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Steady State Stability Analysis of a CSI-fed synchronous motor drive using Digital modeling

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Abstract

This paper develops a digital model of a current source Inverter fed three phase synchronous motor drive system from the view point of steady state stability aspect. The motivation lies in the fact that to control any electrical drive system digital controller is needed. To develop the software and hardware of such controller, a suitable digital model of the original drive system becomes necessary. Approach to develop the model in s-domain has been outlined and then z-transform has been applied. Different aspects of the model like the stability assessment using pole-zero mapping, Jury's test, range of coefficients of characteristic equation for stability etc., have been computed leading to various graphical plots. Furthermore perturbation of machine design parameters have been modeled from the view point of stability assessment with necessary computational results.

Keywords : CSI fed Synchronous Motor, Z transform, Jury's test, Impulse response, stability analysis.

1. Introduction

Permanent magnet Synchronous Motor (PMSM) drive systems are becoming more popular due to their advantages such as utility of self control, good efficiency and operation near to unity power factor, small inertia etc. Due to these advantages synchronous motors are also serious competitors to both dc motor drives and induction motor drives [XI].When compared to an AC induction motor, PMSM has superior advantage to achieve higher efficiency as it produces the rotor magnetic flux with permanent magnets. For this reason PMSMs are used in high end appliances and equipments that require high efficiency and reliability [XII].

PMSMs are gaining increasing popularity and demand in various areas such as automobiles, robotics and aerospace engineering [X]. In addition, Z Source Based Permanent Magnet Synchronous Motor Drive System [IX] is also gaining importance. Design of controller for PMSM has been reported in [XIII]. PMSM has many attractive characteristics such as high-power density, high-torque-to-inertia ratio, wide speed operation range, and free from maintenance, it is suitable for many industrial applications[VI,VII]. So far the literature review is concerned, digital implementation of an adaptive speed regulator for a PMSM has been reported in [VIII]. However this work does not focus on the stability analysis from the digital domain point of view To evaluate the field performance accurately we need accurate digital simulation tools. Hence design of digital controllers merges to be the main area of interest for such drive systems. One disadvantage with PMSM is that for accurate analysis of the performance of the drive systems, circuit theory loses its advantage whereas in such problem, electromagnetic field theory becomes quite successful to analyze and then to come to a concrete conclusion.

That's why analysis and control of a normal synchronous motor drive system (without PMSM) becomes advantageous. A normal synchronous motor drive system does not have variety of applications due to the feature of constant speed, but it has some critical applications in steel industries [III] in the form of drives for big blowers and rolling mills. Without using power electronics devices, classical control of synchronous motors have not much practical importance. However, voltage source inverter (VSI) or current source inverter (CSI) can be used as a specific driver for the synchronous motor depending on the necessity. Based on these facts, the authors of this paper have been motivated to take up the digital modeling of a classical analysis performed in the complex frequency domain for a current source inverter fed Synchronous motor drive system [I,II]. Such problem carries utmost practical importance because: (a) Due to the reduction of a problem of two variables (armature current and field angle) to a problem of single variable(only, field angle). (b) Non-uniformity in air-gap within the synchronous motor can be modeled with sufficient accuracy. Therefore the concept in point (a) as stated above clearly indicates that inverter fed synchronous motor drive system leads to a more simpler mathematical model.

Controlling of electrical machines by means of power electronics is one of the most important concerns of many engineers, whether it concerns the designing of complex control algorithms or turning on or off the control parameters. Parameter variations, error measurements and distortions need to be taken into account while designing phase. A model of control loop in discrete time domain or Z-domain needs to be obtained as it involves power electronics and as we are going to use digital controllers. Many studies have been carried out for this purpose in the past [IV,V].

Steady State Stability analysis of a CSI fed Synchronous Motor drive system with

Damper Windings in continuous time domain was done in [I,II]. The authors of [I,II]. expressed the transfer function of a Current Source fed synchronous motor drive system in a convenient manner after developing the voltage-balance equations and torque-balance equations in time domain. The transfer function obtained in this study was the ratio of Laplace transform of change in angle (β) between field and armature axis to the Laplace transform of change in load torque (T_L). The detailed mathematical analysis was presented [I,II]. The base paper used a block diagram of the CSI (current source inverter) fed Synchronous motor drive system as shown in Fig.1 [I].

