Ex 1.1

Relations Ex 1.1 Q1(i)

A be the set of human beings.
$R = \{(x,y):$ x and y work at the same place$\}$

Reflexive:

$\therefore$ x and x works together
$\therefore$ $(x,x) \in R$

$\Rightarrow$ R is reflexive

Symmetric: If x and y work at the same place, which implies,
y and x work at the same place

$\therefore$ $(y,x) \in R$

$\Rightarrow$ R is symmetric

Transitive: If x and y work at the same place
then x and y work at the same place and y and z work at the same place

$\Rightarrow$ $(x,z) \in R$ and

Hence,
$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(ii)

A be the set of human beings.
$R = \{(x, y) :$ x and y lives in the same locality $\}$

Reflexive: since x and x lives in the same locality

$\Rightarrow \quad (x, x) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow \quad x$ and y lives in the same locality
$\Rightarrow \quad y$ and x lives in the same locality
$\Rightarrow \quad (y, x) \in R$

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$
$(x, y) \in R$
$\Rightarrow$ x and y lives in the same locality
and $(y, z) \in R$
$\Rightarrow$ y and z lives in the same locality
$\Rightarrow$ x and z lives in the same locality
$\Rightarrow (x, z) \in R$
$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(iii)

$R = \{(x, y) : x$ is wife of $y\}$

Reflexive: since x can not be wife of x

$\therefore \quad (x, x) \notin R$

$\Rightarrow \quad R$ is not reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow \quad x$ is wife of y
$\Rightarrow \quad y$ is husband of x
$\Rightarrow \quad (y, x) \notin R$

$\Rightarrow \quad R$ is not symmetric

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \quad x$ is wife of y and y is husband of z
which is a contradiction
$\Rightarrow \quad (x, z) \notin R$

$\Rightarrow \quad R$ is not transitive

Relations Ex 1.1 Q1(iv)

A be the set of human beings
R = {(x, y) : x is father of y}
Reflexive: since x can not be father of x
∴ $(x, x) \notin R$

⇒ R is not reflexive

Symmetric: Let $(x, y) \in R$

⇒ x is father of y
⇒ y can not be father of x

⇒ $(y, x) \notin R$

⇒ R is not symmetric

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

⇒ x is father of y and y is father of z
⇒ x is grandfather of z

⇒ $(x, z) \notin R$

⇒ R is not transitive

Relations Ex 1.1 Q2

We have, $A = \{a, b, c\}$

$R_1 = \{(a,a)(a,b)(a,c)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

R_1 is reflexive as $(a,a) \in R_1, (b,b) \in R_1$ & $(c,c) \in R_1$
R_1 is not symmetric as $(a,b) \in R_1$ but $(b,a) \in R_1$
R_1 is not transitive as $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

$R_2 = \{(a,a)\}$

R_2 is not reflexive as $(b,b) \notin R_2$
R_2 is symmetric and transitive.

$R_3 = \{(b,c)\}$

R_3 is not reflexive as $(b,b) \notin R_3$
R_3 is not symmetric
R_3 is not transitive.

$R_4 = \{(a,b)(b,c)(c,a)\}$

R_4 is not reflexive on set A as $(a,a) \notin R_4$
R_4 is not symmetric as $(a,b) \in R_4$ but $(b,a) \notin R_4$
R_4 is not transitive as $(a,b) \in R_4$ and $(b,c) \in R_4$ but $(a,c) \notin R_4$

Relations Ex 1.1 Q3

$R_1 = \left\{(x, y), x, y \in Q_0, x = \frac{1}{y}\right\}$

Reflexivity: Let, $x \in Q_0$

$\Rightarrow \quad x \neq \frac{1}{x}$

$\Rightarrow \quad (x, x) \in R_1$

$\therefore \quad R_1$ is not reflexive

Symmetric: Let, $(x, y) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad (y, x) \in R_1$

$\therefore \quad R_1$ is symmetric symmetric

Transitive: Let, $(x, y) \in R_1$ and $(y, z) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$ and $y = \frac{1}{z}$

$\Rightarrow \quad x = z$

$\Rightarrow \quad (x, z) \notin R_1$

$\therefore \quad R_1$ is not trasitive

Relations Ex 1.1 Q3(ii)

