Temporal Emergence Model Aligned with the Projection Rendering Theorem (PRT)

Overview

The Projection Rendering Theorem (PRT) posits that **reality is a recursive projection** from a *singular, incompressible informing state* under entropy constraints. In this model, **time is not a fundamental dimension** but an **emergent property** of the projection process. We develop a formal temporal-emergence model consistent with PRT, integrating quantum relational time (Page–Wootters mechanism) and entropy-driven time arrows. This model defines time as an internal correlation within the underlying state, explains the forward-directed sequence of events via entropy increase, and reconciles a static atemporal base with our ordered experience of time. Both **subjective (psychological)** time and **objective (physical)** time are accounted for as emergent phenomena in this framework.

Key Variables and Symbols

- **$\mathcal{I}_0$ (Informing State):** The atemporal, singular base state containing all information. It is “incompressible,” meaning it cannot be reduced or compressed without loss of information (maximal entropy/information content). This state represents the static seed of reality in PRT.
- **t (Emergent Time Parameter):** A label or parameter *not present fundamentally* but arising from correlations in $\mathcal{I}_0$. It indexes the sequence of projected states (e.g. $t_0, t_1, t_2,\dots$) and is identified with what we perceive as time.
- **$\mathcal{P}_t$ (Projection Operator/Mapping):** An operation that yields the state of the system at emergent “time” t. Formally, $\mathcal{P}t(\mathcal{I}_0)$ produces the projected reality state at t. The sequence ${\mathcal{P}}(\mathcal{I0), \mathcal{P}}(\mathcal{I0), \ldots}$ represents the evolving universe. Recursively, $\mathcal{P}_0$) to produce the next state. }$ acts on the prior state (and $ \mathcal{I
- **Clock System (C) and Evolving System (S):** In the quantum formalism (Page–Wootters mechanism), the total system is split into a clock C and the rest of the universe S. The clock provides a reference against which the state of S is measured.
- **$|t\rangle_C$ (Clock Basis State):** An eigenstate of the clock indicating a specific reading (e.g. a “tick” label t). These states form an orthonormal basis for the clock’s Hilbert space.
- **$|\psi(t)\rangle_S$ (System State at t):** The state of the system S correlated with the clock reading t. This is the *conditional state* of reality when the clock shows time t. It emerges from projecting the total state onto $|t\rangle_C$.
- **$H_C,\;H_S$ (Clock and System Hamiltonians):** The self-Hamiltonians of the clock and system respectively. A global *Hamiltonian constraint* $H_C + H_S = 0$ is imposed on the total state (ensuring a time-independent, stationary total state as per Wheeler–DeWitt).
- **$\rho_S(t)$ or $\rho(t)$ (Density Matrix at t):** The density operator describing the system at emergent time t. If the total state is pure $|\Psi\rangle$, then $\rho_S(t) = |\psi(t)\rangle_S\langle\psi(t)|$.

- **$S_{\text{entropy}}(t)$ (Entropy at t):** The thermodynamic or information entropy of the state at time t. For a given $\rho_S(t)$, one may define $S_{\text{entropy}}(t) = -\mathrm{Tr}[\rho_S(t)\ln \rho_S(t)]$ (von Neumann entropy) or the classical thermodynamic entropy of the corresponding macrostate. The **entropy gradient** $dS_{\text{entropy}}/dt \ge 0$ imposes the arrow of time.

Atemporal Base State and Projection Mechanics

Atemporal Base: In the PRT framework, the entire cosmos is described by a static, atemporal state $ \mathcal{I}_0$. This base state can be thought of as a timeless "block" of information or the wavefunction of the universe containing all possible correlations. Notably, such a state is analogous to the Wheeler–DeWitt quantum cosmology state, which is *stationary* and does not explicitly depend on time [1]. In other words, **fundamental reality lacks an intrinsic time coordinate** – the Wheeler–DeWitt equation yields a "frozen" universe with no built-in time progression [1]. All dynamics and temporal order must therefore emerge from within this static picture.

