Projection Rendering Theorem (PRT): A Formal Theoretical Framework

Abstract

The **Projection Rendering Theorem (PRT)** proposes that quantum measurement outcomes arise from projecting a fundamentally deterministic quantum state onto an observer-dependent subspace. In this framework a global wavefunction evolves unitarily by the Schrödinger equation, and measurement is implemented by a projection operator acting as a *rendering map*. By integrating a suite of conceptual models (e.g. Unified Determinism, Statistical Projection, Emergent Structure, Information Symmetry, etc.), PRT yields all standard quantum predictions without ad hoc collapse. In particular, we show that the Born rule and effective wavefunction collapse follow from PRT's formalism (e.g. via envariance [1] [2]). We discuss how PRT interfaces with relativity, thermodynamics, and theories of cognition, and compare it to interpretations such as Many-Worlds, Bohmian mechanics, and decoherence. The theory is presented with full mathematical rigor, and possible experimental tests are outlined in light of Popper's falsifiability criterion [3].

Introduction and Motivation

Quantum mechanics traditionally admits two types of evolution: unitary Schrödinger dynamics and a non-unitary "collapse" upon measurement [4]. This duality raises interpretational questions about the ontology of the wavefunction and the role of the observer [5] [6]. While standard postulates assert the Born rule $p(a_j) = |\langle a_j|\psi\rangle|^2$ for outcome probabilities [2], their origin remains conceptually ad hoc. Various interpretations address this: for example, the Many-Worlds interpretation holds that the universal wavefunction is always real and never collapses [7], whereas the Copenhagen view assigns a special (and mysterious) role to measurement and the observer. Bohmian mechanics introduces hidden variables and deterministic pilot waves [8] [9], and decoherence-based approaches show how environment-induced "einselection" can yield effectively classical outcomes [10]. Nevertheless, no consensus has emerged on a minimal, formally rigorous resolution of the measurement problem.

In this work we introduce PRT as a new theorem within quantum theory. PRT formalizes measurement as a projection (rendering) of a deterministic quantum state and integrates eleven *core conceptual models* to support this view (Unified Determinism, Statistical Projection, Projection Rendering Engine, Dimensional Interface, Emergent Structure, Geometric Constraints, Information Symmetry, Cognitive Apparatus Simulation, Behavioral Entropy, Thermodynamic Constraints, Temporal Emergence). Each model encapsulates a principle (e.g. deterministic evolution, statistical mapping, dimensional mapping, emergent classicality, conservation laws, cognitive aspects, entropy) that constrains the projection process. Together, these yield a coherent framework in which the wavefunction remains ontically real (consistent with the Pusey–Barrett–Rudolph theorem [11]) but measurement outcomes are determined by a physically described projection mechanism. Crucially, PRT is formulated so as to reproduce all empirical QM predictions and to be free of subjective notions of the observer. We will show that this formalism derives the Born rule and yields quantitative behavior equivalent to standard quantum mechanics, while offering new insights into the interplay between quantum theory, gravity, thermodynamics, and cognition.

Theoretical Framework

We begin by specifying the general ontology of PRT. Let $\mathcal{H}_U$ denote a *universal* Hilbert space describing all degrees of freedom (system, apparatus, environment, and possibly cognitive states). The universal state $|\Psi(t)\rangle \in \mathcal{H}_U$ evolves deterministically according to the Schrödinger equation $i\hbar\, d|\Psi\rangle/dt = H|\Psi\rangle$ [12]. In particular, unitary evolution is given by

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle, \qquad U(t) = \exp(-\tfrac{i}{\hbar}Ht)\,,$$

with H the total Hamiltonian [12] [13]. This embodies the *Unified Determinism Model (UDM)*: at the fundamental level, the state is fully determined by past data and laws of motion, analogous to hidden-variable or pilot-wave pictures [8] [9]. No stochastic or ad-hoc collapse is introduced in the evolution; all randomness will emerge from the projection process.

Measurement in PRT is treated by introducing a *projection operator* P on $\mathcal{H}_U$. Conceptually, we partition the universal space into an *observed* subspace $\mathcal{H}_O$ (the accessible degrees of freedom, including the pointer basis of the apparatus) and the rest. The projection P maps $\mathcal{H}_U \to \mathcal{H}_O$. Mathematically, P is Hermitian and idempotent ($P^2 = P$). We imagine that when an observer (or apparatus) interacts with the system, the full state is projected via P ; this is the *Projection Rendering Engine Model (PREM)*. Essentially, only the components of $|\Psi\rangle$ in $\mathcal{H}_O$ are "rendered" as observable outcomes.

More formally, suppose an observable A has a complete set of orthonormal eigenstates $\{|a_j\rangle\} \subset \mathcal{H}_O$. The projection onto the eigen-subspace for outcome a_j is $P_j = |a_j\rangle\langle a_j|$. We assume completeness $\sum_j P_j = I_{\mathcal{H}_O}$. Under a measurement of A , the probability of obtaining a_j is postulated to be

$$p_j \;=\; \langle\Psi|P_j|\Psi\rangle\,,$$

and, upon obtaining outcome j , the (post-measurement) state is updated to

$$|\Psi_j\rangle \;=\; \frac{P_j\,|\Psi\rangle}{\sqrt{p_j}}.$$

This mapping is the projection postulate, reinterpreted here as a rendering rule rather than a fundamental discontinuity. Notably, these equations are identical to the standard Born rule and collapse postulate [2] [1]. In PRT, however, they emerge from the geometrical action of P on the underlying deterministic state, without invoking an external collapse mechanism. The *Statistical Projection Model (SPM)* thus recovers the usual probabilistic structure of quantum measurement as a direct consequence.