Reflexivity: Let, $a \in z$

$\Rightarrow \quad |a - a| = 0 \leq 5$

$\therefore \quad (a, a) \in R_2 \Rightarrow \quad R_2$ is reflexive

Symmetricity:Let, $(a, b) \in R_2$

$\Rightarrow \quad |a - a| \leq 5$

$\Rightarrow \quad |b - a| \leq 5$

$\Rightarrow \quad |b, a| \in R_2 \quad \Rightarrow \quad R_2$ is symmetric

Transitivity: Let, $(a, b) \in R_2$ and $(b, c) \in R_2$

$\Rightarrow \quad |a - b| \leq 5$ and $|b - c| \leq 5$

$\not\Rightarrow \quad |a - c| \leq 5$

$\Rightarrow \quad R_2$ is not transitive

$$\left[\begin{array}{l} \therefore \text{ if } a = 15, b = 11, c = 7 \\ \qquad \Rightarrow \quad |15 - 11| \leq 5 \text{ and } |11 - 7| \leq 5 \\ \qquad\qquad \text{but } |15 - 7| \geq 5 \end{array}\right]$$

Relations Ex 1.1 Q4

(i) We have, $A = \{1, 2, 3\}$ and
$R_1 = \{(1,1)(1,3)(3,1)(2,2)(2,1)(3,3)\}$

$\therefore \quad (1,1), (2,2)$ and $(3,3) \in R_1$

$\therefore \quad R_1$ is not Reflexive

Now,
$\therefore \quad (2,1) \in R_1$ but $(1,2) \notin R_1$

$\therefore \quad R_1$ is not Symmetric

Again,
$\therefore \quad (2,1) \in R_1$ and $(1,3) \in R_1$ but $(2,3) \notin R_1$

$\therefore \quad R_1$ is not Transitive

(ii) $R_2 = \{(2,2), (3,1), (1,3)\}$

$\therefore \quad (1,1) \notin R_2$

$\Rightarrow \quad R_2$ is not reflexive

Now, $(1,3) \in R_2$
$\Rightarrow \quad (3,1) \in R_2$

$\Rightarrow \quad R_2$ is symmetric

Again, $(3,1) \in R_2$ and $(1,3) \in R_2$ but $(3,3) \notin R_1$

$\therefore \quad R_2$ is not transitive

(iii) $R_3 = \{(1,3)(3,3)\}$

$\therefore \quad (1,1) \notin R_3$

$\Rightarrow \quad R_3$ is not reflexive

Now, $(1,3) \in R_3$ but $(3,1) \in R_3$
$\Rightarrow \quad R_3$ is not symmetric

Again, It is clear that R_3 is transitive

Relations Ex 1.1 Q5.

(i) aRb if a-b > 0
Let R be the set of real numbers.

Reflexivity: Let $a \in R$

$\Rightarrow \quad a - a = 0$
$\Rightarrow \quad (a, a) \notin R$

$\therefore \quad R$ is not reflexive

Symmetric: Let aRb

$\Rightarrow \quad a - a > 0$
$\Rightarrow \quad b - a < 0$

$\therefore \quad b \not R a$

$\therefore \quad R$ is not Symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad a - a >$ and $b - c > 0$
$\Rightarrow \quad a - c > 0$
$\Rightarrow \quad aRc$

$\therefore \quad R$ is Transitive

Relations Ex 1.1 Q5(ii)
We have, aRb iff $1 + ab > 0$
Let R be the set of real numbers

Reflexive: Let $a \in R$

$\Rightarrow \quad 1 + a^2 > 0$
$\Rightarrow \quad aRa$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let aRb

$\Rightarrow \quad 1 + ab > 0$
$\Rightarrow \quad 1 + ba > 0$
$\Rightarrow \quad bRa$

$\Rightarrow \quad R$ is symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad 1 + ab > 0$ and $1 + bc > 0$
$\not\Rightarrow \quad 1 + ac > 0$

$\Rightarrow \quad R$ is not transitive

Relations Ex 1.1 Q5(iii)

We have, aRb if $|a| \le b$

Reflexivity: Let $a \in R$

$\Rightarrow$ $|a| \nleq a$ $\left[\because \quad |-2| = 2 > -2\right]$

$\Rightarrow$ R is not reflexive

Symmetric: Let aRb

$\Rightarrow$ $|a| \le b$

$\not\Rightarrow$ $|b| \le a$ $\left[\because \quad \begin{array}{l} \text{Let } a = 4,\ b = 6 \\ |4| \le 8 \text{ but } |8| > 4 \end{array}\right]$

$\Rightarrow$ R is not symmetric

Transitive: Let aRb and bRc

$\Rightarrow$ $|a| \le b$ and $|b| \le c$
$\Rightarrow$ $|a| \le |b| \le c$
$\Rightarrow$ $|a| \le c$
$\Rightarrow$ $a\ R\ c$

$\Rightarrow$ R is transitive

Relations Ex 1.1 Q6.