Projection Mechanism: The theorem asserts that reality at any given emergent time is obtained by *projecting* from $\mathcal{I}*0$. Formally, one may define a one-parameter family of projection operators $ {\mathcal{P}}_t}$ such that the state of the world at emergent time t is $|\Psi(t)\rangle = \mathcal{P} _t(\mathcal{I}_0)$. The "**rendering**" is recursive: given $|\Psi(t)\rangle$, the next state $|\Psi(t+\Delta t)\rangle$ is determined by applying the projection with an incremented parameter, $\mathcal{P}*_0$ as encoding all possible configurations, and the projection process }$, under the constraint that the transition respects entropy increase (discussed below). Intuitively, we can imagine $\mathcal{I**selects an ordered sequence** of configurations that constitute a consistent history. This selection is not arbitrary: it is constrained such that each subsequent state follows naturally (causally) from the previous and **never violates the entropy condition** (the sequence's overall entropy must increase or remain constant).

Because $\mathcal{I}_0$ is *singular* and all-encompassing, it contains the information to generate every possible "moment" of reality. The emergent time parameter t merely *indexes* these projections; it does not exist outside the process. In the absence of projection (or if a system is isolated from any sequence), no change would be observed – **time would effectively not exist for that system**. For example, in a completely isolated, unentangled state (no projection or correlation with a clock), the system would perceive no passage of time (the universe would appear "frozen") [2]. Thus, **time in PRT is defined by the ordering of projected states, not by any fundamental Newtonian or spacetime coordinate**.

Causality and Sequence: Even though $\mathcal{I}*0$ is static, it encodes relations between states that give rise to causality in the projected sequence. Each projection $\mathcal{P}*}(\mathcal{I*0)$ is built upon the prior state $\mathcal{P}_t(\mathcal{I}_0)$, preserving information about past configurations. This recursive dependency means the **state at $t+\Delta t$ contains records or effects of the state at t**, establishing a cause-effect linkage along the emergent timeline. In formal terms, the projection operators can be designed to satisfy $ \mathcal{P}}* \circ \mathcal{P*{t} = \mathcal{P}*$ (consistency when projecting in steps), and the dynamics can be described by an effective evolution operator $U(\Delta t)$ acting on the state: $|\Psi(t+\Delta t)\rangle = U(\Delta t)\,|\Psi(t)\rangle$. This $U(\Delta t)$ will be derived from the underlying state's structure (for instance, via an effective Hamiltonian, see below). The key point is that **the projection sequence introduces a one-way ordering**: given $|\Psi(t)\rangle$, one can compute $|\Psi(t+\Delta t)\rangle$, but not vice versa without loss of generality, because reversing the projection would require decreasing entropy (forbidden by the constraints). Thus, a **partial order** of events is defined within $\mathcal{I}_0$ that corresponds to the familiar forward flow of time in the projected reality.

Quantum Relational Time via the Page–Wootters Mechanism

To formalize emergent time, we incorporate the Page–Wootters mechanism, a quantum model where time arises from entanglement and correlations rather than an external parameter. We assume the total, atemporal state $\mathcal{I}0$ can be represented as a **stationary entangled state** $|\Psi}}\rangle$ in a bipartite Hilbert space $\mathcal{HC \otimes \mathcal{H}_S$. Here C is a clock subsystem and S is the rest of the universe (system). The total Hamiltonian is constrained by $(H_C + H_S)|\Psi_0$) [1] .}}\rangle = 0$, ensuring the combined state is an energy eigenstate with no net evolution (this reflects the static nature of $\mathcal{I

Entangled Clock–System State: We expand the total state in the clock's basis ${|t\rangle_C}$ (which might be e.g. eigenstates of the clock's time indicator observable or an idealized continuous basis). The total state can be written as an entangled superposition or integral:

$$ |\Psi_{\text{total}}\rangle \;=\; \int dt\; |t\rangle_C \otimes |\psi(t)\rangle_S\,, $$

for some set of (unnormalized) states $|\psi(t)\rangle_S$ in the system's Hilbert space. In a simpler discrete picture, one could write $|\Psi_{\text{total}}\rangle = \sum_{t} |t\rangle_C \otimes |\psi(t)\rangle_S$. The entanglement between C and S ensures that **when the clock is found in state $|t\rangle_C$, the system is *simultaneously* found in state $|\psi(t)\rangle_S$** [2]. This correlation establishes *time for the system*: the label t serves as the emergent time of state $|\psi(t)\rangle$. In an unentangled scenario (the clock and system in a product state), there would be no such correlation and thus no notion of "before" or "after" – *time would not exist for that system* [2]. It is the entanglement that endows one subsystem with a sense of progression relative to the other.

