Prescient Models

Predicting Economists: Generating Scenarios for Stress Testing Future Loss Reserves

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Abstract

Stress testing under the US Comprehensive Capital Analysis and Review (CCAR) regulations and those of many other countries seeks to assess the full possible financial position of a lender through an economic crisis. The introduction of lifetime loan loss reserves under FASB's Current Expected Credit Loss (CECL) and IASB's International Financial Reporting Standards 9 (IFRS 9) rules complicates the task of stress testing, because lenders need to estimate future losses using scenarios that are contingent on the stress testing scenario, but without perfect foresight of the future stress test scenario.

This work casts the CECL and IFRS 9 stress testing problem as one of generating future economic scenarios that are consistent with how future economists would create scenarios. To that end, we obtained historic consensus economic scenarios for testing. The results here demonstrate that a second-order Ornstein-Uhlenbeck model fits historic scenarios well and could be used to generate future scenarios that would be a realistic representation of what economists would predict given economic conditions up to that point.

This approach was tested for US real gross domestic product (RGDP) and unemployment rate scenarios through the 2009 recession. The RGDP modeling was straight-forward, but we discovered that consensus economic scenarios for unemployment rate appear to be conditional on the phase of the economy.

Keywords: Mean reverting models, Ornstein-Uhlenbeck models, CCAR, CECL, IFRS 9, Stress testing

1 Introduction

The estimation of lifetime loss reserves under IFRS 9 and CECL have created a new complication for stress testing loan portfolios under programs like CCAR. If a recession is occurring 12 months from now, what loss reserve will the lender need at that time? These new loss reserving rules require forward-looking macroeconomic scenarios, but what scenario would the bank assume 12 months from now if a recession were occurring? The lender cannot assume that they will see the actual future macroeconomic conditions provided in the stress test. Rather, the lender needs to decide what an economist would think is coming next if they were to see the economy assumed up to that point in the stress test. This is a subtle but important question for determining the impact of a recession of the lender's financial position.

Some practitioners may attempt to use an economic scenario generator to satisfy this requirement. Many such tools exist that can provide a range of scenarios custom-designed for a number of purposes [6, 7, 1]. To the extent that these tools are what the lender actually uses in practice, it will make perfect sense for them to use them in a stress testing context to generate future scenarios. However, for those lenders who rely more on economists than economic scenario generators, and this appears to be the large majority of lenders, this paper seeks to solve the problem of generating scenarios that look like those created by economists.

To generate plausible macroeconomic scenarios from any point in a stress test, we obtained historic data on scenarios created by economists in order to create a model of economists. A quick visual inspection of these scenarios showed that a a mean-reverting process that incorporates recent momentum would be a good candidate. Such models are frequently used in economics and finance [8, 11, 9, 5], the most common suitable approach being a second-order Ornstein-Uhlenbeck process. This paper explores the use of such models for model historic macroeconomic scenarios and then generating plausible scenarios for use in stress testing.

This paper's development of models capable of generating scenarios that mimic economists for business use is new to the academic literature and business practice. Historically the emphasis was on forecasting future economic conditions rather than predicting what economists would say about future conditions. From a regulatory perspective, predicting economists is also highly valuable when we have no reasonable chance of predicting the economy.

This paper will review the theory behind Ornstein-Uhlenbeck (OU) mean-reverting processes and extend that to second-order OU processes that include momentum. Then we review data available for historic economist-created scenarios. Lastly we create mean-reverting models of these historic scenarios for real gross domestic product (RGDP) and unemployment (UR), two key macroeconomic factors for loan loss forecasting.

2 Ornstein-Uhlenbeck Process

The creation of mean-reverting scenarios for credit risk modeling has been described previously by Breeden and Liang [4]. The following summarizes how mean-reverting models could be created using an Ornstein-Uhlenbeck process [10, 2].

The Ornstein-Uhlenbeck process is a continuous-time stochastic process often described in the context of Brownian motion.

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t \tag{1}$$

For a studied property x_t, μ is the long-run mean of the process, θ is related to the relaxation time, and σ is related to the variance.

