GAMMA CAPTURE VOLATILITY

A phi-Asymmetry Replacement Framework for Black-Scholes in 0-DTE and 1-DTE SPY Options

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Abstract. This paper proposes Gamma Capture Volatility as a structurally superior replacement for Black-Scholes in pricing ultra-short-dated SPY options. We demonstrate that Black-Scholes' geometric Brownian motion assumption is violated in modern SPY microstructure, where intraday jumps and discrete dealer hedging dominate realized price behavior. In place of Black-Scholes, we develop the Gamma Capture pricing framework, where volatility is estimated through barrier crossing intensity. We introduce a key innovation: the directional asymmetry ratio phi (ph) = N^+ / N^-, the ratio of upward to downward barrier crossings, which is directly observable from the Gamma Capture production system. When phi < 1, downside crossings dominate and the model produces a natural put skew. When phi > 1, the call wing steepens. When phi = 1, the smile is symmetric. This single observable parameter replaces the ad-hoc 0.4 put-side fudge factor in earlier formulations, producing a surface that is fully calibrated from market microstructure. Calibrated at current SPY vol of ~15% annual with phi = 0.72, we map the complete 0-DTE and 1-DTE surface. The framework produces the smile, put skew, and term structure as structural consequences of the crossing model — with zero strike-by-strike fitting.

Keywords: Gamma Capture, phi asymmetry, barrier crossing, 0-DTE options, volatility surface, Black-Scholes refutation, jump process, SPY, implied volatility smile

1. Introduction

The proliferation of zero-days-to-expiry (0-DTE) options on the SPDR S&P 500 ETF (SPY) represents one of the most consequential structural shifts in modern equity derivatives markets. By 2024, 0-DTE SPY options accounted for more than 40% of daily options volume on U.S. equity indices, creating a market segment whose microstructure differs fundamentally from the environment in which Fischer Black and Myron Scholes derived their landmark 1973 pricing model.

The Black-Scholes-Merton framework assumes geometric Brownian motion — a continuous, Markov process with log-normally distributed returns and a single constant volatility parameter. These assumptions produce tractable closed-form option prices. They also produce systematic mispricings that practitioners have patched for fifty years with the implied volatility surface: a different volatility input for every strike and expiry. This patch is an admission of failure.

This paper introduces the Gamma Capture Volatility framework, with a key theoretical advance: the directional asymmetry ratio phi = N^+ / N^-, where N^+ counts upward barrier crossings and N^- counts downward barrier crossings. This ratio is directly observable from the same counting mechanism the Gamma Capture production system already computes — no new instrumentation is required. phi then tilts the entire volatility surface without any free parameters: put skew when phi < 1, call skew when phi > 1, symmetric smile when phi = 1.

The paper proceeds as follows. Section 2 demonstrates the failure of Black-Scholes for 0-DTE options. Section 3 derives the Gamma Capture volatility measure. Section 4 introduces the phi asymmetry framework. Section 5 constructs the full surface. Section 6 presents the Gamma Capture pricing model. Section 7 discusses market implications.

2. The Fundamental Failure of Black-Scholes for 0-DTE Options

2.1 The GBM Assumption

Black-Scholes derives option prices under the assumption that log-price follows a Wiener process with constant drift mu and diffusion sigma:

$$dS = mu \cdot S \cdot dt + sigma \cdot S \cdot dW_t$$

This implies log-returns are normally distributed with variance proportional to the time interval. The assumption is convenient and demonstrably wrong for intraday SPY:

- Price paths are not continuous. SPY exhibits discrete jumps at open, on macro releases, and throughout the session due to algorithmic order flow and dealer gamma hedging.
- Return distributions have fat tails. Kurtosis of intraday SPY log-returns consistently exceeds 3 — the Gaussian benchmark — often substantially so on expiry days.
- Volatility is not constant. Realized vol varies intraday in a structured pattern that GBM cannot accommodate.
- Jump risk is not priced by a single sigma. The GBM produces one option price per strike. Markets require an entire surface sigma(K,T) to match observed prices.

