

FUNCTIONS (XII, R. S. AGGARWAL)

EXERCISE 2A (Pg. No.: 34)

1. Define a function. What do you mean by the domain and range of a function? Give examples.

Sol. Let A and B be two non empty sets. Then, a rule f which associates to each element $x \in A$, a unique element, denoted by $f(x)$ of B , is called a function from A to B and we write $f: A \rightarrow B$

Domain, co-domain and range of function let $f: A \rightarrow B$, then, A is called the domain of f and B is called the co-domain of f . And $f(A) = \{f(x) : x \in A\}$ is called the range of f .

Example : let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16, 25\}$

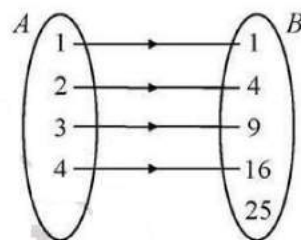
Consider the rule $f: A \rightarrow B: f(x) = x^2 \forall x \in A$.

Then, each element in A has its unique image in B . So, f is a function from A to B .

$$f(1) = (1)^2 = 1, f(2) = (2)^2 = 4, f(3) = (3)^2 = 9, f(4) = (4)^2 = 16$$

Domain $(f) = \{1, 2, 3, 4\} = A$, Co-domain $(f) = \{1, 4, 9, 16, 25\} = B$ and range $(f) = \{1, 4, 9, 16\}$.

Clearly, $25 \in B$ does not have its pre-image in A .



2. Define each of the following:

- (i) injective function (ii) surjective function (iii) bijective function (iv) many-one function (v) into function

Sol. (i) Injective function : A function $f: A \rightarrow B$ is said to be one-one if distinct element in A have distinct image in B .

Example: let N be the set of all natural number let $f: N \rightarrow N: f(x) = 2x \forall x \in N$

Then $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2 \therefore f$ is one- one

(ii) Surjective function : A function $f: A \rightarrow B$ is said to be onto if every element in B has at least one pre-image in A . Thus, if f is onto, then for each $y \in B$ at least one element $x \in A$ such that $y = f(x)$. Also, f is onto $\Leftrightarrow$ range $(f) = B$.

Example : let N be the set of all natural number at least E be the set of all even natural numbers.

Let $f: N \rightarrow E: f(x) = 2x \forall x \in N$. Then $y = 2x \Rightarrow x = \frac{1}{2}y$.

Thus, for each $y \in E$, there exists $\frac{1}{2}y \in N$ such that $f\left(\frac{1}{2}y\right) = \left(2 \times \frac{1}{2}y\right) = y \therefore$ its onto.

(iii) Bijective function: A one-one onto function is said to be bijective. If there is a one-to-one correspondence between A and B .

(iv) Many one function : A function $f: A \rightarrow B$ is said to be many one if two or more then two element in A have the same image in B .

Example. Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$

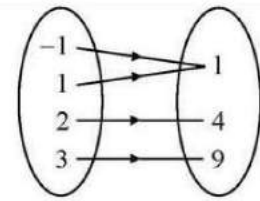
Let $f: A \rightarrow B: f(x) = x^2 \forall x \in A$.

Then each element in A has a unique image under f in B .

$\therefore f$ is a function from A to B such that

$$f(-1) = (-1)^2 = 1, f(1) = (1)^2 = 1$$

$$f(2) = (2)^2 = 4, \text{ and } f(3) = (3)^2 = 9$$



Clearly, two different elements, namely -1 and 1 have the same image $1 \in B$.

Hence f is many one.

(v) Into function : A function $f: A \rightarrow B$ is said to be into if there exists even a single element in B having no pre-image in A . Clearly, f is into $\Leftrightarrow \text{range}(f) \subset B$.

Example : Let $A = \{2, 3, 5, 7\}$ and $B = \{0, 1, 3, 5, 7\}$. Let $f: A \rightarrow B: f(x) = (x-2)$.

Then, $f(2) = (2-2) = 0$, $f(3) = (3-2) = 1$, $f(5) = 5-2 = 3$ and $f(7) = 7-2 = 5$.

Thus, every element in A has a unique image in B .

Now, $\exists 7 \in B$ having no pre-image in A . $\therefore f$ is into. **Note :** $\text{Range}(f) = \{0, 1, 3, 5\} \subset B$

3. Give an example of a function which is

(i) one-one but not onto

(ii) one-one and onto

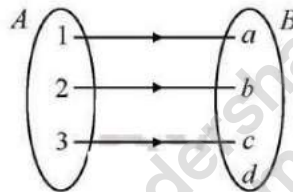
(iii) neither one-one nor onto

(iv) onto but not one-one.

Sol. (i) $A = \{1, 2, 3\}$

$$B = \{a, b, c, d\}$$

$$f = \{(1, a), (2, b), (3, c)\}$$



(ii) We observe the following properties of $f(x) = 2x$.

Injectivity: Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$. then, $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

So, $f: R \rightarrow R$ is one-one

Subjectivity: Let y be any real number in R (co-domain)

Then, $f(x) = y \Rightarrow 2x = y \Rightarrow x = \frac{y}{2}$ clearly, $\frac{y}{2} \in R$ for $y \in R$ such that $f\left(\frac{y}{2}\right) = 2\left(\frac{y}{2}\right) = y$.

Thus, for each $y \in R$ (co-domain) there exists $x = \frac{y}{2} \in R$ (domain) such that $f(x) = y$.

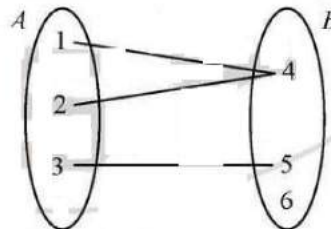
This means that each element in co-domain has its pre-image in domain.

Hence $f: R \rightarrow R$ is a bijective.