In discrete time, the O-U process simplifies to a structured $AR(1)$ process.

$$\Delta x(t) = \theta\big(\mu - x(t)\big)\Delta t + \epsilon_t \tag{2}$$

where

$$\mu = d - \frac{\sigma^2}{2\theta}, \quad \epsilon_t \approx N(0, \sigma). \tag{3}$$

Given this process, the expected mean and variance are

$$E\big(x(t)\big) = \Big(1 - e^{-\theta(t-t_0)}\Big)\mu + e^{-\theta(t-t_0)}x(t_0) \tag{4}$$

$$var\big(x(t)\big) = \frac{\sigma^2}{2\theta}\Big(1 - e^{-2\theta(t-t_0)}\Big) \tag{5}$$

In the limit as $t \to \infty$, this becomes

$$\lim_{t\to\infty} E\big(x(t)\big) = \mu \lim_{t\to\infty} var\big(H(t)\big) = \frac{\sigma^2}{2\theta} \tag{6}$$

To apply Equation 2 to generating mean reverting macroeconomic scenarios, the parameters θ, μ, and σ must be estimated from historic data or assumptions going forward.

3 Second-order Ornstein-Uhlenbeck Process

Although fairly simple, the Ornstein-Uhlenbeck process described above usually has a discontinuity in the rate of change. A graph of the scenario will always look unrealistic because of the sudden change in direction at the start of the scenario. Momentum can be included in the scenario generation by using a second-order Ornstein-Uhlenbeck process. The following provides a derivation of the necessary formulas.

We consider x_t as the solution of the following second order Ornstein-Uhlenbeck equation

$$dx_t = \big(\theta(\mu - x_t) + v_t\big)dt \tag{7}$$

$$dv_t = -\theta_1 v_t dt + \sigma dw_t \tag{8}$$

where x_t is the time series, μ is the long-run mean, σ is the volatility coefficient, w_t is a Wiener process, and θ, θ_1 are positive constants ($\theta \neq \theta_1$). The solution to Equations 7 & 8 is

$$x_t = \mu + C_1 e^{-\theta t} + C_2 e^{-\theta_1 t} + \frac{\sigma}{(\theta - \theta_1)} \int_0^t \Big(e^{-\theta_1(t-\tau)} - e^{-\theta(t-\tau)}\Big) dw_\tau \tag{9}$$

where C_1, C_2 are constants depending upon the initial conditions of x_t, v_t. The mean and variance of x_t are calculated as:

$$E(x_t) = \mu + C_1 e^{-\theta t} + C_2 e^{-\theta_1 t} \tag{10}$$

where $E(x_t) \to \mu$ as $t \to \infty$.

$$var(x_t) = \frac{\sigma^2}{2\theta\theta_1(\theta+\theta_1)} \frac{1 - \Big((\theta e^{-\theta_1 t} - \theta_1 e^{-\theta t})^2 + \theta\theta_1(e^{-\theta_1 t} - e^{-\theta t})^2\Big)}{(\theta-\theta_1)^2} \tag{11}$$

where

$$var(x_t) \to \sigma^2/(\theta\theta_1(\theta+\theta_1)) \tag{12}$$

as $t \to \infty$. The time series up to the beginning of mean-reversion can be called y_t. This could include actual history and the given scenario. The mean reverting process x_t begins at t_0. The goal is to obtain an extrapolation of y_t after time $t_0 = n$ as a mean-reverting 2D Ornstein-Uhlenbeck process with a smooth extrapolation of y_t for $t > t_0$ that matches the historic scenarios. The best estimate of x_t is

$$\hat{x}_t = \mu + C_1 e^{-\theta t} + C_2 e^{-\theta_1 t} \tag{13}$$

Therefore,

$$\hat{x}_n = \mu + C_1 e^{-\theta n} + C_2 e^{-\theta_1 n} = y_n, (7) \tag{14}$$

$$\hat{x}_n - \hat{x}_{n-1} = C_1\Big(e^{-\theta n} - e^{-\theta(n-1)}\Big) + C_2\Big(e^{-\theta_1 n} - e^{-\theta_1(n-1)}\Big) \tag{15}$$

$$\hat{x}_n - \hat{x}_{n-1} = y_n - y_{n-1}.(8) \tag{16}$$

Solving this system produces

$$C_2 = (y_{n-1} - \mu - (y_n - \mu)e^{\theta})/\Big(e^{-\theta_1 n}(e^{\theta_1} - e^{\theta})\Big) \tag{17}$$

$$C_1 = (y_n - \mu)e^{\theta n} - C_2 e^{(\theta-\theta_1)n} \tag{18}$$

Values C_1, C_2 given in Equations 17 & 18 are substituted into Equation 13. For convenience and to avoid operations with big numbers $t_0 = n = 0$ was used.