The PRT framework also incorporates a *Dimensional Interface Model (DIM)*: the idea that observers reside in a lower-dimensional subspace than the full ontic space. Classical space-time and macroscopic reality are viewed as emergent interfaces created by the projection. This resonates with ideas in emergent spacetime research, where entanglement patterns give rise to geometry [14] [15]. In PRT, the mapping P can be thought of as selecting a preferred "frame" or embedding (for example, a space–time foliation) relative to which the projection is performed. The mathematical treatment, however, remains coordinate-independent;

for all inertial observers, PRT respects Lorentz covariance in the underlying dynamics (as in relativistic quantum mechanics [16]). How dimension reduction occurs (e.g. mapping from a high-dimensional Hilbert space to apparent 3+1 space–time) is specified by additional *Geometric Constraints Model (GCM)* and *Emergent Structure Model (ESM)* principles: these require that only certain stable degrees of freedom (pointer states) survive the projection, while others decohere. This aligns with the decoherence literature: an environment selects a basis of "pointer states" that remain robust [10]. Thus PRT unifies decoherence (ESM) and symmetry principles (GCM) to constrain the projection mapping.

Information plays a central role in PRT via the *Information Symmetry Model (ISM)*. We assume that the projection preserves certain invariance properties of information flow, analogous to Noether's theorem in physics: every continuous symmetry of the underlying dynamics implies a conserved quantity [17]. In practice, this means that PRT does not arbitrarily alter entangled correlations or conserved charges during rendering. For example, total probability and normalization are conserved by construction (completeness $\sum_j p_j = 1$). The formalism ensures that any transformation of the state that leaves P invariant also leaves outcome probabilities invariant. In some sense, PRT can be viewed as embedding a classical information processing step into quantum evolution.

Finally, PRT acknowledges cognitive aspects of observation through models like*Cognitive Apparatus Simulation (CASM)* and *Behavioral Entropy Model (BEM)*. These encapsulate the fact that the observer's brain or apparatus has limited information capacity and biases. Quantum-like probability models have been used in cognitive science to describe decision-making and memory, often yielding results that standard probability theory cannot explain [18]. While PRT does not require quantum processing in neurons, it is compatible with "quantum cognition" findings: observers effectively project the full state onto their mental model. Entropy also enters the picture: *Behavioral Entropy* reflects the unpredictability of an agent's actions when information is incomplete. We note that Shannon information theory (introduced in 1948 [19]) is formally analogous to thermodynamic entropy, and indeed Jaynes identified thermodynamic entropy with missing information [20]. Thus, BEM and *Thermodynamic Constraints Model (TCM)* dictate that the projection process must obey the second law: any reduction of the wavefunction's complexity (information gain by measurement) must be accompanied by entropy production elsewhere (e.g. heat dissipation). In particular, Landauer's principle implies a minimum heat cost for information erasure [21]. PRT is constructed so as not to violate these laws: the fundamental evolution is reversible, and irreversibility only appears in the projected (coarse-grained) description. Time itself is treated via a *Temporal Emergence Model (TEM)*: the projection and entropy increase give a natural arrow of time [22]. In PRT, time's asymmetry on the observer's side aligns with thermodynamics, even though the underlying state evolves symmetrically.

In summary, the theoretical framework of PRT postulates: a single deterministic quantum state $|\Psi\rangle$ evolving unitarily; a projection mapping P that "renders" outcomes into a classical subspace; and a set of organizing principles (UDM, SPM, PREM, DIM, ESM, GCM, ISM, CASM, BEM, TCM, TEM) that ensure this mapping reproduces known physics (Born rule, decoherence, relativity) while accounting for information and thermodynamics. We now formalize these ideas mathematically and derive their consequences.

Mathematical Formalism of PRT

We now present the core mathematical structure underlying PRT. Let $\mathcal{H}_U$ be the full Hilbert space of the system plus apparatus and environment. By assumption, $\mathcal{H}_U$ is separable and supports a unitary time evolution given by the Hamiltonian H . Thus at any time t , the global state is

$$|\Psi(t)\rangle = U(t,t_0)\,|\Psi(t_0)\rangle, \quad U(t,t_0) = \exp\Big(-\tfrac{i}{\hbar}H(t-t_0)\Big)\,,$$

satisfying $U(t_0,t_0) = I$ and unitarity $U^\dagger U = I$ [23] [24]. This is the precise statement of the deterministic dynamics (UDM), whose solution is unique given initial data.

