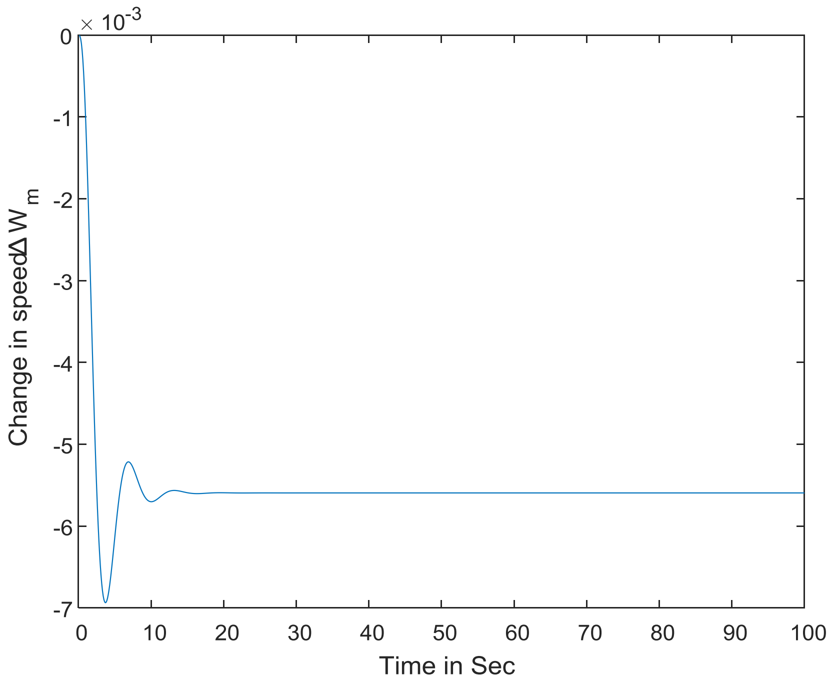

Fig.4(c). Unit step response of $\Delta W_m(t)$ for $\alpha_0 = 9.6°$

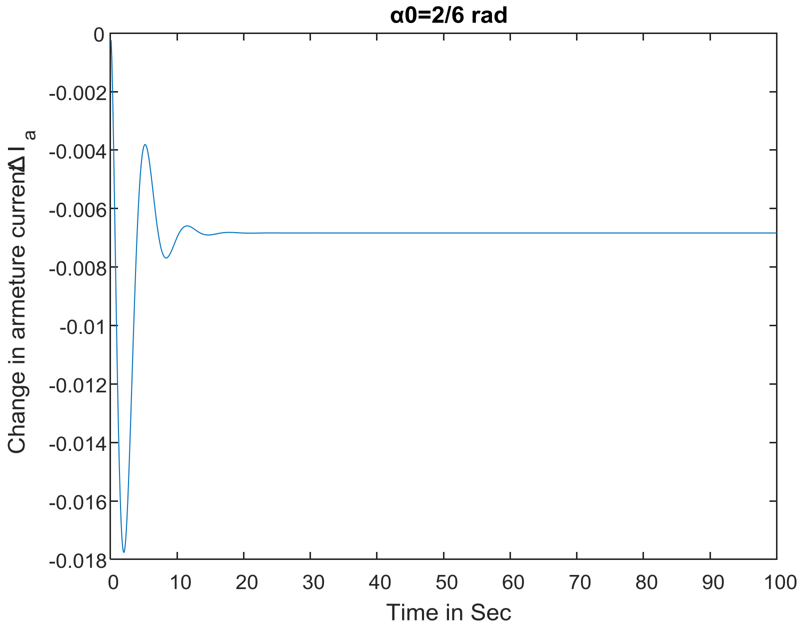

Fig.5(a). Unit step response of $\Delta I_a(t)$ for $\alpha_0 = 19.2°$

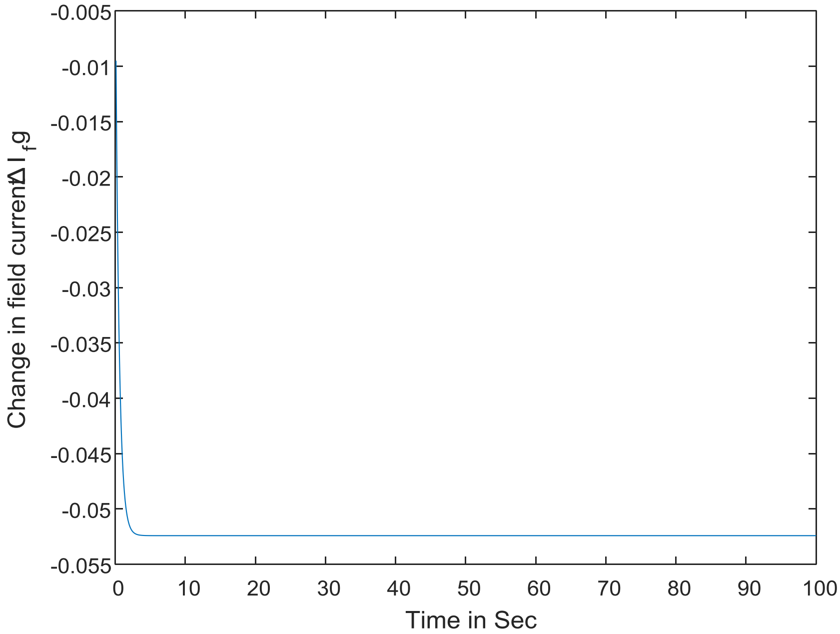

Fig.5(b). Unit step response of $\Delta I_{fg}(t)$ for $\alpha_0 = 19.2°$

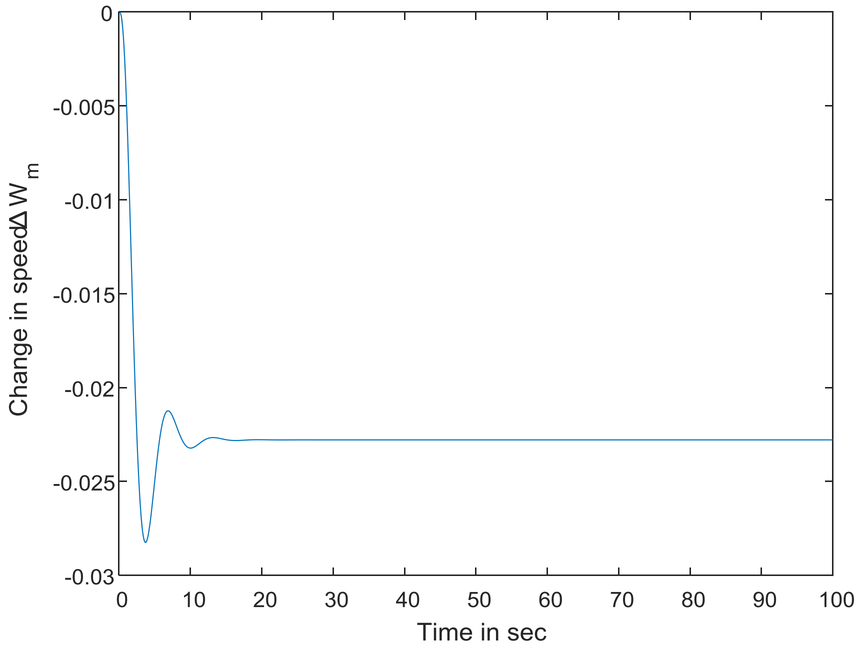

Fig5(c). Unit step response of $\Delta W_m(t)$ for $\alpha_0 = 19.2°$

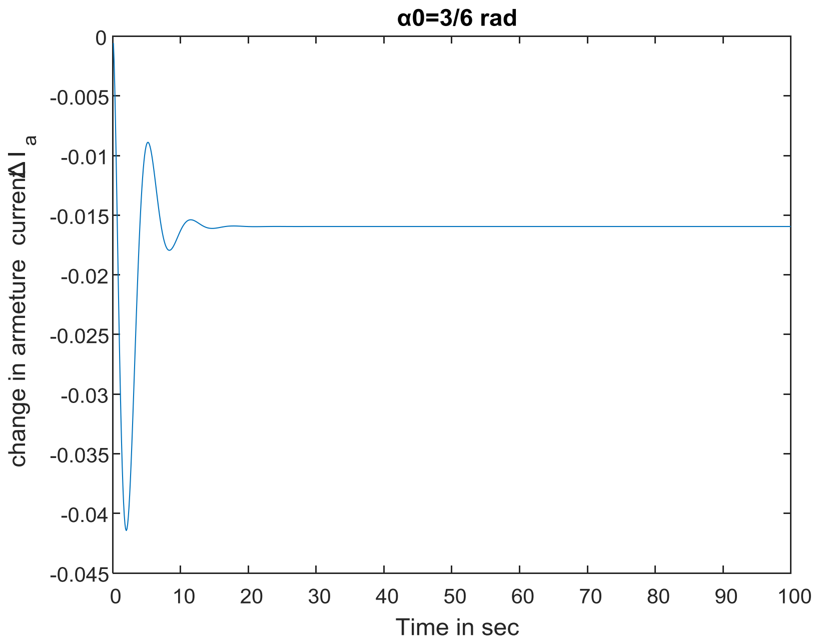

Fig.6(a). Unit step response of $\Delta I_a(t)$ for $\alpha_0 = 28.8°$,

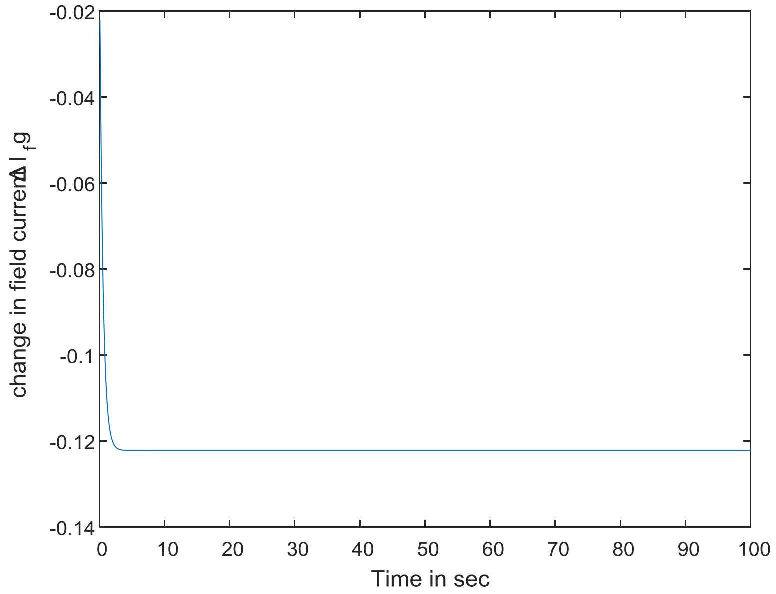

Fig.6(b). Unit step response of $\Delta I_{fg}(t)$ for $\alpha_0 = 28.8°$,

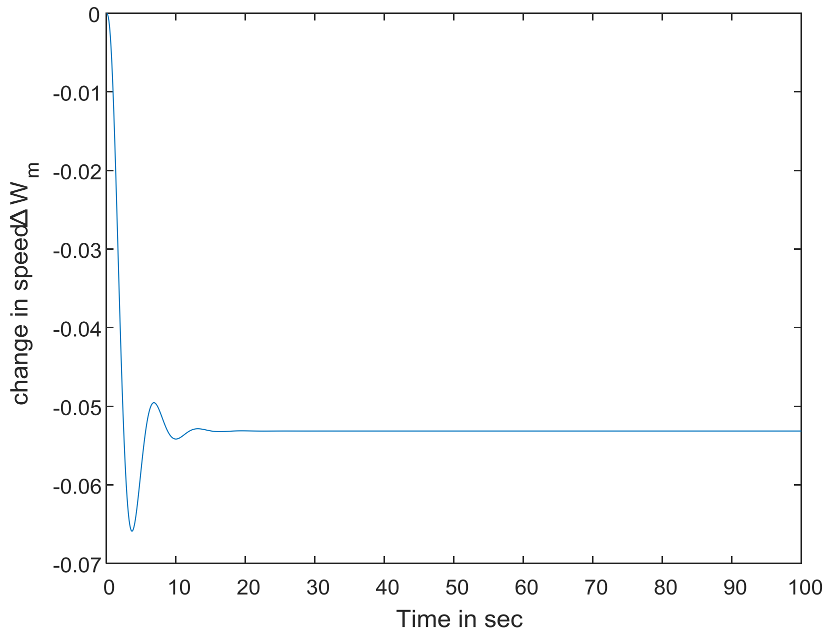

Fig6(c). Unit step response of $\Delta W_m(t)$ for $\alpha_0 = 28.8°$,

V. Conclusion:

- This particular circuit and modeling show that the effect of parameter control on the variation of speed of the motor can be obtained in a tabular form. Even though the mapping is non linear, a