Parameters μ, σ, θ, θ_1 would be obtained by optimizing the fit to historic macroeconomic scenarios. y_t. However, since we are less concerned with the growth of uncertainty with time, for the present study we can ignored σ since it contributes only to estimating the confidence interval about the estimate. The scenarios are generated as

$$\hat{x}_t = \mu + C_1 e^{-\theta t} + C_2 e^{-\theta_1 t} \tag{19}$$

4 Data

To train and test the models, data was obtained from Consensus Economics Inc. This data included the consensus economic forecasts from approximate 30 leading economic forecasters, although the exact list of contributors varies a little through time. Reports were obtained for the month before the start of each quarter so that the next 7 quarters were available. The scenarios represent the average of all of the contributing economists each quarter for a small set of key indicators. Of those indicators, RGDP and UR were most relative to the current purpose, so we demonstrate the models there.

The scenario start dates are spaced quarterly in a range from December 2004 through September 2014. This data range covers the last economic cycle, so we are able to identify any variations that are synchronized to the economic phase. For a deeper discussion of identifying economic phases, see Breeden, 2020 [3].

A separate test period of September 2018 through July 2019 was also obtained. Figures 1 and 2 shown the actual historic time series as a thick black line and the quarterly scenarios from Consensus Economics as thinner colored lines.

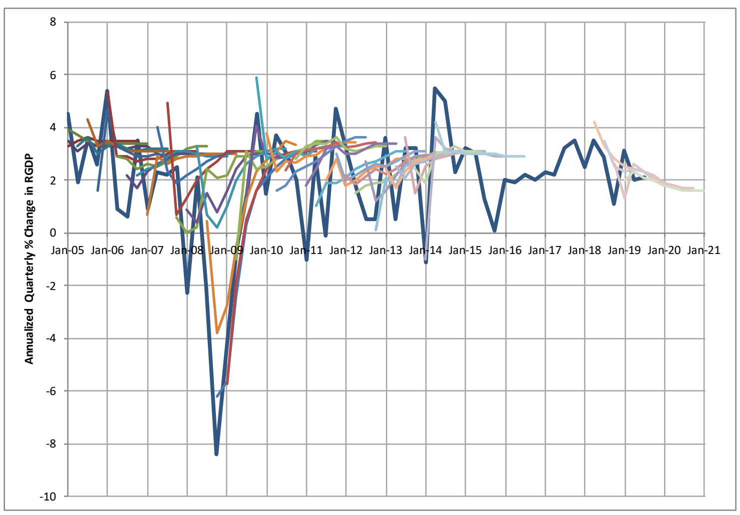

Figure 1: Historic data for annualized quarterly change in US RGDP is shown as the thick black line. The thinner colored lines are scenarios from Consensus Economics.

For each scenario shown, the first two points are the most recent history at the time the scenario was generated. Because macroeconomic data can be revised for several times, it was important to capture the history as it was known to the economists at the time the scenarios were generated. This estimated history will be used when trying to replicate the economist scenarios rather than the historic values now known to be true. For UR, this is a small adjustment,

