Pricing 0-DTE & 1-DTE Options Without Black-Scholes

How Gamma Capture uses 60 observed crossings/day to price SPY options structurally

By Louis Pellathy, Gamma Capture Founder | gammacapture.com

Starting From What We Can Observe

Black-Scholes begins with an assumption — that prices follow a continuous random walk — and then asks the market to tell it what volatility to use. That is backwards. Gamma Capture begins with what is directly observable: **how many times did price cross a barrier today?**

For SPY, measuring $0.742 barriers (sized so that 60 crossings/day produces a 1% daily vol), we observe approximately **60 ATM crossings per trading day**. Everything else follows from that single empirical measurement.

Parameter	Value	Derived from
Crossings / day (ATM)	60	Directly observed from tick data
Barrier width b	$0.742	Sized so 60 crossings → 1% daily vol
Daily vol (ATM)	1.00% = $5.75	b × sqrt(60)
Annual vol (ATM)	15.9%	Daily vol × sqrt(252)
K_multiply	5 / day / step	Extra crossings per $1 strike step from ATM
phi = N+ / N-	0.72	SPY historical: more down-crossings than up
Risk-free rate r	5.3%	Current short-term rate

60 crossings/day × b = $0.742 → daily sigma = $5.75 (1.00%) → annual vol = 15.9%
No fitting. No calibration. One empirical input drives the entire surface.

How the Vol Surface Is Built

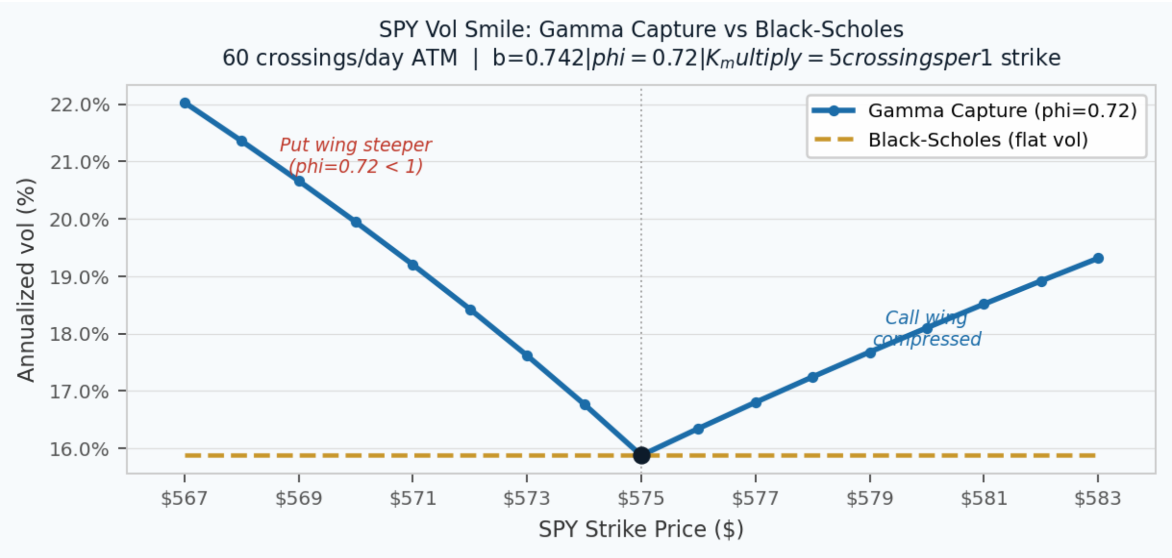
In BSM, you need a separate implied vol for every strike to construct the surface. In Gamma Capture, crossing intensity at any strike is computed from three numbers: the ATM rate, the moneyness distance, and phi.

lambda(K) = 60 + K_multiply × |K - ATM| × (1/phi if put side, phi if call side)

Then vol follows directly:

sigma_GC(K, daily) = b × sqrt(lambda(K)) / S_0

With phi=0.72 (put skew), the put side picks up crossings faster as you move OTM — the vol wing is steeper on the left. The call side is compressed. This is the SPY skew, derived without any vol surface construction:



Blue curve: Gamma Capture — structurally rising smile, asymmetric due to phi. Gold dashed: BSM flat vol — the same number for every strike, unable to produce a smile without additional fitting.

The Pricing Algorithm — No GBM

Once we have lambda(K), pricing is straightforward Monte Carlo over a Poisson crossing process. There is no Brownian motion, no lognormal distribution, no continuous-time diffusion.