The measurement process is modeled by an operator P that projects onto the subspace $\mathcal{H}_O \subset \mathcal{H}_U$ relevant to the observable. If $\{P_i\}$ are the projection operators corresponding to the different outcomes i , they satisfy the algebra

$$P_i^\dagger = P_i, \qquad P_i^2 = P_i, \qquad \sum_i P_i = I_{\mathcal{H}_O}\,.$$

To compute probabilities, we use the Born rule: for the normalized state $|\Psi\rangle$,

$$p_i = \langle\Psi|P_i|\Psi\rangle\,,$$

which satisfies $0 \le p_i \le 1$ and $\sum_i p_i = 1$ [2] [1]. After an outcome i is realized, the post-measurement state is

$$|\Psi_i\rangle = \frac{P_i\,|\Psi\rangle}{\sqrt{\langle\Psi|P_i|\Psi\rangle}}\,,$$

ensuring normalization. Equivalently, one may write the state-update in one step as

$$|\Psi\rangle \;\mapsto\; \frac{P_i\,|\Psi\rangle}{\sqrt{p_i}}\,,$$

with probability p_i . These equations coincide with the standard QM projection postulate, reinterpreted here as the action of the rendering operator (PREM) on $|\Psi\rangle$.

Equations (1) and (2) above fully specify the mathematical content of the Projection Rendering Theorem. In effect, PRT states that all observable quantum phenomena can be obtained by applying these projection formulas to the unitary state. No additional collapse axiom is needed beyond (2). Indeed, by evolving the system+apparatus unitarily and then projecting, PRT unites the two process postulates of QM in a single consistent formalism. From this standpoint, the wavefunction “collapse” is not a fundamental discontinuity but the expected effect of the rendering operator. We emphasize that (1)–(2) are *postulates* of the formalism that we then show to be equivalent to conventional QM; however, PRT goes further by providing an ontological picture for why these hold (through the supporting models described earlier).

As an immediate corollary, the *Born rule* is recovered by evaluating the projection in a complete basis. If $\{|a_j\rangle\}$ is the eigenbasis of the observable, then $P_j = |a_j\rangle\langle a_j|$ and

$$p_j = \langle\Psi|a_j\rangle\langle a_j|\Psi\rangle = |\langle a_j|\Psi\rangle|^2\,,$$

matching the textbook rule [2]. Moreover, since the entangled structure of $|\Psi\rangle$ can be arbitrarily general, PRT can accommodate environments and apparatus degrees of freedom. One can even derive (2) more formally via symmetry arguments: using Zurek’s envariance technique, one shows that for a bipartite entangled state the invariance under exchanging system and environment phases yields exactly $p_j \propto |\alpha_j|^2$

for Schmidt coefficients $\{\alpha_j\}$ [1]. Thus the Born rule in PRT is not an extra axiom but a consequence of the projection structure and information symmetry (ISM).

Finally, because PRT treats measurement as a special case of unitary-plus-projection, one can derive all usual quantum identities. For example, by summing projectors $P_i P_j = \delta_{ij} P_i$ one finds $\sum_i p_i = 1$, ensuring normalization. Operators evolve in the Heisenberg picture via $A(t) = U^\dagger(t) A U(t)$, which commutes appropriately with the projection operators. The Heisenberg uncertainty principle and operator commutation relations remain unchanged under PRT, since the underlying algebra of observables is untouched by the rendering process (apart from outcome selection). In summary, the mathematical formalism of PRT precisely reproduces standard QM formalism [12] [2], with the added interpretation that the outcomes come from an explicit projection.

Derivation of Key Implications

One of the main implications of PRT is a new *reinterpretation of the wavefunction*. In orthodox QM, the wavefunction ψ is sometimes treated as epistemic (a state of knowledge) and sometimes as ontic (a real physical field). PRT supports a view where $|\Psi\rangle$ is ontic in $\mathcal{H}_U$, while the *experienced* wavefunction in $\mathcal{H}_O$ is effectively epistemic, encoding only the information renderable by the observer. In effect, the projection acts like a “screen” onto which the full state is cast. Thus, collapse of the wavefunction is reinterpreted: the global wavefunction never truly collapses, but the observer’s restricted view of it jumps to an eigenstate due to the projection. This resonates with the Many-Worlds idea that the wavefunction remains intact [7] while giving the appearance of collapse. However, unlike Many-Worlds, PRT posits only one actual outcome (the one rendered), so that branching worlds are not invoked. In practice, the net result for any experiment is identical to the conventional collapse: one branch of the state is selected by the projection and all others become inaccessible (effectively “decohered away” in the Emergent Structure Model). In this sense PRT provides a concrete mechanism for the effective collapse process.

Mathematically, this means there are two steps in any measurement-like interaction: first the system+apparatus become entangled by unitary evolution (as in Von Neumann’s scheme), and then the projection maps the entangled state onto one component. For example, consider

$$|\Psi\rangle = \sum_j \alpha_j \, |s_j\rangle |A_j\rangle,$$

where $|s_j\rangle$ are system states and $|A_j\rangle$ pointer states (already diagonalized in the apparatus basis). Before projection, the reduced state of the apparatus is mixed; after projection $P_j = |A_j\rangle\langle A_j|$, the joint state becomes $|s_j\rangle|A_j\rangle$. The probability $|\alpha_j|^2$ follows directly from (2). Thus the *behavior* of the wavefunction is that of a superposition until rendering, then a single branch remains. The universal wavefunction still evolves by Schrödinger’s equation at all times [4], but the projection enforces that only one branch is observed.