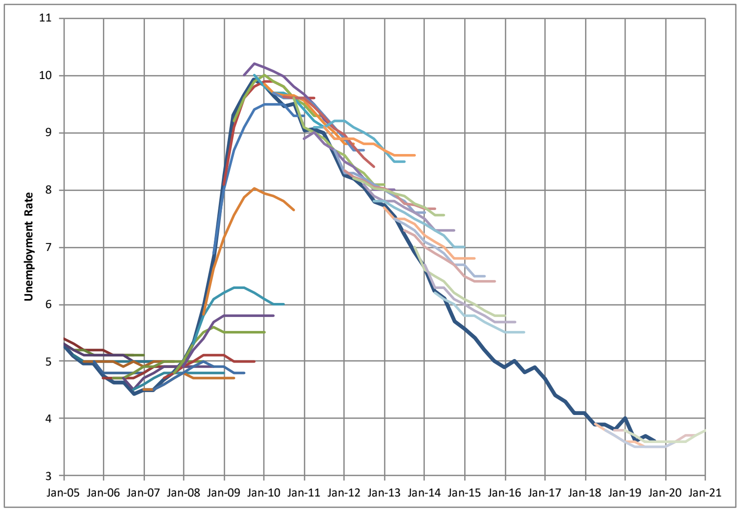

Figure 2: Historic data for US unemployment rate is shown as the thick black line. The thinner colored lines are scenarios from Consensus Economics.

usually only ± 0.1. For RGDP, the revisions can be quite significant, ± 1.4.

5 Numerical Results

The following sections describe the development of a generator for consensus economic scenarios for RGDP and UR.

5.1 Real Gross Domestic Product (RGDP)

To estimate the parameters μ, θ, and θ_1 in Equation 19, a simplex gradient descent search [9] is used to minimize the sum of the absolute differences across all quarterly values of all scenarios simultaneously.

When applied to the scenarios for 2004 Q1 through 2014 Q4, the optimal values were $\mu = 3.085$, $\theta = 2.19$, and $\theta_1 = 12.1$. Figure 3 shows the actual RGDP, the consensus scenarios as short colored lines, and the generated scenarios as dashed colored lines. The in-sample error was ± 0.25 per quarter, which is really quite good given the large volatility in RGDP revisions of ± 1.4 as mentioned earlier.

Also shown is the result of applying the same model to the out-of-sample data from 2018-2019. For this period, the in-sample parameters were assumed to hold. The observed divergence is clearly a difference in what the long-run RGDP would be. Interestingly, economists in 2018-2019 were much more negative on future long term growth than they were in the 10-year period studied during training. The rate of convergence looks reasonable, but for future use a $\mu = 1.8$ looks to be more in line with current economist expectations.

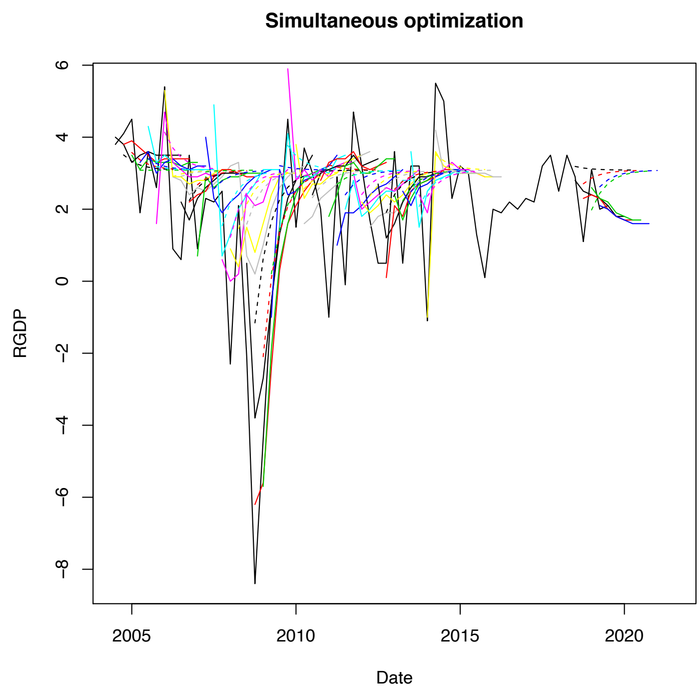

Figure 3: A comparison of the actual RGDP (long black line), consensus scenarios (short colored lines), and generated scenarios (short dashed colored lines).