Conditional States and Schrödinger Evolution: By conditioning on the clock reading, one can recover ordinary time evolution for the system S. Mathematically, the *conditional state* of S given a clock value t is $|\psi(t)\rangle_S = \frac{\langle t|\Psi_{\text{total}}\rangle}{\sqrt{\langle \Psi_{\text{total}}| t\rangle\langle t|\Psi_{\text{total}}\rangle}}$. Page and Wootters showed that if $|\Psi_{\text{total}}\rangle$ satisfies the Hamiltonian constraint and is entangled as above, the conditional state $|\psi(t)\rangle_S$ **obeys the Schrödinger equation** with t as the time parameter [3]. In essence, the stationary total state contains all "frames" of an evolution, and by selecting a particular clock reading we *project out* the system's state at that emergent time. The famous result is that **there is only one time, and it is a manifestation of entanglement** [4]. Time is nothing mysterious here – it is simply a way to parameterize the correlations between two parts of a static quantum state. Recent work confirms that applying this mechanism yields standard quantum dynamics for the subsystem and even reproduces classical equations of motion in appropriate limits [4] [5].

Notably, **the flow of time emerges from within**: an *unentangled* (or only weakly entangled) total state yields no meaningful internal clock and appears frozen, whereas a highly entangled state provides a well-correlated "clock + system" pair that experiences evolution [6]. Time for an object thus emerges **only through its entanglement with another system acting as a clock** [2]. This relational view aligns perfectly with PRT: the base state (which we now equate with $|\Psi_{\text{total}}\rangle$) is static, yet it gives rise to an internal time parameter t by virtue of entangled correlations. The **mathematical time variable t is emergent and relational**, not part of the fundamental backdrop.

Formally, if $H_C |t\rangle_C = i\hbar \frac{d}{dt}|t\rangle_C$ (for a clock with continuous spectrum) and H_S acts on $|\psi(t)\rangle_S$, the global constraint $H_C + H_S = 0$ implies:

$$ H_S |\psi(t)\rangle_S = -H_C |\psi(t)\rangle_S = i\hbar \frac{d}{dt}|\psi(t)\rangle_S\,, $$

which is exactly the Schrödinger equation for $|\psi(t)\rangle_S$ (with time measured by the clock's eigenvalue). In other words, the **system's state changes with respect to the clock state t in the same way it would change with respect to an external time parameter** [3]. This is how the **model recovers ordinary quantum dynamics from a fundamentally timeless picture** – time is an *internal degree of freedom* arising from entanglement.

Entropy and the Thermodynamic Arrow of Time

While the Page–Wootters mechanism gives us a **time parameter and unitary evolution**, it does not by itself explain **why we experience a unidirectional *arrow* of time** (why the sequence runs "forward" and not in reverse). For this, we integrate entropy-based time asymmetry. According to the second law of thermodynamics, **entropy tends to increase with time** in an isolated system [7]. In fact, entropy is essentially the only quantity in physics that unequivocally **requires a particular time-direction for its description** [7]. **As one goes forward in emergent time, the total entropy of a closed system can increase but not decrease** [7]**.** This provides a natural arrow: the *forward direction* in our projection sequence is that direction in which entropy grows (or at least does not decrease).

Entropy Gradient Constraint: We impose an **entropy gradient condition** on the projection recursion: for states in the sequence $|\Psi(t_0)\rangle, |\Psi(t_1)\rangle, ..., |\Psi(t_n)\rangle$, we require $S_{\text{entropy}}(t_{n+1}) \ge S_{\text{entropy}}(t_n)$. In differential form, if t is continuous, $\frac{d}{dt}S_{\text{entropy}}(t) \ge 0$ for the physical (forward) direction of time. This monotonic increase of entropy distinguishes the **future** from the **past** within the model [7]. It effectively selects one direction along the emergent time axis as the *physical* one. If one attempted to run the projections backwards (decreasing t), one would encounter states of lower entropy leading to higher entropy – a statistically **improbable** trajectory. In fact, the entropy gradient provides a way to **distinguish a forward projection from a reversed sequence**: a reversed movie of macroscopic events (e.g. smoke gathering into a cigarette, shattered glass reassembling) would violate the entropy increase condition and thus is recognized as unphysical [8]. Only when the sequence is played forward (entropy non-decreasing) do we obtain the correct "arrow of time" consistent with reality [8].