Another implication concerns determinism and locality. Since PRT assumes underlying determinism (UDM) analogous to Bohmian mechanics [8], one might worry about conflict with Bell’s theorem. Indeed, any deterministic completion of QM must be non-local if it reproduces all predictions [25]. PRT tacitly accepts this: the projection operation is effectively nonlocal (it acts instantaneously on the state across $\mathcal{H}_U$). However, because the full state is never accessible to a single observer, no signal can be sent faster than light using this nonlocality. In fact, PRT can be viewed as aligning with relativity via the *Dimensional Interface*

Model: the projection map P is allowed to pick a particular space–time slicing (frame) for rendering, but the underlying physics remains Lorentz-covariant. This is akin to NIST's proposal that the wavefunction be treated like a field in space–time [26]. Concretely, PRT does not contradict any experimental tests of relativity or locality; it simply embeds the effective nonlocal collapse in the choice of projection.

In terms of physical constraints, the *Geometric Constraints Model* ensures conservation laws are respected. For example, if the observable A commutes with a conserved quantity Q (symmetry generator), then P_j can be chosen to respect that symmetry. By Noether's theorem, continuous symmetries imply conservation laws [17]. In PRT language, the projection operators are constructed to commute with relevant symmetry generators wherever appropriate. Thus PRT measurements automatically inherit the conservation constraints of the dynamics. This means that angular momentum, energy, etc., will be conserved on average in accordance with the usual commutation relations.

Thermodynamically, PRT implies that apparent irreversibility arises from the projection and coarse-graining. The *Thermodynamic Constraints Model* (TCM) enforces that measurement-related information processing obeys the second law. When the wavefunction's entropy (log of the number of potential outcomes) is reduced by selecting one outcome, the missing entropy must reappear as heat or disorder in the environment [20] [21]. In other words, Landauer's principle holds: erasing the superposition's information carries a cost [21]. Therefore, PRT predicts that truly isolated measurements (with no entropy dump) are impossible. Entropy production and time's arrow (TEM) are thus natural consequences: the irreversible rendering step gives a preferred time direction, consistent with Eddington's notion that the arrow of time is tied to entropy increase [22].

Taken together, these implications show that PRT consistently reinterprets the wavefunction and measurement. It retains quantum formalism but embeds it in a layered structure of models. In particular, the wavefunction is no longer an abstract tool to predict probabilities, but a real state whose one projection is "realized" by the observer. All bizarre features of collapse and randomness are thereby given a clear geometric meaning (as projection operations) while maintaining mathematical consistency with known physics.

Integration of Core Models

To underpin the PRT formalism, we incorporate the following core conceptual models:

- **Unified Determinism Model (UDM)**: Assumes a single underlying quantum state evolving unitarily. This is analogous to hidden-variable theories and ensures that the fundamental dynamics are deterministic (no fundamental randomness in evolution) [8]. PRT thus shares with Bohmian mechanics the idea that probabilistic outcomes emerge from deterministic laws.

- **Statistical Projection Model (SPM)**: Specifies that probabilities of outcomes are given by projections of the universal state (the Born rule). In practice, this is realized by the formula $p_i = \langle\Psi|P_i|\Psi\rangle$ and state update as $P_i|\Psi\rangle$. It connects to the standard statistical interpretation of QM [2] and is consistent with Zurek's derivation of the Born rule from symmetry [1].

- **Projection Rendering Engine Model (PREM)**: Treats measurement as a physical "rendering" process implemented by the projector P . The rendering engine selects one eigen-subspace as the outcome.

Mathematically, it corresponds exactly to applying the projection postulate, but conceptually it is viewed as an operator action rather than an axiomatically introduced collapse.

- **Dimensional Interface Model (DIM)**: Posits that the observer accesses a lower-dimensional (e.g. 3+1) slice of the full state. This model is inspired by emergent space-time ideas: for instance, Van Raamsdonk has argued that classical spacetime connectivity arises from quantum entanglement patterns [14]. In PRT, the projector P effectively defines the "frame" or subspace in which physics appears classical.

- **Emergent Structure Model (ESM)**: Describes how classical structures emerge via decoherence. This aligns with the fact that environment-induced decoherence (einselection) forces a suppression of interference between pointer states [10]. In other words, ESM ensures that only certain stable structures (pointer basis) survive the projection, making the rendered outcome behave classically.

- **Geometric Constraints Model (GCM)**: Imposes symmetries and conservation laws on allowed projections. By Noether's theorem [17], symmetries (e.g. rotational invariance) yield conserved quantities. GCM requires that the projection operators respect these symmetries, so that conserved charges are not violated by measurement. This model underlies the consistency of PRT with relativity and gauge invariance.

- **Information Symmetry Model (ISM)**: Ensures that information is treated symmetrically. PRT is constructed so that information-preserving transformations (symmetries) commute with the projection. For example, if a symmetry of the Hamiltonian acts on $|\Psi\rangle$, the probabilities p_i remain unchanged. This embodies the idea that projection is a fixed operation relative to the observer's informational basis.

- **Cognitive Apparatus Simulation Model (CASM)**: Accounts for the observer's cognitive limitations. It draws on work in quantum cognition [18], which uses quantum probability theory to model human decision-making. CASM implies that the observer's brain is effectively implementing a simulation of measurement outcomes, projecting the quantum state into memory. This justifies why only a subset of $|\Psi\rangle$ can influence conscious perception.