(iii) $A = \{1, 2, 3\}$

$$B = \{4, 5, 6\}$$

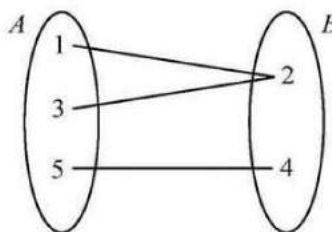
$$\Rightarrow f = \{(1, 4), (2, 4), (3, 5)\}$$



(iv) $A = \{1, 3, 5\}$

$$B = \{2, 4\}$$

$$\Rightarrow f = \{(1, 2), (3, 2), (5, 4)\}$$



4. Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 2x+3, & \text{when } x < -2 \\ x^2-2, & \text{when } -2 \leq x \leq 3 \\ 3x-1, & \text{when } x > 3 \end{cases}$

Find (i) $f(2)$ (ii) $f(4)$ (iii) $f(-1)$ (iv) $f(-3)$

Sol. (i) $f(x) = x^2 - 2 \Rightarrow f(2) = (2)^2 - 2 = 4 - 2 = 2 \therefore f(2) = 2$

(ii) $f(x) = 3x - 1 \Rightarrow f(4) = 3 \times 4 - 1 = 12 - 1 = 11 \therefore f(4) = 11$

(iii) $f(x) = x^2 - 2 \Rightarrow f(-1) = (-1)^2 - 2 = 1 - 2 = -1 \therefore f(-1) = -1$

(iv) $f(x) = 2x + 3 \Rightarrow f(-3) = 2(-3) + 3 = -6 + 3 = -3 \therefore f(-3) = -3$

5. Show that the function $f: R \rightarrow R: f(x) = 1 + x^2$ is many-one into

Sol. Clearly for every negative and positive value of x will get same value of $f(x)$ hence $f(x)$ is not one-one i.e., many one.

Clearly $x^2 + 1 \geq 1$ for all x . Hence, $\text{Range} = [1, \infty) \subset R \therefore$ function is into function.

6. Show that the function $f: R \rightarrow R: f(x) = x^4$ is many-one and into.

Sol. Clearly for every negative and positive value of x we will get same value of $f(x)$.

Hence, $f(x)$ is not one-one i.e., many one. Clearly $x^4 \geq 0$ for all x

Hence $\text{range} = [0, \infty) \subset R \therefore$ function is into function.

7. Show that the function $f: R \rightarrow R: f(x) = x^5$ is one-one and onto.

Sol. Injectivity: Let $x, y \in R$ such that $f(x) = f(y)$

$$\Rightarrow x^5 = y^5 \Rightarrow x^5 - y^5 = 0 \Rightarrow (x^{5/2})^2 - (y^{5/2})^2 = 0 \Rightarrow (x^{5/2} - y^{5/2})(x^{5/2} + y^{5/2}) = 0$$

$$\Rightarrow x^{5/2} - y^{5/2} = 0 \Rightarrow x = y. \text{ Thus } f(x) = f(y) \Rightarrow x = y \text{ for all } x, y \in R. \text{ So, } f \text{ is an injective map.}$$

Surjectivity: Let y be arbitrary element of R . Then $f(x) = y \Rightarrow x^5 = y \Rightarrow x^5 - y = 0$

We know that an odd degree equation has at least one real root. Therefore, for every real value of y , the equation $x^5 - y = 0$ has a real root α such that

$$\alpha^5 - y = 0 \Rightarrow \alpha^5 = y \Rightarrow f(\alpha) = y$$

Thus for every $y \in R$ there exists $\alpha \in R$ such that $f(\alpha) = y$.

So, f is a surjective map. Hence $f: R \rightarrow R$ is a bijective.

8. Let $f: \left[0, \frac{\pi}{2}\right] \rightarrow R: f(x) = \sin x$ and $g: \left[0, \frac{\pi}{2}\right] \rightarrow R: g(x) = \cos x$. Show that each one of f and g is one-one but $(f+g)$ is not one-one.

Sol. We observe that for any two distinct elements x_1 and x_2 in $\left[0, \frac{\pi}{2}\right]$

$$\sin x_1 \neq \sin x_2 \text{ and } \cos x_1 \neq \cos x_2$$

$$\Rightarrow f(x_1) \neq f(x_2) \text{ and } g(x_1) \neq g(x_2) \Rightarrow f \text{ and } g \text{ are one-one}$$

We have, $(f+g)(x) = f(x) + g(x) = \sin x + \cos x$

$$\Rightarrow (f+g)(0) = \sin 0 + \cos 0 = 1 \text{ and } (f+g)\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = 1$$

$$\therefore 0 \neq \frac{\pi}{2}, \text{ but } (f+g)(0) = (f+g)\left(\frac{\pi}{2}\right). \text{ So, } f+g \text{ is not one-one.}$$

9. Show that the function

$$(i) f: N \rightarrow N: f(x) = x^2 \text{ is one-one into} \quad (ii) f: Z \rightarrow Z: f(x) = x^2 \text{ is many-one into.}$$

Sol. (i) Let N be the set of all natural numbers. Let $f: N \rightarrow N: f(x) = x^2 \forall x \in N$

$$\text{Then } f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1^2 - x_2^2 = 0 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$$

$$\Rightarrow (x_1 - x_2) = 0 \Rightarrow x_1 = x_2. \therefore f \text{ is one-one.}$$

$$\text{Let } A = \{1, 2, 3, 4\} \text{ and } B = \{1, 4, 9, 16, 25\}. \text{ Let } f: A \rightarrow B, f(x) = x^2$$

$$\text{Then, } f(1) = (1)^2 = 1, f(2) = (2)^2 = 4, f(3) = (3)^2 = 9, f(4) = (4)^2 = 16.$$

Thus, every element in A has a unique image in B . Now, $\exists 25 \in B$ having no pre-image in $A \therefore f$ is into.

$$(ii) \text{ Let } A = \{-1, 1, 2, 3\} \text{ and } B = \{1, 4, 9\}.$$

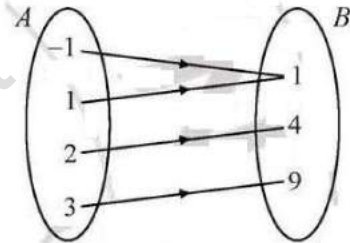
$$\text{Let } f: A \rightarrow B: f(x) = x^2 \forall x \in A$$

Then, each element in A has a unique image under f in B .

$\therefore f$ is a function from A to B such that

$$f(-1) = (-1)^2 = 1, f(1) = (1)^2 = 1$$

$$f(2) = (2)^2 = 4, f(3) = (3)^2 = 9$$



Clearly, two different elements, namely -1 and 1 have the same image $1 \in B$.

Hence f is many-one.