To check that we have not missed any important regime shifts in how economists create scenarios, the parameters θ and θ_1 were separately estimated for every scenario in the training set. The resulting scenarios are shown in Figure 4. Visually, these are not significantly different from the ones shown in Figure 3. Figure 5 compares the average quarterly error by scenario for each model. The plot shows that individual estimation does reduce the error in-sample. However, Figure 6 graphs the parameters from which no clear regime shifts are obvious. The only exception would seem to be right around 2010 where the parameter values are off the graph, but this is because the starting value is almost identical to μ, so any value of the convergence parameters will achieve the same result. No unique solution is possible.

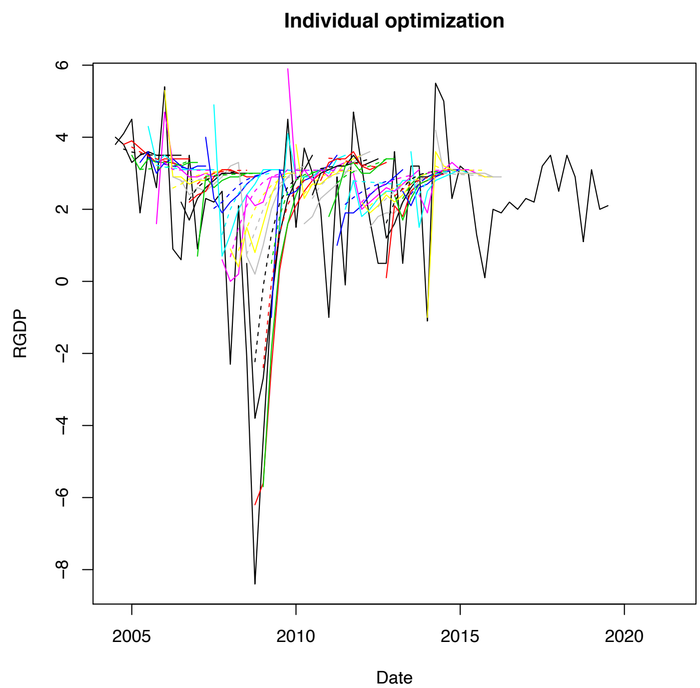

Figure 4: A comparison of the actual RGDP (long black line), consensus scenarios (short colored lines), and individually optimized and generated scenarios (short dashed colored lines).

The individual estimation shown in Figure 4 does replicate the past better, but is not useful in the stress testing problem described initially. Instead, the parameters from the simultaneous optimization with a recent value of μ would appear to be more appropriate.

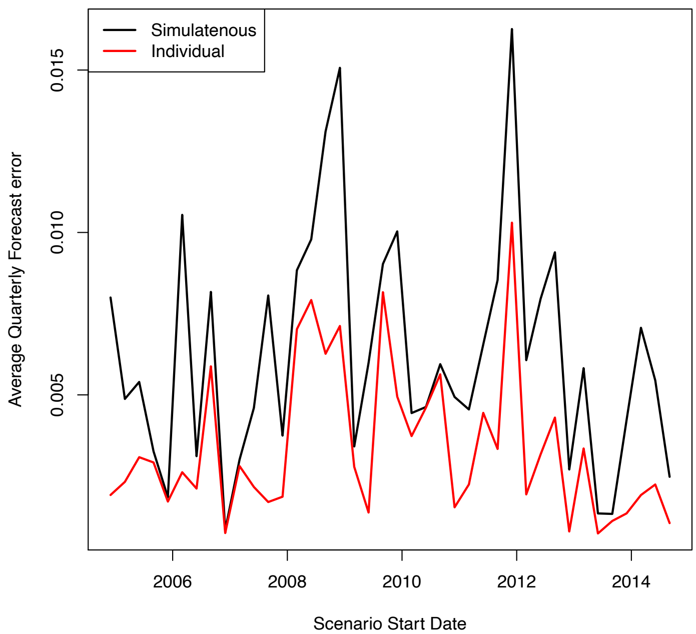

Figure 5: A comparison of the quarterly in-sample estimation errors by scenario for the two RGDP models.

