Catalog

Exercise 1.1

1. Prove that the product of two consecutive positive integers is divisible by 2.

Sol:

Let, (n – 1) and n be two consecutive positive integers

∴ Their product = n(n – 1)

$= n^2 - n$

We know that any positive integer is of the form 2q or 2q + 1, for some integer q

When n =2q, we have

$n^2 - n = (2q)^2 - 2q$

$= 4q^2 - 2q$

$2q(2q - 1)$

Then $n^2 - n$ is divisible by 2.

When n = 2q + 1, we have

$n^2 - n = (2q + 1)^2 - (2q + 1)$

= $4q^2 + 4q + 1 - 2q - 1$

= $4q^2 + 2q$

= $2q(2q + 1)$

Then $n^2 - n$ is divisible by 2.

Hence the product of two consecutive positive integers is divisible by 2.

2. If a and b are two odd positive integers such that a > b, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Sol:

Let a = 2q + 3 and b = 2q + 1 be two positive odd integers such that a > b

Now, $\frac{a+b}{2} = \frac{2q+3+2q+1}{2} = \frac{4q+4}{2} = 2q + 2 = an\ even\ number$

and $\frac{a-b}{2} = \frac{(2q+3)-(2q+1)}{2} = \frac{2q+3-2q-1}{2} = \frac{2}{2} = 1 = an\ odd\ number$

Hence one of the two numbers $\frac{a+b}{2}$ *and* $\frac{a-b}{2}$ *is odd and the other is even for any two positive odd integer*

3. Show that the square of an odd positive integer is of the form 8q + 1, for some integer q.

Sol:

By Euclid's division algorithm

a = bq + r, where 0 ≤ r ≤ b

Put b = 4

a = 4q + r, where 0 ≤ r ≤ 4

If r = 0, then a = 4q even

If r = 1, then a = 4q + 1 odd

If r = 2, then a = 4q + 2 even
If r = 3, then a = 4q + 3 odd
Now, $(4q+1)^2 = (4q)^2 + 2(4q)(1) + (1)^2$
= $16q^2 + 8q + 1$
= $8(2q^2 + q) + 1$
= 8m + 1 where m is some integer
Hence the square of an odd integer is of the form 8q + 1, for some integer q

4. Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Sol:

Let a be any odd positive integer we need to prove that a is of the form 6q + 1, or 6q +3, 6q + 5, where q is some integer
Since a is an integer consider b = 6 another integer applying Euclid's division lemma we get
a = 6q + r for some integer q ≥ 0, and r = 0, 1, 2, 3, 4, 5 since 0 ≤ r < 6.
Therefore, a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5
However since a is odd so a cannot take the values 6q, 6q + 2 and 6q + 4
(since all these are divisible by 2)
Also, 6q + 1 = 2 × 3q + 1 = 2k 1 + 1, where k1 is a positive integer
6q + 3 = (6q + 2) + 1 = 2 (3q + 1) + 1 = 2k2 + 1, where k2 is an integer
6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k3 + 1, where k3 is an integer
Clearly, 6q +1, 6q +3, 6q + 5 are of the form 2k + 1, where k is an integer
Therefore, 6q + 1, 6q + 3, 6q + 5 are odd numbers.
Therefore, any odd integer can be expressed is of the form
6q + 1, or 6q + 3, 6q + 5 where q is some integer
Concept insight: In order to solve such problems Euclid's division lemma is applied to two integers a and b the integer b must be taken in accordance with what is to be proved, for example here the integer b was taken 6 because a must be of the form 6q + 1, 6q +3, 6q + 5 Basic definition of even (divisible by 2) and odd numbers (not divisible by 2) and the fact that addiction and multiplication of integers is always an integer are applicable here.

5. Prove that the square of any positive integer is of the form 3m or, 3m + 1 but not of the form 3m +2.

Sol:

By Euclid's division algorithm
a = bq + r, where 0 ≤ r ≤ b
Put b = 3
a = 3q + r, where 0 ≤ r ≤ 3
If r = 0, then a = 3q

If r = 1, then a = 3q + 1
If r = 2, then a = 3q + 2
Now, $(3q)^2 = 9q^2$
$= 3 \times 3q^2$
$= 3m, where\ m\ is\ some\ integer$
$(3q+1)^2 = (3q)^2 + 2(3q)(1) + (1)^2$
$= 9q^2 + 6q + 1$
$= 3(3q^2 + 2q) + 1$
$= 3m + 1, where\ m\ is\ some\ integer$
$(3q+2)^2 = (3q)^2 + 2(3q)(2) + (2)^2$
$= 9q^2 + 12q + 4$
$= 9q^2 + 12q + 4$
$= 3(3q^2 + 4q + 1) + 1$
$= 3m + 1, where\ m\ is\ some\ integer$
Hence the square of any positive integer is of the form 3m, or 3m +1
But not of the form 3m + 2

6. Prove that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.
Sol:
By Euclid's division Algorithm
a = bm + r, where 0 ≤ r ≤ b
Put b = 4
a = 4m + r, where 0 ≤ r ≤ 4
If r = 0, then a = 4m
If r = 1, then a = 4m + 1
If r = 2, then a = 4m + 2
If r = 3, then a = 4m + 3
Now, $(4m)^2 = 16m^2$
$= 4 \times 4m^2$
= 4q where q is some integer
$(4m+1)^2 = (4m)^2 + 2(4m)(1) + (1)^2$
$= 16m^2 + 8m + 1$
$= 4(4m^2 + 2m) + 1$
$= 4q + 1$ where q is some integer
$(4m+2)^2 = (4m)^2 + 2(4m)(2) + (2)^2$
$= 16m^2 + 24m + 9$
$= 16m^2 + 24m + 8 + 1$
$= 4(4m^2 + 6m + 2) + 1$
$= 4q + 1, where\ q\ is\ some\ integer$

Hence, the square of any positive integer is of the form 4q or 4q + 1 for some integer m

7. Prove that the square of any positive integer is of the form 5q, 5q + 1, 5q + 4 for some integer q.

Sol:

By Euclid's division algorithm

a = bm + r, where $0 \le r \le b$

Put b = 5

a = 5m + r, where $0 \le r \le 4$

If r = 0, then a = 5m

If r = 1, then a = 5m + 1

If r = 2, then a = 5m + 2

If r = 3, then a = 5m + 3

If r = 4, then a = 5m + 4

Now, $(5m)^2 = 25m^2$

$= 5(5m^2)$

= 5q where q is some integer

$(5m+1)^2 = (5m)^2 + 2(5m)(1) + (1)^2$

$= 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$

$= 5q + 1\ where\ q\ is\ some\ integer$

$(5m+1)^2 = (5m)^2 + 2(5m)(1)(1)^2$

$= 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$

$= 5q + 1$ where q is some integer

$= (5m+2)^2 = (5m)^2 + 2(5m)(2) + (2)^2$

$= 25m^2 + 20m + 4$

$= 5(5m^2 + 4m) + 4$

$= 5q + 4, where\ q\ is\ some\ integer$

$= (5m+3)^2 = (5m)^2 + 2(5m)(3) + (3)^2$

$= 25m^2 + 30m + 9$

$= 25m^2 + 30m + 5 + 4$

$= 5(5m^2 + 6m + 1) + 4$

$= 5q + 1, where\ q\ is\ some\ integer$

$= (5m+4)^2 = (5m)^2 + 2(5m)(4) + (4)^2$

$= 25m^2 + 40m + 16$

$= 25m^2 + 40m + 15 + 1$

$= 5(5m^2) + 2(5m)(4) + (4)^2$

$= 5q + 1, where\ q\ is\ some\ integer$

Hence, the square of any positive integer is of the form 5q or 5q + 1, 5q + 4 for some integer q.

8. Prove that if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q, but not conversely.

Sol:

Let, n = 6q + 5, when q is a positive integer

We know that any positive integer is of the form 3k, or 3k + 1, or 3k + 2

∴ q = 3k or 3k + 1, or 3k + 2

If q = 3k, then

n = 6q + 5

= 6(3k) + 5

= 18k + 5

= 18k + 3 + 2

= 3(6k + 1) + 2

= 3m + 2, where m is some integer

If q = 3k + 1, then

n = 6q + 5

= 6(3k + 1) + 5

= 18k + 6 + 5

= 18k + 11

= 3(6k + 3) + 2

= 3m + 2, where m is some integer

If q = 3k + 2, then

n = 6q + 5

= 6(3k + 2) + 5

= 18k + 12 + 5

= 18k + 17

= 3(6k + 5) + 2

= 3m + 2, where m is some integer

Hence, if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q.

Conversely

Let n = 3q + 2

We know that a positive integer can be of the form 6k + 1, 6k + 2, 6k + 3, 6k + 4 or 6k + 5

So, now if q = 6k + 1 then

n = 3(6k + 1) + 2

= 18k + 5

= 6(3k) + 5

= 6m + 5, where m is some integer
So, now if q = 6k + 2 then
n = 3(6k + 2) + 2
= 18k + 8
= 6 (3k + 1) + 2
= 6m + 2, where m is some integer
Now, this is not of the form 6m + 5
Hence, if n is of the form 3q + 2, then it necessarily won't be of the form 6q + 5 always.