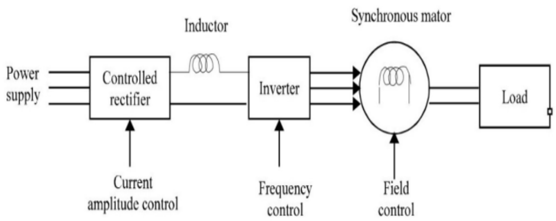

Fig.1: Drive configuration for open loop current fed synchronous motor

In continuation with the investigation in [I]. the proposed analysis in the present paper targets to convert the transfer function from continuous domain to discrete domain. Z transform is used to analyze the transfer function in discrete domain. The main objective of this paper is to conduct a steady state stability analysis of CSI fed synchronous motor in discrete time domain and to calculate the range of stability. A detailed mathematical analysis in this direction is presented in this paper.

The main advantage of having a transfer function in digital domain is that it becomes easier to design a digital controller. Since digital control offers more flexibility, it can perform really complicated control activities keeping scope of modification as per necessity compared to analog control which is slow to develop and difficult to make accurate designs. Furthermore such digital modeling of Synchronous motor can be used as a design tool to investigate the range of certain machine design parameters from the view point of steady state stability phenomenon. Basically the facts can be looked upon on the following lines:

- If there occurs some small perturbations of load torque the corresponding change in the field angle is to be noticed. As, conceptually the field angle is very near to our traditional concept of torque angle (or load angle), such problem leads to the Steady State stability problem of a Synchronous motor drive system.

- Field angle is the space angle between armature m.m.f position (axis) and the direct axis(d-axis).

- Based on such concept, the result and discussion section shows the complete calculation methods for range of parameter like R_{kd}, R_f, R_{kq} and L_{md} on the basis of maintaining steady state stability performance of the said drive system. Such analysis demands for Z-transform applications along with application of Euler's equation involving partial derivative concepts. These detailed treatments along with appropriate numerical results have been presented in the concerned section.

The next section has been presented, based on the above said philosophy. In the Problem Formulation section, for a better clarity of understanding the proposed research work has been decoupled into two parts. The first part gives a brief outline of the modeling in s-domain with the expression for transfer function. The second part deals with the digital modeling of the said system. The digital modeling initiates with the mathematical process of the derivation for obtaining transfer function in Z-domain. The transfer function obtained in complex frequency domain (s- domain) in [I,II]. did take care of air-gap saliency of the motor and that's why the machine equations were developed in d-q frame (using axis model, not phase model of the machine). After this, conceptually the subsequent analytical parts like computation of the range of stability, effect of unit impulse input on stability aspect etc. have been taken up in subsequent sections.

2. Problem Formulation:

Primitive machine model of the Synchronous motor pertaining to the model shown in Fig.1 has been developed and the detailed mathematical analysis is presented in [I].

2.1 Problem Formulation Part-1: Development of Transfer Function

The torque dynamic equation of a synchronous motor can be written as,

$$T_e - T_L = J\frac{d\omega}{dt} \tag{2.1.1}$$

where,

T_e is electromagnetic torque in N-m

T_L is Load Torque in N-m

ω is Motor speed in mechanical rad./sec.

J isPolar moment of inertia of motor and load (combined) kg-m^2

$\Delta T_e(s)$ is perturbed quantity (transformed) of electromagnetic torque and the expression for it is derived from [I] is given as

$$\Delta T_e(s) = \left[\frac{x_1 s^3 + x_2 s^2 + x_3 s + x_4}{l_1 s^3 + l_2 s^2 + l_3 s + l_4} \right] \Delta\beta(s) \tag{2.1.2}$$

The small change in speed 'ω' equal to Δω can be related to small change in field angle, Δβ as given by,

$$\Delta\omega = -\frac{d(\Delta\beta)}{dt} \tag{2.1.3}$$

The negative sign in equation physically indicates a drop in speed (ω) due to increase in field angle (β).

Based on equation (2.1.3), the following expression can be written,

$$J\,\frac{d(\Delta\omega)}{dt} = J\,\frac{d}{dt}\left[-\frac{d}{dt}(\Delta\beta) \right] = -J\,\frac{d^2}{dt^2}(\Delta\beta) \tag{2.1.4}$$

The small-perturbation model of equation (2.1.4) can be written as,

$$\Delta \mathrm{T_e} - \Delta \mathrm{T_L} = J\,\frac{d(\Delta\omega)}{dt} \tag{2.1.5}$$

Combining equations (2.1.4) and (2.1.5), it yields

$$\Delta \mathrm{T_e} - \Delta \mathrm{T_L} = -J\,\frac{d^2}{dt^2}(\Delta\beta) \tag{2.1.6}$$

The transformed version of eqn. (2.1.6), with initial condition relaxed, comes out to be

$$\Delta T_e(s) - \Delta T_L(s) = -Js^2\,\Delta\beta(s) \tag{2.1.7}$$

Substituting the expression for ΔTe(s) from eqn. (2.1.2) in eqn. (2.1.7), it yields

$$\left[\frac{x_1 s^3 + x_2 s^2 + x_3 s + x_4}{l_1 s^3 + l_2 s^2 + l_3 s + l_4} \right] \Delta\beta(s) + Js^2 \Delta\beta(s) = \Delta T_L(s) \tag{2.1.8}$$