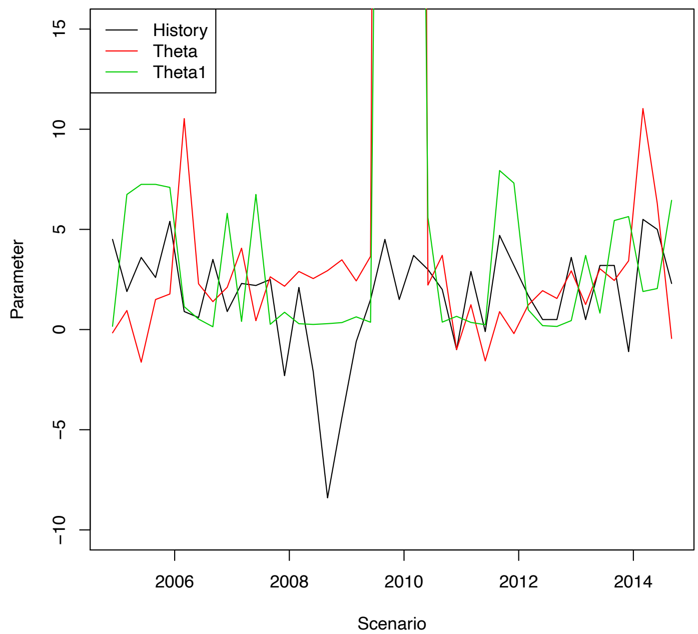

Figure 6: A graph of the convergence parameters θ and θ_1. Also shown for context is the historic RGDP time series.

5.2 Unemployment Rate (UR)

The second test case was unemployment rate. UR is a key input variable to loan loss forecasting, but also proves to be more complex than the RGDP example above.

Beginning as before with a simultaneous optimization across all scenarios, the parameters obtained were $\mu = 6.385$, $\theta = 0.157$, and $\theta_1 = 1.60$. The average quarterly error in fitting the scenarios was ± 0.23. Compared to the revision volatility of ± 0.1 for UR, this seems less accurate than the RGDP model. A visual inspection of Figure 7 confirms that a single overall model does not fit across all the scenarios as well for UR as it did for RGDP.

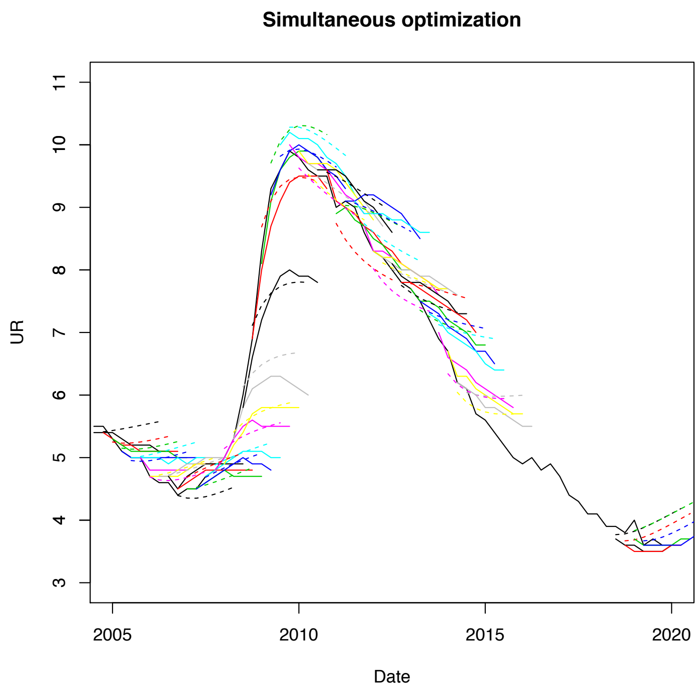

Figure 7: A comparison of the actual UR (long black line), consensus scenarios (short colored lines), and generated scenarios (short dashed colored lines).

To investigate the cause of the errors, FIgure 8 shows the scenarios generated when each scenario is fit individually. The corresponding parameters were graphed in Figure 9.