suitable calibration can be done such that the future researchers can design the control circuit based on the fact that how much change in R2 will lead to how much change in the speed of motor.
- This particular work done does not exactly fall in to the category of adaptive parameter control, but the present research work can be modified so that the voltage proportional to change in speed during the dynamics of the machine can be tapped and fed back to the circuit separately.

Reference

I. A. Bara, S. Dale, C. Rusu and H. Silaghi, "DC electrical drive control with fuzzy systems," 2015 13th International Conference on Engineering of Modern Electric Systems (EMES), Oradea, 2015, pp. 1-4. doi: 10.1109/EMES.2015.7158437.

II. A. Choudhary, S. A. Singh, M. F. Malik, A. Kumar, M. K. Pathak and V. Kumar, "Virtual lab: Remote access and speed control of DC motor using Ward-Leonard system," 2012 IEEE International Conference on Technology Enhanced Education (ICTEE), Kerala, 2012, pp. 1-7. doi: 10.1109/ICTEE.2012.6208666.

III. A.S.Avila Balula, M.S. “Nonlinear Control of an inverted pendulum”Thesis in engineering physics,and Technology, School of engineering University of Lisbon, Portugal September,2016.

IV. F. P. A. Vaccaro, M. Janusz and K. Kuhn, "Digital control of a Ward Leonard drive system," 3D Africon Conference. Africon '92 Proceedings (Cat. No.92CH3215), Ezulwini Valley, Swaziland, 1992, pp. 123-127. doi: 10.1109/AFRCON.1992.624433.

IV. G. A. Biacs and M. S. Adzic, "Modeling of the thyristor controlled rectifiers for control of Ward - Leonard system," 2009 7th International Symposium on Intelligent Systems and Informatics, Subotica, 2009, pp. 193-196.doi: 10.1109/SISY.2009.5291167.

V. Meng Cno, X. jin & R.E. White, “Nonlinear state-variable method for solving physical based Li-Ion cell model with High frequency inputs” journal of Electrochimical Society, Feb 2017.

VI. Q. Zhong, "Speed-sensorless AC Ward Leonard drive systems," SPEEDAM 2010, Pisa, 2010, pp. 1512-1517. doi: 10.1109/SPEEDAM.2010.5544901.