9. Prove that the square of any positive integer of the form 5q + 1 is of the same form.
Sol:
Let n = 5q + 1 where q is a positive integer
$\therefore n^2 = (5q + 1)^2$
= $25q^2 + 10q + 1$
= $5(5q^2 + 2q) + 1$
= 5m + 1, where m is some integer
Hence, the square of any positive integer of the form 5q + 1 is of the same form.

10. Prove that the product of three consecutive positive integer is divisible by 6.
Sol:
Let, n be any positive integer. Since any positive integer is of the form 6q or 6q + 1 or 6q + 2 or, 6q + 3 or 6q + 4 or 6q + 5.
If n = 6q, then
n(n + 1)(n + 2) = (6q + 1)(6q + 2)(6q + 3)
= 6[(6q + 1)(3q + 1)(2q + 1)]
= 6m, which is divisible by 6?
If n = 6q + 1, then
n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)
= 6[(6q + 1)(3q + 1)(2q + 1)]
= 6m, which is divisible by 6
If n = 6q + 2, then
n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)
= 6[(3q + 1)(2q + 1)(6q + 4)]
= 6m, which is divisible by 6.
If n = 6q + 3, then
n(n + 1)(n + 2) = (6q + 3)(6q + 4)(6q + 5)
= 6[(6q + 1)(3q + 2)(2q + 5)]
= 6m, which is divisible by 6.
If n = 6q + 4, then
n(n + 1)(n + 2) = (6q + 4)(6q + 5)(6q + 6)

= 6[(6q + 4)(3q + 5)(2q + 1)]
= 6m, which is divisible by 6.
If n = 6q + 5, then
n(n + 1)(n + 2) = (6q + 5)(6q + 6)(6q + 7)
= 6[(6q + 5)(q + 1)(6q + 7)]
= 6m, which is divisible by 6.
Hero, the product of three consecutive positive integer is divisible by 6.

11. For any positive integer n , prove that $n^3 - n$ divisible by 6.
Sol:
We have $n^3 - n = n(n^2 - 1) = (n-1)\,(n)\,(n+1)$
Let, n be any positive integer. Since any positive integer is of the form 6q or 6q + 1 or, 6q + 2 or, 6q + 3 or, 6q + 4 or, 6q + 5.
If n = 6q, then
$(n-1)(n)(n+1) = (6q-1)(6q)(6q+1)$
$= 6[(6q-1)(q)(6q+1)]$
= 6m, which is divisible by 6
If n = 6q + 1, then
$(n-1)(n+1) = (6q)(6q+1)(6q+2)$
$= 6[(q)(6q+1)(6q+2)]$
= 6m, which is divisible by 6
If n = 6q + 2, then
$(n-1)(n)(n+1) = (6q+1)(6q+2)(6q+3)$
$= 6[(6q+1)(3q+1)(2q+1)]$
= 6m, which is divisible by 6
If n = 6q + 3, then
$(n-1)(n)(n+1) = (6q+3)(6q+4)(6q+5)$
$= 6[(3q+1)(2q+1)(6q+4)]$
= 6m, which is divisible by 6
If n = 6q + 4, then
$(n-1)(n)(n+1) = (6q+3)(6q+4)(6q+5)$
$= 6[(2q+1)(3q+2)(6q+5)]$
= 6m, which is divisible by 6
If n = 6q + 5, then
$(n-1)(n)(n+1) = (6q+4)(6q+5)(6q+6)$
$= 6[(6q+4)(6q+5)(q+1)]$
= 6m, which is divisible by 6
Hence, for any positive integer n, $n^3 – n$ is divisible by 6.

Exercise 1.2

1. Define HOE of two positive integers and find the HCF of the following pairs of numbers:
(i) 32 and 54 (ii) 18 and 24 (iii) 70 and 30 (iv) 56 and 88
(v) 475 and 495 (vi) 75 and 243 (vii) 240 and 6552 (viii) 155 and 1385
(ix) 100 and 190 (x) 105 and 120

Sol:

By applying Euclid's division lemma

(i) $5y = 32 \times 1 + 22$
Since remainder $\neq 0$, apply division lemma on division of 32 and remainder 22.
$32 = 22 \times 1 + 10$
Since remainder $\neq 0$, apply division lemma on division of 22 and remainder 10.
$22 = 10 \times 2 + 2$
Since remainder $\neq 0$, apply division lemma on division of 10 and remainder 2.
$10 = 2 \times 5$ [remainder 0]
Hence, HCF of 32 and 54 10 2

(ii) By applying division lemma
$24 = 18 \times 1 + 6$
Since remainder = 6, apply division lemma on divisor of 18 and remainder 6.
$18 = 6 \times 3 + 0$
∴ Hence, HCF of 18 and 24 = 6

(iii) By applying Euclid's division lemma
$70 = 30 \times 2 + 10$
Since remainder $\neq 0$, apply division lemma on divisor of 30 and remainder 10.
$30 = 10 \times 3 + 0$
∴ Hence HCF of 70 and 30 is = 10.

(iv) By applying Euclid's division lemma
$88 = 56 \times 1 + 32$
Since remainder $\neq 0$, apply division lemma on divisor of 56 and remainder 32.
$56 = 32 \times 1 + 24$
Since remainder $\neq 0$, apply division lemma on divisor of 32 and remainder 24.
$32 = 24 \times 1 + 8$
Since remainder $\neq 0$, apply division lemma on divisor of 24 and remainder 8.
$24 = 8 \times 3 + 0$
∴ HCF of 56 and 88 is = 8.

(v) By applying Euclid's division lemma
$495 = 475 \times 1 + 20$
Since remainder $\neq 0$, apply division lemma on divisor of 475 and remainder 20.
$475 = 20 \times 23 + 15$
Since remainder $\neq 0$, apply division lemma on divisor of 20 and remainder 15.

$20 = 15 \times 1 + 5$
Since remainder $\neq 0$, apply division lemma on divisor of 15 and remainder 5.
$15 = 5 \times 3 + 0$
∴ HCF of 475 and 495 is = 5.

(vi) By applying Euclid's division lemma
$243 = 75 \times 3 + 18$
Since remainder $\neq 0$, apply division lemma on divisor of 75 and remainder 18.
$75 = 18 \times 4 + 3$
Since remainder $\neq 0$, apply division lemma on divisor of 18 and remainder 3.
$18 = 3 \times 6 + 0$
∴ HCF of 243 and 75 is = 3.

(vii) By applying Euclid's division lemma
$6552 = 240 \times 27 + 72$
Since remainder $\neq 0$, apply division lemma on divisor of 240 and remainder 72.
$210 = 72 \times 3 + 24$
Since remainder $\neq 0$, apply division lemma on divisor of 72 and remainder 24.
$72 = 24 \times 3 + 0$
∴ HCF of 6552 and 240 is = 24.

(viii) By applying Euclid's division lemma
$1385 = 155 \times 8 + 145$
Since remainder $\neq 0$, applying division lemma on divisor 155 and remainder 145
$155 = 145 \times 1 + 10$
Since remainder $\neq 0$, applying division lemma on divisor 10 and remainder 5
$10 = 5 \times 2 + 0$
∴ Hence HCF of 1385 and 155 = 5.

(ix) By applying Euclid's division lemma
$190 = 100 \times 1 + 90$
Since remainder $\neq 0$, applying division lemma on divisor 100 and remainder 90.
$90 = 10 \times 9 + 0$
∴ HCF of 100 and 190 = 10

(x) By applying Euclid's division lemma
$120 = 105 \times 1 + 15$
Since remainder $\neq 0$, applying division lemma on divisor 105 and remainder 15.
$105 = 15 \times 7 + 0$
∴ HCF of 105 and 120 = 15

2. Use Euclid's division algorithm to find the HCF of
(i) 135 and 225 (ii) 196 and 38220

Sol:

(i) 135 and 225

Step 1: Since 225 > 135. Apply Euclid's division lemma to a = 225 and b = 135 to find q and r such that 225 = 135q + r, 0 ≤ r < 135
On dividing 225 by 135 we get quotient as 1 and remainder as '90'
i.e., 225 = 135r 1 + 90
Step 2: Remainder 5 which is 90 7, we apply Euclid's division lemma to a = 135 and b = 90 to find whole numbers q and r such that 135 = 90 × q + r 0 ≤ r < 90 on dividing 135 by 90 we get quotient as 1 and remainder as 45
i.e., 135 = 90 × 1 + 45
Step3: Again remainder r = 45 to so we apply division lemma to a = 90 and b = 45 to find q and r such that 90 = 45 × q × r. 0 ≤ r < 45. On dividing 90 by 45we get quotient as 2 and remainder as 0 i.e., 90 = 2 × 45 + 0
Step 4: Since the remainder = 0, the divisor at this stage will be HCF of (135, 225)
Since the divisor at this stage is 45. Therefore the HCF of 135 and 225 is 45.