10. Show that the function

$$(i) f: N \rightarrow N: f(x) = x^3 \text{ is one-one into} \quad (ii) f: Z \rightarrow Z: f(x) = x^3 \text{ is one-one into}$$

Sol. (i) Let N be the set of all natural number, let $f: N \rightarrow N, f(x) = x^3 \forall x \in N$

$$\text{Then } f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3 \therefore f \text{ is one-one.}$$

$$\text{Here } f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3$$

$\Rightarrow x_1 = x_2 \therefore$ function is one-one and there is no pre-image for the numbers which are not perfect cubes in N e.g., 7 in co-domain has no pre-image in domain.

not onto. Hence, function is not onto.

$$(ii) \text{ Let } Z \text{ be the set of all integers. Let } f: Z \rightarrow Z, f(x) = x^3 \forall x \in Z$$

$$\text{Then } f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow x_1 = x_2. \therefore f \text{ is one-one. The explanation is same as in pre-image question}$$

11. Show that the function $f: R \rightarrow R: f(x) = \sin x$ is neither one-one nor onto.

Sol. As range is $[-1, 1]$ and Co-domain is R

$$\therefore \text{Range } \subset \text{co-domain} \therefore f(x) \text{ is onto, also } f(0) = f(\pi) = 0$$

Hence, $f(x)$ is many-one function.

12. Prove that the function $f: N \rightarrow N: f(n) = (n^2 + n + 1)$ is one-one but not onto.

Sol. $f: N \rightarrow N, f(x) = n^2 + n + 1$

$$\begin{aligned} \text{Let } f(x_1) &= f(x_2) \Rightarrow n_1^2 + n_1 + 1 = n_2^2 + n_2 + 1 \Rightarrow (n_1^2 - n_2^2) + (n_1 - n_2) = 0 \\ &\Rightarrow (n_1 - n_2)(n_1 + n_2) + (n_1 - n_2) = 0 \Rightarrow (n_1 - n_2) \{n_1 + n_2 + 1\} = 0 \\ &\Rightarrow (n_1 - n_2) = 0, (n_1 + n_2 + 1) \neq 0 \Rightarrow n_1 = n_2 \end{aligned}$$

13. Show that the function $f: N \rightarrow Z$, defined by $f(n) = \begin{cases} \frac{1}{2}(n-1), & \text{when } n \text{ is odd} \\ -\frac{1}{2}n, & \text{when } n \text{ is even} \end{cases}$

is both one-one and onto.

Sol. f is an injection: let $N, m \in N$ if n and m are even then $f(n) = f(m)$

$$\Rightarrow \frac{-n}{2} = \frac{-m}{2} \Rightarrow -n = -m \Rightarrow n = m$$

If n and m are odd then $f(n) = f(m)$

$$\Rightarrow \frac{n-1}{2} = \frac{m-1}{2} \Rightarrow n-1 = m-1 \Rightarrow n = m$$

Thus in both case we have $f(n) = f(m) \Rightarrow n = m$

And if n is even and m is odd then the $f(n) = f(m)$

$$\Rightarrow \frac{n}{2} = \frac{m-1}{2} \Rightarrow n = m-1$$

Which is not possible because m, n are natural number. Now for every +ve integer n there is a number $2n+1$ in domain show that $f(2n+1) = \frac{1}{2}(2n+1-1) = n$ and for every negative number n in Z there is a number $-2n$ in N such that $f(-2n) = -\frac{1}{2}2n = n$ and for 0 in Z the number in N is 1 such that $f(1) = 0$

Hence, the function is one-one and onto.

14. Find the domain and range of the function $f: R \rightarrow R: f(x) = x^2 + 1$.

Sol. Domain $(f) = R$, Also $y = x^2 + 1 \Rightarrow x = \pm\sqrt{y-1}$

x is defined when $y-1 \geq 0$ i.e. $y \geq 1$. $\therefore$ Range $(f) = \{y \in R: y \geq 1\}$

15. Which of the following relations are functions? Give reasons. In case of a function, find its domain and range.

(i) $f = \{(-1, 2), (1, 8), (2, 11), (3, 14)\}$

(ii) $g = \{(1, 1), (1, -1), (4, 2), (9, 3), (16, 4)\}$

(iii) $h = \{(a, b), (b, c), (c, b), (d, c)\}$

Sol. (i) $f = \{(-1, 2), (1, 8), (2, 11), (3, 14)\}$

f is a function, domain $(f) = \{-1, 1, 2, 3\}$ and range $(f) = \{2, 8, 11, 14\}$

(ii) $g = \{(1, 1), (1, -1), (4, 2), (9, 3), (16, 4)\}$. g is not a function.

(iii) h is a function, domain $(h) = \{a, b, c, d\}$ and range $(h) = \{b, c\}$.

16. Find the domain and range of the real function, defined by $f(x) = \frac{x^2}{(1+x^2)}$. Show that f is many-one.

Sol. When x is real, $1+x^2 \neq 0$, so, domain $(f) = R$

$$y = \frac{x^2}{1+x^2} \Rightarrow y + x^2 y = x^2 \Rightarrow y = x^2 - x^2 y \Rightarrow y = x^2(1-y) \Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \sqrt{\frac{y}{1-y}}$$

For x to be real, $\frac{y}{1-y} \geq 0$ and $(1-y) \neq 0$. $\therefore y(1-y) \geq 0 \Rightarrow 0 \leq y \leq 1$

$$\therefore \text{Range } (f) = \{y \in R : 0 \leq y \leq 1\}$$

Also 1 and -1 have the same image $\left(\frac{1}{2}\right)$. Hence f is many-one

17. Show that the function $f: R \rightarrow R: f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$

is many-one into. Find (i) $f\left(\frac{1}{2}\right)$ (ii) $f(\sqrt{2})$ (iii) $f(\pi)$ (iv) $f(2+\sqrt{3})$.

Sol. Since all rational have the same image, namely 1, so f is many one.

Range $(f) = \{-1, 1\} \subset R$, so f is into

$$(i) f\left(\frac{1}{2}\right) = 1 \quad (ii) f(\sqrt{2}) = -1 \quad (iii) f(\pi) = -1 \quad (iv) f(2+\sqrt{3}) = -1$$

EXERCISE 2B (Pg. No.: 42)

1. Let $A = \{1, 2, 3, 4\}$. Let $f: A \rightarrow A$ and $g: A \rightarrow A$, defined by $f = \{(1, 4), (2, 1), (3, 3), (4, 2)\}$ and $g = \{(1, 3), (2, 1), (3, 2), (4, 4)\}$. Find (i) gof (ii) fog (iii) fof .

Sol. (i) Here range $(f) = \{1, 2, 3, 4\}$ and domain $(g) = \{1, 2, 3, 4\}$. Clearly, range $(f) \subseteq$ domain (g) .