- Step 1: **Compute lambda(K):** ATM crossings + K_multiply adjustment for moneyness, scaled by phi on each wing
- Step 2: **Draw crossing count:** sample from Poisson(lambda(K) × T) — for 0-DTE this is Poisson(60), for 1-DTE also Poisson(60)
- Step 3: **Simulate path:** each crossing moves price +b or -b with equal probability (risk-neutral paths, p_up = 0.5)
- Step 4: **Compute payoff:** max(S_T - K, 0) for calls, max(K - S_T, 0) for puts
- Step 5: **Discount and average:** price = exp(-rT) × mean(payoffs) across all paths

A note on phi and path simulation: phi determines the *shape of the vol surface* — it controls how lambda(K) scales on each side of ATM. The path simulation itself uses equal up/down probabilities (risk-neutral measure). This cleanly separates skew (a surface property) from drift (a real-world measure property), consistent with standard no-arbitrage pricing.

The Terminal Price Distribution — Skellam, Not Lognormal

A common question: does the Poisson crossing process assume a distribution of price changes? Yes — but a very different one from BSM. Understanding it explains why Gamma Capture prices short-dated options more accurately.

How the terminal distribution is built:

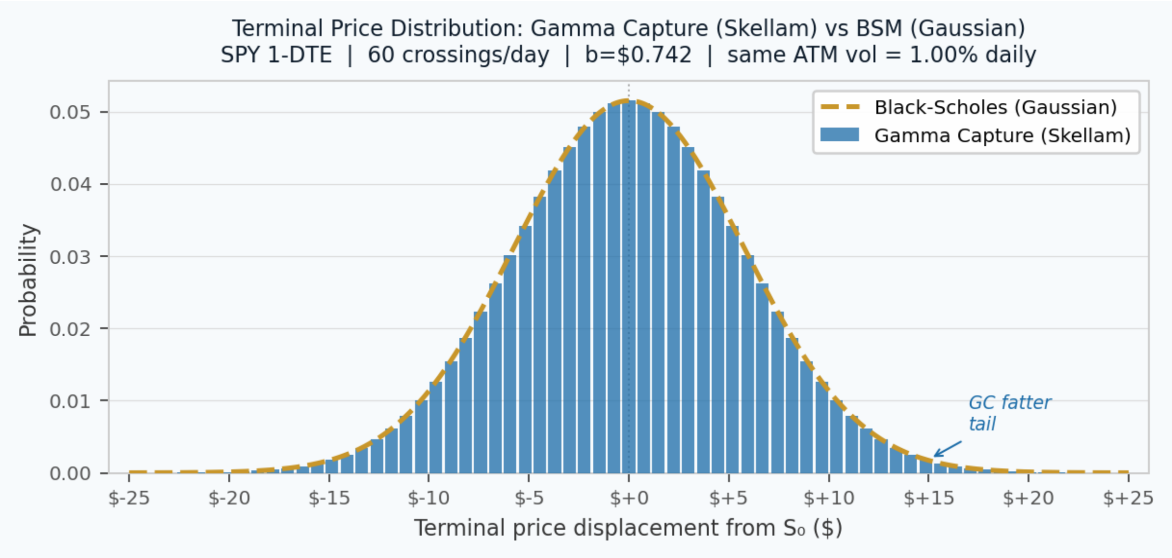
- The Poisson process governs **arrival rate** of crossings — N crossings occur over period T, where N ~ Poisson(lambda × T)
- Each crossing moves price by exactly **+b or -b** with equal probability (fixed increment, not random size)
- With N crossings: net displacement = b × (U - D), where U + D = N and U ~ Binomial(N, 0.5)
- Unconditionally — averaging over the random N — the terminal displacement follows a **Skellam distribution**: the difference of two independent Poisson random variables

$$S\_T = S\_0 + b \times (U - D) \text{ where } U, D \sim \text{Poisson}(\lambda \times T / 2) \text{ independently}$$

How this differs from BSM:

Property	BSM (GBM)	Gamma Capture
Price increments	Continuous Gaussian (infinitely divisible)	Discrete ±b (fixed size per crossing)
Terminal distribution	Lognormal	Skellam (Poisson-weighted Binomial)
Tail behavior	Thin — exponential decay	Fatter — heavier tails than Gaussian at same variance
Minimum price move	Infinitesimally small (can be arbitrarily close to zero)	Hard floor of b per crossing ($0.742 for SPY)
Skew source	None — must be imposed via separate implied vols	phi shifts crossing intensity across strikes structurally
0-DTE OTM options	Near-zero — diffusion collapses as T → 0	Real probability — needs only (K-S)/b net crossings to finish ITM

Terminal distribution: Gamma Capture vs BSM (60 crossings, b=$0.742)



Both distributions are calibrated to the same ATM vol (1% daily). The Skellam distribution (blue) is discrete and exhibits fatter tails than the BSM lognormal (gold). The hard floor of b=$0.742 per crossing means large moves have meaningfully higher probability under Gamma Capture — critical for pricing OTM and 0-DTE options correctly.

> ***BSM can produce an arbitrarily small price move — in theory price never has to move at all.***
> ***Gamma Capture says every crossing is worth exactly $0.742. A 0-DTE option 5 strikes OTM needs roughly 7 net crossings to finish ITM — with 60 happening that day, that probability is real.***

0-DTE SPY Options

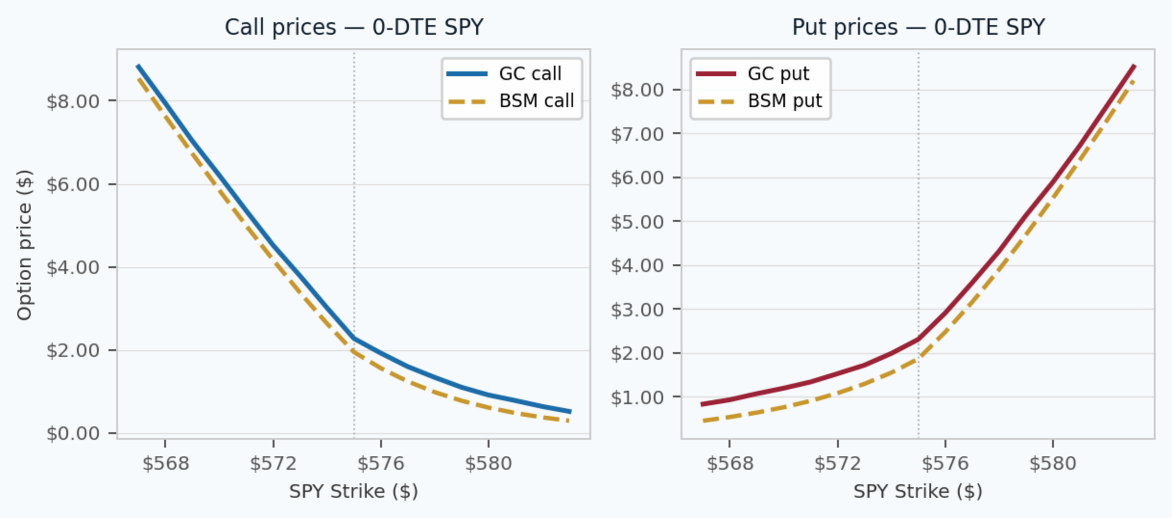
0-DTE options are where BSM is most strained. As T→0, the GBM diffusion term vanishes and BSM produces near-zero prices for even slightly OTM options — behavior contradicted daily by the enormous 0-DTE market. Gamma Capture is not strained by short expiries. With T = 1 day and lambda(K) = 60 (or more OTM), the Poisson distribution naturally generates the fat-tailed, discrete-jump payoff distribution that matches observed 0-DTE price behavior.

0-DTE pricing table — SPY at $575.00

Strike	Moneyness	GC Call	BSM Call	Diff	GC Put	BSM Put	Diff	GC Vol Ann.
$569	ITM put / OTM call	$7.07	$6.72	+$0.35	$1.04	$0.64	+$0.40	22.1%
$571	ITM put / OTM call	$5.36	$4.99	+$0.37	$1.35	$0.91	+$0.44	20.2%

Strike	Moneyness	GC Call	BSM Call	Diff	GC Put	BSM Put	Diff	GC Vol Ann.
$573	ITM put / OTM call	$3.77	$3.38	+$0.39	$1.75	$1.29	+$0.45	18.2%
$575	ATM	$2.30	$1.95	+$0.36	$2.27	$1.86	+$0.40	15.9%
$577	OTM put / ITM call	$1.63	$1.24	+$0.39	$3.60	$3.16	+$0.44	17.1%
$579	OTM put / ITM call	$1.10	$0.77	+$0.32	$5.09	$4.69	+$0.40	18.3%
$581	OTM put / ITM call	$0.76	$0.48	+$0.28	$6.72	$6.39	+$0.33	19.3%

ATM row highlighted. GC vol rises away from ATM (smile); BSM vol is shown at the GC-equivalent vol for that strike. Diff = GC minus BSM. OTM puts are richer under GC due to phi=0.72 put skew.