Equation (2.1.8) gives after manipulation, a Transfer Function, T(s) expressed as,

$$\mathrm{T}(\mathrm{s}) = \frac{\Delta\beta(s)}{\Delta T_L(s)} = \frac{l_1 s^3 + l_2 s^2 + l_3 s + l_4}{K_1 s^5 + K_2 s^4 + K_3 s^3 + K_4 s^2 + K_5 s + K_6} \tag{2.1.9}$$

where,

$$\mathrm{K}_1 = \mathrm{J} l_1$$

$$\mathrm{K}_2 = \mathrm{J} l_2$$

$$K_3 = (Jl_3 + x_1)$$

$$K_4 = (Jl_4 + x_2)$$

$$K_5 = x_3$$

$$K_6 = x_4$$

Once the transfer function has been developed in s-domain, the z-transfer function has been obtained using MATLAB as explained in the next section.

2.2 Problem Formulation Part-2 : Transfer Function Development in Discrete domain

The transfer function in the continuous time domain was given as [I]:

$$T(s) = \frac{\Delta\beta(s)}{\Delta T_L(s)} = \frac{l_1 s^3 + l_2 s^2 + l_3 s + l_4}{K_1 s^5 + K_2 s^4 + K_3 s^3 + K_4 s^2 + K_5 s + K_6} \quad (2.2.1)$$

Where

I. The algebraic expressions of l_1, l_2, l_3, l_4 and K_1, K_2, K_3, K_4, K_5, K_6 are available in [I]

II. $\Delta\beta(s)$ is the transformed field angle (small perturbation).

III. $\Delta T_L(s)$ is the transformed load torque (small perturbation).

Here the terminology "transformed" means that the mathematical tool Laplace Transform has been applied.

The polynomial coefficients (K_1 to K_6) [I] were calculated using the machine data present in the appendix. The values of K_1 to K_6 are as follows.

$K_1 = 2.2871$

$K_2 = 0.3585$

$K_3 = 0.5577$

$K_4 = 0.0776$

$K_5 = 0.0023$

$K_6 = 2.419 \times 10^{-6}$

The numerator polynomial coefficients were calculated using the equations and the machine data [I]. The values of l_1 to l_4 are as follows:

$l_1 = 0.2857$, $l_2 = 0.0447$, $l_3 = 1.615*10^{-3}$, $l_4 = 1.755*10^{-6}$.

The above equation in continuous form is converted to discrete form (i.e. converted to z domain). The discrete domain conversion was done using MATLAB software with sampling time as 0.1 seconds. Z transform was taken after substituting the coefficient values. The transfer function obtained is represented as T(z).

$$T(z) = \frac{\Delta\beta(z)}{\Delta T_L(z)} = \frac{0.0006245z^4 - 0.001239z^3 - 9.676 * 10^{-6}z^2 + 0.001239z - 0.0006148}{z^5 - 4.982z^4 + 9.931z^3 - 9.899z^2 + 4.935z - 0.9844} \quad (2.2.2)$$

2.2.1 Stability assessment using POLE-ZERO mapping

The stability analysis of the discrete time equation is done by plotting a pole zero map using MATLAB software.