(ii) $\therefore gof$ is defined and domain $(gof) = \text{domain}(f) = \{1, 2, 3, 4\}$

$$\text{Now, } (gof)(1) = g\{f(1)\} = g(4) = 4; \quad (gof)(2) = g\{f(2)\} = g(1) = 3$$

$$(gof)(3) = g\{f(3)\} = g(3) = 2; \quad (gof)(4) = g\{f(4)\} = g(2) = 1$$

$$\text{Hence } gof = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

(iii) $\therefore fog$ is defined and domain $(fog) = \text{domain}(g) = \{1, 2, 3, 4\}$

$$\text{Now, } (fog)(1) = f\{g(1)\} = f(3) = 3; \quad (fog)(2) = f\{g(2)\} = f(1) = 4$$

$$(fog)(3) = f\{g(3)\} = f(2) = 1; \quad (fog)(4) = f\{g(4)\} = f(4) = 2$$

$$\text{Hence } fog = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$$

(iv) $\therefore fof$ is defined and domain $(fof) = \text{domain}(f) = \{1, 2, 3, 4\}$

$$\text{Now, } (fof)(1) = f\{f(1)\} = f(4) = 2; \quad (fof)(2) = f\{f(2)\} = f(1) = 4$$

$$(fof)(3) = f\{f(3)\} = f(3) = 3; \quad (fof)(4) = f\{f(4)\} = f(2) = 1$$

$$\therefore fof = \{(1, 2), (2, 4), (3, 3), (4, 1)\}$$

2. Let $f: \{3, 9, 12\} \rightarrow \{1, 3, 4\}$ and $g: \{1, 3, 4, 5\} \rightarrow \{3, 9\}$ be defined as $f = \{(3, 1), (9, 3), (12, 4)\}$ and $g = \{(1, 3), (3, 3), (4, 9), (5, 9)\}$. Find (i) $(g \circ f)$ (ii) $(f \circ g)$.

Sol. (i) $\therefore g \circ f$ is defined and domain $(g \circ f) = \text{domain}(f) = \{3, 9, 12\}$

$$(g \circ f)(3) = g\{f(3)\} = g\{1\} = 3; \quad (g \circ f)(9) = g\{f(9)\} = g\{3\} = 3$$

$$(g \circ f)(12) = g\{f(12)\} = g\{4\} = 9; \quad \therefore g \circ f = \{(3, 3), (9, 3), (12, 9)\}$$

(ii) $\therefore f \circ g$ is defined and domain $(f \circ g) = \text{domain}(g) = \{1, 3, 4, 5\}$

$$(f \circ g)(1) = f\{g(1)\} = f\{3\} = 1; \quad (f \circ g)(3) = f\{g(3)\} = f\{3\} = 1$$

$$(f \circ g)(4) = f\{g(4)\} = f\{9\} = 3; \quad (f \circ g)(5) = f\{g(5)\} = f\{9\} = 3$$

$$\therefore f \circ g = \{(1, 1), (3, 1), (4, 3), (5, 3)\}$$

3. Let $f: R \rightarrow R: f(x) = x^2$ and $g: R \rightarrow R: g(x) = (x+1)$. Show that $(g \circ f) \neq (f \circ g)$.

Sol. Let $f(x) = x^2$, and $g(x) = x+1$

$$\text{L.H.S., } g \circ f = g \circ f(x) = g\{f(x)\} = g\{x^2\} = (x^2) + 1 = x^2 + 1$$

$$\text{R.H.S., } f \circ g = f \circ g(x) = f\{g(x)\} = f\{x+1\} = (x+1)^2 = x^2 + 2x + 1$$

Hence $g \circ f \neq f \circ g$

4. Let $f: R \rightarrow R: f(x) = (2x+1)$ and $g: R \rightarrow R: g(x) = (x^2-2)$. Write down the formulae for

(i) $(g \circ f)$ (ii) $(f \circ g)$ (iii) $(f \circ f)$ (iv) $(g \circ g)$.

Sol. Let $f(x) = 2x+1$ and $g(x) = x^2-2$

$$(i) \quad g \circ f = g \circ f(x) = g\{f(x)\} = g\{2x+1\} = (2x+1)^2 - 2 = 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$$

$$(ii) \quad f \circ g = f \circ g(x) = f\{g(x)\} = f\{x^2-2\} = 2\{x^2-2\} + 1 = 2x^2 - 4 + 1 = (2x^2 - 3)$$

$$(iii) \quad f \circ f = f \circ f(x) = f\{f(x)\} = f\{2x+1\} = 2(2x+1) + 1 = 4x + 2 + 1 = 4x + 3$$

$$(iv) \quad g \circ g(x) = g\{g(x)\} = g\{x^2-2\} = (x^2-2)^2 - 2 = x^4 - 4x^2 + 4 - 2 = x^4 - 4x^2 + 2$$

5. Let $f: R \rightarrow R: f(x) = x^2 + 3x + 1$ and $g: R \rightarrow R: g(x) = 2x - 3$ write down the formula for

(i) $g \circ f$ (ii) $f \circ g$ (iii) $g \circ g$

Sol. Let $f(x) = x^2 + 3x + 1$, and $g(x) = 2x - 3$

$$(i) \quad g \circ f = g \circ f(x) = g\{f(x)\} = g\{x^2 + 3x + 1\} = 2(x^2 + 3x + 1) - 3$$

$$= 2x^2 + 6x + 2 - 3 = 2x^2 + 6x - 1$$

$$(ii) \quad f \circ g = f \circ g(x) = f\{g(x)\} = f\{2x-3\} = (2x-3)^2 + 3(2x-3) + 1$$

$$= 4x^2 - 12x + 9 + 6x - 9 + 1 = 4x^2 - 6x + 1$$

$$(iii) \quad g \circ g(x) = g\{g(x)\} = g\{2x-3\} = 2(2x-3) - 3 = 4x - 6 - 3 = 4x - 9$$

6. Let $f: R \rightarrow R: f(x) = |x|$, prove that $f \circ f = f$

Sol. $f(x) = |x|$, $f \circ f = f \{f(x)\} = f(|x|) = |x|$... (i)

$f = |x|$... (ii)

Hence from (i) and (ii) $f \circ f = f$

7. Let $f: R \rightarrow R: f(x) = x^2$, $g: R \rightarrow R: g(x) = \tan x$ and $h: R \rightarrow R: h(x) = \log x$.

Find a formula for $h \circ (g \circ f)$. Show that $\left[h \circ (g \circ f) \right] \sqrt{\frac{\pi}{4}} = 0$.