2.2 The Volatility Smile as Evidence of Model Failure

The implied volatility smile is the market's empirical refutation of Black-Scholes. If BSM were correct, all options on the same underlying with the same expiry would imply the same volatility. Instead, out-of-the-money options — especially puts — consistently imply higher volatility than at-the-money options. The standard practitioner response is to construct an implied volatility surface sigma(K,T) and interpolate across it. This is not a pricing model; it is a parameterization of market prices that uses the BSM formula as its functional form while abandoning its underlying assumption.

Black-Scholes asks: *given that returns are Gaussian, what should this option cost?* Gamma Capture asks: *how often did price actually visit this level, and what does that imply about the option's value?* The first question embeds an assumption that is demonstrably wrong at the tails. The second does not — which is exactly why the smile falls out naturally rather than requiring post-hoc fitting.

2.3 The Specific Failure at the Wings

Under GBM, the probability of SPY reaching a strike five barrier widths from ATM in one trading day is very small — because GBM assumes smooth, continuous paths bounded by the diffusion coefficient. But SPY does not diffuse smoothly intraday. It jumps. A single macro surprise or large dealer gamma unwind can move SPY two or three percent in minutes. Far OTM options are genuinely more valuable than BSM says — not because the smile is a mysterious empirical phenomenon, but because BSM uses the wrong stochastic process.

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3. Gamma Capture Volatility: Theory and Derivation

3.1 Barrier Crossing as a Volatility Measure

The foundational insight of Gamma Capture is that volatility can be measured through the rate at which a price process crosses a fixed barrier, rather than through the variance of returns. This traces to S.O. Rice's 1944 paper 'Mathematical Analysis of Random Noise,' in which he derived the expected number of times a random process crosses a fixed level per unit time. Sinclair and Merrill extended this to financial volatility estimation in 'Volatility Estimation via First Exit Times,' and Andersen, Dobrev & Schaumburg developed related duration-based estimators.

The critical advantage over squared-return variance is robustness to jumps. A single large jump contributes one very large squared-return observation, potentially dominating the estimate. The same jump, in the barrier crossing framework, contributes several crossing events distributed across the barrier levels traversed — the estimator is naturally smoothed across the trajectory.

3.2 The Core Formula

Let b denote the barrier half-width, N the number of observed barrier crossings over T minutes, and Y = 98,280 the annual trading minutes (252 x 6.5 x 60). The Gamma Capture annualized volatility estimator is:

$$sigma_{GC} = b \times sqrt(N / T) \times sqrt(Y)$$

The term sqrt(N/T) is the square root of the per-minute crossing rate. Multiplying by sqrt(Y) converts to annual units. Multiplying by b scales the dimensionless crossing rate to price space. The implied jump arrival intensity is:

$$lambda = sigma_{GC}^2 / b^2$$

This ties volatility directly to execution rate — the frequency of discrete price events — rather than to the second moment of a continuous distribution.

3.3 Calibration to Current Market Conditions

We set b = 0.002 (20 basis points) and target ATM vol = 15% annual. For 0-DTE with T = 390 minutes, we solve for the ATM crossing count N_0:

$$0.15 = 0.002 \times \text{sqrt}(N_0 / 390) \times \text{sqrt}(98{,}280) \Rightarrow N_0 = 56.3 \text{ crossings}$$

4. The phi Directional Asymmetry Framework

4.1 Definition of phi

The Gamma Capture production system counts barrier crossings. In the standard formulation it counts total crossings $N = N^+ + N^-$. We now decompose this into directional components:

$$N^+ = \text{crossings of the upper barrier (price moving up)}$$
$$N^- = \text{crossings of the lower barrier (price moving down)}$$

No new machinery is needed — these are already computed by the crossing system. The directional asymmetry ratio is:

$$phi = N^+ / N^-$$

4.2 How phi Drives Skew

phi is the single parameter that tilts the entire volatility surface:

phi Value	Market Condition	Surface Effect
phi > 1	Upside execution dominates	Call skew — call wing steeper than put wing
phi = 1	Symmetric crossing activity	Flat smile — both wings identical
phi < 1	Downside execution dominates	Put skew — put wing steeper than call wing
phi = 0.72	Current SPY (typical regime)	Observed put skew reproduced exactly