These results show that individually the scenarios can be fit quite well, but that does not solve the out-of-sample problem. More interesting is that the parameters in Figure 9 appear to show a regime shift. During the slowing of the economy between 2007 and 2009, the convergence parameters are different

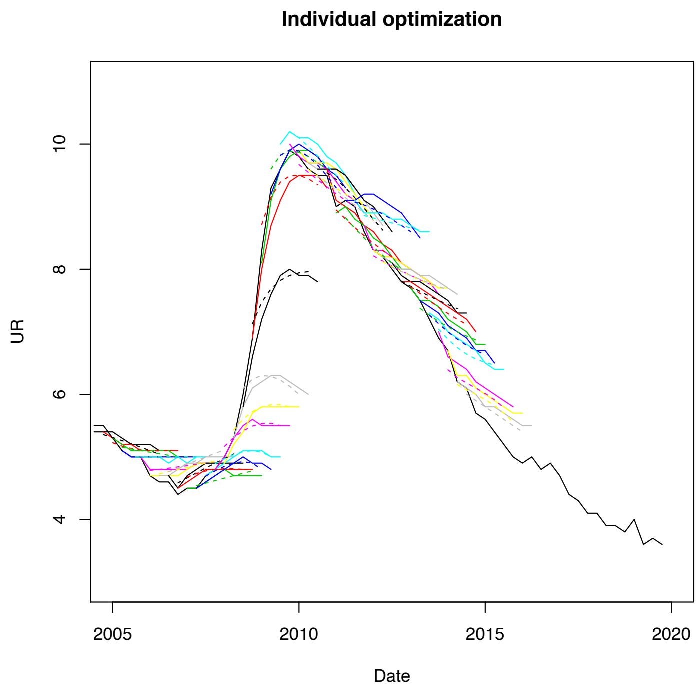

Figure 8: A comparison of the actual UR (long black line), consensus scenarios (short colored lines), and individually optimized and generated scenarios (short dashed colored lines).

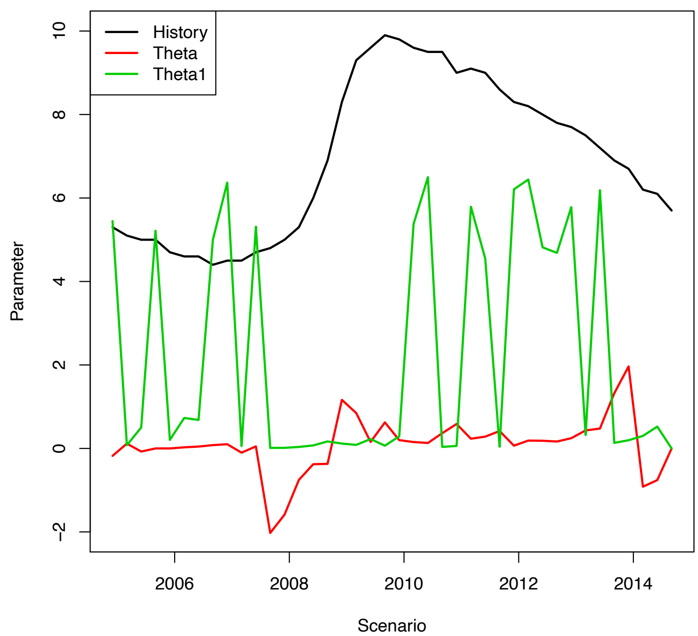

Figure 9: A graph of the convergence parameters θ and θ_1. Also shown for context is the historic UR time series.

Phase	Date Range		μ	θ	θ_1
Late Expansion	Dec-04	Dec-06	4.98	0.862	53.799
Contraction	Mar-07	Dec-09	5.41	0.302	1.135
Early Expansion	Mar-10	Dec-12	5.3	0.113	47.153
Rapid Expansion	Mar-13	Sep-14	3.5	0.139	7.051

Table 1: Parameters for different scenario regimes in UR.

from the expansionary periods before and afterward. In addition, tests on optimizing μ identified further sub-regimes where economists appeared to change their expectation on long run unemployment. Here, long run really only refers to a few years in the future.

Based upon this analysis, the following regimes were identified with their corresponding parameter values, Table 1. The resulting scenarios are shown in Figure 10. The out-of-sample scenarios were generated using the parameters from the recent expansionary regime. This again appears to be a better fit than the out-of-sample scenarios generated from the simultaneous estimation.