Sol. Let $f(x) = x^2$, $g(x) = \tan x$, $h(x) = \log x$

$$\begin{aligned} h \circ (g \circ f) &= h \{g \circ f(x)\} = h \{g(f(x))\} \\ &= h \{g(x^2)\} = h \{\tan(x^2)\} = \log \{\tan(x^2)\} \end{aligned}$$

$$\text{Now, } \left[h \circ (g \circ f) \right] \sqrt{\frac{\pi}{4}} = \log \left\{ \tan \left(\sqrt{\frac{\pi}{4}} \right)^2 \right\} = \log \left(\tan \frac{\pi}{4} \right) = \log(1) = 0$$

8. Let $f: R \rightarrow R: f(x) = (2x-3)$ and $g: R \rightarrow R: g(x) = \frac{1}{2}(x+3)$. Show that $(f \circ g) = I_R = (g \circ f)$.

Sol. Let $f(x) = 2x-3$, and $g(x) = \frac{x+3}{2}$

$$f \circ g = f \circ g(x) = f \{g(x)\} = f \left\{ \frac{x+3}{2} \right\} = 2 \left(\frac{x+3}{2} \right) - 3 = x+3-3 = x$$

$$g \circ f = g \circ f(x) = g \{f(x)\} = g \{2x-3\} = \frac{2x-3+3}{2} = \frac{2x}{2} = x. \quad \therefore g \circ f = f \circ g = I_R$$

9. Let $f: Z \rightarrow Z: f(x) = 2x$. Find $g: Z \rightarrow Z: g \circ f = I_Z$.

Sol. Let $f(x) = 2x$, $g \circ f = g \circ f(x) = g \{f(x)\} = g \{2x\}$.

$$\text{Let } x \in Z \text{ then } x, g \circ f = I_Z \Rightarrow (g \circ f)(x) = I_Z(x) \Rightarrow g[f(x)] = x$$

$$\Rightarrow g(2x) = x \Rightarrow g(y) = \frac{1}{2}y, \text{ where } 2x = y. \text{ Thus } g: Z \rightarrow Z: g(y) = \frac{1}{2}y.$$

10. Let $f: N \rightarrow N: f(x) = 2x$, $g: N \rightarrow N: g(y) = 3y+4$ and $h: N \rightarrow N: h(z) = \sin z$.

Show that $h \circ (g \circ f) = (h \circ g) \circ f$.

Sol. Let $f(x) = 2x$, $g(y) = 3y+4$, $h(z) = \sin z$

$$\begin{aligned} \text{L.H.S. } h \circ (g \circ f) &= h \{g \circ f(x)\} = h \{g(f(x))\} = h \{g(2x)\} = h \{3(2x)+4\} \\ &= h(6x+12) = \sin(6x+12) \end{aligned}$$

$$\begin{aligned} \text{R.H.S. } (h \circ g) \circ f &= (h \circ g) f(x) = (h \circ g)(2x) \\ &= h \{g(2x)\} = h \{3(2x)+4\} = h(6x+12) = \sin(6x+12) \end{aligned}$$

Hence $h \circ (g \circ f) = (h \circ g) \circ f$

11. If f be a greatest integer function and g be an absolute value function, find the value of $(f \circ g)\left(\frac{-3}{2}\right) + (g \circ f)\left(\frac{4}{3}\right)$.

Sol. $(f \circ g)\left(\frac{-3}{2}\right) = f\left\{g\left(\frac{-3}{2}\right)\right\} = f\left\{\left|\frac{-3}{2}\right|\right\} = f\left(\frac{3}{2}\right) = \left[\frac{3}{2}\right] = 1$

$(g \circ f)\left(\frac{4}{3}\right) = g\left\{f\left(\frac{4}{3}\right)\right\} = g\left[\frac{4}{3}\right] = g(1) = |1| = 1$. Required sum $= (1+1) = 2$

12. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2 + 2$ and $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = \frac{x}{x-1}, x \neq 1$. Find the $f \circ g$ and $g \circ f$ and hence find $(f \circ g)(2)$ and $(g \circ f)(-3)$

Sol. $(f \circ g)x = f\{g(x)\}$

$$= f\left\{\frac{x}{x-1}\right\} = \left\{\frac{x}{x-1}\right\}^2 + 2 = \frac{x^2 + 2(x-1)^2}{(x-1)^2} = \frac{x^2 + 2(x^2 - 2x + 1)}{(x-1)^2} = \frac{x^2 + 2x^2 - 4x + 2}{(x-1)^2} = \frac{3x^2 - 4x + 2}{(x-1)^2}$$

$(g \circ f)x = g\{f(x)\}$

$$\Rightarrow (g \circ f)x = g\{x^2 + 2\} \Rightarrow (g \circ f)x = \frac{x^2 + 2}{x^2 + 2 - 1} = \frac{x^2 + 2}{x^2 + 1}$$

Now, $(f \circ g)(2)$

$$= \left(\frac{2}{2-1}\right)^2 + 2$$

$= 6$

and, $(g \circ f)(2)$

$$= \frac{2^2 + 2}{2^2 + 1} = \frac{6}{5}$$

EXERCISE 2C (Pg.No.: 53)

Very-short-Answer Questions

1. Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = 2x$ is one-one and onto.

Sol. $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$. So, f is one-one

Let $y = 2x$ Then $x = \frac{1}{2}y$.

Thus for each y in co domain $\mathbb{R}$, there exists $\frac{1}{2}y$ such that $f\left(\frac{1}{2}y\right) = \left(2 \times \frac{1}{2}y\right) = y$

$\therefore f$ is onto.

2. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = 3x$ is one-one and into.

Sol. $f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$, so f is one-one.

If we consider 2 in co domain $\mathbb{N}$, there is no natural whose image is 2. So, f is into.

3. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2$ is neither one-one nor onto.

Sol. Clearly $f(1) = (1)^2 = 1$ and $f(-1) = (-1)^2 = 1$. So, f is many one.

If we consider -1 in the co domain $\mathbb{R}$, then there is no elements in $\mathbb{R}$, whose square is -1 .

$\therefore -1 \in \mathbb{R}$ has no pre-image in $\mathbb{R}$, so f is many one into.

4. Show that the function $f: N \rightarrow N: f(x) = x^2$ is one-one and into.

Sol. $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$
 $\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2 \quad [\because x_1, x_2 \in N] \quad \therefore f$ is one-one.