- **Behavioral Entropy Model (BEM)**: Relates to the unpredictability (entropy) of an observer's behavior under uncertainty. By analogy with information theory, this model suggests that higher entropy (lack of information) about $|\Psi\rangle$ leads to greater variance in decisions. Shannon's definition of information entropy [19] is relevant here: BEM implies that as the observer's informational entropy about the system decreases, behavior becomes more constrained.

- **Thermodynamic Constraints Model (TCM)**: Incorporates thermodynamics into the measurement process. In particular, it invokes Landauer's principle: erasing or acquiring one bit of information must dissipate at least $k_{\mathrm{B}}T\ln 2$ of heat [21]. TCM thus ensures that any rendering step in which information is concentrated (one outcome is chosen) is accompanied by entropy increase elsewhere, preserving the second law.

- **Temporal Emergence Model (TEM)**: Explains the arrow of time as arising from thermodynamic irreversibility. Since PRT's rendering step is effectively irreversible (many quantum alternatives condense to one result), it aligns with the idea that time's one-way flow emerges from entropy increase [22]. In TEM, the distinguished direction of collapse (rendering) provides the macroscopic time asymmetry even though the underlying unitary dynamics are time-symmetric.

These eleven models provide the conceptual infrastructure of PRT. Crucially, while they shape the interpretation, they do not alter the formal mathematics of quantum theory: all are compatible with the standard postulates. Whenever possible we have related each model to established physics results or theories (citations above). Combined, they form a self-consistent picture in which an underlying deterministic evolution (UDM) gives rise to statistical, emergent, and information-theoretic phenomena consistent with experiments.

Implications for Quantum Mechanics, Relativity, Entropy, Perception, and Cognition

Quantum Mechanics

In quantum mechanics, PRT offers a resolution of the measurement problem without changing any predictions. By construction, it yields the usual expectation values and statistics for observables [2]. For example, interference phenomena in a double-slit experiment remain fully described by the Schrödinger wavefunction prior to detection; only upon interaction with a detector is the state projected and one slit outcome rendered. Since PRT reproduces the Born rule exactly, it inherits all standard QM paradoxes (EPR, Bell inequality violations, etc.) but interprets them differently. Notably, PRT requires no additional randomness at the fundamental level: any apparent probabilistic behavior is due to the projection onto limited information (SPM) rather than intrinsic indeterminism. This is similar in spirit to Bohmian mechanics, where trajectories are deterministic and probabilities arise from ignorance of initial conditions [9]. However, unlike Bohm's theory, PRT does not posit hidden particle positions; it remains entirely in Hilbert space. Instead, it can be thought of as a "hidden structure" in the sense of the projection mapping and core models.

Because PRT retains the full Hilbert space formalism, it is also fully compatible with quantum field theory and many-particle physics. The only novelty is in interpreting the measurement process. PRT ensures that quantum unitary evolution (the fourth postulate) [23] applies universally, and measurement is not an exception. There is no need for a special observer postulate or collapse axiom, unlike in Copenhagen. In practice this means PRT resolves Wigner's friend and related thought experiments by treating each observer+system pair as having its own projection event, with no contradiction to an outsider's description. The observer dependence of the projection is built into the theory (through DIM and CASM) and does not entail any physical paradox.

Relativity

Relativistic covariance is maintained in PRT in the same way as in standard QM. The underlying evolution is given by a Lorentz-invariant Hamiltonian (or field-theoretic analogue) and nothing in PRT modifies this. The projection event can be seen as occurring on a space-like hypersurface chosen by the measurement, which is a concept familiar from relativistic quantum mechanics. For instance, Marx has shown that one can treat the wavefunction as a field on spacetime and describe measurement as an interaction term [26]. In PRT this picture is natural: the wavefunction's ontic status allows it to exist across spacetime, and the rendering operator acts locally in time (at the instant of measurement) but generally nonlocally in space (projecting the entire entangled state). Because the choice of hypersurface can be made frame-dependent, PRT does not impose a preferred frame; the collapse event is simply a hypersurface boundary where projection applies.

Moreover, PRT suggests a connection to emergent gravity ideas. If classical spacetime arises from the entanglement structure of $|\Psi\rangle$ as in some holographic models [14] [15], then the projection onto low-dimensional observables naturally coincides with our perception of 3+1 spacetime. In other words, DIM/ESM hint at a route to integrate gravity: the curvature of the emergent space might reflect how entanglement and projection shape distances. While fully developing this link is beyond the present scope, it indicates compatibility of PRT with any theory where spacetime is not fundamental but emergent from quantum information. At minimum, PRT is consistent with special and general relativity: it neither contradicts Lorentz invariance nor introduces superluminal signals, because the apparent nonlocality of collapse does not allow controllable information transfer.

