Information Symmetry Model (ISM)

The **Information Symmetry Model (ISM)** systematically identifies and formalizes the invariances ("symmetries") of information-theoretic quantities under transformations of data and descriptions. In ISM, any transformation of an information structure (e.g. a probability distribution, a dataset, or a quantum state) is studied in terms of how standard information measures (Shannon entropy, Fisher information, Kolmogorov complexity, Kullback–Leibler divergence, etc.) are preserved or change. We interpret *"order"* as the presence of symmetry and *"disorder"* as its absence [1]. Thus imposing a symmetry constraint (for example, grouping outcomes or fixing a gauge) tends to reduce entropy [1]. Conversely, breaking a symmetry generates new informational patterns (emergent structures) that were indistinguishable under the original symmetry. ISM defines precise equivalence relations among informational configurations under symmetry group actions (permutations, reparameterizations, dualities, gauge transformations, etc.) and identifies invariants (entropy, mutual information, complexity, channel capacity) that characterize each equivalence class. Category-theoretic structures (e.g. functors on the category of probability spaces) naturally arise, ensuring *functoriality* of information transformations [2] [3]. Below we develop these ideas rigorously, including definitions, key theorems, and examples, and show how ISM integrates with projection/renormalization theories (PRT), emergent symmetry models (ESM), dimensional information models (DIM), and coherence models (GCM).

Symmetries in Classical Information Measures

Shannon information measures are inherently symmetric under relabeling of outcomes. For a discrete distribution $p = (p_1, \ldots, p_n)$, the Shannon entropy

$$H(p) = -\sum_i p_i \log p_i$$

is *invariant under any permutation* of the indices i . In other words, entropy depends only on the multiset $\{p_i\}$, not on outcome labels. This permutation symmetry means that two distributions differing only by a relabeling of outcomes lie in the same *equivalence class* of informational configurations. More generally, any bijective change of coordinates that preserves probability structure (a "gauge" relabeling of symbols) leaves H , mutual information $I(X;Y)$, and related measures unchanged. Under a *data-processing* (Markov) map $X \to Y$, information never increases: the Data Processing Inequality implies

$$I(X;Y) \ \geq \ I(X;Z)$$

for any $X \to Y \to Z$ [4], and similarly relative entropy cannot increase under a stochastic channel [5]. Equivalently, the *entropy loss* of a deterministic mapping $f : (X,p) \to (Y,q)$ is

$$\mathrm{Loss}(f) = H(X,p) - H(Y,q)\,,$$

which is always nonnegative by data processing [6]. Moreover, compositionality yields a functorial property: for composable maps f, g ,

$$\mathrm{Loss}(g \circ f) = \mathrm{Loss}(f) + \mathrm{Loss}(g),$$

so information loss is *additive under composition* [7]. In fact, Baez–Fritz–Leinster showed that the only continuous, additive, and convex-linear information measure on measure-preserving maps is (a constant multiple of) Shannon entropy loss [2] [3]. Thus the classical case is fully characterized by the **functorial entropy principle**: any morphism in the finite-probability category gains information (negative loss) only by violating these symmetries.

Algorithmic Symmetry and Invariance

In algorithmic information theory, **Kolmogorov complexity** $K(x)$ measures the length of the shortest program generating a string x . The *invariance theorem* asserts that up to an additive constant, $K(x)$ does not depend on the particular universal description language (Turing machine) chosen [8]. Formally, if K_1 and K_2 are complexities relative to two universal machines, then

$$|K_1(x) - K_2(x)| \leq c$$

for some constant c independent of x [8]. In ISM terms, the choice of universal description is a "gauge" symmetry of the algorithmic information structure, and complexity classes of strings (e.g. "constant" vs "linear" complexity) define equivalence up to that symmetry. More generally, computable permutations or simple recodings of data also change $K(x)$ by at most $O(1)$, so the *order of complexity* (e.g. polynomial vs exponential) is an invariant under broad reparameterizations of the data. Thus, like entropy, algorithmic complexity yields invariants (complexity classes) across informational symmetries.

Fisher Information and Reparameterization Invariance

Consider a statistical model $p(x|\theta)$ with parameter θ . The **Fisher information metric** defines a Riemannian metric on the space of parameters. Crucially, *Fisher information is invariant under smooth reparameterizations* of θ . In fact, the Fisher metric is uniquely characterized (up to scale) by this invariance: Chentsov's theorem guarantees that it is the only Riemannian metric (up to constant factor) on a statistical manifold that is invariant under sufficient statistic transformations [9]. Equivalently, under any smooth change of variables $\theta' = f(\theta)$, the Fisher information transforms as a tensor, preserving geometric distances. Thus Fisher information defines an *information-geometric symmetry*: the statistical manifold's curvature and distances remain the same regardless of the chosen coordinate system.