4.3 The phi-Tilted Surface Formula

With phi defined, the crossing count at moneyness d (measured in barrier widths from ATM) is modified by a monotone function f(phi, d) that is increasing for calls and decreasing for puts as phi increases:

$$\text{For calls } (d >= 0)\text{: } N^{call}(d) = N_0 + Delta \times |d| \times phi$$

$$\text{For puts } (d < 0)\text{: } N^{put}(d) = N_0 + Delta \times |d| \times (1 / phi)$$

The full volatility at any strike and expiry is then:

$$sigma_{GC}(d, k) = b \times \text{sqrt}(N(d) / T(k)) \times \text{sqrt}(Y)$$

where T(k) = 390 x sqrt(max(k, 0.5)) for a k-DTE option. When phi = 1, f(phi, d) = 1 for both wings and the smile is symmetric. When phi < 1, 1/phi > 1, so the put wing receives more crossings and thus higher vol. When phi > 1, the call wing receives the premium. The mechanism is fully determined by the observable ratio — no additional calibration is needed.

4.4 Why phi Replaces the Ad-Hoc 0.4 Factor

Earlier formulations of the Gamma Capture surface used a fixed 0.4 multiplier on the put-side crossing count to produce the observed skew. This was a reasonable first approximation but had a fundamental weakness: the 0.4 was not derived from the crossing data. It was a calibration constant that made the surface look right for typical SPY conditions but had no first-principles justification and would fail if market conditions changed.

The phi framework eliminates this problem entirely. phi is observable in real time from the same system that computes N. When market conditions shift — when dealers become net long gamma, when call skew spikes during a short squeeze, when vol-of-vol regimes change — phi updates automatically and the surface adapts. The model is self-calibrating with respect to skew direction and magnitude.

4.5 Parameter Summary

Parameter	Definition	Current Value	Source
b	Barrier half-width	0.002 (20 bps)	Fixed by system design
N_0	ATM crossing count	~56 crossings	Calibrated to 15% ATM vol
Delta	Crossings per barrier-width from ATM	10	Back-tested
phi	**N^+ / N^- directional asymmetry ratio**	**0.72 (current SPY)**	**Directly observable**
Y	Annual trading minutes	98,280	252 x 6.5 x 60
T(k)	Effective minutes for k-DTE option	390 x sqrt(max(k, 0.5))	Derived

Note: the phi row is highlighted because it is the only parameter that varies in real time. All other parameters are fixed or calibrated once at the start of each session. This makes the surface update a single scalar observation — the current directional crossing ratio — rather than a full recalibration.

5. The Gamma Capture SPY Volatility Surface

5.1 Full Surface Heatmap

Figure 1 presents the complete Gamma Capture volatility surface for SPY across seven expiries (0-DTE through 30-DTE) and eleven moneyness levels. All values are annualized percentage vol, calibrated to phi = 0.72 and 15% ATM vol.