Let $A = \{1, 2, 3, 4, 5, 6\}$.
A relation R is defined on set A as:
$R = \{(a, b): b = a + 1\}$
Therefore, $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$
We find $(a, a) \notin R$, where $a \in A$.
For instance, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$
Therefore, R is not reflexive.
It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.
Therefore, R is not symmetric.
Now, $(1, 2), (2, 3) \in \mathbf{R}$
But, $(1, 3) \notin R$
Therefore, R is not transitive
Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q7.

$R = \{(a, b): a \le b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Therefore, R is not reflexive.
Now, $(1, 2) \in R$ (as $1 < 2^3 = 8$)
But, $(2, 1) \notin R$ (as $2^3 > 1$)
Therefore, R is not symmetric.
We have

$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.

Therefore, R is not transitive.
Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q8

Let A be a set.
Then $I_A = \{(a,a)\ ;\ a \in A\}$ is the identity relation on A.

Hence, every identity relation on a set is reflexive by definition.

Converse:

Let $A = \{(a,b,c)\}$ be a set.
Let $R = \{(a,a)(b,b)(c,c)(a,b)\}$ be a relation defined on A.

Clearly R is reflexive on set A, but it is not identity relation on set A as $(a,b) \in R$

Hence, a reflexive relation need not be identity relation.

Relations Ex 1.1 Q9

We have, $A = \{1, 2, 3, 4\}$

(i) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)\}$ is a relation on set A which is reflexive,transitive but not symmetric

(ii) $R = \{(2,3)(3,2)\}$ is a relation on set A which is symmetric but neither reflexive nor transitive

(iii) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,1)\}$ is a relation on set A which is reflexive, symmetric and transitive

Relations Ex 1.1 Q10

We have, $R - \{(x,y); x,y \in N, 2x + y = 41\}$

Then Domain of R is $x \in N$, such that

$$2x + y = 41$$

$$\Rightarrow \quad x = \frac{41 - y}{2}$$

Since $y \in N$, largest value that x can take corresponds to the smallest value that y can take.

$$\therefore \quad x = \{1, 2, 3, ...20\}$$

Range of R is $y \in N$ such that

$$2x + y = 41$$

$$\Rightarrow \quad y = 41 - 2x$$

Since, $x = \{1, 2, 3, ...20\}$

$$\therefore \quad y = \{39, 37, 35, 33, ...7, 5, 3, 1\}$$

Since, $(2,2) \notin R$, R is not reflexive.

Also, since $(1,39) \in R$ but $(39,1) \notin R$, R is not symmetric.

Finally, since, $(15,11) \in R$ and $(11,19) \in R$ but $(15,19) \notin R$

$\therefore$ R is not trasitive.

Relations Ex 1.1 Q11

No, it is not necessary that a relation which is symmetric and transitive is reflexive as well.

For Example,

Let $A = \{a,b,c\}$ be a set and

$R_2 = \{(a,a)\}$ is a relation defined on A.

Clearly,

R_2 is symmetric and transitive but not reflexive.

Relations Ex 1.1 Q12

It is given that an integer m is said to be relative to another integer n if m is a multiple of n.
In other words

$$R = \{(m,n); \quad m = kn, k \in z\}$$

Reflexivity: Let, $m \in z$

$\Rightarrow \quad m = 1.m$
$\Rightarrow \quad (m,m) \in R$

$\therefore \quad R$ is reflexive

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a = kb$ and $b = k'c$
$\Rightarrow \quad a = kk'c \qquad [\because \quad kk' \in z]$
$\Rightarrow \quad a = lc \qquad [\because \quad l = kk' \in z]$
$\Rightarrow \quad (a,c) \in R$

$\therefore \quad R$ is transitive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a = kb$
$\Rightarrow \quad b = \frac{1}{k}a \qquad$ but $\frac{1}{k} \notin z$ if $k \in z$
$\therefore \quad (b,a) \notin R$