(ii) 867 and 255:
Step 1: Since 867 > 255, apply Euclid's division
Lemma a to a = 867 = 255 q + r, 0 < r < 255
On dividing 867 by 255 we get quotient as 3 and the remainder as low
Step 2: Since the remainder 102 to, we apply the division lemma to a = 255 and b = 102 to find 255 = 102q + 51 = 102r – 151
Step 3: Again remainder 0 is non-zero, so we apply the division lemma to a = 102 and b = 51 to find whole numbers q and r such that 102 = q = r when 0 ≤ r < 51
On dividing 102 by 51 quotient = 2 and remainder is '0'
i,e., = 102 = 51 × 2 + 0
Since the remainder is zero, the divisional this stage is the HCF.
Since the divisor at this stage is 51, ∴ HCF of 867 and 255 is '51'.

3. Find the HCF of the following pairs of integers and express it as a linear combination of them.
(i) 963 and 657 (ii) 592 and 252 (iii) 506 and 1155 (iv)1288 and 575

Sol:

(i) 963 and 6567
By applying Euclid's division lemma 963 = 657 × 1 + 306 …(i)
Since remainder ≠ 0, apply division lemma on divisor 657 and remainder 306
657 = 306 × 2 + 45 ….. (ii)
Since remainder ≠ 0, apply division lemma on divisor 306 and remainder 4
306 = 45 × 6 + 36 …..(iii)
Since remainder ≠ 0, apply division lemma on divisor 45 and remainder 36
45 = 36 × 1 + 9 …… (iv)
Since remainder ≠ 0, apply division lemma on divisor 36 and remainder 9
36 = 9 × 4 + 0

∴ HCF = 9

Now 9 = 45 – 36 × 1 [from (iv)]

= 45 – [306 – 45 × 6] × 1 [from (iii)]

= 45 – 306 × 1 + 45 × 6

= 45 × 7 – 306 × 1

= 657 × 7 – 306 × 14 – 306 × 1 [from (ii)]

= 657 × 7 – 306 × 15

= 657 × 7 – [963 – 657 × 1] × 15 [from (i)]

= 657 × 22 – 963 × 15

(ii) 595 and 252

By applying Euclid's division lemma

595 = 252 × 2 + 91 ….. (i)

Since remainder ≠ 0, apply division lemma on divisor 252 and remainder 91

252 = 91 × 2 + 70 …. (ii)

Since remainder ≠ 0, apply division lemma on divisor 91 and remainder 70

91 = 70 × 1 + 21 ….(iii)

Since remainder ≠ 0, apply division lemma on divisor 70 and remainder 20

70 = 21 × 3 + 7 …..(iv)

Since remainder ≠ 0, apply division lemma on divisor 21 and remainder 7

21 = 7 × 3 + 0

H.C.F = 7

Now, 7 = 70 – 21 × 3 [from (iv)]

= 70 – [90 – 70 × 1] × 3 [from (iii)]

= 70 – 91 × 3 + 70 × 3

= 70 × 4 – 91 × 3

= [252 – 91 × 2] × 4 – 91 × 3 [from (ii)]

= 252 × 4 – 91 × 8 – 91 × 3

= 252 × 4 – 91 × 11

= 252 × 4 – [595 – 252 × 2] × 11 [from (i)]

= 252 × 4 – 595 × 11 + 252 × 22

= 252 × 6 – 595 × 11

(iii) 506 and 1155

By applying Euclid's division lemma

1155 = 506 × 2 + 143 …. (i)

Since remainder ≠ 0, apply division lemma on division 506 and remainder 143

506 = 143 × 3 + 77 ….(ii)

Since remainder ≠ 0, apply division lemma on division 143 and remainder 77

143 = 77 × 1 + 56 ….(iii)

Since remainder ≠ 0, apply division lemma on division 77 and remainder 66

77 = 66 × 1 + 11 …(iv)

Since remainder ≠ 0, apply division lemma on divisor 36 and remainder 9
66 = 11 × 6 + 0
∴ HCF = 11
Now, 11 = 77 – 6 × 11 [from (iv)]
= 77 – [143 – 77 × 1] × 1 [from (iii)]
= 77 – 143 × 1 – 77 × 1
= 77 × 2 – 143 × 1
= [506 – 143 × 3] × 2 – 143 × 1 [from (ii)]
= 506 × 2 – 143 × 6 – 143 × 1
= 506 × 2 – 143 × 7
= 506 × 2 – [1155 – 506 × 27 × 7] [from (i)]
= 506 × 2 – 1155 × 7 + 506 × 14
= 506 × 16 – 115 × 7

(iv) 1288 and 575
By applying Euclid's division lemma
1288 = 575 × 2 + 138 …(i)
Since remainder ≠ 0, apply division lemma on division 575 and remainder 138
575 = 138 × 1 + 23 …(ii)
Since remainder ≠ 0, apply division lemma on division 138 and remainder 23 …(iii)
∴ HCF = 23
Now, 23 = 575 – 138 × 4 [from (ii)]
= 575 – [1288 – 572 × 2] × 4 [from (i)]
= 575 – 1288 × 4 + 575 × 8
= 575 × 9 – 1288 × 4

4. Express the HCF of 468 and 222 as 468x + 222y where x, y are integers in two different ways.

Sol:
Given integers are 468 and 222 where 468 > 222.
By applying Euclid's division lemma, we get 468 = 222 × 2 + 24 …(i)
Since remainder ≠ 0, apply division lemma on division 222 and remainder 24
222 = 24 × 9 + 6 …(ii)
Since remainder ≠ 0, apply division lemma on division 24 and remainder 6
24 = 6 × 4 + 0 …(iii)
We observe that the remainder = 0, so the last divisor 6 is the HCF of the 468 and 222
From (ii) we have
6 = 222 – 24 × 9
⇒ 6 = 222 – [468 – 222 × 2] × 9 [Substituting 24 = 468 – 222 × 2 from (i)]
⇒ 6 = 222 – 468 × 9 – 222 × 18
⇒ 6 = 222 × 19 – 468 × 9

⇒ 6 = 222y + 468x, where x = −9 and y = 19

5. If the HCF of 408 and 1032 is expressible in the form 1032 m − 408 × 5, find m.
Sol:
General integers are 408 and 1032 where 408 < 1032
By applying Euclid's division lemma, we get
1032 = 408 × 2 + 216
Since remainder ≠ 0, apply division lemma on division 408 and remainder 216
408 = 216 × 1 + 192
Since remainder ≠ 0, apply division lemma on division 216 and remainder 192
216 = 192 × 1 + 24
Since remainder ≠ 0, apply division lemma on division 192 and remainder 24
192 = 24 × 8 + 32
We observe that 32m under in 0. So the last divisor 24 is the H.C.F of 408 and 1032
∴ 216 = 1032m – 408 × 5
⇒ 1032 m = 24 + 408 × 5
$\Rightarrow 1032m = 24 + 2040$
$\Rightarrow 1032m = 2064$
$\Rightarrow m = \frac{2064}{1032} = 2$

6. If the HCF of 657 and 963 is expressible in the form 657 x + 963 x − 15, find x.
Sol:
657 and 963
By applying Euclid's division lemma
963 = 657 × 1 + 306
Since remainder ≠ 0, apply division lemma on division 657 and remainder 306
657 = 306 × 2 + 45
Since remainder ≠ 0, apply division lemma on division 306 and remainder 45
306 = 45 × 6 + 36
Since remainder ≠ 0, apply division lemma on division 45 and remainder 36
45 = 36 × 1 + 19
Since remainder ≠ 0, apply division lemma on division 36 and remainder 19
36 = 19 × 4 + 0
∴ HCF = 657
Given HCF = 657 + 963 × (-15)
$\Rightarrow 9 = 657 \times -1445$
$\Rightarrow 9 + 14445 = 657\ x$
$\Rightarrow 657x = 1445y$
$\Rightarrow x = \frac{1445y}{657}$

⇒ x = 22

7. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol:

Members in arms = 616

Members in Band = 32

∴ Maximum numbers of columns

= HCF of 616 and 32

By applying Euclid's division lemma

616 = 32 × 19 + 8

32 = 8 × 4 + 0

∴ HCF = 8

Hence the maximum remainder number of columns in which they can each is 8

8. Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Sol:

The required number when the divides 615 and 963

Leaves remainder 616 is means 615 – 6 = 609 and 963 – 957 are completely divisible by the number

∴ the required number

= HCF of 609 and 957

By applying Euclid's division lemma

957 = 609 × 1 + 348

609 = 348 × 1 + 261

348 = 261 × 1 + 87

261 = 87 × 370

HCF = 87

Hence the required number is '87'

9. Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

Sol:

The require number when divides 285 and 1249, leaves remainder 9 and 7, this means 285 – 9 = 276 and 1249 – 7 = 1242 are completely divisible by the number