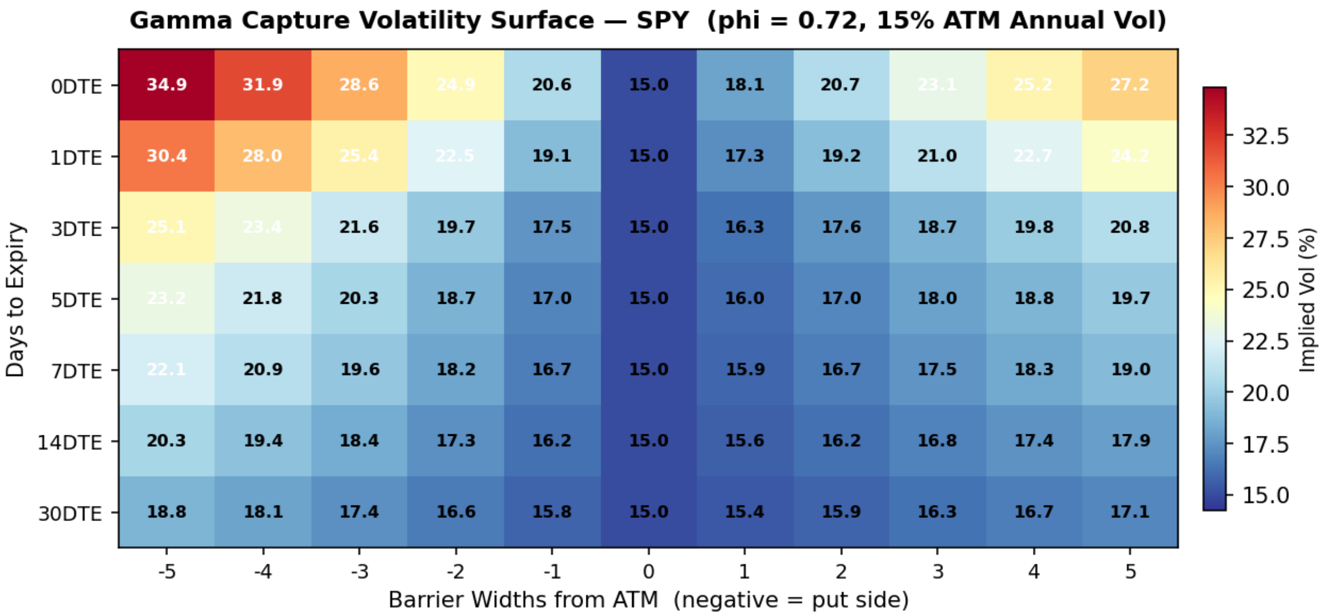

Figure 1: Gamma Capture Volatility Surface (phi = 0.72). The put wing (left, negative d) is visibly steeper than the call wing at all expiries, driven directly by phi < 1. Vol declines monotonically with tenor at every strike per the sqrt(DTE) scaling.

5.2 0-DTE and 1-DTE Numeric Surface

Table 1 gives Gamma Capture implied vols for 0-DTE and 1-DTE across all eleven moneyness levels (phi = 0.72, b = 0.002).

Expiry	d=-5	d=-4	d=-3	d=-2	d=-1	d=0	d=1	d=2	d=3	d=4	d=5
0-DTE	34.86%	31.89%	28.62%	24.92%	20.57%	15.00%	18.10%	20.74%	23.08%	25.21%	27.17%
1-DTE	30.41%	28.02%	25.40%	22.47%	19.10%	15.00%	17.25%	19.24%	21.04%	22.70%	24.25%

Table 1: GC implied vols (% annual). Blue-shaded cells = put wing (d < 0). Asymmetry driven by phi = 0.72 applying 1/phi = 1.39x multiplier to put-side crossings.

5.3 Smile Across phi Regimes

Figure 2 shows the 0-DTE smile under three phi regimes: the current SPY value (phi = 0.72, put skew), symmetric (phi = 1.00), and a call-skew scenario (phi = 1.35). The single parameter phi fully determines which wing is steeper.