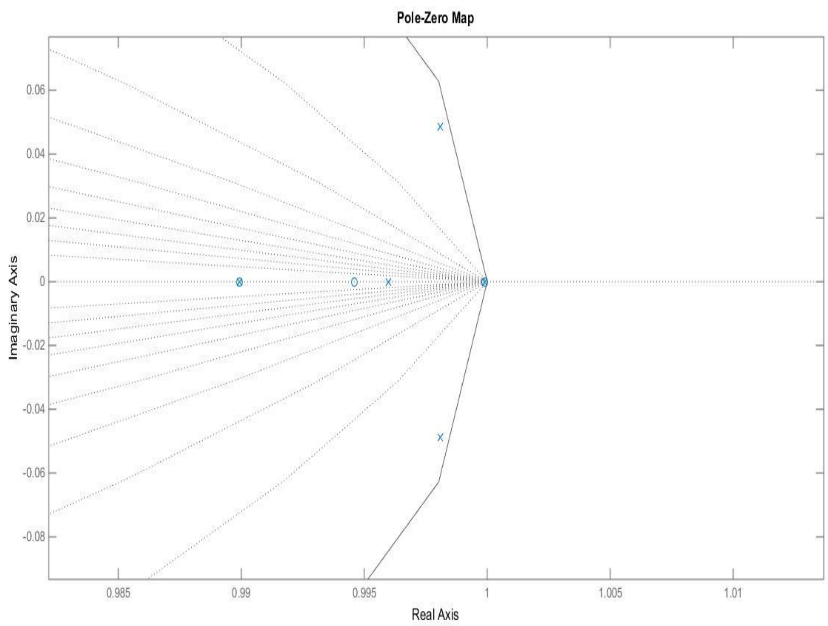

Fig: 2 pole zero map of the transfer function

Fig: 2 shows the pole zero map obtained for the z-transfer function. Since all the poles lie inside the unit circle, the steady state stability is assured for the system in discrete domain. Even though pole-zero locations give a basic information about the system stability, investigations on the coefficients of the characteristic equation of the same system gives a more clear picture on the stability assessment. Such investigation is known as Jury's test and it is taken up in the next section for a through result and discussion.

2.2.2 Jury's Test

Jury's stability criterion is a method to analyze the stability of discrete time system using the coefficients of the characteristic equation derived from the transfer function. Using Jury's test we can do stability analysis of a discrete time system without having to calculate the poles of the system.

Detailed mathematical analysis of jury test on the system is given below.

The characteristic equation is:

$$f(z) = z^5 - 4.982z^4 + 9.931z^3 - 9.899z^2 + 4.935z - 0.9844 \quad (2.2.3)$$

The necessary and sufficient conditions to satisfy for the system to be considered stable are:

Rule 1

If z is 1, the system output must be positive:

$$f(1) > 0$$

Rule 2

If z is -1, then the following relationship must hold:

$$(-1)^n f(-1) > 0$$

Where n is highest power of the characteristic equation.

Rule 3

The absolute value of the constant term (a_0) must be less than the value of the highest coefficient (a_n):

$|a_0| < |a_n|$ **, where the polynomial is given as**

$$f(z) = a_n + a_{n-1}z^1 + a_{n-2}z^2 + a_{n-3}z^3 ...... + a_0 z^n$$

If Rule 1 Rule 2 and Rule 3 are satisfied, construct the Jury Array.

Rule 4

Once the Jury Array has been formed, all the following relationships must be satisfied until the end of the array:

$|b_0| > |b_{n-1}|$

$|c_0| > |c_{n-2}|$

$|d_0| > |d_{n-3}|$

And so on until the last row of the array. If all these conditions are satisfied, the system is stable. Jury array for the system is shown in Table.1.

Table.1: Jury's array pertaining to the stability assessment in discrete time domain

Z^0	Z^1	Z^2	Z^3	Z^4	Z^5
-0.9844	4.935	-9.899	9.931	-4.982	1
1	-4.982	9.931	-9.899	4.935	-0.9844
-0.0309	0.1239	-0.186	0.1229	-0.0307	x
-0.0307	0.1229	-0.186	0.1239	-0.0309	x
$1.232x10^{-5}$	$1.594x10^{-4}$	$0.0372x10^{-3}$	$6.1x10^{-6}$	x	x
$6.1x10^{-6}$	$0.0372x10^{-3}$	$1.594x10^{-4}$	$1.232x10^{-5}$	x	x
$1.458x10^{-10}$	$1.6814x10^{-9}$	$-0.0514x10^{-8}$	x	x	x

The results in Table.1, hereby assures the stability of the system. It is well known that if the coefficients of a characteristic equation are perturbed , the system may move to the unstable zone. This will give a rough indication of the degree of robustness of the system, Such treatment is taken up in the next section.

2.3 Range of coefficients for stability:

The stability ranges for coefficients were found by changing the value of one coefficient while keeping the other coefficients constant. The stability analysis was done by plotting the pole zero maps using MATLAB for the transfer function. If the poles of the equation lie inside the unit circle then the system was concluded to be stable and if the poles lie outside the unit circle then the system was concluded to be unstable. Table 2 represents the range of co efficient.

Table 2: Range for the coefficients

Range	Original Values(stable)
$K_1 \leq 2.4$	(2.2871)
$K_2 \leq 15.9$	(0.3585)
$K_3 \leq 52.75$	(0.5577)
$K_4 \leq 0.081$	(0.0776)

The original value of the coefficients is represented in the brackets. For example when K_1 is given value more than 2.4 the system becomes unstable. Similar analysis was done for rest of the coefficients as well.

