# The Modified Signed Likelihood Ratio Test and its Application to Detect Cheating

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#### Outline



2 Existing Approaches











## Cheating Detection

- Score differencing is one of the six categories of cheating detection methods listed in Wollack and Schoenig (2018)
  - gain score analysis
  - item preknowledge detection
  - erasure analysis
  - ....



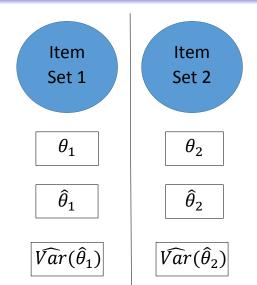


Conference On Test Security Methods

Application

Conclusions

#### The Problem Setup





## Existing Frequentist Methods

- Z or Wald test (e.g., Guo & Drasgow, 2010)
- Likelihood ratio test (LRT; e.g., Finkelman et al., 2010)
- Signed likelihood ratio test (Sinharay, 2017)
- Score test (Klauer & Rettig, 1990; Sinharay, 2017)
- Other methods such as erasure detection index (Wollack et al., 2015)





## The Z Statistic

• For testing  $H_0: \theta_1 = \theta_2$  vs  $H_1: \theta_2 > \theta_1$ , the Wald/Z statistic is given by

$$Z = \frac{\hat{\theta}_2 - \hat{\theta}_1}{\sqrt{\widehat{\operatorname{Var}}(\hat{\theta}_2) + \widehat{\operatorname{Var}}(\hat{\theta}_1)}}$$

• For the 2PL model,

$$\widehat{\operatorname{Var}}(\widehat{\theta}_1) = \left[\sum_i a_i^2 \frac{\exp[a_i(\widehat{\theta}_1 - b_i)]}{(1 + \exp[a_i(\widehat{\theta}_1 - b_i)])^2}\right]^{-1}$$



#### Log-likelihood of $\theta_1$ and $\theta_2$

- Log-likelihood of  $\theta_1$  and  $\theta_2$ :  $\ell(\theta_1, \theta_2)$
- For dichotomous items,  $\ell(\theta_1, \theta_2) = \sum_i [X_i \log p_i(\theta_1) + (1 X_i) \log(1 p_i(\theta_1))] + \sum_j [Y_j \log p_j(\theta_2) + (1 Y_j) \log(1 p_j(\theta_2))]$
- Under the 2PL model,  $\ell(\theta_1, \theta_2) = \sum_i [X_i a_i(\theta_1 - b_i) - \log(1 + e^{a_i(\theta_1 - b_i)})] + \sum_j [Y_j \tilde{a}_j(\theta_2 - \tilde{b}_j) - \log(1 + e^{\tilde{a}_j(\theta_2 - \tilde{b}_j)})]$





Applications

Conclusions

#### Likelihood Ratio Test Statistic

To test H<sub>0</sub>: θ<sub>1</sub> = θ<sub>2</sub> vs H<sub>1</sub>: θ<sub>1</sub> ≠ θ<sub>2</sub>, the LRT statistic is given by

$$\Gamma = 2[\ell(\hat{ heta}_1,\hat{ heta}_2) - \ell(\hat{ heta}_0,\hat{ heta}_0)]$$

• The LRT not appropriate for  $H_1: \theta_2 > \theta_1$ 





#### Signed Likelihood Ratio Statistic

To test H<sub>0</sub>: θ<sub>1</sub> = θ<sub>2</sub> vs H<sub>1</sub>: θ<sub>2</sub> > θ<sub>1</sub>, one can use the signed LRT statistic (Sinharay, 2017)

$$\mathcal{L}_{s} = \left\{ \begin{array}{l} \sqrt{\Gamma} \text{ if } \hat{\theta}_{2} \geq \hat{\theta}_{1}, \\ -\sqrt{\Gamma} \text{ if } \hat{\theta}_{2} < \hat{\theta}_{2} \end{array} \right.$$

• The  $L_s$  statistic  $\sim \mathcal{N}(0,1)$  for long subtests under  $H_0: \theta_1 = \theta_2$ 





## Higher-order Asymptotics

Methods based on *higher-order asymptotics* 

- Modified signed likelihood ratio (MSLR) test (Barndorff-Nielson, 1986)
- Lugannani-Rice approximation (Lugannani & Rice, 1980)

have excellent properties (e.g., Pierce & Peters, 1992), especially for small samples, and can be used for score differencing





## MSLR Statistic

- Derivation is simple only for the exponential family of distributions
- The MSLR statistic for 2PL+GPCM:

$$L_s + rac{1}{L_s}\lograc{Z'}{L_s}$$

• The MSLR statistic  $\sim \mathcal{N}(0,1)$  for long subtests under  $H_0$ :  $\theta_1 = \theta_2$ 

 R code for computing the statistic is publicly Conference On available

#### Results for Simulated Data

 In simulation studies, the Type I error rate of the MSLR statistic was very close to the nominal level and the power was satisfactory in <u>comparison to Z and L<sub>s</sub></u> <u>Level Z stat MSLR stat</u>

Level	Z Stat	MOLK Stat
0.001	0.0233	0.0007
0.01	0.0340	0.0096
0.05	0.0727	0.0500





#### Results for a Language Test

- Scores of 629 repeaters were available on two forms of an English language test
- 34 dichotomous items in each form
- The 2PL model (operationally used) was used
- The operational item parameter estimates used
- Computed the Z and MSLR statistic





# Significance (At 1% level)

Z	MSLR Statistic					
Statistic	Significant	Not Significant				
Significant	35	6				
	(6%)	(1%)				
Not	0	588				
Significant	(0%)	(93%)				





#### Two Examinees

Examinee	Item Set 1		Item Set 2		Z	MSLR
	Raw-1	$\theta_1$	Raw-2	$\theta_2$	stat	stat
1	14	-1.1	24	.5	2.46	2.38
2	18	9	25	.6	2.36	2.29





#### Conclusions

- A new statistic based on higher-order asymptotics suggested for score differencing
- It has a  $\mathcal{N}(0,1)$  null distribution for long tests
- The Type I error rate and power of the statistic were satisfactory in simulations
- A real data example was discussed
- Promises to be useful in cheating detection
- Reference: Sinharay and Jensen (2018), Psychometrika Online First.