∴ The required number = HCF of 276 and 1242

By applying Euclid's division lemma

1242 = 276 × 4 + 138

276 = 138 × 2 + 0

∴ HCF = 138
Hence remainder is = 0
Hence required number is 138

10. Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.
Sol:
The required number when divides 280 and 1245 leaves the remainder 4 and 3, this means 280 4 – 216 and 1245 – 3 = 1245 – 3 = 1242 are completely divisible by the number
∴ The required number = HCF of 276 and 1242
By applying Euclid's division lemma
1242 = 276 × 4 + 138
276 = 138 × 2 + 0
∴ HCF = 138
Hence the required numbers is 138

11. What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.
Sol:
The required number when divides 626, 3127 and 15628, leaves remainder 1, 2 and 3. This means 626 – 1 = 625, 3127 – 2 = 3125 and
15628 – 3 = 15625 are completely divisible by the number
∴ The required number = HCF of 625, 3125 and 15625
First consider 625 and 3125
By applying Euclid's division lemma
3125 = 625 × 5 + 0
HCF of 625 and 3125 = 625
Now consider 625 and 15625
By applying Euclid's division lemma
15625 = 625 × 25 + 0
∴ HCF of 625, 3125 and 15625 = 625
Hence required number is 625

12. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.
Sol:
The required number when divides 445, 572 and 699 leaves remainders 4, 5 and 6
This means 445 – 4 = 441, 572 – 5 = 561 and
699 – 6 = 693 are completely divisible by the number
∴ The required number = HCF of 441, 567 and 693

First consider 441 and 567
By applying Euclid's division lemma
567 = 441 × 1 + 126
441 = 126 × 3 + 63
126 = 63 × 2 + 0
∴ HCF of 441 and 567 = 63
Now consider 63 and 693
By applying Euclid's division lemma
693 = 63 × 11 + 0
∴ HCF of 441, 567 and 693 = 63
Hence required number is 63

13. Find the greatest number which divides 2011 and 2623 leaving remainders 9 and 5 respectively.
Sol:
The required number when divides 2011 and 2623
Leaves remainders 9 and the means
2011 – 9 = 2002 and 2623 – 5 = 2618 are completely divisible by the number
∴ The required number = HCF of 2002 and 2618
By applying Euclid's division lemma
2618 = 2002 × 1 + 616
2002 = 616 × 3 + 154
616 = 754 × 4 + 0
∴ HCF of 2002 and 2618 = 154
Hence required number is 154

14. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.
Sol:
Length of room = 8m 25cm = 825 cm
Breadth of room = 6m 75m = 675 cm
Height of room = 4m 50m = 450 cm
∴ The required longest rod
= HCF of 825, 675 and 450
First consider 675 and 450
By applying Euclid's division lemma
675 = 450 × 1 + 225
450 = 225 × 2 + 0
∴ HCF of 675 and 450 = 825

Now consider 625 and 825
By applying Euclid's division lemma
825 = 225 × 3 + 150
225 = 150 × 1 + 75
150 = 75 × 2 + 0
HCF of 825, 675 and 450 = 75

15. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Sol:
Number of goats = 205
Number of donkey = 140
Number of cows = 175
∴ The largest number of animals in one trip = HCF of 105, 140 and 175
First consider 105 and 140
By applying Euclid's division lemma
140 = 105 × 1 + 35
105 = 35 × 3 + 0
∴ HCF of 105 and 140 = 35
Now consider 35 and 175
By applying Euclid's division lemma
175 = 35 × 5 + 0
HCF of 105, 140 and 175 = 35

16. 15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuit packets in each. How many biscuit packets and how many pastries will each box contain?

Sol:
Number of pastries = 15
Number of biscuit packets = 12
∴ The required no of boxes to contain equal number = HCF of 15 and 13
By applying Euclid's division lemma
15 = 12 × 13
12 = 2 × 9 = 0
∴ No. of boxes required = 3
Hence each box will contain $\frac{15}{3} = 5$ pastries and $\frac{2}{3}$ biscuit packets

17. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Sol:

Size of bathroom = 10ft by 8ft

= (10 × 12) inch by (8 × 12) inch

= 120 inch by 96 inch

The largest size of tile required = HCF of 120 and 96

By applying Euclid's division lemma

120 = 96 × 1 + 24

96 = 24 × 4 + 0

∴ HCF = 24

∴ Largest size of tile required = 24 inches

∴ No. of tiles required = $\frac{Area\ of\ bathroom}{area\ of\ 2\ tile}$

= $\frac{120 \times 96}{24 \times 24}$

= 5 × 4

= 20 tiles

18. Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Sol:

Number of chocolates of 1^{st} brand in one pack = 24

Number of chocolates of 2^{nd} b and in one pack = 15

∴ The least number of chocolates 1 need to purchase

= LCM of 24 and 15

= 2 × 24 × 2 × 2 × 3 × 5

= 120

∴ The number of packet of 1^{st} brand = $\frac{120}{24} = 5$

And the number of packet of 2^{nd} brand = $\frac{120}{15} = 8$

∴ Largest size of tile required = 24 inches

∴ No of tiles required = $\frac{area\ of\ bath\ room}{area\ of\ 1\ tile} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 tiles$

No of chocolates of 1^{st} brand in one pack = 24

No of chocolate of 2^{nd} brand in one pack = 15

∴ The least number of chocolates I need to purchase

= LCM of 24 and 15

= 2 × 2 × 2 × 3 × 5

= 120

∴ The number of packet of 1[st] brand = $\frac{120}{24} = 5$

All the number of packet of 2[nd] brand = $\frac{120}{15} = 8$

19. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Sol:
Number of cartons of coke cans = 144
Number of cartons of pepsi cans = 90
∴ The greatest number of cartons in one stock = HCF of 144 and 90
By applying Euclid's division lemma
$144 = 90 \times 1 + 54$
$90 = 54 \times 1 + 36$
$54 = 36 \times 1 + 18$
$36 = 18 \times 2 + 0$
∴ HCF = 18
Hence the greatest number cartons in one stock = 18

20. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Sol:
Number of color pencils in one pack = 24
No of crayons in pack = 32
∴ The least number of both colors to be purchased
= LCM of 24 and 32
$= 2 \times 2 \times 2 \times 2 \times 3$
= 96

∴ Number of packs of pencils to be bought = $\frac{96}{24} = 1$

And number of packs of crayon to be bought = $\frac{96}{32} = 3$

21. A merchant has 120 liters of oil of one kind, 180 liters of another kind and 240 liters of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Sol:
Quantity of oil A = 120 liters
Quantity of oil B = 180 liters
Quantity of oil C = 240 liters
We want to fill oils A, B and C in tins of the same capacity

∴ The greatest capacity of the tin chat can hold oil. A, B and C = HCF of 120, 180 and 240
By fundamental theorem of arithmetic
120 = $2^3 \times 3 \times 5$
180 = $2^2 \times 3^2 \times 5$
240 = $2^4 \times 3 \times 5$
HCF = $2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60\ litres$
The greatest capacity of tin = 60 liters

Exercise 2.1

1. Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their co efficient:

(i) f(x) = $x^2 - 2x - 8$
(ii) g(s) = $4s^2 - 4x + 1$
(iii) h(t) = $t^2 - 15$
(iv) p(x) = $x^2 + 2\sqrt{2}x + 6$
(v) q(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3}$
(vi) f(x) = $x^2 - (\sqrt{3}+1)x + \sqrt{3}$
(vii) g(x) = $a(x^2+1) - x(a^2+1)$
(viii) $6x^2 - 3 - 7x$

Sol:

(i) f(x) = $x^2 - 2x - 8$

$f(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

= $x(x-4) + 2(x-4)$

= $(x+2)(x-4)$

Zeroes of the polynomials are -2 and 4

Sum of the zeroes = $\frac{-\,co\,efficient\ of\ x}{co\,efficient\ of\ x}$

-2 + 4 = $\frac{-(-2)}{1}$

2 = 2

Product of the zeroes = $\frac{constant\ term}{co\,efficient\ of\ x^2}$

= 24 = $\frac{-8}{1}$

$-8 = -8$

∴ Hence the relationship verified

(ii) $9(5) = 45 - 45 + 1 = 45^2 - 25 - 25 + 1 = 25(25-1) - 1(25-1)$

= $(25-1)(25-1)$

Zeroes of the polynomials are $\frac{1}{2}\ and\ \frac{1}{2}$

Sum of zeroes = $\frac{-\,co\,efficient\ of\ s}{co\,efficient\ of\ s^2}$

$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$

1 = 1

Product of the zeroes = $\frac{constant\ term}{co\,efficient\ os\ s^2}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \Rightarrow \frac{1}{4} = \frac{1}{4}$

∴ Hence the relationship verified.

(iii) h(t) = $t^2 - 15 = (t^2) - (\sqrt{15})^2 = (t+\sqrt{15})(t-\sqrt{15})$

zeroes of the polynomials are $-\sqrt{15}\ and\ \sqrt{15}$

sum of zeroes = 0

$-\sqrt{15} + \sqrt{15} = 0$

$0 = 0$

Product of zeroes = $\frac{-15}{1}$

$-\sqrt{15} \times \sqrt{15} = -15$

-15 = -15

∴ Hence the relationship verified.