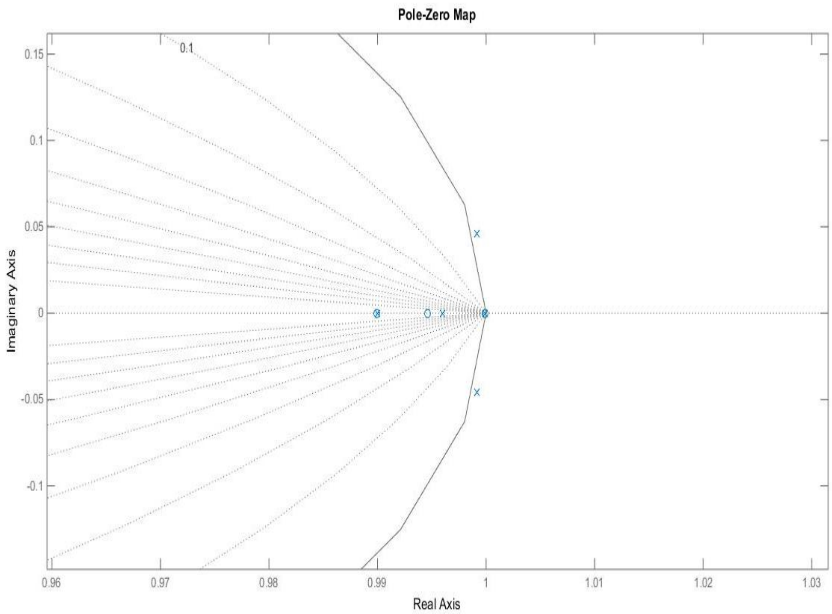

Fig 3: pole zero map for stable system

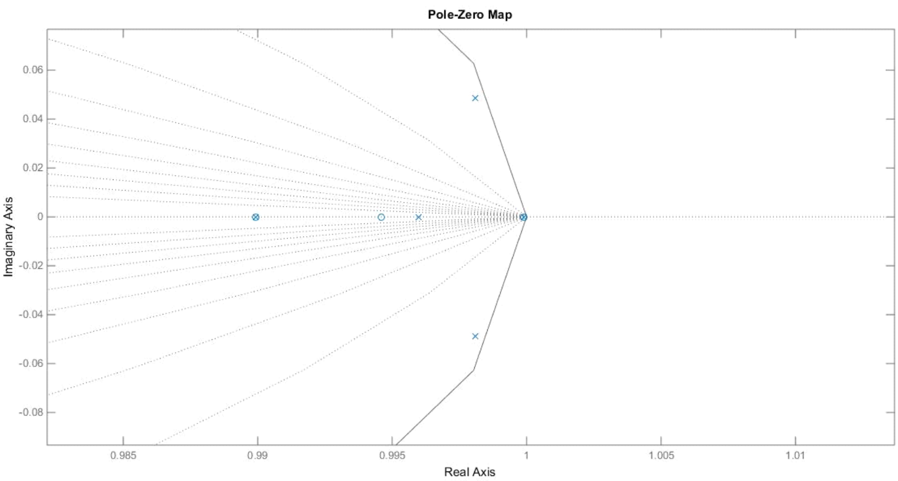

Fig 4: pole zero map for unstable system

Fig 3 represents the pole zero map of the stable system when there was no change in the coefficient values.

Fig 4 represents the pole zero map of unstable system. In this transfer function the K1 value was taken as 2.5 which clearly exceeds the maximum range which is 2.4. Therefore the system becomes unstable. Once the range of coefficients of the characteristic equations are known from the view point of stability, it becomes necessary to find the impulse response of the system as an important part of the whole digital modeling. The analysis involving the impulse response is presented in the next section.

3. Impulse response:

It is well known that an Impulse response of a system can be looked upon as Inverse Laplace Transform of the Transfer Function. Hence it is a very important analysis of a control system. For example if we consider an AVR control system that controls the output voltage of the generator, when designing a controller for this system, one has to consider worst case scenarios or conditions. For example if we consider the worst case scenario can be a lightning for a very short period of time (in microseconds). This causes a very big voltage to be impressed at the terminals of the generator. In this case the control system must be able to secure the generator from the worst possible scenario. In such case stability analysis using impulse response method becomes a strong tool for analysis.

In the following section the stability analysis of the system with input as unit impulse is performed and subsequently a detailed mathematical analysis is represented

3.1 Unit impulse input in s-domain:

With reference to Fig. 1, there may occur so many types of variations in the magnitude of change in load torque. Due to abnormal behaviour of the mechanical coupling system or any other reason there may appear a sudden change in load torque. In other words change in load torque may be considered as a unit impulse function and our objective in this subsection is to investigate the stability assessment of the particular system with input as δ(t) through the method of Pole-Zero mapping.

$$\Delta T_L(S) = L\delta(t) = 1 \qquad (3.1.1)$$

$$\Delta\beta(t) = \quad L^{-1}\Delta\beta(S) \qquad (3.1.2)$$

Where L and L^{-1} are Laplace and inverse Laplace transform operator respectively.