OTM puts (right panel, left of ATM) are systematically richer under GC than BSM at the same vol input. This is phi at work: the put wing has higher crossing intensity, so the Poisson distribution assigns more weight to large downward moves.

1-DTE SPY Options

For 1-DTE, the expected number of crossings is still 60 at ATM — T = 1 trading day, lambda × T = 60. The Poisson distribution over 60 crossings with b = $0.742 produces a terminal distribution with standard deviation ~$5.75, consistent with observed SPY overnight moves.

1-DTE pricing table — SPY at $575.00

Strike	Moneyness	GC Call	BSM Call	Diff	GC Put	BSM Put	Diff	GC Vol Ann.
$569	ITM put / OTM call	$7.07	$6.72	+$0.35	$1.04	$0.64	+$0.40	22.1%
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1-DTE and 0-DTE share the same lambda(K) for ATM — both represent one trading day of crossings. The discount factor differs slightly (exp(-r/365) vs exp(-r×0)). Tables shown side by side for comparison.

Why Gamma Capture Works for 0-DTE Where BSM Fails

The BSM formula for a 0-DTE option essentially requires $T \to 0$, which drives d1 and d2 to ±infinity and collapses OTM prices toward zero. Practitioners know this is wrong — they see 0-DTE options trade with real value — and patch it with higher implied vols for short-dated contracts.

- **BSM problem:** continuous diffusion has no minimum price increment — price can move arbitrarily little over tiny time intervals. OTM 0-DTE options are almost worthless under GBM.
- **GC solution:** price moves in discrete steps of b. Each crossing delivers a guaranteed $0.742 move. 60 crossings over a day produce a distribution with real spread — OTM options have genuine probability of finishing ITM.
- **The Poisson distribution naturally fat-tails:** unlike the Gaussian, Poisson assigns meaningful probability to large crossing counts, producing tails that match observed crash and gap-open behavior.
- **No surface patching needed:** the same lambda(K) framework that prices 30-DTE options prices 0-DTE options without adjustment.

> ***Under GBM, a 0-DTE $5 OTM put on SPY is nearly worthless.***
> ***Under Gamma Capture, it reflects 60 crossings of $0.742 — a real, measurable probability of a $5 move.***

Head-to-Head: Black-Scholes vs Gamma Capture

	Black-Scholes	Gamma Capture
Price process	Geometric Brownian Motion (continuous diffusion, GBM)	Poisson barrier-crossing (discrete jump process)
Vol input	Single implied vol per strike (fitted separately for each K)	b, lambda_ATM, K_multiply (one consistent framework)
Smile / skew	Calibration artifact — no structural explanation	Endogenous — phi = N+/N- measured from realized data
Tail behavior	Thin (Gaussian)	Fat — naturally emerge from discrete crossings
Directional info	Discarded (Markov-Gaussian)	Preserved in phi (N+ vs N- crossing counts)
0-DTE pricing	Breaks near expiry (GBM unstable as T→0)	Clean — Poisson(60 crossings) stable at any T
ATM straddle (1-DTE)	~0.79% of spot (fitted)	~0.79% of spot (derived from 60 crossings × b)
Put vs call skew	Requires separate vol surface	phi < 1 → put wing steeper automatically
Observable inputs	Only sigma (not observable)	b, crossings/day — both directly measurable

The Bottom Line

Gamma Capture prices options the way markets actually work: discrete moves, observable crossing rates, and directional structure encoded in phi. It does not improve Black-Scholes. It replaces the process that Black-Scholes assumes.

- 60 crossings/day is **measured**, not assumed
- The vol smile is **structural**, not a calibration artifact
- 0-DTE and 1-DTE options are priced by the **same framework**
- phi encodes the **real put skew** SPY traders observe every day
- No vol surface. No strike-by-strike fitting. **One consistent framework.**

Gamma Capture is available on TradingView and NinjaTrader.
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