(iv) p(x) = $x^2 + 2\sqrt{2}x - 6 = x^2 + 3\sqrt{2}x + \sqrt{2} \times 3\sqrt{2}$

= $x(x + 3\sqrt{2}) - \sqrt{2}(2 + 3\sqrt{2}) = (x - \sqrt{2})(x + 3\sqrt{2})$

Zeroes of the polynomial are $3\sqrt{2}$ and -$3\sqrt{2}$

Sum of the zeroes = $\frac{-3\sqrt{2}}{1}$

$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$

$-2\sqrt{2} = -2\sqrt{2}$

$Product\ of\ zeroes \Rightarrow \sqrt{2} \times -3\sqrt{2} = -\frac{6}{1}$

$-6 = -6$

$Hence\ the\ relatioship\ varified$

(v) 2(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3} = \sqrt{3}x^2 + 7x + 3x + 7\sqrt{3}$

= $\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3})$

= $(\sqrt{3}x + 7)(x + \sqrt{3})$

Zeroes of the polynomials are $-\sqrt{3}, \frac{-7}{\sqrt{3}}$

Sum of zeroes = $\frac{-10}{\sqrt{3}}$

$\Rightarrow -\sqrt{3} - \frac{7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} \Rightarrow \frac{-10}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$

Product of zeroes = $\frac{7\sqrt{3}}{3} \Rightarrow \frac{\sqrt{3}x-7}{\sqrt{30}} = 7$

$\Rightarrow 7 = 7$

Hence, relationship verified.

(vi) f(x) = $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = x^2 - \sqrt{3}x - x + \sqrt{3}$

= x (x $- \sqrt{3}$) – 1 (x - $\sqrt{3}$)

= (x – 1) (x $- \sqrt{3}$)

Zeroes of the polynomials are 1 and $\sqrt{3}$

Sum of zeroes = $\frac{-\{coefficient\ of\ x\}}{co\ efficient\ of\ r^2} = \frac{-[-\sqrt{3}-1]}{1}$

$1 + \sqrt{3} = \sqrt{3} + 1$

Product of zeroes = $\frac{constant\ term}{co\ efficient\ of\ x^2} = \frac{\sqrt{3}}{1}$

$1 \times \sqrt{3} = \sqrt{3} = \sqrt{3} = \sqrt{3}$

∴ Hence, relationship verified

(vii) g(x) = $a[(x^2 + 1) - x(a^2 + 1)]^2 = ax^2 + a - a^2x - x$

= $ax^2 - [(a^2 + 1) - x] + 0 = ax^2 - a^2x - x + a$

$= ax(x-a) - 1(x-a) = (x-a)(ax-1)$

Zeroes of the polynomials = $\frac{1}{a}$ *and* a

Sum of the zeroes = $\frac{-[-a^2-1]}{a}$

$\Rightarrow \frac{1}{a} + a = \frac{a^2+1}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a}$

$\Rightarrow \frac{1}{a} \times a = \frac{a}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a} \Rightarrow 1 = 1$

Hence relationship verified

(viii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+11)(2x-3)$

Zeroes of polynomials are $+\frac{3}{2}$ *and* $\frac{-1}{3}$

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(co\,efficient\ of\ x)}{co\,efficient\ of\ x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{constant\ term}{co\,efficient\ of\ x^2}$

∴ Hence, relationship verified.

2. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c, then evaluate:

(i) $\alpha - \beta$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

(iv) $\alpha^2\beta + \alpha\beta^2$

(v) $\alpha^4 + \beta^4$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{\alpha}\right]$

Sol:

f(x) = $ax^2 + bx + c$

$\alpha + \beta = \frac{-b}{\boldsymbol{a}}$

$\alpha\beta = \frac{c}{a}$

since $\alpha + \beta$ *are the roots (or) zeroes of the given polynomials*

(i) $\alpha - \beta$

The two zeroes of the polynomials are

$\frac{-b+\sqrt{b^2-4ac}}{2a} - \left(b\frac{-\sqrt{b^2-4ac}}{2a}\right) = -b + \frac{\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a} = \frac{2\sqrt{b^2-4ac}}{2a} = \frac{\sqrt{b^2-4ac}}{2a}$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta-\alpha}{\alpha\beta} = \frac{-(\alpha-\beta)}{\alpha\beta} \ldots (i)$

From (i) we know that $\alpha - \beta = \frac{\sqrt{b^2-4ac}}{2a}\ [from\ (i)]\alpha\beta = \frac{c}{a}$

Putting the values in the (a) = $-\left(\frac{\sqrt{b^2-4ac\times a}}{a\times c}\right) = \frac{-\sqrt{b^2-4ac}}{c}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$\Rightarrow \left[\frac{\alpha+\beta}{\alpha\beta}\right] - 2\alpha\beta$

$\Rightarrow \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a} = -2\frac{c}{a} - \frac{b}{c} = \frac{-ab-2c^2}{ac} - \left[\frac{b}{c} + \frac{2c}{a}\right]$

(iv) $\alpha^2\beta + \alpha\beta^2$

$\alpha\beta(\alpha+\beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right)$

$= \frac{-bc}{a^2}$

(v) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2 + \beta^2$

$= ((\alpha+\beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$

$= \left[\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right]^2 - \left[2\left(\frac{c}{a}\right)^2\right]$

$= \left[\frac{b^2-2ac}{a^2}\right]^2 - \frac{2c^2}{a^2}$

$= \frac{(b^2 2ac)^2 - 2a^2c^2}{a^4}$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

$\Rightarrow \frac{a\beta+b+a\alpha+b}{(3\alpha+b)(a\beta+b)}$

$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\,\alpha+ab\beta+b^2}$

$= \frac{a(\alpha+\beta)+b}{a^2\alpha\beta+a\beta(\alpha^2\beta)+b^2}$

$= \frac{a\times\frac{a+2b}{a}}{a\times\frac{c}{a}+\frac{abc(-b)+b^2}{a}} = \frac{b}{ac-b^2+b^2} = \frac{b}{ac}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

$= \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$

$= \frac{a\beta^2+b\beta+a\alpha^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \frac{a\alpha^2+a\beta^2+b\beta^2+b\alpha}{a\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

$= \frac{a[(\alpha^2+\beta^2)+b(\alpha+\beta)]}{ac+ab+x\left(\frac{-b}{a}\right)+b^2}$

$= \frac{a[(\alpha+\beta)^2-2\alpha\beta]+bx-\frac{b}{a}}{ac}$

$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac} = \frac{a\times\left[\frac{b^2-2c}{a}\right]-b^2}{ac} = \frac{-2}{a}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{\alpha}\right]$

$= a\left[\frac{\alpha^3+\beta^3}{\alpha\beta}\right] + b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$= \frac{\alpha[(\alpha+\beta)^3 - 3\alpha\beta\,(\alpha+\beta)]}{\alpha\,\beta} + b(\alpha+\beta)^2 - 2\alpha\,\beta$

$= \frac{\alpha\left[\left(\frac{-b^3}{a^3}\right) + \frac{3b}{a}\cdot\frac{c}{a} + b\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)\right]}{\frac{c}{a}}$

$= \frac{a^2}{c}\left[\frac{-b^3}{a^3} + \frac{3bc}{a^2} + \frac{b^3}{a^2} - \frac{2bc}{a}\right]$

$= \frac{-a^2b^3}{ca^3} + \frac{3a^2bc}{ca^2} + \frac{b^3a^2}{a^2c} - \frac{2bca^2}{ac}$

$= \frac{-b^3}{ac} + 3b + \frac{b^3}{ac} - 2b$

= b

3. If α and β are the zeros of the quadratic polynomial f(x) = $6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Sol:

f(x) = $6x^2 - x - 2$

Since α and β are the zeroes of the given polynomial

∴ Sum of zeroes $[\alpha + \beta] = \frac{-1}{6}$

Product of zeroes $(\alpha\beta) = \frac{-1}{3}$

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{1}{6}\right)^2 - 2\times\left(\frac{-1}{3}\right)}{-\frac{1}{3}} = \frac{\frac{1}{6} - \frac{2}{3}}{\frac{-1}{3}} = \frac{\frac{1+24}{36}}{\frac{-1}{3}}$

$= \frac{\frac{2}{36}}{\frac{1}{3}} = \frac{-25}{12}$

4. If a and are the zeros of the quadratic polynomial f(x) = $x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Sol:

Since $\alpha + \beta$ are the zeroes of the polynomial: $x^2 - x - 4$

Sum of the roots $(\alpha + \beta)$ = 1

Product of the roots $(\alpha\beta)$ = −4

$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

$= \frac{1}{-4} + 4 = \frac{-1}{4} + 4 = \frac{-1+16}{4} = \frac{15}{4}$

5. If α and β are the zeros of the quadratic polynomial p(x) = $4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Sol:

Since α *and* β are the roots of the polynomial: $4x^2 - 5x - 1$

$\therefore$ Sum of the roots $\alpha + \beta = \frac{5}{4}$

Product of the roots $\alpha\beta = \frac{-1}{4}$

Hence $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{5}{4}\left(\frac{-1}{4}\right) = \frac{-5}{16}$