Hence

$$\Delta ß(t)= e^{-(p1)t}+Be^{-(p2)t}+Ce^{-(p3)t}+De^{-(p4)t}+Ee^{-(p5)t} \qquad (3.1.3)$$

Where p1,p2,p3,p4,p5 are the roots of the characteristic equation and A,B,C,D,E are the coefficient values.

p1= -0.0071+0.4874i

p2= -0.0071-0.4874i

p3= -0.1011

p4= -0.0403

p5= -0.0011

A= -0.0036-0.1284i

B= -0.0036+0.1284i

C= 0

D= 0.0072

E= 0

Since all the poles of the characteristic equation have negative real parts assured stability is here by assured. In this context plot of change in amplitude versus time has been presented in Fig. 5. This plot clearly indicates a decaying sinusoid having both upper envelope and lower envelope. It is also a point of interest that such plot is treated as a symmetrical wave form leading to the major physical conclusion , i.e., the DC component is absent. Such concept appears to be automatically correct because of the ordinate of Fig.5 is a change in variable (not absolute value).

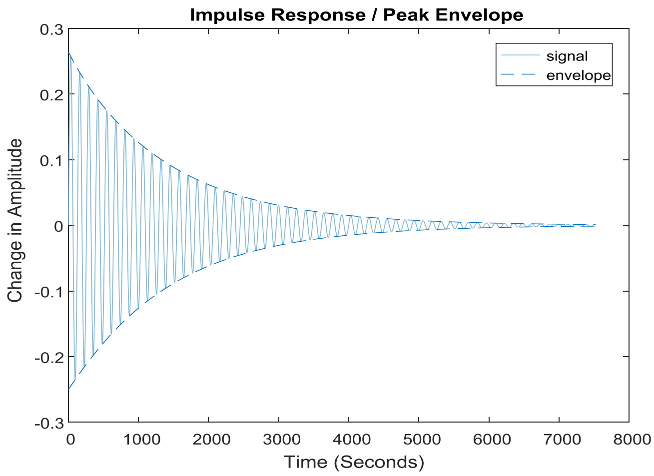

Fig.5: Impulse response of the transfer function in continuous time domain

3.2 Unit impulse input in Z-domain:

It may be noted that the concept put in section 2.1 was valid in continuous time domain and that is why Laplace Transform operator was used to investigate the satiability assessment phenomena. However, as the present paper deals with the digital modelling of the system, at this stage it becomes necessary to convert the above said philosophy in section 2.2 to discrete time domain. As a natural responsibility the use of 'Z' transform must be involved in such case to investigate the stability phenomenon in discrete time domain. Based on such philosophy the related mathematical equations are presented as follows:

Since the Z[δ(n)] =1,

$$\Delta \beta (z)=T(z) * 1 \tag{3.2.1}$$

$$\Delta \beta(n) = \boldsymbol{Z^{-1}}[T(z)] \tag{3.2.2}$$

Hence

$$\Delta \beta(n) = A\,\delta(n) + B\boldsymbol{l1^n} + C\boldsymbol{l2^n} + D\boldsymbol{l3^n} + E\boldsymbol{l4^n} + F\boldsymbol{l5^n} \tag{3.2.3}$$

Here l_1,l_2,l_3,l_4,l_5 are the roots of the characteristic equation and A,B,C,D,E,F are the coefficients.

l1= 1.1875+0.1692i

l2= 1.1875-0.1692i

l3= 0.9014+0.1956i

l4= 0.9014-0.1956i

l5= 0.8041

A= 0.0006

B= 0.0008-0.0007i

C= 0.0008+0.0007i

D= -0.0005-0.001i

E= -0.0005+0.001i

F= -0.0012

The explanation behind the presentation of Fig.5 can be reproduced in similar lines for simulations in discrete time domain. Such approach leads to the development of Fig.6; where impulse response of transfer function is presented in discrete domain.

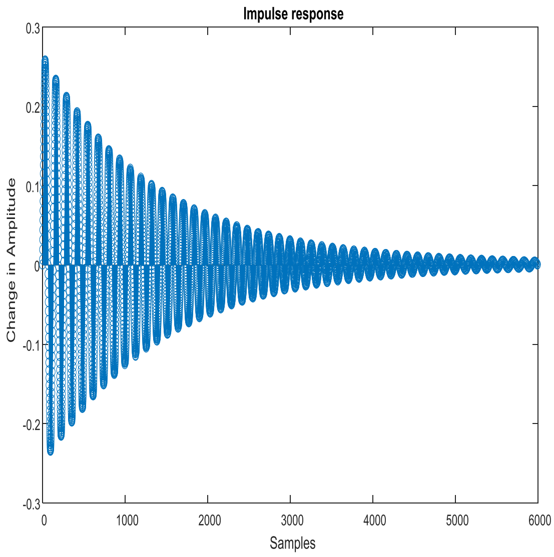

Fig 6: Impulse response of transfer function in time discrete domain

It is natural to put a question on the fact that Δß(t) or Δß(n) are considered to be variables of importance, and why not ß(t) or ß(n).