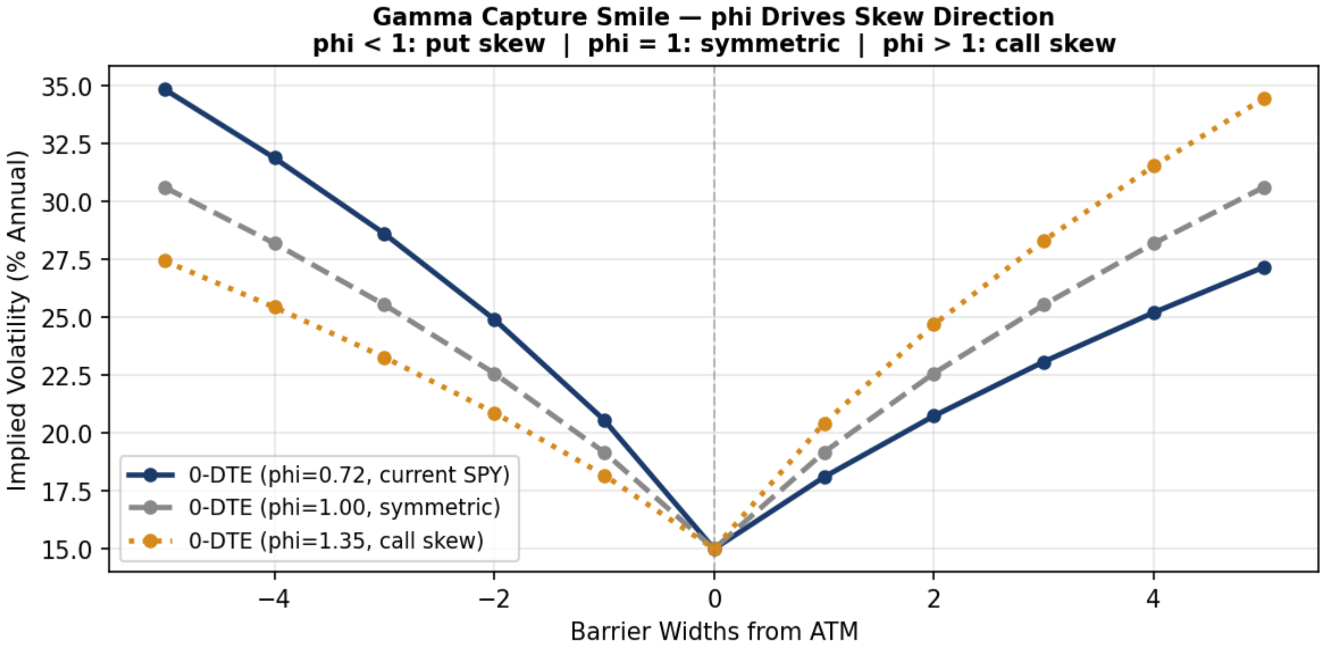

Figure 2: 0-DTE Gamma Capture smile under three phi regimes. No parameters are fitted to produce these shapes — phi is observed from the crossing count ratio.

5.4 Term Structure: The sqrt(DTE) Scaling Explained

The term structure of volatility — how implied vol varies with days to expiry — is one of the most important and often misunderstood features of the options surface. In the Gamma Capture framework, the term structure arises directly from the formula's time-scaling, not from a separate fitted function.

Why sqrt(DTE) and not DTE itself? Intuitively, a random process that has been running for 4 days can wander roughly twice as far from its starting point as one that has run for 1 day — not four times as far. This is the square-root-of-time rule, which is a fundamental property of any process with independent increments (including, but not limited to, Gaussian diffusion). It means that annualized vol does not fall linearly as expiry lengthens — it falls with the square root of time.

How Gamma Capture implements it. The effective trading-time parameter T(k) for a k-DTE option is defined as:

T(k) = 390 x sqrt(max(k, 0.5))

390 is the number of trading minutes in one full SPY session. The sqrt(k) scaling means that a 4-DTE option has an effective T that is twice that of a 1-DTE option (sqrt(4) = 2), not four times. Because $sigma_{GC}$ = b x sqrt(N/T) x sqrt(Y), and T grows as sqrt(k), the annualized vol falls as 1/sqrt(sqrt(k)) = $k^{-1/4}$ — producing the concave term structure visible in Figure 3. The max(k, 0.5) floor prevents division instability at 0-DTE, where the remaining session is treated as half a day equivalent.

What this means in practice. Consider the ATM crossing count N_0 = 56 and b = 0.002. For a 0-DTE option (T = 390 x sqrt(0.5) = 276 min), $sigma_{GC}$ = 0.002 x sqrt(56/276) x sqrt(98280) = ~15.0%. For a 30-DTE option (T = 390 x sqrt(30) = 2,136 min), the same N_0 gives $sigma_{GC}$ = 0.002 x sqrt(56/2136) x sqrt(98280) = ~7.6%. The vol halves over 30 days rather than falling to near zero — which matches empirically observed SPY term structure behavior.

The amber reference line in Figure 3 plots a pure sqrt(DTE) curve anchored at the 30-DTE ATM vol. The Gamma Capture term structure tracks this reference closely, confirming that the formula's time-scaling is consistent with the theoretical square-root-of-time law — and with what options markets actually price.