It's important to note that this thermodynamic arrow is fundamentally a **statistical statement** rather than an absolute law carved into the microscopic equations. The **underlying dynamics (quantum or classical Hamiltonian)** are time-symmetric and information-preserving (unitary), which means *fundamental equations alone do not pick a direction for time* [9]. In a closed system, if we had complete information, evolution is reversible and entropy would be constant (no preferred direction) [9]. The arrow arises because the **universe started in a special low-entropy configuration** (the Past Hypothesis) and because observers have incomplete information (coarse-graining leads to apparent irreversibility) [10] [11]. In the PRT model, we incorporate this by assuming $\mathcal{I}_0$ projects an initial state $|\Psi(t_0)\rangle$ of extremely low entropy (a highly ordered macrostate). From there, almost every subsequent projection will tend toward higher entropy simply because **high-entropy states are vastly more numerous (more probable) than low-entropy states** [12] [10]. The progression from order to disorder is thereby "built into" the unfolding of $\mathcal{I}_0$. Stated differently, **the forward direction of emergent time is the direction in which the**

system moves to more probable (higher entropy) macrostates [12] [10]. This statistical bias ensures irreversibility: while microscopic equations permit reversed evolution, the *model's entropy constraint renders such trajectories negligibly probable*. Forward projection is the only viable history we observe.

Information-Theoretic Irreversibility: Entropy increase is equivalent to *loss of information* about the system's microstate (from the perspective of an observer or subsystem) [9]. Each projection step that increases entropy corresponds to dispersal of information into correlations or inaccessible degrees of freedom (often the environment). In quantum terms, as the system S entangles with its environment or the clock, information about its precise state becomes delocalized (entropy of S increases). This yields an effective *irreversibility*. For instance, the **"collapse" of the wavefunction** (a non-unitary, irreversible process in quantum mechanics interpretations) can be understood as arising from entanglement with an environment and subsequent **decoherence/disentanglement**, which produces an effectively irreversible evolution for the subsystem [13]. In our model, each step forward in time involves *entanglement and subsequent information dilution* into broader degrees of freedom, making the reverse process (re-assembling all information to a past state) effectively impossible. This aligns with the thermodynamic arrow: **any process that increases entropy (e.g. diffusion, thermalization) cannot spontaneously reverse because that would require a conspiratorial gathering of information/energy** that is statistically precluded [14].

If at some stage the system (plus environment) reaches maximum entropy (equilibrium), the arrow of time would effectively **fade** [11] – projections past that point would all look the same (no further change). Indeed, in a hypothetical heat-death equilibrium of the universe, the model predicts time would cease to *have meaning*, since $\frac{dS}{dt}=0$ and no differentiable "moments" can be projected that are physically distinguishable. Thus the **emergence of time is tightly linked to entropy gradients**: time "flows" when there is an entropy gradient to define a direction [15], and it ceases to flow (or flows imperceptibly) when equilibrium is reached.

Formal Structure: Sequence, Causality, and Forward Direction

Bringing the pieces together, we outline the mathematical structure that produces an ordered, causal, forward-moving reality from the static base state:

1. **Global Static State ($\mathcal{I}_0$):** Represented in quantum formalism as $|\Psi_{\text{total}}\rangle \in \mathcal{H}_C \otimes \mathcal{H}_S$ satisfying $(H_C + H_S)|\Psi\rangle = 0$. This is the PRT informing state, containing all correlations. It has no external time dependence (atemporal). In classical terms, one could imagine an analogue: a timeless phase-space distribution encompassing all of history at once.}