If we consider 2 is the co domain N , then $\sqrt{2} \notin N$ and $f(\sqrt{2}) = (\sqrt{2})^2 = 2$. So, f is into.

5. Show that the function $f: R \rightarrow R: f(x) = x^4$ is neither one-one nor onto.

Sol. $f(1) = (1)^4 = 1$ and $f(-1) = (-1)^4 = 1$. So f is many-one

If we consider -1 in the co-domain in the co-domain R ,
 then there exists no $x \in R$ such that $f(x) = x^4 = -1$ so, f is into.

6. Show that the function $f: Z \rightarrow Z: f(x) = x^3$ is one-one and into.

Sol. Let $x_1, x_2 \in Z$ and $f(x_1) = f(x_2)$
 $\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one. Let $2 \in Z$. Then, there exists no $x \in Z$ such that $x^3 = 2$.
 Thus $2 \in Z$ has no pre-image in Z . so f is into.

7. Let R_0 be the set of all nonzero real numbers. Then, show that the function $f: R_0 \rightarrow R_0: f(x) = \frac{1}{x}$ is one-one and onto.

Sol. $f(x_1) = f(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$. So, f is one-one.

$$y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$$

Thus, for each y in co domain R_0 , there exists $\frac{1}{y}$ in domain R_0 such that $f\left(\frac{1}{y}\right) = \frac{1}{1/y} = y$

So, f is onto.

8. Show that the function $f: R \rightarrow R: f(x) = 1 + x^2$ is many-one into.

Sol. $f(-1) = (-1)^2 + 1 = 2$ and $f(1) = 1 + (1)^2 = 2$, $\therefore$ so, f is many one.

$$y = (1 + x^2) \Rightarrow x = \sqrt{y-1}$$

So, when $y < 1$, then $\sqrt{y-1}$ is imaginary in particular, $0 \in R$ has no pre image in R . $\therefore f$ is into.

9. Let $f: R \rightarrow R: f(x) = \frac{2x-7}{4}$ be an invertible function. Find f^{-1}

Sol. $y = \frac{2x-7}{4} \Rightarrow 4y = 2x-7 \Rightarrow 4y+7 = 2x \Rightarrow x = \frac{4y+7}{2}$

$$f^{-1}(y) = \frac{4y+7}{2} \text{ for all } y \in R.$$

10. Let $f: R \rightarrow R: f(x) = 10x+3$. Find f^{-1} .

Sol. $f(x) = 10x+3 \Rightarrow y = 10x+3 \Rightarrow y-3 = 10x \Rightarrow x = \frac{y-3}{10} \Rightarrow f^{-1}(y) = \frac{y-3}{10}$

11. $f: R \rightarrow R: f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$. Show that f is many-one and into.

Sol. Since all rationals have the same image namely 1, so f is many one.

Range $(f) = \{-1, 1\} \subset R$ so f is onto.

12. Let $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$. Find $(f \circ g)(7)$.

Sol. Let $f(x) = x + 7$ and $g(x) = x - 7$

$$f \circ g = f \circ g(x) = f\{g(x)\} = f(x - 7) = x - 7 + 7 \Rightarrow f \circ g = x \Rightarrow f \circ g(7) = 7$$

13. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ defined by $f(x) = x^2$ and $g(x) = (x + 1)$. Show that $g \circ f \neq f \circ g$.

Sol. $f(x) = x^2$, and $g(x) = x + 1$

$$g \circ f = g \circ f(x) = g\{f(x)\} = g\{x^2\} = x^2 + 1; \quad f \circ g = f \circ g(x) = f\{g(x)\} = f(x + 1) = (x + 1)^2$$

Hence $g \circ f \neq f \circ g$.

14. Let $f: R \rightarrow R: f(x) = (3 - x^3)^{\frac{1}{3}}$. Find $f \circ f$.

Sol. $f(x) = (3 - x^3)^{\frac{1}{3}}$

$$\Rightarrow f \circ f(x) = f\{f(x)\} = f\left\{(3 - x^3)^{\frac{1}{3}}\right\} = \left[3 - \left\{(3 - x^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{3}} = \left[3 - (3 - x^3)\right]^{\frac{1}{3}} = [x^3]^{\frac{1}{3}} = x$$

15. Let $f: R \rightarrow R: f(x) = 3x + 2$, find $f \circ f(x)$

Sol. $f(x) = 3x + 2$

$$\Rightarrow f \circ f(x) = f(3x + 2)$$

$$= 3(3x + 2) + 2 = 9x + 6 + 2 = 9x + 8$$

16. Let $A = \{1, 3, 4\}$, $B = \{1, 2, 5\}$ and $C = \{1, 3\}$. Let $f: A \rightarrow B$ and $g: B \rightarrow C$, defined as

$$f = \{(1, 2), (3, 5), (4, 1)\} \text{ and } g = \{(1, 3), (2, 3), (5, 1)\}. \text{ Write down } (g \circ f).$$

Sol. $\text{Dom}(g \circ f) = \text{Dom}(f) = \{1, 3, 4\}$

$$\Rightarrow g \circ f(1) = g\{f(1)\} = g(2) = 3 \Rightarrow g \circ f(3) = g\{f(3)\} = g(5) = 1$$

$$\Rightarrow g \circ f(4) = g\{f(4)\} = g(1) = 3 \therefore (g \circ f) = \{(1, 3), (3, 1), (4, 3)\}.$$

17. Let $A = \{1, 2, 3, 4\}$ and $f = \{(1, 4), (2, 1), (3, 3), (4, 2)\}$. Write down $(f \circ f)$.

Sol. $\text{Domain}(f \circ f) = \text{Dom}(f) = \{1, 2, 3, 4\}$

$$f \circ f(1) = f\{f(1)\} = f(4) = 2 \Rightarrow f \circ f(2) = f\{f(2)\} = f(1) = 4$$

$$f \circ f(3) = f\{f(3)\} = f(3) = 3 \Rightarrow f \circ f(4) = f\{f(4)\} = f(2) = 1$$

$$\therefore f \circ f = \{(1, 2), (2, 4), (3, 3), (4, 1)\}$$

18. Let $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$. Find $g \circ f$ and $f \circ g$.

Sol. $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

$$g \circ f = g \circ f(x) = g\{f(x)\} = g(8x^3) = (8x^3)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot x = 2x$$