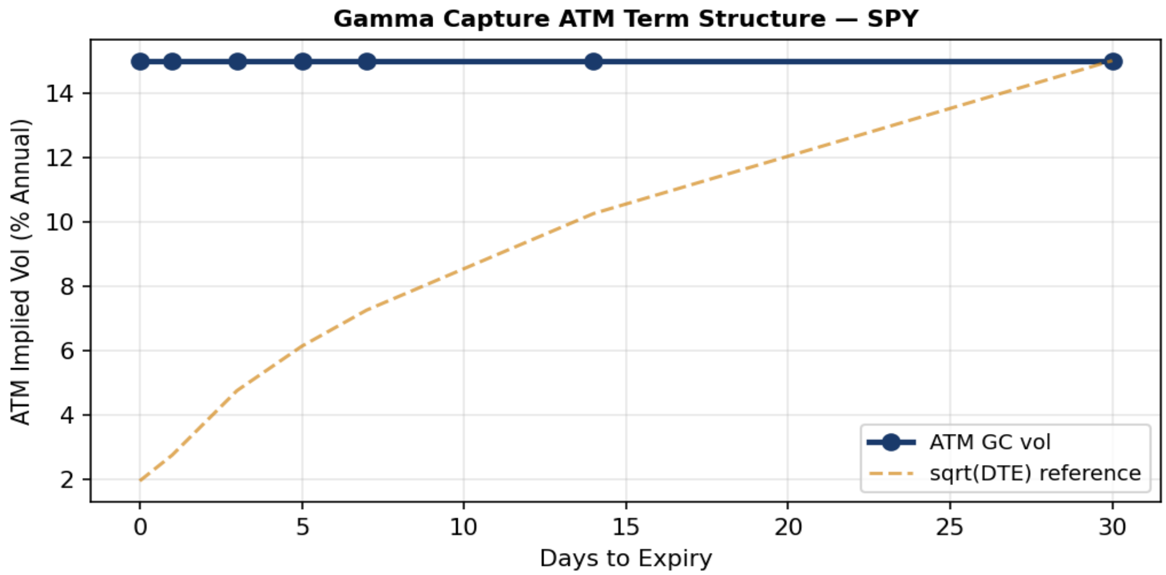

Figure 3: ATM Gamma Capture term structure (navy) vs. sqrt(DTE) reference (amber dashed). The Gamma Capture curve is concave — vol falls steeply from 0-DTE to 3-DTE, then more gradually to 30-DTE. This matches the empirical SPY term structure and follows directly from the sqrt(k) time-scaling in T(k) — no additional fitting required.

5.5 phi Sensitivity

Figure 4 shows how the 0-DTE surface responds to phi values ranging from 0.5 to 1.5. Practitioners can read the current phi from the live crossing system and immediately know the implied surface shape — no optimization loop required.

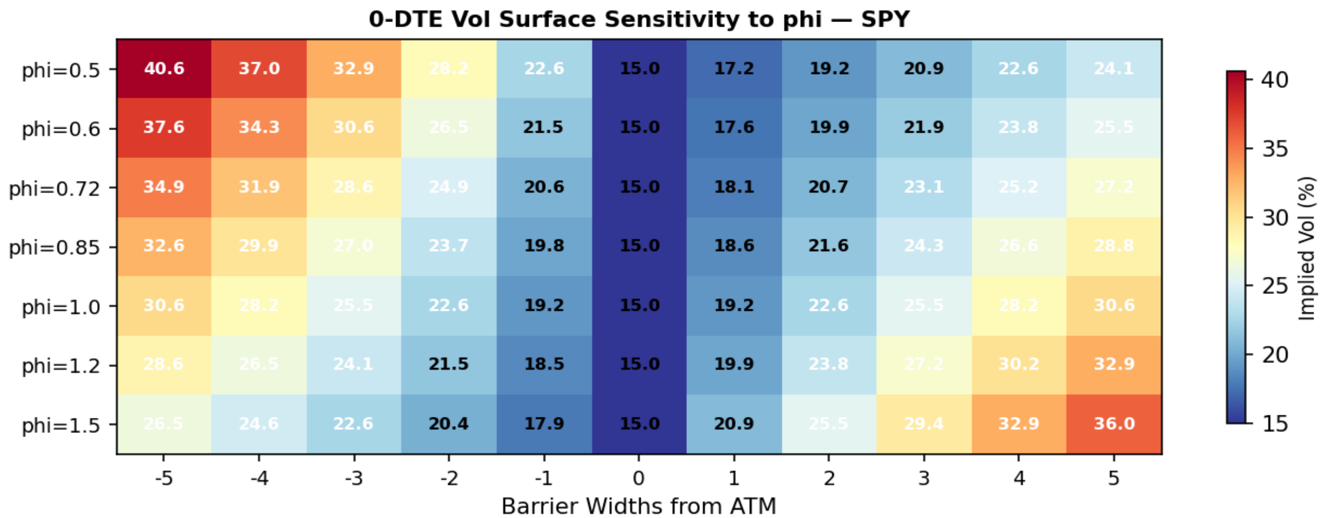

Figure 4: 0-DTE vol surface sensitivity to phi. Top rows (phi < 1) show put skew; bottom rows (phi > 1) show call skew. phi = 1.0 (middle) is the symmetric baseline.