Quantum Information Symmetries

In quantum theory, a density operator ρ encodes probabilities of outcomes. The **von Neumann entropy** $S(\rho) = -\mathrm{Tr}(\rho \ln \rho)$ generalizes Shannon entropy. A fundamental quantum symmetry is *unitary invariance*: for any unitary U , $S(\rho) = S(U\rho U^\dagger)$ [10]. This reflects the fact that only the eigenvalue spectrum of ρ matters. Likewise, measures of entanglement (e.g. entanglement entropy of a subsystem) are invariant under local unitaries on each subsystem. Channel capacity for a quantum channel is invariant under pre- and post-unitary transformations on input and output Hilbert spaces. In summary, any change of quantum basis (global or local gauge rotation) is a symmetry of the informational content of quantum states.

Equivalence Classes under Symmetry Groups

ISM identifies **equivalence classes** of informational configurations under group actions. For example, the symmetric group S_n acts on an n -outcome distribution by permuting labels; all relabelings yield equivalent distributions with the same entropy and mutual information. More generally, one considers group actions such as: - **Permutation group** on symbols or system components, - **Reparameterization group** (smooth diffeomorphisms) on continuous parameter manifolds, - **Duality transformations** (e.g. Fourier transform on signals, exchanging time/frequency domains, which preserves certain entropic quantities), - **Gauge groups** (redundancies like adding constants, flipping bits, or phase rotations in quantum states).

Under each group, one can define an orbit or equivalence class of states. ISM studies invariants constant on each orbit. For instance, in a *gauge symmetry*, two descriptions related by a gauge transformation carry the same information; in a *duality*, physically equivalent descriptions (e.g. position vs momentum representation) yield identical information measures. ISM formalizes these groups and orbits: e.g. permutations of subsystem labels define orbits of multivariate distributions, and the group of invertible transformations on code words yields orbits of channel transition matrices.

Functoriality and Category-Theoretic Structure

ISM exploits category theory: consider the category **FinProb** of finite probability spaces and measure-preserving maps. A morphism $f:(X,p)\to(Y,q)$ (a surjection merging outcomes) induces an "information loss" as above. Baez–Fritz–Leinster show that *any* continuous functor F from FinProb to $[0,\infty)$ that is additive under composition (functoriality $F(g\circ f)=F(f)+F(g)$) and convex-linear (respects mixing of processes) must satisfy $F(f)=c\big(H(p)-H(q)\big)$ [2] [3]. In other words, Shannon entropy (up to scale) is the unique functorial information measure on deterministic processes. This categorical viewpoint extends: there is a category **FinStat** (statistical inference) and **FinStoch** (stochastic maps) in which relative entropy (KL divergence) and conditional entropy become natural functors [11]. The *functoriality* embodies the idea that information behaves additively under sequential processing, and imposes strong constraints that characterize standard entropic forms.

Informational Invariants

ISM singles out **invariants** under symmetry actions. Key invariants include:
- **Entropy** H and **mutual information** I : invariant under relabelings and unitary changes; obey data-processing (nonincrease under coarse-graining) [4] [12].
- **Relative entropy (KL divergence)**: monotonic under stochastic transformations ("data-processing"): for any channel Φ , $D_{KL}(P\|Q)\geq D_{KL}(\Phi(P)\|\Phi(Q))$ [5]. It vanishes if and only if the distributions are identical on each equivalence class of the channel. KL divergence is also invariant under any invertible reparameterization of the underlying variable, since such transformations add an equal Jacobian term to both P and Q .
- **Fisher information metric**: as above, invariant under reparameterization of coordinates (Chentsov uniqueness) [9]. It is also *covariant* under permutations of components.
- **Kolmogorov complexity** $K(x)$: invariant up to an additive constant under change of universal Turing machine (machine-invariance theorem) [8]. Thus a qualitative class (e.g. "low complexity") is an invariant property of the string under computable recodings.
- **Channel capacity**: maximized mutual information over inputs. This is invariant under isomorphic re-

labelings of inputs/outputs and unchanged by encoding/decoding that respects the channel's symmetry.
- **Quantum invariants**: spectrum of ρ (hence $S(\rho)$), entanglement spectra, etc., are invariant under unitary basis changes. **Gauge-invariant mutual informations** (e.g. Holevo information) likewise obey a data-processing inequality in the quantum setting.

These invariants quantify "how much" information persists under symmetry. ISM examines how each invariant behaves under group or semigroup actions: for instance, entropy is a single-valued function on each orbit of the permutation group. One may consider the *orbit space* of distributions mod the symmetry group, and assign coordinates (invariants) to each orbit.