19. Let $f: R \rightarrow R: f(x) = 10x + 7$. Find the function $g: R \rightarrow R: g \circ f = f \circ g = I_R$.

Sol. Let $f(x) = 10x + 7$

$$g \circ f = I_R \Rightarrow g \circ f(x) = Ig(x) = x \Rightarrow g\{f(x)\} = x \Rightarrow g\{10x + 7\} = x$$

$$\text{Put } 10x + 7 = y \text{ then } x = \frac{y-7}{10}. \therefore g(y) = \frac{y-7}{10}. \text{ Hence } g: R \rightarrow R: g(x) = \frac{x-7}{10} \text{ for all } x \in R.$$

20. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. State whether f is one-one

Sol. Here, $f(1) = 4$,

$$f(2) = 5$$

$$\text{and } f(3) = 6$$

$\therefore$ different elements in A have different images in B,

Hence f is one - one

EXERCISE 2D (Pg.No.: 56)

1. Let $A = \{2, 3, 4, 5\}$ and $B = \{7, 9, 11, 13\}$, and let $f = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$.

Show that f is invertible and find f^{-1} .

Sol. We have $f(2) = 7$, $f(3) = 9$, $f(4) = 11$, $f(5) = 13$.

$$\text{Dom}(f) = \{2, 3, 4, 5\} = A \text{ and range}(f) = \{7, 9, 11, 13\} = B.$$

Clearly, different element in A have different image. $\therefore f$ is one-one.

$\text{Rang}(f) = B \Rightarrow f$ is onto. Thus f is one - one onto and there fore invertible.

$$\text{Now } f(2) = 7, f(3) = 9, f(4) = 11, f(5) = 13$$

$$\Rightarrow f^{-1}(7) = 2, f^{-1}(9) = 3, f^{-1}(11) = 4, f^{-1}(13) = 5. \text{ Hence } f^{-1}\{(7, 2), (9, 3), (11, 4), (13, 5)\}.$$

2. Show that the function $f: R \rightarrow R: f(x) = 2x + 3$ is invertible and find f^{-1} .

Sol. We have $f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2. \therefore f$ is one-one.

$$\text{Again } y = 2x + 3 \Rightarrow y - 3 = 2x \Rightarrow x = \frac{y-3}{2}$$

$$\text{Now, if } y \in R \text{ (co-domain of } f), \text{ then there exists } x = \frac{y-3}{2} \in R.$$

$$\text{Such that } f(x) = f\left(\frac{y-3}{2}\right) = 2\left(\frac{y-3}{2}\right) + 3 = y. \therefore f \text{ is onto.}$$

Thus f is one-one onto and there fore invertible.

$$\text{Now } y = f(x) \Rightarrow y = 2x + 3 \Rightarrow x = \frac{y-3}{2} \Rightarrow f^{-1}(y) = \frac{y-3}{2} \quad [\because f(x) = y \Rightarrow x = f^{-1}(y)]$$

Thus, we define : $f^{-1} : R \rightarrow R : f^{-1}(y) = \frac{y-3}{2}$ for all $y \in R$

3. Let $f : Q \rightarrow Q : f(x) = 3x - 4$. Show that f is invertible and find f^{-1} .

Sol. We have $f(x_1) = f(x_2) \Rightarrow 3x_1 - 4 = 3x_2 - 4 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$. $\therefore f$ is one-one.

Again $y = 3x - 4 \Rightarrow y + 4 = 3x \Rightarrow \frac{y+4}{3} = x$.

Now, if $y \in R$ (co-domain of f), then there exists $x = \frac{y+4}{3} \in R$.

Such that $f(x) = f\left(\frac{y+4}{3}\right) = 3\left(\frac{y+4}{3}\right) - 4 = y$. $\therefore f$ is onto.

Thus, f is one one onto and therefore invertible.

Now $y = f(x) \Rightarrow y = 3x - 4 \Rightarrow x = \frac{y+4}{3} \Rightarrow f^{-1}(y) = \frac{y+4}{3}$ [$\because f(x) = y \Rightarrow x = f^{-1}(y)$]

Thus we define : $f^{-1} : R \rightarrow R : f^{-1}(y) = \frac{y+4}{3}$ for all $y \in R$.

4. Let $f : R \rightarrow R : f(x) = \frac{1}{2}(3x+1)$. Show that f is invertible and find f^{-1} .

Sol. We have $f(x_1) = f(x_2) \Rightarrow \frac{3x_1+1}{2} = \frac{3x_2+1}{2} \Rightarrow 3x_1+1 = 3x_2+1 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$

$\therefore f$ is one-one. Again $y = \frac{3x+1}{2} \Rightarrow 2y = 3x+1 \Rightarrow 2y-1 = 3x \Rightarrow x = \frac{2y-1}{3}$

Now, if $y \in R$ (co-domain of f), then there exists, $x = \frac{2y-1}{3} \in R$.

Such that $f(x) = f\left(\frac{2y-1}{3}\right) = \frac{3\left(\frac{2y-1}{3}\right)+1}{2} = y$. $\therefore f$ is onto.

Thus, f is one - one and therefore invertible.

Now, $y = f(x) \Rightarrow y = \frac{3x+1}{2} \Rightarrow x = \frac{2y-1}{3}$ [$\because f(x) = y \Rightarrow x = f^{-1}(y)$]

Thus we define, $f^{-1} : R \rightarrow R : f^{-1}(y) = \frac{2y-1}{3}$ for all $y \in R$.

5. If $f(x) = \frac{(4x+3)}{(6x-4)}$, $x \neq \frac{2}{3}$, show that $(f \circ f)(x) = x$ for all $x \neq \frac{2}{3}$. Hence, find f^{-1} .

Hint : $f \circ f = I \Rightarrow f^{-1} = f$.

Sol. $f(x) = \frac{4x+3}{6x-4}$; $f \circ f = f \circ f(x) = f \circ \{f(x)\} = f\left\{\frac{4x+3}{6x-4}\right\}$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{\frac{16x+12}{6x-4}+3}{\frac{24x+18}{6x-4}-4} = \frac{\frac{16x+12+18x-12}{6x-4}}{\frac{24x+18-24x+16}{6x-4}} = \frac{24x}{24} = x$$

$\therefore$ Hence $(f \circ f)(x) = x$; $f \circ f = I \Rightarrow f^{-1} = I \Rightarrow f^{-1} = \frac{4y+3}{6y-4}$

6. Show that the function f on $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$, defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}

Sol. one - one $f \rightarrow$

Let, x_1 and $x_2 \in \text{domain}(f)$

Now, $f(x_1) = f(x_2)$

$$\Rightarrow \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4} \Rightarrow (4x_1+3)(6x_2-4) = (4x_2+3)(6x_1-4)$$

$$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 - 16x_2 + 18x_1 - 12 \Rightarrow 34x_2 = 34x_1 \Rightarrow x_1 = x_2$$

Hence, f is one - one.