The answer lies in the fact that the very purpose of this research is to investigate the steady state stability of synchronous motor drive system from various viewpoints. Moreover the concept of the steady state stability is based on the theory of small perturbations. At this stage it is felt that even though necessary analysis has been presented involving the coefficients of the characteristic equation, still some more study from the view point of design aspects of the motor can be done. This study will be based on perturbing the real machine design parameters like R_f (Field winding resistance), R_{kd} (Direct axis damper windings) etc. Due to these changes, the coefficients like K1, K2 etc (2.2.1) will be perturbed and hence steady state stability of the system may get affected. Such analysis with the relevant results are presented in the next section.

3.3: Analysis of parameter perturbation

Based on the stability range for the coefficients given in the table 1, it reveals that changing the parameter affects the design from the view point of steady state stability assessment through digital modeling.

To find out which machine parameter affects the stability of the system most a method is proposed with a detailed explanation in an example below. If consider coefficient K_6 as

$K_6=x_4$ as in (2.1.2)

$$x_4=n_4 i_s \cos\Delta\beta - m_4 i_s \sin\Delta\beta \tag{3.3.1}$$

where,

$n_4 = c_2 f_2 b_3$ (3.3.2)

and

$c_1 = [(L_d - L_q)i_{q0} - L_{mq}i_{kq0}]$ (3.3.3)

$c_2 = [(L_d - L_q)i_{d0} + L_{md}i_{kd0} + L_{md}i_{f0}]$ (3.3.4)

$f_2 = R_{kq}$ (3.3.5)

$b_3 = R_{kd}R_f$ (3.3.6)

Let, $i_s\cos\Delta\beta = P1$ and $i_s\sin\Delta\beta = P2$

Therefore, (3.2.1) can be rewritten as

$x_4 = c_2 f_2 b_3\ P1 - c_1 b_3 f_2 P2$ (3.3.7)

$K_6 = x_4 = R_{kd}R_fR_{kq}\{[(L_d - L_q)i_{d0} + L_{md}i_{kd0} + L_{md}i_{f0}]\ P1 - [(L_d - L_q)i_{q0} - L_{mq}i_{kq0}]\ P2\}$ (3.3.8)

Substituting the constant values given in appendix we get,

$K_6 = R_{kd}R_fR_{kq}\{[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\ P1 - [(L_d - L_q)i_{q0}]\ P2\}$ (3.3.9)

At this stage, it will be a matter of justice if the physical significances of variables with suffice "0" (i.e. $\beta_o, i_{f0}, i_{d0}, i_{q0}, i_{kq0}, i_{kd0}$) are stated clearly. Basically, the theory of calculus states that Taylor's series expression of any function becomes mathematically defined about an operating point (or quiescent point). i_{d0}, i_{q0} etc are the quiescent values. Practically also, one cannot perturb a system until and unless the equilibrium state is known.

We can see that K_6 is a function of ($R_{kd}, R_f, R_{kq}, L_{md}$). Now from the table 1 we know that the system is stable when $K_6 \leq 0.00027$ the system is stable. The system becomes unstable at $K_6 = 0.0003$.

Therefore,

$dK_6 = 0.0003 - 0.00027$

or

$dK_6 = 3*10$^-5. (3.3.10)

Now using the Euler's differential formula we can write it as,

$$dK_6 = \frac{\partial f}{\partial R_{Kd}} dR_{Kd} + \frac{\partial f}{\partial R_{Kq}} dR_{Kq} + \frac{\partial f}{\partial R_f} dR_f + \frac{\partial f}{\partial L_{md}} dL_{md} \quad (3.3.11)$$

Case: 1

When we consider R_{Kd} is changing,

$$dK_6 = \frac{\partial f}{\partial R_{Kd}} dR_{Kd} + 0 \quad (3.3.12)$$

Since only R_{Kd} is changing and the other coefficients are constant, the rest of the equation becomes zero.