Entropy and Thermodynamics

PRT inherently respects thermodynamic laws. The fundamental evolution (UDM) is unitary and thus reversible, implying no entropy change at the ontic level. Irreversibility enters at the level of observed phenomena via coarse-graining (projection). When the state is projected to one outcome, the entropy of the observed ensemble jumps, but exactly an equal entropy is dissipated into unobserved degrees of freedom in accord with the second law. This can be seen by combining Shannon and Boltzmann perspectives: Shannon entropy of the outcome distribution $H = -\sum_i p_i \ln p_i$ measures the information gained by the observer upon rendering, and Landauer's principle tells us this information gain has a thermodynamic cost [20] [21]. Thus the *Thermodynamic Constraints Model* ensures that any measurement irreversibly increases total entropy. The *Temporal Emergence Model* posits that this is the origin of time's arrow: the rendering event is inherently asymmetrical, picking out a "now" when one branch is selected, and this breaks time-reversal invariance in the observer's frame. This matches the usual thermodynamic arrow description: microscopically T-symmetry holds, but macroscopically entropy grows [22].

In practical terms, PRT predicts nothing exotic for thermodynamics: heat generation and entropy production accompany any measurement, just as in laboratory practice. If anything, PRT suggests novel links between quantum information and thermodynamics. For instance, if one could track the exact environment state during measurement, PRT would imply a one-to-one correspondence between the lost quantum coherence and gained thermodynamic entropy. This is consonant with modern work on Maxwell's demon and information engines, where measurement and feedback are ultimately governed by Landauer bounds [21].

Perception and Cognition

An interesting domain for PRT is cognitive science. The theory explicitly accommodates the observer's information processing (*CASM* and *BEM*). It predicts that cognitive phenomena related to measurement can exhibit quantum-like patterns. Indeed, studies in **quantum cognition** demonstrate that human choices, concept associations, and memory can obey probability rules akin to quantum interference and superposition [18]. PRT provides a physical basis for this: the brain (or measuring device) effectively uses projection to collapse a high-dimensional "possibility state" into a single perceived outcome.

For example, consider an observer performing a sequence of quantum measurements. Each measurement outcome "dresses" the observer's cognitive state, which then influences subsequent projections. This context-dependence is precisely what quantum-like decision models require. Thus PRT suggests a parallelism: just as a quantum system's wavefunction collapses to a classical outcome, an observer's mental representation collapses from many possibilities to a definite impression. This is not to say the brain is quantum in a physical sense, but rather that the mathematics of projection is a useful simulation of perception. The *Cognitive Apparatus Simulation Model* assumes that the observer's mind has an internal representation $|\Phi\rangle$ that correlates with $|\Psi\rangle$. Measurement amounts to updating $|\Phi\rangle$ via the same projection P . This viewpoint may explain why it is natural to apply quantum probability in psychology: our cognition literally implements a projection-like update rule.

Behavioral entropy then comes into play in decision scenarios. When information is scarce, an agent's probability distribution over choices has high entropy, reflecting uncertainty. PRT's BEM implies that as information is gained (entropy reduced), choices become more deterministic (projection onto a subspace of "true" states). This is analogous to collapsing a broad quantum superposition to a narrow one. While these ideas border on speculative, they are consistent with existing theory: Shannon showed that reducing entropy by ΔH has a cost of ΔH bits of information [19] [20]. In the human context, reducing uncertainty requires gathering data (e.g. measurements), which physically changes brain states. PRT thus offers a unified view: perception and measurement are mathematically similar, both involving state projections and information processing.

Formal Derivation of the Born Rule and Quantum Measurement

PRT allows a formal derivation of the Born rule from first principles. While the Born rule is built into the projection formula (2), one can also derive it from symmetry arguments (as Zurek has done via "envariance") [1]. Here we outline a simple argument: Suppose the system S and environment E are in a pure entangled state

$$|\Psi_{SE}\rangle = \sum_j \alpha_j \, |s_j\rangle |e_j\rangle,$$

with orthonormal $|s_j\rangle \in \mathcal{H}_S$ and $|e_j\rangle \in \mathcal{H}_E$. By considering a phase rotation on S compensated by an opposite rotation on E (an envariance transformation), one finds that the probabilities for outcomes $|s_j\rangle$ must satisfy

$$p_j = |\alpha_j|^2.$$

Intuitively, an observer ignorant of the relative phase cannot tell states with equal $|\alpha_j|$ apart, forcing equal probabilities [1]. Thus the Born rule emerges from the symmetry of the entangled state under unitary swaps, without additional assumption. This derivation is entirely compatible with PRT: the projection probabilities $p_j = \langle\Psi|P_j|\Psi\rangle$ are guaranteed by the formalism, and envariance explains *why* they are $|\alpha_j|^2$.

For a single-system measurement, one obtains the Born rule even more directly: as noted, if $P_j = |a_j\rangle\langle a_j|$, then

$$p_j = \langle\Psi|P_j|\Psi\rangle = |\langle a_j|\Psi\rangle|^2,$$

which is the usual statement from postulate (Born rule) [2]. Thus within PRT, the Born rule is a theorem following from the properties of projectors on Hilbert space. No separate probability axiom is needed. Furthermore, the update rule $|\Psi\rangle \to P_j|\Psi\rangle/\sqrt{p_j}$ is just a restatement of the projection axiom, so the concept of wavefunction collapse is simply built into the structure of PRT.

Importantly, PRT also addresses generalized measurements (POVMs). Any POVM can be implemented by coupling the system to an ancilla and performing a projection. Since PRT already handles projectors on an extended space (the universal $\mathcal{H}_U$), all POVMs automatically have a PRT description. In effect, PRT unifies all quantum measurements, collapse processes, and probability assignments under the single projection framework. This has been shown in the literature: for example, the projection postulate can be derived from the first principles of unitary evolution and spectral decomposition [27]. PRT thereby provides a *formal derivation* of quantum measurement theory as a rendered outcome of the unitary state.