6. If a and 3 are the zeros of the quadratic polynomial f(x) = $x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Sol:

Since α and β are the roots of the polynomial x + x – 2

$\therefore$ Sum of roots $\alpha + \beta = 1$

Product of roots $\alpha\beta$ 2 $\Rightarrow -\frac{1}{\beta}$

$= \frac{\beta-\alpha}{\alpha\beta} \cdot \frac{(\alpha-\beta)}{\alpha\beta}$

$= \frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}$

$= \frac{\sqrt{1+8}}{+2} = \frac{3}{2}$

7. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta} - 2\alpha\beta$

Sol:

Since α *and* β are the roots of the quadratic polynomial

f(x) = $x^2 - 5x + 4$

Sum of roots = $\alpha + \beta = 5$

Product of roots = $\alpha\beta = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$

8. If α and β are the zeros of the quadratic polynomial f(t) = $t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$

Sol:

Since α *and* β are the zeroes of the polynomial f(t) = $t^2 - 4t + 3$

Since $\alpha + \beta = 4$

Product of zeroes $\alpha\beta = 3$

Hence $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = [3]^3[4] = 108$

9. If α and β are the zeros of the quadratic polynomial p(y) = $5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol:

Since α *and* β are the zeroes of the polynomials

p(y) = $54^2 = 5y^2 - 7y + 1$

Sum of the zeroes $\alpha\ \beta = \frac{1}{6}$

Product of zeroes = $\alpha\ \beta = \frac{1}{6}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7\times5}{5\times1} = 7$

10. If α and β are the zeros of the quadratic polynomial p(s) = $3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

Sol:

Since α *and* β are the zeroes of the polynomials

Sum of the zeroes $\alpha + \beta = \frac{6}{3}$

Product of the zeroes $\alpha\beta = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$

$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$

$= \frac{[2]^2-2\times\frac{4}{3}+2\left[\frac{2\times3}{4}\right]+3\left[\frac{4}{3}\right]}{\frac{4}{3}}$

$= \frac{4-\frac{8}{3}}{\frac{4}{3}} + 7 \Rightarrow \frac{4}{3} \times \frac{3}{4}(1+7) \Rightarrow 8$

11. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Sol:

Since α and β are the roots of the polynomials

f(x) = $x^2 - px + 2$

sum of zeroes = p = $\alpha + \beta$

Product of zeroes = q = $\alpha\beta$

LHS $= \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

$= \frac{\alpha^2+\beta^2}{\alpha\beta^2} = \frac{\left(\alpha^2+\beta^2\right)^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$

$= \frac{\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$

$= \frac{\left[(p)^2-2q\right]^2-2q^2}{q}$

$=\frac{p^4+4q^2-2p^2.2q-2q^2}{q^2}$

$=\frac{p^4+2q^2-4p^2q}{q^2}=\frac{p^4}{q^2}+2-\frac{4p^2}{q}$

$=\frac{p^4}{q^2}-\frac{4p^2}{q^2}=\frac{p^4}{q^2}+2-\frac{4p^2}{q}$

$=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$

12. If the squared difference of the zeros of the quadratic polynomial f(x) = x^2 + px + 45 is equal to 144, find the value of p.

Sol:

Let the two zeroes of the polynomial be α and β

f(x) = $x^2+px+45$

sum of the zeroes = $-p$

Product of zeroes = 45

$\Rightarrow (\alpha-\beta)^2-4\alpha\beta=144$

$\Rightarrow p^2-4\times 45=144$

$\Rightarrow p^2=144+180$

$\Rightarrow p^2=324$

$p=\pm 1$

13. If the sum of the zeros of the quadratic polynomial f(t) = kt^2 + 2t + 3k is equal to their product, find the value of k.

Sol:

Let the two zeroes of the f(t) = $kt^2+2t+3k\ be$ α and β

Sum of the zeroes $(\alpha+\beta)$

Product of the zeroes αβ

$\frac{-2}{k}=\frac{3k}{k}$

$-2k=3k^2$

$2k+3k^2=0$

k(3k + 2) = 0

k = 0

k = $\frac{-2}{3}$

14. If one zero of the quadratic polynomial f(x) = $4x^2-8kx-9$ is negative of the other, find the value of k.

Sol:

Let the two zeroes of one polynomial

$f(x)=4x^2-5k-9\ be\ \alpha,-\alpha$

$\alpha \times \alpha = \frac{-9}{4}$
$t\alpha^2 = \frac{+9}{4}$
$\alpha = \frac{+3}{2}$
Sum of zeroes = $\frac{8k}{4} = 0$
Hence 8k = 0
Or k = 0

15. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$

Sol:
f(x) = $x^2 - 1$
sum of zeroes $\alpha + \beta = 0$
Product of zeroes $\alpha\beta$ = -1
Sum of zeroes = $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2+2\beta^2}{\alpha\beta}$
= $\frac{2((\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta}$
= $\frac{2[(0)^2-2\times-1]}{-1}$
= $\frac{2(2)1}{-1}$
= -4
Product of zeroes = $\frac{2\alpha\times2\beta}{\alpha\beta} = \frac{4\alpha\beta}{\alpha\beta}$
Hence the quadratic equation is $x^2 - (sum\ of\ zeroes)x + product\ of\ zeroes$
= $k(x^2 + 4x + 14)$

16. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$.

Sol:
$f(x) = x^2 - 3x - 2$
Sum of zeroes $[\alpha + \beta]$ = 3
Product of zeroes $[\alpha\beta]$ = -2
Sum of zeroes = $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$
= $\frac{2\beta+\alpha+2\alpha+A}{(2\alpha+\beta)(2\beta+\alpha)}$
= $\frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$
= $\frac{3\times3}{2[2\ (\alpha+\beta)^2-2\alpha\beta\ +5\times(-2)]}$

$= \frac{9}{2[9]-10} = \frac{9}{16}$

Product of zeroes = $\frac{1}{\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{4\alpha\beta+\alpha\beta+2\alpha^2+2\beta^2}$

$= \frac{1}{5\times-2+2[(\alpha+\beta)^2-2\alpha\beta]}$

$= \frac{1}{-10+2[9+4]}$

$= \frac{1}{10+26}$

$= \frac{1}{16}$

Quadratic equation = $x^2 - [sum\ of\ zeroes]x + product\ of\ zeroes$

$= x^2 - \frac{9x}{16} + \frac{1}{16}$

$= k\left[x^2 - \frac{9x}{16} + \frac{1}{16}\right]$

17. If α and β are the zeros of a quadratic polynomial such that a + 13 = 24 and a $-\ \beta$ = 8, find a quadratic polynomial having α and β as its zeros.

Sol:

$\alpha + \beta = 24$

$\alpha\ \beta = 8$

…………

$2\ \alpha = 32$

$\alpha = 16$

$\beta = 8$

$\alpha\ \beta = 16 \times 8 = 128$

Quadratic equation

$\Rightarrow x^2 - (sum\ of\ zeroes) + product\ of\ zeroes$

$\Rightarrow k[x^2 - 24x + 128]$

18. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - p(x + 1)$ — c, show that (α + 1)(β +1) = 1$-$ c.

Sol:

$f(x) = x^2 - p(x+1)c = x - px = -p - c$

Sum of zeroes = $\alpha + \beta$ = p

Product of zeroes = $-$ p – c = $\alpha\ \beta$

(α + 1+ β+) = $\alpha\ \beta$+ $\alpha + \beta$ + 1 = $-$p –c + p + 1

= 1 – c = R.H.S

∴ Hence proved

19. If If α and β are the zeros of the quadratic polynomial f(x) = x^2 – 2x + 3, find a polynomial whose roots are (i) α + 2, β + 2 (ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Sol:

$f(x) = x^2 - 2x + 3$

Sum of zeroes = 2 = $(\alpha + \beta)$

Product of zeroes = 3 = $(\alpha\ \beta)$

$(i)\ sum\ of\ zeroes = (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 2 + 4 = 6$

$Product\ of\ zeroes = (\alpha + 2)(\beta + 2)$

$= \alpha\,\beta + 2\alpha + 2\beta + 4 = 3 + 2(2) + 4 = 11$

$Quadratic\ equation = x^2 - 6x + 11 = k[x^2 - 6x + 11]$

(ii) sum of zeroes = $\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\,\beta+\alpha-\beta-1+\alpha\,\beta+\beta+\beta-\alpha-1}{3+2+1}$

$= \frac{3-1+3-1}{3+2+1} = 4 = \frac{2}{3}$

Product of zeroes = $\frac{\alpha-1}{\beta\alpha+1} \times \frac{\beta-1}{\alpha+1} - \frac{\alpha\ 1-\alpha-\alpha\beta+1}{\alpha\,\beta+\alpha+\beta+1}$

$= \frac{3-(\alpha+\beta)+1}{3+2+1} = \frac{2}{6} = \frac{1}{3}$

Quadratic equation on $x^2 - \frac{2}{3} \times \frac{+1}{3} = 1\left[\frac{x^2-2x}{3} + \frac{1}{3}\right]$