2. **Entangled Basis – Emergent Time:** Choose an observable on the clock system C with a continuous spectrum labeled by t (the clock's "hand" or internal time). Expand $|\Psi_{\text{total}}\rangle = \int dt\, |t\rangle_C \otimes |\psi(t)\rangle_S$. The support of this state in the clock basis defines the range of emergent time that actually has support (e.g. $t \in [t_0, t_{\text{final}}]$ if time has a beginning or end in this model). For all t in that range, a corresponding system state $|\psi(t)\rangle_S$ is defined (up to normalization). Each $|\psi(t)\rangle_S$ is a *projection* of $\mathcal{I}_0$ onto the clock subspace at t: $|\psi(t)\rangle_S \propto \langle t| \Psi\rangle$. These projected states are }**automatically ordered by the parameter t**. We identify the sequence ${|\psi(t)\rangle_S}$ as the *history of the universe*. The continuum parameter t behaves like time because the correlations ensure that for $t_2 > t_1$, $|\psi(t_2)\rangle_S$ represents a later state of

the system than $|\psi(t_1)\rangle_S$. The model does not allow arbitrary re-ordering of these states without also permuting the clock – which would contradict the entangled structure. Thus a natural sequence is built in.

3. **Schrödinger Evolution and Causality:** The conditional evolution of $|\psi(t)\rangle_S$ with respect to t is governed by the system's Hamiltonian H_S: $i\hbar\,\frac{\partial}{\partial t}|\psi(t)\rangle_S = H_S\,|\psi(t)\rangle_S$, derived as shown earlier. This differential equation ensures *continuous, causal evolution*: given the state at time t and the system's Hamiltonian (laws of physics), the state at a slightly later time $t+dt$ is determined. In other words, **the model recovers local causality in time: the state's change is caused by (determined by) the state at the previous instant** (plus any stochastic elements or quantum uncertainties if we extend to density matrices). This satisfies the notion of causality as no future state can arbitrarily influence a past state without being mediated through the intermediate dynamics. The entanglement in $|\Psi_{\text{total}}\rangle$ is such that **correlations respect the dynamical laws** – effectively, $\mathcal{I}*0$ encodes the correct equations of motion. For example, if H_S generates a certain unitary $U(\Delta t)=e^{-iH_S \Delta t/\hbar}$, then one can show $|\psi(t+\Delta t)\rangle_S = U(\Delta t)|\psi(t)\rangle_S$ for all t, consistent with how $|\Psi*\rangle$ is structured. Hence the }*projection at $t+\Delta t$ can be seen as the result of evolving the projection at t forward by Δt under H_S*. This guarantees a causal, sequential consistency (the sequence is not a random jumble of allowed states, but a path in state space following physical law).

4. **Entropy Arrow Condition:** Among the two directions of the t-parameter (which a priori is symmetric in the equations above), we **distinguish one as the physical forward direction by imposing $dS_{\text{entropy}}/dt \ge 0$**. Let $S_{\text{tot}}(t)$ be the total entropy of the universe at the slice t (or more practically, entropy of some large closed system under study). Then our model asserts $S_{\text{tot}}(t_2) \ge S_{\text{tot}}(t_1)$ for $t_2 > t_1$. This is consistent with the second law and is taken as an additional postulate reflecting the special initial state of the universe (low entropy at t_0). We do not derive this inequality from first principles here (as it likely requires a choice of initial conditions and perhaps the assumption of typicality), but we incorporate it to **ensure the model's timeline has the same irreversibility as observed time**. With this condition, *causality* acquires a direction: causes lie in lower-entropy past states, and effects lie in higher-entropy future states. For instance, a memory record in an observer's brain at time t_2 (higher entropy state) can reliably refer to an event at $t_1 < t_2$ (lower entropy state), but not vice versa – because to have a "memory" of t_2 in the state at t_1 would imply information travelling from future to past, violating the thermodynamic arrow. In the model, such scenarios are excluded by the entropy condition.