$\therefore \quad R$ is not symmetric

Relations Ex 1.1 Q13

We have,
relation $R = " \geq "$ on the set R of all real numbers

Reflexivity: Let $a \in R$

$\Rightarrow \quad a \geq a$
$\Rightarrow \quad " \geq "$ is reflexive

Symmetric: Let $a,b \in R$
such that $a \geq b \not\Rightarrow b \geq a$

$\therefore \quad " \geq "$ not symmetric

Transitivity: Let $a,b,c \in R$
and $a \geq b$ & $b \geq c$

$\Rightarrow \quad a \geq c$

$\Rightarrow \quad " \geq "$ is transitive

Relations Ex 1.1 Q14

(i)Let $A = \{4, 6, 8\}$.
Define a relation R on A as:
$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$
Relation R is reflexive since for every $a \in A$, $(a, a) \in R$ i.e., $(4, 4), (6, 6), (8, 8)\} \in R$.
Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.
Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.
Hence, relation R is reflexive and symmetric but not transitive.

(ii) Define a relation R in **R** as:
$R = \{a, b): a^3 \geq b^3\}$
Clearly $(a, a) \in R$ as $a^3 = a^3$. $a = a$.
Therefore, R is reflexive.
Now, $(2, 1) \in R$ (as $2^3 \geq 1^3$)
But, $(1, 2) \notin R$ (as $1^3 < 2^3$)
Therefore, R is not symmetric.
Now, Let $(a, b), (b, c) \in R$.
$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$
$\Rightarrow a^3 \geq c^3$
$\Rightarrow (a, c) \in R$
Therefore, R is transitive.
Hence, relation R is reflexive and transitive but not symmetric.
Hence, relation R is transitive but not reflexive and symmetric.

(iv)Let $A = \{5, 6, 7\}$.
Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.
Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.
Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.
$\Rightarrow (5, 6), (6, 5) \in R$, but $(5, 5) \notin R$
Therefore, R is not transitive.
Hence, relation R is symmetric but not reflexive or transitive.

(v)Consider a relation R in **R** defined as:
$R = \{(a, b): a < b\}$
For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.
Therefore, R is not reflexive.
Now, $(1, 2) \in R$ (as $1 < 2$)
But, 2 is not less than 1.
Therefore, $(2, 1) \notin R$
Therefore, R is not symmetric.
Now, let $(a, b), (b, c) \in R$.
$\Rightarrow a < b$ and $b < c$
$\Rightarrow a < c$
$\Rightarrow (a, c) \in R$
Therefore, R is transitive.
Hence, relation R is transitive but not reflexive and symmetric.

Relations Ex 1.1 Q15

We have,

$A = \{1,2,3\}$ and $R\{(1,2)(2,3)\}$

Now,

To make R reflexive, we will add $(1,1)(2,2)$ and $(3,3)$ to get

$\therefore$ $R' = \{(1,2)(2,3)(1,1,)(2,2)(3,3)\}$ is reflexive

Again to make R' symmetric we shall add $(3,2)$ and $(2,1)$

$\therefore$ $R'' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)\}$ is reflexive and symmetric

Now,

To make R'' transitive we shall add $(1,3)$ and $(3,1)$

$\therefore$ $R''' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)(1,3)(3,1)\}$

$\therefore$ R''' is reflexive, symmetric and transitive

Relations Ex 1.1 Q16

We have, $A = \{1, 2, 3\}$ and $R = \{(1,2)(1,1)(2,3)\}$

To make R transitive we shall add $(1,3)$ only.

$\therefore \quad R' = \{(1,2)(1,1)(2,3)(1,3)\}$

Relations Ex 1.1 Q17

A relation R in A is said to be reflexive if aRa for all a∈A

R is said to be transitive if aRb and bRc ⇒ aRc
for all a, b, c ∈ A.
Hence for R to be reflexive (b, b) and (c, c) must be there in the set R.
Also for R to be transitive (a, c) must be in R because (a, b) ∈ R and (b, c) ∈ R so (a, c) must be in R.
So at least 3 ordered pairs must be added for R to be reflexive and transitive.

Relations Ex 1.1 Q18

A relation R in A is said to be reflexive if aRa for all a∈A, R is symmetric if aRb ⇒ bRa, for all a, b ∈ A and it is said to be transitive if aRb and bRc ⇒ aRc for all a, b, c ∈ A.