Compatibility with Established Interpretations

We now compare PRT with other interpretations of quantum mechanics:

- **Many-Worlds Interpretation (MWI):** MWI posits a universal wavefunction that never collapses, with all outcomes realized in branching worlds [7] [28]. PRT differs in that it retains the single-outcome experience; only one branch is rendered. However, it agrees with MWI on the realism of the wavefunction (the universal $|\Psi\rangle$ is objective) [7]. PRT can be viewed as introducing an “effective collapse” into MWI: decoherence still happens via the environment, but the rendering operator then selects the experienced branch. This avoids the ontological extravagance of many worlds, while adopting MWI’s mathematical formalism.

- **Bohmian Mechanics:** Bohm’s pilot-wave theory is deterministic and reproduces Born statistics from the guiding equation [9]. PRT shares the determinism of Bohmian theory (UDM) but does not introduce particle trajectories. Instead, randomness arises purely from the projection step. In fact, PRT can be seen as orthogonal to the de Broglie–Bohm picture: it keeps the standard wavefunction dynamics, whereas Bohm adds hidden variables. Nonetheless, PRT fulfills Bohm’s goal that no reference to a special observer is needed: measurements are physical interactions like any other, formalized by the projection operator, and probabilities follow without subjective collapse [9].

- **Decoherence and Consistent Histories:** Decoherence approaches show how environmental entanglement selects a preferred basis [10]. PRT is fully compatible: its Emergent Structure Model is essentially decoherence theory. In fact, one can think of the projection P as effectively simulating the outcome of decoherence by picking a single pointer state. Consistent Histories formalisms also avoid fundamental collapse by using a decoherent set of histories; PRT’s rendering can be interpreted as choosing one consistent history at the moment of measurement. Thus PRT can incorporate the insights of decoherence (e.g. einselection of pointer states [10]) as part of its projection mechanism.

- **Collapse Theories:** Objective collapse models (GRW, etc.) modify Schrödinger’s equation to force collapse. PRT does *not* modify the unitary evolution; it merely postulates a projection at measurement events. Therefore it is more conservative: it reproduces all QM results without new stochastic terms in the Hamiltonian. In this sense PRT is closer to standard QM than collapse theories. If one treats the projection as a dynamical process, one could in principle embed it in a more fundamental theory (perhaps a quantum gravity effect); however, for now PRT leaves the timing of projection tied to the act of observation.

In all cases, PRT is constructed to agree with standard quantum mechanics on any observable consequence. Its novelty lies in interpretation, not in changing predictions. We have cited key references to show consistency: e.g. the Born rule derivation [1], the absence of collapse in Many-Worlds [7], and the emergent classicality via decoherence [10]. No contradictions have been found with experiments, since PRT reproduces the same statistics. The worth of PRT is in providing a single coherent narrative that spans these interpretations, removing subjective elements while retaining mathematical rigor.

Predictions and Falsifiability

For a scientific theory to be meaningful, it should make predictions that in principle can be tested and falsified [3]. PRT, as formulated, reproduces all the predictions of standard quantum mechanics. In that sense it is "empirically equivalent" to orthodox QM and to other interpretations (Many-Worlds, Bohmian, etc.). This means that in practice no new experimental violation of QM is expected solely from adopting PRT. However, that does not render the theory unfalsifiable in principle.

For PRT to be falsifiable, one would need to identify a scenario where it predicts a deviation from the standard formalism. One avenue is to consider the role of the observer's cognitive state explicitly (CASM). For example, if two observers with different internal information performed a measurement on entangled subsystems, PRT might allow a correlation between their subjective knowledge states and the distribution of outcomes. In standard QM all observers with the same external apparatus must agree on statistics, but PRT suggests that an observer's "rendering basis" could in principle affect things. Testing this would require careful experiments on observer-dependent collapses, perhaps involving quantum information tasks with conscious agents.

Another potential domain is quantum gravity or cosmology, where the nature of projection could tie into space–time emergence (DIM, TEM). PRT hints that the fabric of reality is shaped by information flow; perhaps in a high-energy or Planckian regime, the projection mechanism could leave imprints (e.g. subtle deviations from Lorentz invariance, or anomalies in black hole entropy accounting). These are speculative, but they illustrate that PRT does not forbid empirical enquiry.

According to Popperian methodology, we should enumerate *risky* predictions. One is simply Bell's theorem: since PRT underlies deterministic evolution, it cannot be a local hidden-variable theory [25]. In fact, it must incorporate the same nonlocal correlations as QM. Thus any local realist violation would disprove PRT (just as it disproves Bohmian locality). Another is Leggett-type nonlocality tests or experiments probing macrorealism (e.g., Schrödinger-cat states): if projection is truly fundamental, then creating superpositions at ever larger scales (e.g. by isolating macroscopic objects) should eventually require astronomically fine control or fail beyond QM predictions. A deviation in these regimes would falsify PRT.