5. **Causal Structure and Conditional Probability:** We can frame the emergence of causality also in terms of conditional probabilities. The probability that the system was in configuration X at time t_1 *given* that it is in configuration Y at a later time t_2 is not the same as the probability of being in Y at t_2 given X at t_1. In fact, due to the vastly larger phase space volume corresponding to higher entropy, **$P(\text{State at }t_2 = Y \mid \text{State at }t_1 = X)$ can be large, whereas $P(\text{State at }t_1 = X \mid \text{State at }t_2 = Y)$ is extremely small unless X already had high entropy**. This asymmetry in conditional probabilities is another way to see the arrow of time and causality: knowing the present, one cannot precisely retrodict the past (many past microstates could lead to the same present), but knowing the past one can, in principle, predict the present within probabilistic laws. Our emergent time model encodes this asymmetry because

the projection $\mathcal{P}_{t_2}$ maps many prior micro-configurations into one macro-state, losing information (increase in entropy), whereas going backward would require *gaining* information – disallowed in practice [9]. Thus, the **sequence is inherently oriented**: it's a one-way map from fewer micro possibilities (ordered state) to many micro possibilities (disordered state).

In summary, the formal structure is that of a *static universe state* that yields an **ordered series of states** $|\psi(t)\rangle$ (governed by standard physics) with an imposed entropy condition to pick out the **forward-time direction**. This satisfies the requirement to explain **sequence (the indexing by t), causality (laws of evolution relating states, with past influencing future), and forward direction (entropy growth defining an arrow)** within the projection mechanics of PRT.

Subjective vs Objective Time Compatibility

Our model distinguishes between **objective (or relational) time** – the physical parameter t that orders states and correlates with clocks – and **subjective time** – the psychological feeling of "flow" or the perception of duration by conscious observers. Any viable theory of time must account for both, and the PRT-aligned model does so naturally:

- **Objective/Relational Time:** In this model, time is essentially *clock readings correlated to system states*. It is entirely relational: one physical variable (the clock C) changes in concert with another (the system S). This is how time is operationally defined in physics – by correlations (e.g., the Earth rotates as the caesium clock ticks). Here, t is just a label for the state of the clock, and objectively, time is measured by the relation $|\psi(t)\rangle_S$ has to the clock basis $|t\rangle_C$. This objective time is what appears in physical laws and what an external observer would measure with instruments. It emerges from the fundamental state as described above and is fully consistent with known physics (recovering Schrödinger dynamics and, in classical limits, Newtonian or relativistic evolution when combined with space). Notably, because it is relational, this notion of time dovetails with relativity's idea that there is no absolute time – only relationships between systems (clocks and events). Our model's time is a local parameter defined within the context of the entangled system; it could be generalized to multiple clocks and reference frames by considering multiple subsystems in entanglement.

- **Subjective/Psychological Time:** The sense of time passing that we experience – often called the *stream of consciousness* – is an emergent phenomenon of brain processes, which themselves are physical processes. In this PRT model, an observer (with a brain, memory, cognition) is part of the physical system S (or a subsystem of it). The **subjective flow of time corresponds to the sequence of cognitive states** the observer's brain assumes, which are indexed by the same t (the same emergent time that governs all physical evolution). Critically, **memory formation and information processing in the brain are thermodynamic processes that obey the arrow of time**. For instance, forming a memory of an event at time t_1 involves creating a lasting record in the brain at a later time $t_2 > t_1$. This memory encoding increases local order in the brain (a more organized neural state) at the cost of expending energy and generating heat – hence increasing overall entropy [16]. As one author explains: when we form a memory, our neurons achieve a bit more order (the memory trace), but only while the body dissipates heat and increases entropy overall [16]. Our brains can only get more ordered in *accord* with the universe's larger trend toward disorder [16]. This means **our ability to remember the past but not the future is directly tied to the thermodynamic arrow**: we can create records of past states (lower entropy) in the present (higher entropy), but we have no records of future states because those states don't exist yet and entropy would have to decrease to record them. The psychological arrow of time – the fact that we feel time moving forward and accumulate memories of the past – **is thus contingent on the entropic arrow** and is fully compatible with the model's emergent time. Our brains function as sub-systems that measure the passage of time by counting internal changes (neural firing, cognitive updates), which correlate with the external clock t. Subjective duration (why time sometimes *feels* faster or slower) is a complex neuropsychological phenomenon (depending on attention, emotion, etc.), but importantly it never violates the ordered sequence dictated by t. The model can accommodate such variations by noting that subjective time is a *reconstructed narrative* by our brains, which themselves run on the physical time t. There is no separate mystical time for the mind – it is riding the same emergent t but with a variable rate of perception.