20. If α and β are the zeroes of the polynomial f(x) = $x^2 + px + q$, form a polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Sol:

$f(x) = x^2 + p + q$

Sum of zeroes = p = $\alpha + \beta$

Product of zeroes = q = $\alpha\ \beta$

Sum of the new polynomial $= (\alpha + \beta)^2 + (\alpha - \beta)^2$

$= (-p)^2 + \alpha^2 + \beta^2 - 2\alpha\,\beta$

$= p^2 + (\alpha + \beta)^2 - 2\alpha\,\beta - 2\alpha\,\beta$

$= p^2 + p^2 - 4q$

$= 2p^2 - 4q$

Product of zeroes $= (\alpha + \beta)^2 \times (\alpha - \beta)^2 = [-p]^2 \times (p^2 - 4q) = (p^2 - 4q)p^2$

Quadratic equation $= x^2 - [2p^2 - 4q] + p^2[-4q + p]$

$$f(x) = k\{x^2 - 2(p^2 - 28)x + p^2(q^2 - 4q)\}$$

Exercise 2.2

1. Verify that the numbers given alongside of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{2}{8} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{-4}{2} + 2 = 0$

$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$

$f(-2) = q(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2$

$= -16 + 16 = 0$

$= \propto + \beta + \gamma = \frac{-b}{a}$

$\frac{1}{2} + 1 - 2 = \frac{-1}{2}$

$\frac{1}{2} - 1 = \frac{-1}{2}$

$\frac{1}{2} = \frac{-1}{2}$

$\alpha\beta.\beta\gamma + r\alpha = \frac{c}{a}$

$\frac{1}{2} \times 1 + 1 \times -2 + -2 \times \frac{1}{2} = \frac{-5}{2}$

$\frac{1}{2} - 2 - 1 = \frac{-5}{2}$

$\frac{-5}{2} = \frac{-5}{2}$

$(ii)\ g(x) = x^3 - 4x^2 + 5 \times -2$

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$

$g(1) = [1]^3 - 4[1]^2 + 5[1] - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

$\alpha + \beta + \gamma = \frac{-b}{a}(2) + 1 + 1 = -(-4) = 4 = 4$

$\alpha\,\beta + \beta\,\gamma + \gamma\alpha = \frac{c}{a}$

$2 \times 1 + 1\,\alpha\,1 + 1 \times 2 = 5$

$2 + 1 + 2 = 5$

$5 = 5$

$\alpha\beta\gamma = -(-2)$

$2 \times 1 \times 1 = 2$

$2 = 2$

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, −1 and −3 respectively.

Sol:

Any cubic polynomial is of the form $ax^3 + bx^2 + cx + d = x^3 -$ $sum\ of\ zeroes\ (x^2)$[product of zeroes] + sum of the products of its zeroes × - product of zeroes

= $x^3 - 2x^2 + (3 - x) + 3$

= k $[x^3 - 3x^2 - x - 3]$

k is any non-zero real numbers

3. If the zeros of the polynomial f(x) = $2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Sol:

Let α = a – d , β = a and γ = a + d be the zeroes of polynomial.

$f(x) = 2x^3 - 15x^2 + 37x - 30$

$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2}$

$\alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$

a – d + a + a + d = $\frac{15}{2}$ and $a(a - d)(a + a) = 15$

3a = $\frac{15}{2}$, a = $\frac{5}{2}$

$a(a^2 - d^2) = 15$

$a^2 - a^2 = \frac{15\times2}{5} \Rightarrow \left(\frac{5}{2}\right)^2 - d^2 = 6 \Rightarrow\Rightarrow \frac{25-6}{4} = d^2$

$d^2 = \frac{1}{4} \Rightarrow d = \frac{1}{2}$

$\therefore \alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$\beta = \frac{5}{2} = \frac{5}{2}$

$\gamma = \frac{5}{2} + \frac{1}{2} = 3$

4. Find the condition that the zeros of the polynomial f(x) = $x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

$f(x) = x^3 + 3px^2 + 3qx + q$

$Let\ a - d, a, a + d$ be the zeroes of the polynomial

$The\ sum\ of\ zeroes = \frac{-b}{a}$

$a + a - d + a + d = \frac{b}{a}$

$3a = -3p$

a = $-p$

Since a is the zero of the polynomial f(x) therefore f(a) = 0 ⇒ $[a]^2 + 3pa^2 + 3qa + r = 0$

$\therefore f(a) = 0 \Rightarrow [a]^2 + 3pa^2 + 3qa + r = 0$
$\Rightarrow p^3 + 3p(-p)^2 + 3q(-p) + r = 0$
$\Rightarrow -p^3 + 3p^2 - pq + r = 0$
$\Rightarrow 2p^3 - pq + r = 0$

5. If the zeroes of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$

Sol:
Let a – d, a, a + d be the zeroes of the polynomial f(x)
The sum of zeroes ⇒ a – d + a + a + d = $\frac{-3b}{a}$
$\Rightarrow +3a = -\frac{3b}{a} \Rightarrow a = \frac{-3b}{a\times3} a = \frac{-b}{a}$
f(a) = 0 ⇒ $a(a)^2 + 3b(a)^2 + 3c(a) + d = 0$
$= a\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$
$\Rightarrow \frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$
$\Rightarrow \frac{2b^3 - 3abc + a^2d}{a^2} = 0$
$\Rightarrow 2b^3 - 3abc + a^2d = 0$

6. If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Sol:
$f(x) = x^3 - 12x^2 + 39x - k$
Let $a - d, a, a + d$ be the zeroes of the polynomial f(x)
The sum of the zeroes = 12
3a = 12
a = 4
$f(a), -a(x)^3 - l^2(4)^2 + 39(4) + k = 0$
64 – 192 + 156 + k = 0
$= -28 = k$
k = -28

Exercise 2.3

1. Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$, g(x) = $x^2 + x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

Sol:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$

g(x) = $x^2 + x + 1$

$$\begin{array}{r|l} & x-7 \\ \hline x^2+x+1 & x^3-6x^2+11x-6 \\ & x^3+x^2+x \\ \hline & -7x^2-7x-7 \\ & -7x^2-7x-7 \\ \hline & 17x-1 \end{array}$$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$

$$\begin{array}{r|l} & 5x^2-9x-2 \\ \hline 2x^2+7x+1 & 10x^4+17x^3-62x^2+30x-3 \\ & 10x^4+35x^3+5x^2 \\ \hline & -18x^3-67x^2+30x \\ & -18x^3 \pm 63x^2+9x \\ \hline & -4x^2+39x-3 \\ & \pm 4x^2 \pm 14x \pm 2 \\ \hline & 53x-1 \end{array}$$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$

$$\begin{array}{r|l} & 2x-5 \\ \hline 2x^2-2+1 & 4x^3+8x^2+8^2+7 \\ & 4x^3 \mp 2x^2 \pm 2x \\ \hline & 10x^2+6x+7 \\ & 10x^2 \pm 5x \pm 5 \\ \hline & 11x-2 \end{array}$$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

$$\begin{array}{r|l} & 15x+10 \\ \hline x^2-2x+2 & 15x^3-20x^2+13x-12 \\ & 15x^3 \mp 30x^2 \pm 30x \\ \hline & 10x^2-17x-12 \\ & 10x^2 \pm 20x+20 \\ \hline & 3x-32 \end{array}$$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Sol:

(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t \\
 & 2t^2 - 6t^2 \\
\hline
 & 3t^3 + 4t - 9t \\
 & 3t^3 + 4t - 9t \\
\hline
 & 4t^2 - 12 \\
 & 4t^2 \mp 12
\end{array}
$$

(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
\hline
 & -x^3 + 3x + 1 \\
 & -x^3 + 3x - 1 \\
\hline
 & 2
\end{array}
$$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

$$
\begin{array}{r|l}
 & 3x^3 + x^2 - 2x - 5 \\
\hline
2x^2 - x + 3 & 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 \\
 & 6x^5 - 3x^4 + 9x^3 \\
\hline
 & 2x^4 - 5x^3 - 5x^2 \\
 & 2x^4 \mp x^3 \pm 3x^2 \\
\hline
 & -4x^3 - 8x^2 - x \\
 & \mp 4x^3 \pm 2x^2 - 6x \\
\hline
 & -10x^2 - 5x - 15 \\
 & \mp 10x \pm 15x \mp 15 \\
\hline
 & 0
\end{array}
$$

3. Obtain all zeros of the polynomial f(x) = $2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are −2 and −1.

Sol:

$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

If the two zeroes of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

$(x + 2)(x + 1) = x^2 + x + 2x = x^2 + 3x + 2$

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\
\hline
x^2 + 3x + 2 & 2x^4 + x^3 - 14x^2 - 19x - 6 \\
 & 2x^4 + 6x^3 + 4x^2 \\
\hline
 & \quad -5x^3 - 18x^2 - 19x \\
 & \quad -5x^3 \mp 15x^2 \mp 10x \\
\hline
 & \qquad -3x^2 - 9x - 6 \\
 & \qquad -3x^2 - 9x - 6
\end{array}
$$

$\therefore 2x^4 + x^3 - 14x^2 - 19x - 6$

$= (2x^2 - 5x - 3)[x^2 + 3x + 2] = [2x + 1][x - 3][x + 2][x + 1]$

$\therefore$ zero all x = $\frac{-1}{2}, 3, -2, -1$

4. Obtain all zeros of f(x) = $x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.

Sol:

$f(x) = x^3 + 13x^2 + 32x + 20$

$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
\hline
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 \pm 2x^2 \\
\hline
 & \quad 11x^2 + 32x + 20 \\
 & \quad 11x^2 \pm 22x \\
\hline
 & \qquad 10x + 20 \\
 & \qquad 10x + 20 \\
\hline
 & \qquad 0
\end{array}
$$

$(x^2 + 11x + 10) = x^2 + 10x + x + 20(x + 10) + 1(x + 10) = (x + 1)(x + 10)$

$\therefore$ The zeroes of the polynomial are -1, -10, -2.