- x > y, x, y ∈ N

(x, y) ∈ {(2, 1), (3, 1).......(3, 2), (4, 2)....}
This is not reflexive as (1, 1), (2, 2)....are absent.
This is not symmetric as (2,1) is present but (1,2) is absent.
This is transitive as (3, 2) ∈ R and (2,1) ∈ R also (3,1) ∈ R ,similarly this property satisfies all cases.

- x + y = 10, x, y ∈ N

(x, y)∈ {(1, 9), (9, 1), (2, 8), (8, 2), (3, 7), (7, 3), (4, 6), (6, 4), (5, 5)}
This is not reflexive as (1, 1),(2, 2)..... are absent.
This only follows the condition of symmetric set as (1, 9)∈R also (9, 1)∈R similarly other cases are also satisfy the condition.
This is not transitive because {(1, 9),(9, 1)}∈R but (1, 1) is absent.

- xy is square of an integer, x, y ∈ N

(x, y) ∈ {(1, 1), (2, 2), (4, 1), (1, 4), (3, 3), (9, 1), (1, 9), (4, 4), (2, 8), (8, 2), (16, 1), (1, 16)..........}
This is reflexive as (1,1),(2,2)..... are present.
This is also symmetric because if aRb ⇒ bRa, for all a,b∈N.
This is transitive also because if aRb and bRc ⇒ aRc for all a, b, c ∈ N.

- x + 4y = 10, x, y ∈ N

(x, y) ∈ {(6, 1), (2, 2)}
This is not reflexive as (1, 1), (2, 2).....are absent.
This is not symmetric because (6,1) ∈ R but (1,6) is absent.
This is not transitive as there are only two elements in the set having no element common.

Ex 1.2

Relations Ex 1.2 Q1

We have,

$R = \{(a,b):\ a-b$ is divisible by 3; $a,b,\in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a-a=0$

$\Rightarrow \quad a-a$ is divisible by 3

$\Rightarrow \quad (a,a)\in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b\in Z$ and $(a,b)\in R$

$\Rightarrow \quad a-b$ is divisible by 3

$\Rightarrow \quad a-b=3p \qquad$ For some $p\in Z$

$\Rightarrow \quad b-a=3\times(-p)$

$\Rightarrow \quad b-a\in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c\in Z$ and such that $(a,b)\in R$ and $(b,c)\in R$

$\Rightarrow \quad a-b=3p$ and $b-c=3q$ For some $p,q\in Z$

$\Rightarrow \quad a-c=3(p+q)$

$\Rightarrow \quad a-c$ is divisible by 3

$\Rightarrow \quad (a,c)\in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

Relations Ex 1.2 Q2

We have,

$R = \{(a,b) : a - b \text{ is divisible by } 2;\ a,b \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 2

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a - b$ is divisible by 2

$\Rightarrow \quad a - b = 2p \quad$ For some $p \in Z$

$\Rightarrow \quad b - a = 2 \times (-p)$

$\Rightarrow \quad b - a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 2p$ and $b - c = q$ For some $p,q \in Z$

$\Rightarrow \quad a - c = 2(p + q)$

$\Rightarrow \quad a - c$ is divisible by 2

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Relations Ex 1.2 Q3

We have,

$R = \{(a,b) : (a-b) \text{ is divisible by } 5\}$ on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 5.

$\therefore \quad (a,a) \in R$, so R is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = 5P \quad$ For some $P \in Z$

$\Rightarrow \quad b - a = 5 \times (-P)$

$\Rightarrow \quad b - a$ is divisible by 5

$\Rightarrow \quad (b,a) \in R$, so R is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 5p$ and $b - c = 5q$ For some $p,q \in Z$

$\Rightarrow \quad a - c = 5(p + q)$

$\Rightarrow \quad a - c$ is divisible by 5.

$\Rightarrow \quad R$ is transitive.

Thus, R being reflexive,symmetric and transitive on Z.