Onto: $\rightarrow$

Let, $y \in \text{co-domain}(f)$

Now, $y = f(x)$

$$\Rightarrow y = \frac{4x+3}{6x-4} \Rightarrow 6xy = 4y = 4x+3 \Rightarrow 6xy - 4x = 3+4y \Rightarrow x(6y-4) = 3+4y$$

$$\Rightarrow x = \frac{3+4y}{6y-4} \in \text{domain} \forall y \in \text{co-domain}$$

Hence, f is an on to function

$$\text{Since, } x = \frac{3+4y}{6y-4}, y \neq \frac{2}{3} \therefore f^{-1}(y) = \frac{3+4y}{6y-4}, y \neq \frac{2}{3} \text{ Ans.}$$

7. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = x^2$ is one-one and into.

Sol. $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$
 $\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2 \quad [\because x_1, x_2 \in \mathbb{N}] \therefore f \text{ is one-one.}$

If we consider 2 is the co domain $\mathbb{N}$, then $\sqrt{2} \notin \mathbb{N}$ and $f(\sqrt{2}) = (\sqrt{2})^2 = 2$. So, f is into.

8. Let R_0 be the set of all nonzero real numbers. Then, show that the function $f: R_0 \rightarrow R_0: f(x) = \frac{1}{x}$ is one-one and onto.

Sol. $f(x_1) = f(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$. So, f is one-one.

$$y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$$

Thus, for each y in co domain R_0 , there exists $\frac{1}{y}$ in domain R_0 such that $f\left(\frac{1}{y}\right) = \frac{1}{1/y} = y$

So, f is onto.

9. Let $f: \mathbb{N} \rightarrow \mathbb{R}: f(x) = 4x^2 + 12x + 15$. show that $f: \mathbb{N} \rightarrow \text{range}(f)$ is invertible. Find f^{-1}

Sol. Let, x_1 and $x_2 \in \mathbb{N}$ (domain f)

Now, $f(x_1) = f(x_2)$

$$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15 \Rightarrow 4x_1^2 - 4x_2^2 + 12x_1 - 12x_2 = 0$$

$$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow 4(x_1 - x_2)(x_1 + x_2) + 12(x_1 - x_2) = 0$$

$$\Rightarrow (x_1 - x_2) \{4(x_1 + x_2) + 12\} = 0$$

$$\because x_1 \& x_2 \in \mathbb{N} \quad \therefore 4(x_1 + x_2) + 12 \neq 0 \quad \therefore x_1 - x_2 = 0$$

$$\Rightarrow x_1 = x_2$$

Hence, f is one-one

$\therefore$ co-domain = range (given) $\therefore f$ is an onto function.

$\therefore f$ is one-one onto function $\therefore f$ is an invertible function.

Let, $y = f(x)$

$$\Rightarrow y = 4x^2 + 12x + 15 \Rightarrow 4x^2 + 12x + (15 - y) = 0$$

$$\Rightarrow x = \frac{-12 \pm \sqrt{12^2 - 4 \times 4 \times (15 - y)}}{2 \times 4} \Rightarrow x = \frac{-12 \pm \sqrt{144 - 240 + 16y}}{8}$$

$$\Rightarrow x = \frac{-12 + \sqrt{16y - 96}}{8} \Rightarrow x = \frac{-12 + 4\sqrt{y - 6}}{8} \Rightarrow x = \frac{\sqrt{y - 6} - 3}{2} \Rightarrow f^{-1}(y) = \frac{\sqrt{y - 6} - 3}{2} \text{ Ans.}$$

10. Let $f: N - \{2\} \rightarrow B = R - \{1\}$. If $f: A \rightarrow B: f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto.

Hence, find f^{-1}

Sol. Let, x_1 & $x_2 \in \text{domain}(f)$

Now, $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2} \Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$$

$$\Rightarrow x_1 \cdot x_2 - 2x_1 - x_2 + 2 = x_1 \cdot x_2 - x_1 - 2x_2 + 2 \Rightarrow -2x_1 + x_1 = -2x_2 + x_2 \Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$$

Hence, f is one-one function

Let, $y \in \text{co-domain } f$.

Now, $y = f(x)$

$$\Rightarrow y = \frac{x-1}{x-2} \Rightarrow xy - 2y = x - 1 \Rightarrow xy - x = 2y - 1 \Rightarrow x(y - 1) = 2y - 1$$

$$\Rightarrow x = \frac{2y-1}{y-1} \in \text{domain}, \forall y \neq 1$$

Hence, f is an onto function.

$$\text{Here, } x = \frac{2y-1}{y-1} \Rightarrow f^{-1}(y) = \frac{2y-1}{y-1} \forall y \neq 1$$

11. Let f and g be two functions from R into R , defined by $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in R$. Find $f \circ g$ and $g \circ f$

Sol. Given, $f(x) = |x| + x$ & $g(x) = |x| - x$

Now, $(f \circ g)x = f\{g(x)\}$

$$= f\{|x| - x\} = ||x| - x| + |x| - x$$

$$\text{If, } x < 0. \text{ Then, } |x| = -x, \text{ we have } (f \circ g)x = |-x - x| - x - x = |-2x| - 2x = -2x - 2x = -4x$$

$$\text{If, } x = 0$$

$$(f \circ g)x = 0$$

$$\text{If } x > 0, \text{ then, } |x| = x, \text{ we have } (f \circ g)x = |x - x| + x - x = 0$$

$$\text{Hence, } (f \circ g)x = \begin{cases} -4x, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$$

$$\text{Now, } (g \circ f)x = g\{f(x)\} = g\{|x| + x\} = ||x| + x| - \{|x| + x\}$$

$$\text{If, } x < 0, \text{ we have } (g \circ f)x = |-x + x| - \{-x + x\} = 0$$

$$\text{If, } x \neq 0, \text{ we have } (g \circ f)x = 0 \text{ If, } x > 0, \text{ we have, } (g \circ f)x = |x + x| - \{x + x\}$$

$$= |2x| - 2x = 2x - 2x = 0$$

$$\text{Hence, } (g \circ f)x = 0$$