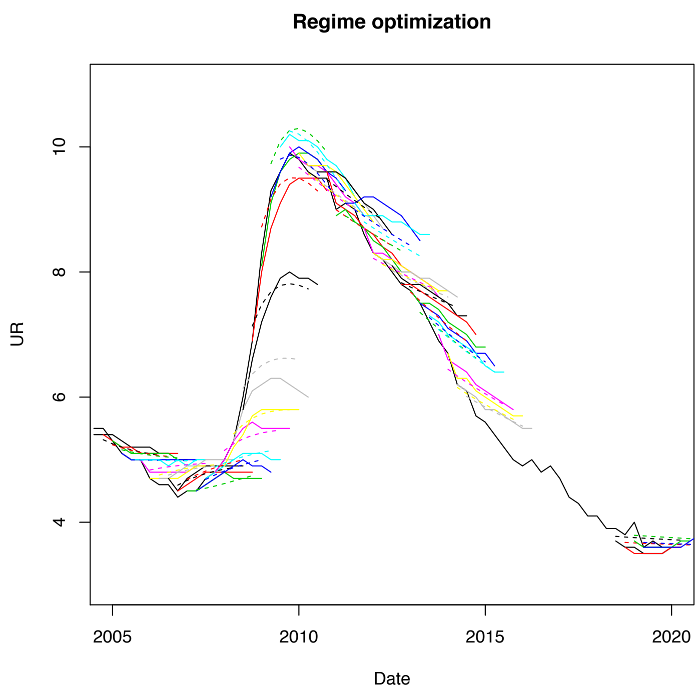

Figure 10: A comparison of the actual UR (long black line), consensus scenarios (short colored lines), and generated scenarios optimized over different regimes (short dashed colored lines).

Figure 11 summarizes the error by scenario for the three estimation methods shown. The regime-based model appears to fill a good middle ground between the overall model and the individual model. It also lends itself to use out-of-sample in a stress testing context where regimes can be identified and corresponding scenario generators applied.

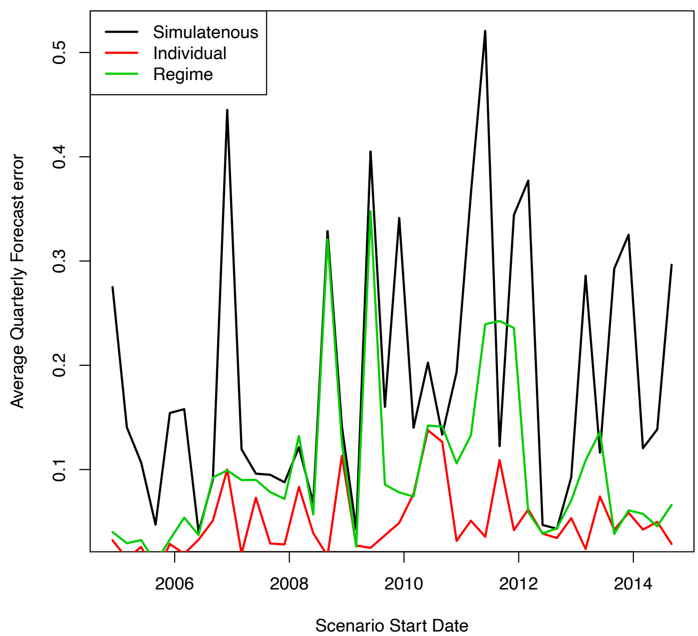

Figure 11: A comparison of the quarterly in-sample estimation errors by scenario for the three UR models.

6 Conclusions

From these results we can see that predicting economists is much easier than predicting the economy. Also, the threshold for success is lower. We do not need a perfect model of economists historically or out-of-sample. Rather, lenders need a model that can generate scenarios that plausibly replicate what a future economist might set as a scenario give the history up to that future point. With that objective in mind, the second-order mean-reverting (Ornstein-Uhlenbeck) model is clearly suitable for the variables tested.

Also, training the parameters of this model against historic scenarios is shown to be a reasonable approach to obtaining the needed calibration, keeping in mind that economists appear to go through different regimes in their planning. These regimes as observed empirically from the analysis align with the economic phases one would expect and might be anticipated in the future using an economic phase measurement [3].

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