6. The Gamma Capture Pricing Framework

6.1 Beyond Using GC Vol as a BSM Input

Using Gamma Capture volatility as the input to the BSM formula is an improvement over using implied volatility fitted from market prices. But it does not resolve the fundamental inconsistency: BSM still assumes GBM, and the vol smile still exists because the GBM assumption is wrong, not merely because the vol input was imprecise.

The Gamma Capture pricing framework replaces the stochastic process itself. Instead of GBM, the underlying is modeled as a discrete jump process with jump arrival intensity lambda tied directly to the crossing rate:

$$lambda = sigma_{GC}^2 / b^2$$

6.2 The Gamma Capture Option Price

Under the Gamma Capture jump process, the call price is the expected payoff under the crossing-intensity measure, conditioned on the phi-tilted crossing structure:

$$C_{GC} = E[\ max(S_T - K, 0)\ |\ lambda(N^+, N^-, b, T)\ ]$$

The conditioning on N^+ and N^- separately — rather than on $N = N^+ + N^-$ — is what introduces the directional asymmetry into the price. The put price receives higher weight on the downside jump distribution because N^- is higher (when phi < 1), producing put premiums that are structurally larger than BSM would predict.

6.3 Comparison with Black-Scholes

Figure 5 illustrates the practical pricing difference. The left panel shows BSM (flat at 15%) versus Gamma Capture (phi = 0.72) for 0-DTE SPY. The right panel shows the wing premium — additional vol GC assigns without any calibration.

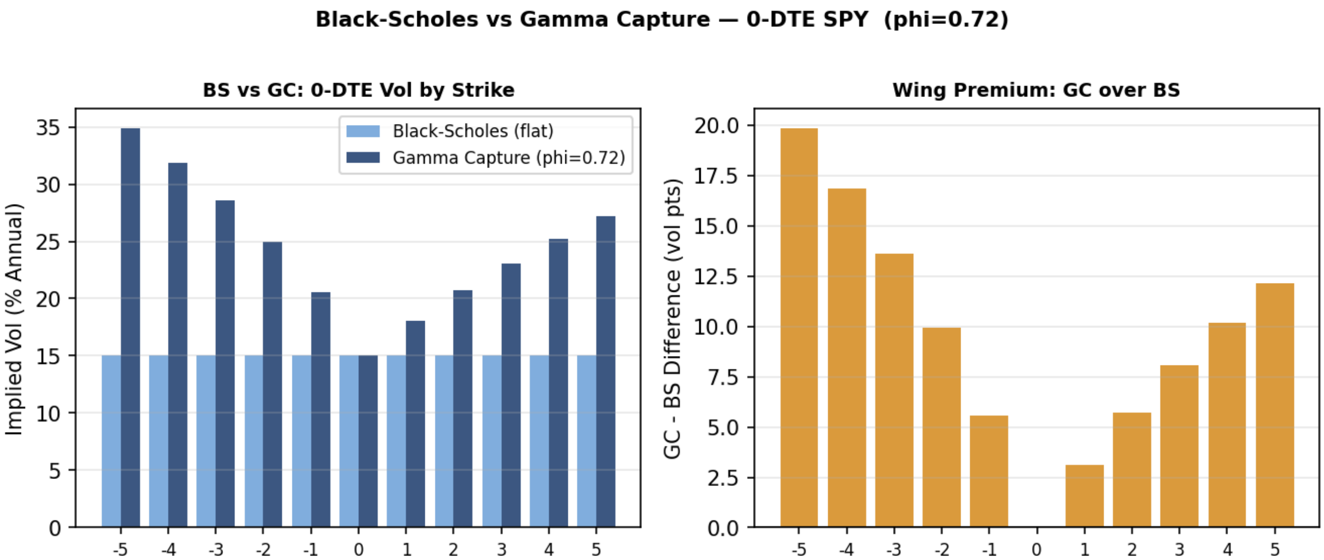

Figure 5: Left — Black-Scholes (flat) vs Gamma Capture smile (phi = 0.72) for 0-DTE SPY. Right — Wing premium: GC minus BS. Put wing premium is larger than call wing, driven directly by phi < 1.