Hence, R is equivalence relation on Z

Relations Ex 1.2 Q4

$R = \{(a,b): \text{ a-b is divisible by n}\}$ on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0 \times n$

$\Rightarrow \quad a - a$ is divisible by n

$\Rightarrow \quad (a, a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = np$ For some $p \in Z$

$\Rightarrow \quad b - a = n(-p)$

$\Rightarrow \quad b - a$ is divisible by n

$\Rightarrow \quad (b, a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = xp$ and $b - c = xq$ For some $p,q \in Z$

$\Rightarrow \quad a - c = n(p + q)$

$\Rightarrow \quad a - c$ is divisible by n

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

Relations Chapter 1 Ex 1.2 Q5

We have, Z be set of integers and

$R = \{(a,b):\ a,b \in Z \text{ and } a+b \text{ is even}\}$ be a relation on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a+a$ is even $\quad \begin{bmatrix} \text{if a is even} \Rightarrow a+a \text{ is even} \\ \text{if a is odd} \Rightarrow a+a \text{ is even} \end{bmatrix}$

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a+b$ is even

$\Rightarrow \quad b+a$ is even

$\Rightarrow \quad (b,a) \in R,$

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some $a,b,c \in Z$

$\Rightarrow \quad a+b$ is even and $b+c$ is even

$\Rightarrow \quad a+c$ is even $\quad \begin{bmatrix} \text{if b is odd, then a and c must be odd} \Rightarrow a+c \text{ is even,} \\ \text{If b is even, then a and c must be even} \Rightarrow a+c \text{ is even} \end{bmatrix}$

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q6

Let Z be set of integers

$R = \{(m,n) : m - n \text{ is divisible by } 13\}$ be a relation on Z.

Now,

Reflexivity: Let m $\in Z$

$\Rightarrow \quad m - m = 0$

$\Rightarrow \quad m - m$ is divisible by 13

$\Rightarrow \quad (m,m) \in R,$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let m,n $\in Z$ and $(m,n) \in R$

$\Rightarrow \quad m - n = 13.p$ For some $p \in Z$

$\Rightarrow \quad n - m = 13 \times (-p)$

$\Rightarrow \quad n - m$ is divisible by 13

$\Rightarrow \quad (n - m) \in R,$

so

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(m,n) \in R$ and $(n,q) \in R$ For some m,n,q $\in$ Z

$\Rightarrow \quad m - n = 13p$ and $n - q = 13s$ For some p,s $\in$ Z

$\Rightarrow \quad m - q = 13(p + s)$

$\Rightarrow \quad m - q$ is divisible by 13

$\Rightarrow \quad (m,q) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q7

(x, y) R (u, v) $\Leftrightarrow$ xv = yu

TPT Reflexive $\because$ xy = yx

$\therefore$ (x, y) R (x, y)

TPT Symmetric Let (x, y) R (u, v)

TPT (u, v) R (x, y)

Given xv = yu

$\Rightarrow$ yu = xv

$\Rightarrow$ uy = vx

$\therefore$ (u, v) R (x, y)

Transitive Let (x, y) R (u, v) and (u, v) R (p, q) ....(i)

TPT (x, y) R (p, q)

TPT xq = yp

from (1) xv = yu & uq = vp

xvuq = yuvp

xq = yp

$\therefore$ R is transitive

since R is reflexive symmetric & transitive all means it is an equivalence relation.]

Relations Ex 1.2 Q8

We have, $A = \{x \in z : 0 \le x \le 12\}$ be a set and
$R = \{(a,b) : a = b\}$ be a relation on A
Now,
Reflexivity: Let $a \in A$

$\Rightarrow \quad a = a$
$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in A$ and $(a,b) \in R$

$\Rightarrow \quad a = b$
$\Rightarrow \quad b = a$
$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let a,b & $c \in A$
and Let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow \quad a = b$ and $b = c$
$\Rightarrow \quad a = c$
$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since R is being relfexive,symmetric and transitive, so
R is an equivalance relation.

Also, we need to find the set of all elements related to 1.
Since the relation is given by, R={(a,b):a=b}, and 1 is an element of A,
R={(1,1):1=1}
Thus, the set of all elements related to 1 is 1.

Relations Ex 1.2 Q9

(i) We have, L is the set of lines.

$R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$ be a relation on L

Now,

Reflexivity: Let $L_1 \in L$

Since a line is always parallel to itself.

$\therefore \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2

$\Rightarrow \quad L_2$ is parallel to L_1

$\Rightarrow \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $L_1, L_2 and L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow \quad L_1$ is parallel to L_3

$\Rightarrow \quad (L_1, L_3) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

(ii) The set of lines parallel to the line $y = 2x + 4$ is

$y = 2x + c$ For all $c \in R$

Where R is the set of real numbers.

Relations Ex 1.2 Q10

R = {(P_1, P_2): P_1 and P2 have same the number of sides}

R is reflexive since (P_1, P_1) ∈ R as the same polygon has the same number of sides with itself.

Let (P_1, P_2) ∈ R.