Now equating (3.3.10) and (3.3.12) we get

$$3*10^{-5} = \frac{\partial f}{\partial R_{Kd}} dR_{Kd} + 0$$

$$= \frac{\partial}{\partial R_{Kd}}\{R_{Kd}R_f R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \frac{\partial}{\partial R_{Kd}}\{R_{Kd}R_f R_{Kq}P2(L_d - L_q)i_{q0}\}$$

$$= [R_f R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0} - R_f R_{Kq}P2(L_d - L_q)i_{q0}]\Delta R_{Kd} \quad (3.3.13)$$

By substituting the values for the parameters given in the appendix we get

$$\Delta R_{Kd} = 0.372 \quad (3.3.14)$$

Case 2:

When we consider R_f is changing,

$$dK_6 = \frac{\partial f}{\partial R_f} dR_f + 0 \quad (3.3.15)$$

Since only R_f is changing and the other coefficients are constant, the rest of the equation becomes zero.

Now equating (3.3.10) and (3.3.15) we get

$$3*10^{-5} = \frac{\partial f}{\partial R_f} dR_f + 0$$

$$= \frac{\partial}{\partial R_f}\{R_{Kd}R_f R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \frac{\partial}{\partial R_f}\{R_{Kd}R_f R_{Kq}P2(L_d - L_q)i_{q0}\}$$

$$= [R_{Kd}R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \{R_{Kd}R_{Kq}P2(L_d - L_q)i_{q0}]\Delta R_f \quad (3.3.16)$$

By substituting the values for the parameters given in the appendix we get.

$$\Delta R_f = 1.86*10^{-2} \quad (3.3.17)$$

Case 3:

Now considering R_{Kq} is changing,

$$dK_6 = \frac{\partial f}{\partial R_{Kq}} dR_{Kq} + 0 \quad (3.3.18)$$

Since only R_{Kq} is changing and the other coefficients are constant, the rest of the equation becomes zero.

Now equating (3.3.10) and (3.3.18) we et

$$3*10^{-5} = \frac{\partial f}{\partial R_{Kq}} dR_{Kq} + 0$$

$$= \frac{\partial}{\partial R_{Kq}}\{R_{Kd}R_f R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \frac{\partial}{\partial R_{Kq}}\{R_{Kd}R_f R_{Kq}P2(L_d - L_q)i_{q0}\}$$

$$= [R_{Kd}R_f P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \{R_{Kd}R_f P2(L_d - L_q)i_{q0}]\Delta R_{Kq} \quad (3.3.19)$$

By substituting the values for the parameters given in the appendix we get.

$$\Delta R_{Kq} = 0.527 \quad (3.3.20)$$

Case 4:

Similarly considering L_{md} is changing,

$$dK_6 = \frac{\partial f}{\partial L_{md}} dL_{md} + 0 \quad (3.3.21)$$

Since only R_{Kq} is changing and the other coefficients are constant, the rest of the equation becomes zero.

Now equating (3.3.10) and (3.3.21)

$$3*10^{-5} = \frac{\partial f}{\partial L_{md}} dL_{md} + 0$$

$$= \frac{\partial}{\partial L_{md}}\{R_{Kd}R_f R_{Kq}P1[(L_d - L_q)i_{d0} + L_{md}i_{f0}]\} - \frac{\partial}{\partial L_{md}}\{R_{Kd}R_f R_{Kq}P2(L_d - L_q)i_{q0}\}$$

$$=[[R_{Kd}R_f R_{Kq}P1L_{md}i_{f0}]\Delta L_{md}] \quad (3.3.22)$$

By substituting the values for the parameters given in the appendix we get.

$$\Delta L_{md} = 17.89 \quad (3.3.23)$$

The physical interpretation of such change can be explained as follows, with reference to (3.3.23), the specified value of the change in L_{md} basically indicates a change in direct-axis air-gap. If the said value of perturbation in L_{md} is not acceptable, then the direct axis air-gap will have to be changed and it will in turn affect the input power factor of the drive system. If, the particular value of change in input power factor is not allowed in practice, then field winding current of the motor will have to be adjusted.

4. Conclusion

This research paper concludes the following salient points:-

- Assessment of steady state stability has been performed in discrete time domain , by the method of "pole-zero" mapping and " Jury's Test". The result of both the methods do not conflict.
- As changing the coefficients of characteristic equation shows a direct impact on the stability of the system, a new view point based on machine design aspect is developed, which is written under the next point.
- The machine design parameters (R_f , R_{Kd} , L_{md} etc.) can be perturbed. This aspects has been studied using necessary mathematical formulations based on " Partial Differentiation".
- It was observed that a minimum amount of perturbation on field winding resistance affects the stability most. This is due to the fact that a change in field winding resistance changes the field current. As a result the flux linkage changes and ultimately the electromagnetic torque is affected. If this effect happens in a negative direction, stability is affected and the motor may finally cease to take the load.

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