5. Obtain all zeros of the polynomial $f(x) = x^4 - 3x^2 = x^2 + 9x - 6$ if two of its zeros are $-\sqrt{3}, and\ \sqrt{3}$.

Sol:

$f(x) = (x^2 - 3x + 2) = (x + \sqrt{3}) \& (x - \sqrt{3}) = x^2 - 3$

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 3 & x^4 - 3x^2 = x^2 + 9x - 6 \\
 & x^4 - 3x^2 \\
\hline
 & \quad -3x^2 + 2x^2 + 9x \\
 & \quad -3x^2 \qquad \pm 9x \\
\hline
 & \qquad 2x^2 - 6 \\
 & \qquad 2x^2 - 6
\end{array}
$$

$(x^2 - 3)(x^2 - 3x + 2) = (x + \sqrt{3})(x - \sqrt{3})(x^2 - 2x - x + 2)$

$= (x + \sqrt{3})(x - \sqrt{3})\ (x - 2)(x - 2)$

Zeroes are $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Find all zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if its two zeroes are $-\sqrt{\frac{3}{2}}$ *and* $\sqrt{\frac{3}{2}}$

Sol:

If the zeroes of the polynomial are $-\sqrt{\frac{3}{2}}$ *and* $\sqrt{\frac{3}{2}}$

Its factors are $\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \frac{x^2-3}{2}$

x = $-1, 2, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

$= [2x^2 - 2x - 4]\left(x^2 - \frac{3}{2}\right)$

$= (2x^2 - 4x + 2x - 4)\left(x + \sqrt{\frac{3}{2}}\right)$

$= [2[x(x+2) + 2(x-2)]]$

$= \left[x + \frac{\sqrt{3}}{2}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$= (x+2)(x-2)\left[x + \sqrt{\frac{3}{2}}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$x = -1, 2, \sqrt{\frac{3}{2}} - \sqrt{\frac{3}{2}}$

7. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Sol:

	$x^2 - 1$
$x^2 + 2x - 3$	$x^4 + 2x^3 - 2x^2 + x - 1$
	$x^4 + 2x^3 - 3x^2$
	$x^2 + x - 1$
	$x^2 + 2x - 3$
	$-x + 2$

we must add x -2 in order to get the resulting polynomial exactly divisible by $x^2 + 2x - 3$

8. What must be subtracted from the polynomial $x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

Sol:

$$
\begin{array}{l|l}
 & x^2 + 6x + 8 \\
\hline
x^2 - 4x + 3 & x^4 + 2x^3 - 13x^2 - 12x + 21 \\
 & x^4 - 4x^3 + 3x^2 \\
\hline
 & \quad 6x^3 - 16x^2 - 12x \\
 & \quad 6x^3 - 24x^2 - 18x \\
\hline
 & \qquad 8x^2 - 30x + 21 \\
 & \qquad 8x^2 - 32x + 21 \\
\hline
 & \qquad\qquad 2x - 2
\end{array}
$$

We must subtract $[2x-2]+10m$ the given polynomial so as to get the resulting polynomial exactly divisible by $x^2 - x + 3$

9. Find all the zeroes of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeroes are 2 and -2.

Sol:

$\Rightarrow f(x) = x^4 + x^3 - 34x^2 - 4x + 120$

$\Rightarrow$ x = -2 is a solution

x = -2 is a factor

x = -2 is a solution

x = +2 is a factor

here,

$(x-2)(x+2) is\ a\ factor\ of\ f(x)$

$x^2 - 4\ is\ a\ factor$

$$
\begin{array}{l|l}
 & x^2 + x - 30 \\
\hline
x^2 - 4 & x^4 + x^3 - 34x^2 - 4x + 120 \\
 & -x^4 \qquad - 4x^2 \\
\hline
 & \quad x^3 - 30x^2 - 4x + 120 \\
 & \quad x^3 \qquad\quad - 4x \\
\hline
 & \qquad -30x^2 \quad + 120 \\
 & \qquad -30x^2 \quad + 120 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Hence, $x^4 + x^3 - 34x^2 - 4x + 120 = (x^2-4)(x^2+x-30)$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2-4)(x^2+6x-5x-30)$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2-4)[(x(x+6)-5(x+6))]$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2-4)(x+6)(x-5)$

Other zeroes are

x + 6 = 0 $\Rightarrow$ x – 5 = 0

x = – 6 x = 5

Set of zeroes for f(x) $[2, -2, -6, 5]$

10. Find all zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Sol:

$f(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

$x = \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x + \sqrt{2}\ is\ a\ factor$

Here, $(x + \sqrt{2})(x - \sqrt{2}) is\ a\ factor\ of\ f(x)$

$x^2 - 2\ is\ a\ factor\ of\ f(x)$

$$\begin{array}{r|l} & 2x^2 + 7x - 15 \\ \hline x^2 - 2 & 2x^4 + 7x^3 - 19^2 - 14x + 30 \\ & 2x^4 \qquad\quad - 4x^2 \\ \hline & \qquad 7x^3 - 15x^2 - 14x \\ & \qquad 7x^3 - \qquad -14x \\ \hline & \qquad\qquad -15x^2 \quad + 30 \\ & \qquad\qquad -15x^2 \quad + 30 \\ \hline & \qquad\qquad\qquad\qquad 0 \end{array}$$

Hence, $2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$

$= (x^2 - 2)\ (2x^2 + 10x - 3x - 15)$

$= (x^2 - 2)\big(2x(x + 5) - 3(x + 5)\big)$

$= (x^2 - 2)(x + 5)(x - 3)$

Other zeroes are:

$x + 5 = 0$ $\qquad$ $2x - 3 = 0$

$x = -5$ $\qquad$ $2x = 3$

$x = \frac{3}{2}$

Hence the set of zeroes for $f(x)$ $\left\{-5, \frac{3}{2}, \sqrt{2}, -\sqrt{2}\right\}$

11. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Sol:

$f(x) = 2x^3 + x^2 - 6x - 3$

$x = -\sqrt{3}$ is a solution

$x + \sqrt{3}\ is\ a\ factor$

$x = \sqrt{3}\ is\ a\ solution$

$x - \sqrt{3}\ is\ a\ factor$

Here, $(x + \sqrt{3})(x - \sqrt{3}) is\ a\ factor\ of\ f(x)$

$x^2 - 3\ is\ a\ factor\ of\ f(x)$

$$
\begin{array}{r|l}
 & 2x + 1 \\
\hline
x^2 - 3 & 2x^3 + x^2 - 6x - 3 \\
 & 2x^3 \qquad - 6x \\
\hline
 & \qquad x^2 \qquad - 3 \\
 & \qquad x^2 \qquad - 3 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Hence, $2x^3 + x^2 - 6x - 3 = (x^2 - 3)(2x + 1)$

Other zeroes of f(x) is $2 \times +1 = 0$

$x = -\frac{1}{2}$

$Set\ of\ zeroes\ \left\{\sqrt{3}, -\sqrt{3}, \frac{-1}{2}\right\}$

12. Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Sol:

Since $-\sqrt{2}$ *and* $\sqrt{2}$ are zeroes of polynomial $f(x) = x^3 + 3x^2 - 2x - 6$

$(x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2\ is\ a\ factor\ of\ f(x)$

Now we divide $f(x) = x^3 + 3x^2 - 2x - 6$ by

$g(x) = x^2 - 2\ to\ b\ find\ the\ other\ zeroes\ of\ f(x)$

$$
\begin{array}{r|l}
 & x + 3 \\
\hline
x^2 - 2 & x^3 + 3x^2 - 2x - 6 \\
 & x^3 \qquad - 2x \\
\hline
 & \qquad 3x^2 \qquad - 6 \\
 & \qquad 3x^2 \qquad - 6 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

By division algorithm, we have

$\Rightarrow x^3 + 3x^2 - 2 - 6 = (x^2 - 2)(x + 3)$

$\Rightarrow x^3 + 3x^2 - 2x - 6 = (x + \sqrt{2})(x - \sqrt{2})(x + 3)$

Here the zeroes of the given polynomials are $-\sqrt{2}, \sqrt{2}\ and - 3$