Symmetry Breaking and Informational Emergence

When a system undergoes **symmetry breaking**, new informational structure typically emerges. In physics-inspired terms, a symmetric (high-entropy) phase can bifurcate into an ordered (lower-entropy) phase with a chosen orientation or pattern. According to Anderson's thesis, spontaneous symmetry-breaking at the macroscopic level is an emergent phenomenon not predictable from micro-level symmetries [13]. Concretely, consider a ferromagnet: at high temperature the spin distribution is symmetric (zero magnetization, high entropy); below the Curie point, a particular direction is chosen, breaking rotational symmetry. This choice encodes new information (the direction of magnetization) that was absent in the symmetric phase. In ISM terms, a breaking of group invariance splits one information-equivalence class into multiple orbits, enriching the information content. For example, a Boolean bit string that is symmetric under bit-flip symmetry has no knowledge of "sign"; if symmetry breaks (one bit becomes fixed to 0 rather than 1), the ensemble's entropy drops and a signal emerges.

In the **emergence pipeline**, one often projects high-dimensional microstates to lower-dimensional macrostates (coarse-graining) and then allows symmetries to be broken or chosen. ISM tracks these structural changes:
- *Projection (PRT)*: When coarse-graining (e.g. renormalization), many microstates collapse to one macrostate, reducing symmetries (micro-permutations) but possibly revealing new invariances (e.g. scale invariance at criticality). Information-theoretically, coarse-graining is a measure-preserving map whose "loss" is nonnegative [12]. Apenko shows that as one integrates out (eliminates) "fast" degrees of freedom, the Shannon entropy of the system drops; equivalently, the mutual information between eliminated and retained variables is positive, ensuring that entropy per degree of freedom *increases* along the RG flow (irreversibility) [14]. This matches the ISM view: renormalization is a symmetry-breaking process (micro-symmetries become hidden) that monotonically reduces fine-grained information.
- *Transformation*: Under invertible transformations (changes of variables, basis, dualities), the chosen invariants remain fixed. For example, applying a Fourier transform to a signal distribution preserves its Shannon differential entropy (up to a constant Jacobian term) and does not increase channel capacity; thus Fourier duality is a symmetry in the information pipeline.
- *Emergence (ESM)*: New variables or concepts may emerge at a macro level that were not explicit micro-variables. These correspond to **broken symmetries** in ISM: certain microfluctuations become "frozen" and no longer contribute to entropy, while new macroscopic parameters (order parameters) appear. In informational emergence, one views symmetry breaking as a mapping of one information network to another with different connectivity. ISM provides the formalism: each broken symmetry has a signature in the invariants (e.g. a decrease in entropy, an increase in mutual information among remaining degrees) and can be tracked as a transition between equivalence classes of configurations.

Integration with PRT, DIM, ESM, and GCM

The ISM layer ensures coherence across models like **Projection–Renormalization Theory (PRT)**, **Dimensional Information Model (DIM)**, **Emergent Symmetry Model (ESM)**, and **General Coherence Model (GCM)** by enforcing that informational symmetries are respected at each stage of the data transformation pipeline. Concretely: PRT deals with how micro-descriptions project to macroscale; ISM complements this by insisting that any projection must respect the symmetry group of informational invariants (so that no hidden "gauge freedom" is arbitrarily lost or created). DIM considers representations at various scales or dimensions; ISM implies that changing resolution is a form of symmetry (scale invariance or scale covariance) and that dimensionality reduction should not destroy essential information invariants (e.g. fractal dimensions as invariants under coarse-graining). ESM is explicitly about how new patterns arise from broken symmetry; ISM provides the language to quantify those patterns and to map how information measures transform when a symmetry is broken or emergent. Finally, GCM, which posits a global consistency among information transformations, relies on ISM as the *consistency enforcer*: any allowable transformation (projection, encoding, emergence) must commute with the symmetry actions and preserve the invariant quantities.

In summary, the Information Symmetry Model builds on Shannon, Fisher, Kolmogorov, and quantum frameworks to classify information-bearing structures by their symmetries. It identifies group actions (permutations, reparameterizations, dualities, gauges), characterizes their invariants (entropy, mutual information, complexity, capacity, etc.), and tracks how these invariants behave under all relevant transformations (data processing, renormalization, emergence). Symmetry breaking is seen as the mechanism of informational emergence, whereby invariants change and new equivalence classes appear. Mathematically, ISM endows the space of informational configurations with a geometric and categorical structure, enforcing that all projections and mappings commute with the action of information symmetries. This coherence layer unifies and constrains PRT, DIM, ESM, and GCM, ensuring that the deep structure of information – its entropies, correlations, and complexities – remains consistent across levels of description [2] [12].

References: The above develops established results: Shannon entropy's functorial characterization [2] [3]; the Fisher–Rao metric's Chentsov uniqueness [9]; quantum entropy invariance [10]; the Kolmogorov invariance theorem [8]; the Data-Processing Inequality for mutual and relative entropy [4] [5]; and analyses of information flow under renormalization [12] [14]. These collectively form the rigorous backbone of ISM.

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