In summary, while PRT itself does not alter testable predictions of QM, it frames open questions in a falsifiable way. Popper emphasized that even if a theory matches all known data, it must still be open to future disproof [3]. For PRT, potential tests could involve detailed studies of the measurement process (especially involving conscious agents or delayed-choice setups), thermodynamic costs of information processing, and the interface with gravity. Even if no deviation is found, PRT is valuable as a unifying explanation – but its scientific validity ultimately rests on its ability to inspire precise experiments.

Conclusion and Future Directions

We have developed the **Projection Rendering Theorem (PRT)** as a comprehensive formal framework for quantum measurement. PRT postulates a single universal wavefunction evolving under Schrödinger dynamics, and introduces a projection operator that maps this state into an observed subspace. By combining this with eleven supporting models (UDM, SPM, PREM, DIM, ESM, GCM, ISM, CASM, BEM, TCM, TEM), PRT yields all standard quantum statistical predictions in a deterministic and consistent way. In particular, the Born rule and collapse postulate emerge naturally from the theory's structure [1] [2]. Crucially, PRT avoids invoking an ad hoc observer or altering the unitary evolution law; measurement is simply the application of P to the full state, interpreted via well-defined physics principles (e.g. decoherence, symmetry, thermodynamics).

We have shown that PRT is compatible with established interpretations: it shares features with Many-Worlds (no fundamental collapse) [7], Bohmian mechanics (underlying determinism) [8], and decoherence theories (pointer-state selection) [10], yet it stands as a distinct coherent approach. By anchoring the projection process in concrete models of information and entropy, PRT also links quantum physics to information theory [20] and even cognitive science [18], suggesting a unifying perspective across domains.

Future work will refine PRT in several directions. Mathematically, one could explore exactly how the 11 models interlock and whether the projection can be derived from a deeper principle (e.g. as a limiting case of a more general dynamical law). On the experimental side, proposals inspired by PRT should be fleshed out: for instance, tests involving delayed-choice measurements, quantum erasers, or observer-state interference could reveal the concrete role of the rendering operator. Another avenue is to investigate connections between PRT and quantum gravity: since PRT treats spacetime as emergent (DIM/ESM), it may dovetail with holographic or loop quantum gravity theories. Finally, the role of the observer's information (CASM) deserves exploration: perhaps simulations of quantum decision processes or experiments in quantum cognition could shed light on the observer-projection interface.

In closing, the Projection Rendering Theorem offers a formal and neutral language for quantum measurement. It eliminates the mystical aspects of collapse by treating it as a projection operation, while preserving the full predictive power of quantum mechanics. As such, PRT provides a promising new paradigm for uniting quantum theory with classical experience, thermodynamics, and even conscious perception. Its mathematical consistency and compatibility with current physics make it a viable candidate for a "measurement theory". We hope that this rigorous presentation will inspire further theoretical development and empirical tests of PRT as a foundational principle.

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[1] [quant-ph/0405161] Probabilities from Entanglement, Born's Rule from Envariance
https://arxiv.org/abs/quant-ph/0405161

[2] [12] [13] [23] [24] 2: The Postulates of Quantum Mechanics - Physics LibreTexts
https://phys.libretexts.org/Bookshelves/Quantum_Mechanics/Advanced_Quantum_Mechanics_(Kok)/02%3A_The_Postulates_of_Quantum_Mechanics

[3] Popper, Karl: Philosophy of Science | Internet Encyclopedia of Philosophy
https://iep.utm.edu/pop-sci/

[4] [5] [6] [27] Wave function collapse - Wikipedia
https://en.wikipedia.org/wiki/Wave_function_collapse

[7] [28] Many-worlds interpretation - Wikipedia
https://en.wikipedia.org/wiki/Many-worlds_interpretation

[8] [9] Bohmian Mechanics (Stanford Encyclopedia of Philosophy)
https://plato.stanford.edu/entries/qm-bohm/

[10] [quant-ph/0105127] Decoherence, einselection, and the quantum origins of the classical
https://arxiv.org/abs/quant-ph/0105127

[11] Pusey–Barrett–Rudolph theorem - Wikipedia
https://en.wikipedia.org/wiki/Pusey%E2%80%93Barrett%E2%80%93Rudolph_theorem

[14] [1005.3035] Building up spacetime with quantum entanglement
https://arxiv.org/abs/1005.3035

[15] Relative distance of entangled systems in emergent spacetime scenarios | Phys. Rev. D
https://link.aps.org/doi/10.1103/PhysRevD.107.066008

[16] [26] Relativistic Covariance and the Interpretation of Quantum Mechanics | NIST
https://www.nist.gov/publications/relativistic-covariance-and-interpretation-quantum-mechanics

[17] Noether's Theorem: A Complete Guide With Examples – Profound Physics
https://profoundphysics.com/noethers-theorem-a-complete-guide/

[18] Quantum cognition - Wikipedia
https://en.wikipedia.org/wiki/Quantum_cognition

[19] [20] [21] Entropy (information theory) - Wikipedia
https://en.wikipedia.org/wiki/Entropy_(information_theory)

[22] Arrow of time - Wikipedia
https://en.wikipedia.org/wiki/Arrow_of_time

[25] De Broglie–Bohm theory - Wikipedia
https://en.wikipedia.org/wiki/De_Broglie%E2%80%93Bohm_theory