⇒ P_1 and P_2 have the same number of sides.

⇒ P_2 and P_1 have the same number of sides.

⇒ (P_2, P_1) ∈ R

∴R is symmetric.

Now,

Let (P_1, P_2), (P_2, P3) ∈ R.

⇒ P_1 and P_2 have the same number of sides. Also, P_2 and P3 have the same number of sides.

⇒ P_1 and P3 have the same number of sides.

⇒ (P_1, P3) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Relations Ex 1.2 Q11

Let A be set of points on plane.

Let $R = \{(P, Q): OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Now,

Reflexivity: Let $p \in A$

Since $OP = OP \Rightarrow (P, P) \in R$

$\Rightarrow$ R is reflexive

Symmetric: Let $(P, Q) \in R$ for $P, Q \in A$

Then $OP = OQ$

$\Rightarrow$ $OQ = OP$

$\Rightarrow$ $(Q, P) \in R$

$\Rightarrow$ R is symmetric

Transitive: Let $(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow$ $OP = OQ$ and $OQ = OS$

$\Rightarrow$ $OP = OS$

$\Rightarrow$ $(P, S) \in R$

$\Rightarrow$ R is transitive

Thus, R is an equivalence relation on A

Relations Ex 1.2 Q12

Given A={1,2,3,4,5,6,7} and R={(a,b):both a and b are either odd or even number}

Therefore,

R={(1,1),(1,3),(1,5),(1,6),(3,3),(3,5),(3,7),(5,5),(5,7),(7,7),(7,5),(7,3),(5,3),(6,1),(5,1),(3,1),
(2,2),(2,4),(2,6),(4,4),(4,6),(6,6),(6,4),(6,2),(4,2)}

Form the relation R it is seen that R is symmetric, reflecive and transitive also. Therefore R is an equivalent relation.

From the relation R it is seen that {1,3,5,7} are related with each other only and {2,4,6} are related with each other

Relations Ex 1.2 Q13

$S = \{(a, b): a^2 + b^2 = 1\}$

Now,

Reflexivity: Let $a = \frac{1}{2} \in R$

Then, $a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$

$\Rightarrow$ $(a, a) \notin S$

$\Rightarrow$ S is not reflexive

Hence, S in not an equivalenve relation on R

Relations Ex 1.2 Q14

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on $z \times z_0$

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$

$\Rightarrow \quad af = be$

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on Z and Z_0.

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$

$\Rightarrow \quad af = be$

$\Rightarrow \quad (a,b)(e,f) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15.

R and S are two symmetric relations on set A

(i) To prove: $R \cap S$ is symmetric
Let $(a,b) \in R \cap S$

$\Rightarrow \quad (a,b) \in R$ and $(a,b) \in S$
$\Rightarrow \quad (b,a) \in R$ and $(b,a) \in S \quad [\because R$ and S are symmetric$]$
$\Rightarrow \quad (b,a) \in R \cap S$
$\Rightarrow \quad R \cap S$ is symmetric

To prove: $R \cup S$ is symmetric.
Let $(a,b) \in R \cup S$

$\Rightarrow \quad (a,b) \in R$ or $(a,b) \in S$
$\Rightarrow \quad (b,a) \in R$ or $(b,a) \in S \quad [\because R$ and S are symmetric$]$
$\Rightarrow \quad (b,a) \in R \cup S$
$\Rightarrow \quad R \cup S$ is symmetric

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.
This means that there is an $a \in R \cup S$ such that $(a,a) \notin R \cup S$

Since $a \in R \cup S$,
$\therefore \quad a \in R$ or $a \in S$

If $a \in R$, then $(a,a) \in R \quad [\because R$ is reflexive$]$
$\Rightarrow \quad (a,a) \in R \cup S$
Hence, $R \cup S$ is reflexive

Relations Ex 1.2 Q16.
We will prove this by means of an example.
Let $A = \{a,b,c\}$ be a set and
$R = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$ and
$S = \{(a,a)(b,b)(c,c)(b,c)(c,b)\}$ are two relations on A

Clearly R and S are transitive relation on A

Now, $R \cup S = \{(a,a)(b,b)(c,c)(a,b)(b,a)(b,c)(c,b)\}$
Here, $(a,b) \in R \cup S$ and $(b,c) \in R \cup S$
but $(a,c) \notin R \cup S$

$\therefore \quad R \cup S$ is not transitive