By grounding subjective time in physical processes, we ensure the model's **compatibility with experience**. An **"hour" of objective time** corresponds to a certain amount of change in the physical world (e.g. Earth rotated 15 degrees, a clock ticked a fixed number of cycles). An individual's **perception of that hour** might vary (if bored vs. excited), but that is due to the *brain's information processing* during that interval – the interval itself is delineated by physical t. In PRT terms, both the observer and the clock are part of the same projections from $\mathcal{I}_0$, so they **share the same emergent temporal ordering**. The subjective feeling of time's flow is essentially the brain *observing the sequence of its own states* in time. Because those brain states are ordered and generated under the same entropy-increasing, causal laws as everything else, the subjective arrow aligns with the objective arrow (we remember yesterday and not tomorrow because yesterday's state is of lower entropy and causally influences today's state, not vice versa [16]).

Conclusion

Definition of Time in PRT: Within the Projection Rendering Theorem framework, we define *time* as an **emergent relational parameter** that orders the projection of reality from a timeless fundamental state. It is not a built-in dimension of the informing state, but rather a contextual label arising from quantum correlations (Page–Wootters clock-system entanglement) and the progressive increase of entropy. Time is thus identified with the *conditional evolution* of subsystems relative to each other within $\mathcal{I}_0$ [4]. It manifests as a sequence of states (snapshots of reality) that are distinguished by growing entropy and correlated with an internal clock variable.

The model we presented integrates **quantum relational time** (ensuring that mathematical time corresponds to entangled correlations [6]) with **thermodynamic time** (ensuring that the sequence has a consistent forward direction given by entropy growth [7]). We reconcile a **static, atemporal base state** with **dynamic temporality** by showing that all change is internally encoded – the base state spans a history, and observers within that history perceive the flow from "past" to "future" due to entropy and entanglement. All key elements of time are accounted for: - *Sequence & Causality:* The model yields an ordered sequence of states obeying physical laws (e.g., Schrödinger or Hamilton's equations), thus preserving causality (earlier states produce later states). - *Forward Arrow:* A built-in entropy gradient picks out a forward direction for that sequence, aligning with the irreversibility we observe [8]. - *Subjective Alignment:* The psychological experience of time is a higher-level emergent property consistent with the physical arrow – memory formation and perception rely on the same one-way projection [16].

In formal terms, the Temporal Emergence Model of PRT consists of a triple $(\mathcal{I}*0,\; {\mathcal{P}_t},\; S*})$, where $\mathcal{I*0$ is the timeless state (with Hamiltonian constraint $H_C+H_S=0$), ${\mathcal{P}_t}$ is the family of projections generating states $|\psi(t)\rangle$, and $S*$ is a monotonic function along those states enforcing t's arrow. Time is defined }**operationally** as the parameter along this projection that correlates with physical change and increase of entropy. This model can serve as a reference framework in which the mysteries of time's origin and direction are addressed in a unified way: **time emerges from the fabric of correlations and information in a static universe, like a playable movie unfolding from a film reel, with entropy as the guiding principle that makes the movie run forward.**

Sources: The concepts and formalisms used here draw upon established literature for quantum emergent time [6] [4], the thermodynamic arrow [15], and the relationship between memory, entropy, and the arrow of time [16]. This synthesis aligns with the PRT hypothesis by showing how a single underlying state can project a temporally ordered reality under the governance of entropy and entanglement.

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[1] [gr-qc/9906010] Time in Quantum Gravity
https://arxiv.org/abs/gr-qc/9906010

[2] [3] [5] [6] Time might be a mirage created by quantum physics, study suggests | Space
https://www.space.com/time-a-mirage-quantum-physics-suggests

[4] Time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states | Nature Communications
https://www.nature.com/articles/s41467-021-21782-4

[7] [8] [14] [15] Entropy as an arrow of time - Wikipedia
https://en.wikipedia.org/wiki/Entropy_as_an_arrow_of_time

[9] [13] thermodynamics - Quantum entaglement and the arrow of time - Physics Stack Exchange
https://physics.stackexchange.com/questions/109189/quantum-entaglement-and-the-arrow-of-time

[10] [11] [12] [16] Arrow of Time | Internet Encyclopedia of Philosophy
https://iep.utm.edu/arrow-of-time/