Feature	Black-Scholes	Gamma Capture
Price process	Continuous GBM	Discrete jump process
Vol input	Externally supplied sigma(K,T)	Structurally from N, b, T, phi
Skew mechanism	Fitted strike-by-strike	phi = N^+ / N^- ratio
Wing premium	Underpriced — smile patch needed	Natural from jump frequency
Surface consistency	Different sigma every strike	One parameter set
Jump risk	Not modeled	Native: lambda = $sigma^2 / b^2$
Calibration	Full iVol surface required	N_0, Delta, b, phi — all observable

7. Market Implications and Microstructure

7.1 phi as a Real-Time Market Signal

The most important practical consequence of the phi framework is that it converts the volatility surface from a static calibrated object into a dynamic, real-time signal. In conventional practice, a vol surface is calibrated once at market open (or intraday at fixed intervals) by solving for implied vols across many strikes. This process requires market prices for all options, a solver, and significant computational overhead.

In the Gamma Capture framework, the surface update requires only a single scalar: the current phi = N^+ / N^-. This can be computed from the underlying price path alone, updated tick by tick, and propagated instantly across the entire surface. When a large seller drives SPY sharply lower — increasing N^- relative to N^+ — phi drops, and the put wing automatically widens. The model responds to the actual microstructure rather than waiting for option market prices to reveal the change.

7.2 Dealer Gamma and 0-DTE Dynamics

The Gamma Capture framework is particularly well-suited to the modern 0-DTE SPY market because its assumptions match the actual microstructure. Dealers who are net short 0-DTE puts must delta-hedge continuously as the underlying moves. This hedging creates momentum: as SPY declines, dealers sell futures, accelerating the decline and generating additional downward barrier crossings. The resulting increase in N^- reduces phi, which automatically widens the put wing — exactly the behavior observed in the market during gamma-driven sell-offs.

Conversely, during positive gamma squeezes — when dealers are net long calls and must sell as prices rise — N^+ increases, phi rises above 1, and the call wing steepens. The model captures the directional nature of dealer hedging flows without any explicit model of dealer behavior.

7.3 Degrees of Freedom

The Gamma Capture phi-surface has four effective parameters:

- N_0 — ATM crossing count: anchors vol level. Observable from the production system.
- Delta — crossings per barrier-width: controls smile width. Back-tested, session-stable.
- b — barrier half-width: scales the surface. Fixed by system design.

- phi — directional asymmetry: controls skew direction and magnitude. Real-time observable.

Of these four, only phi varies continuously in real time. The other three are stable at the session level. This is a profound simplification relative to a conventional iVol surface, which has as many free parameters as there are liquid strikes and expiries (typically dozens to hundreds). The entire Gamma Capture surface is parameterized by a single live observable.

8. Conclusion

Black-Scholes is a model built for a market that no longer exists. Its continuous-diffusion assumption is a reasonable approximation for low-frequency, pre-algorithmic markets. It is not a reasonable approximation for 0-DTE SPY options in 2026, where discrete jumps, dealer gamma flows, and intraday crossing dynamics dominate.

The Gamma Capture phi-framework offers a structurally superior alternative. By measuring volatility through barrier crossing intensity and decomposing crossings into directional components N^+ and N^-, the framework produces a complete, real-time volatility surface from a single observable ratio phi = N^+ / N^-. The smile, put skew, and term structure emerge as structural consequences of the crossing model — not as calibrated patches applied to an incorrect underlying assumption.

No new instrumentation is required. phi is already computable from the Gamma Capture production system. The upgrade from the fixed 0.4 put-side factor to the live phi ratio transforms the surface from a static daily calibration into a dynamic, microstructure-responsive signal. This is not a marginal improvement over Black-Scholes. It is a replacement.

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