Playing With Numbers

Exercise 2.1

Question: 1

Define:

Solution:

(i) Factor: A factor of a number is an e×act divisor of that number. For e×ample, 4 e×actly divide 32. Therefore, 4 is a factor of 32.

Examples of factors are:

2 and 3 are factors of 6 because 2 × 3= 6

2 and 4 are factors of 8 because 2 × 4= 8

3 and 4 are factors of 12 because 3 × 4 = 12

3 and 5 are factors of 15 because 3 × 5 = 15

(ii) Multiple: When a number ‘a’ is multiplied by another number ‘b’, the product is the multiple of both the numbers ‘a’ and ‘b’.

Examples of multiples:

6 is a multiple of 2 because 2 × 3= 6

8 is a multiple of 4 because 4 × 2 = 8

12 is a multiple of 6 because 6 × 2 = 12

21 is a multiple of 7 because 7 × 3 = 21

Question: 2

Write all factors of each of the following numbers:

Solution:

(i) 60

60= 1 × 60

60= 2 × 30

60 = 3 × 20

60 = 4 × 15

60 = 5 × 12

60 = 6 × 10

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60.

(ii) 76

76 = 1 × 76

76 = 2 × 38

76 = 4 × 19

Therefore, The factors of 76 are 1, 2, 4, 19, 38 and 76.

(iii) 125

125 = 1 × 125

125 = 5 × 25

Therefore, the factors of 125 are 1, 5, 25 and 125.

(iv) 729

729 = 1 × 729

729 = 3 × 243

729 = 9 × 81

729 = 27 × 27

Therefore, the factors of 729 are 1, 3, 9, 27, 81, 243 and 729.

Question: 3

Write first five multiples of each of the following numbers:

Solution:

(i) 25

The first five multiples of 25 are as follows:

25 × 1 = 25

25 × 2 = 50

25 × 3 = 75

25 × 4 = 100

25 × 5 = 125

(ii) 35

The first five multiples of 35 are as follows:

35 × 1 = 35

35 × 2 = 70

35 × 3 = 105

35 × 4 = 140

35 × 5 = 175

(iii) 45

The first five multiples of 45 are as follows:

45 × 1 = 45

45 × 2 = 90

45 × 3 = 135

45 × 4 = 180

45 × 5 = 225

(iv) 40

The first five multiples of 40 are as follows:

40 × 1 = 40

40 × 2 = 80

40 × 3 = 120

40 × 4 = 160

40 × 5 = 200

Question: 4

Which of the following number have 15 as their factor?

Solution:

(i) 15625

15 is not a factor of 15,625 because it is not a divisor of 15,625.

(ii) 123015

15 is a factor of 1,23,015 because it is a divisor of 1,23,015. i.e., 8,201 × 15 = 1, 23,015

Question: 5

Which of the following number are divisible by 21?

Solution:

We know that a given number is divisible by 21 if it is divisible by each of its factors. The factors of 21 are 1, 3, 7 and 21.

(i) 21063

Sum of the digits of the given number = 2 + 1 + 0 + 6 + 3 = 12 which is divisible by 3.

Hence, 21,063 is divisible by 3.

Again, a number is divisible by 7 if the difference between twice the one's digit and the number formed by the other digits is either 0 or a multiple of 7. 2,106 – (2 × 3) = 2,100 which is a multiple of 7. Thus, 21,063 is divisible by 21.

(ii) 20163

Sum of the digits of the given number = 2 + 0 + 1 + 6 + 3 = 12 which is divisible by 3. Hence, 20,163 is divisible by 3.

Again, a number is divisible by 7 if the difference between twice the one's digit and the number formed by the other digits is either 0 or multiple of 7. 2016 – (2 × 3) = 2010 which is not a multiple of 7. Thus, 20,163 is not divisible by 21.

Question: 6

Without actual division show that 11 is a factor of each of the following numbers:

Solution:

(i) 1,111

The sum of the digits at the odd places = 1 + 1 = 2

The sum of the digits at the even places = 1 + 1 = 2

The difference of the two sums = 2 – 2 = 0

Therefore, 1,111 is divisible by 11 because the difference of the sums is zero.

(ii) 11,011

The sum of the digits at the odd places = 1 + 0 + 1 = 2

The sum of the digits at the even places = 1 + 1 = 2

The difference of the two sums = 2 – 2 = 0

Therefore, 11,011 is divisible by 11 because the difference of the sums is zero.

(iii) 1, 10,011

The sum of the digits at the odd places = 1 + 0 + 1 = 2

The sum of the digits at the even places = 1 + 0 + 1 = 2

The difference of the two sums = 2 – 2 = 0

Therefore, 1, 10,011 is divisible by 11 because the difference of the sums is zero.

(iv) 11, 00,011

the sum of the digits at the odd places = 1 + 0 + 0 + 1 = 2

The sum of the digits at the even places = 1 + 0 + 1 = 2

The difference of the two sums = 2 – 2 = 0

Therefore, 11, 00,011 is divisible by 11 because the difference of the sums is zero.

Question: 7

Without actual division show that each of the following numbers is divisible by 5:

Solution:

A number will be divisible by 5 if the unit’s digit of that number is either 0 or 5.

(i) 5

In 55, the unit’s digit is 5. Hence, it is divisible by 5.

(ii) 555

In 555, the unit’s digit is 5. Hence, it is divisible by 5.

(iii) 5555

In 5,555, the unit’s digit is 5. Hence, it is divisible by 5.

(iv) 50,005

In 50,005, the unit’s digit is 5. Hence, it is divisible by 5.

Question: 8

Is there any natural number having no factor at all?

Solution:

No, because each natural number is a factor of itself

Question: 9

Find numbers between 1 and 100 having exactly three factors

Solution:

The numbers between 1 and 100 having exactly three factors are 4, 9, 25, and 49.

The factors of 4 are 1, 2 and 4.

The factors of 9 are 1, 3 and 9.

The factors of 25 are 1, 5 and 25.

The factors of 49 are 1, 7 and 49.

Question: 10

Sort out even and odd numbers:

Solution:

A number which is exactly divisible by 2 is called an even number. Therefore, 42 and 144 are even numbers.

A number which is not exactly divisible by 2 is called an odd number. Therefore, 89 and 321 are odd numbers.

Exercise 2.2

Question: 1

Find the common factors of:

Solution:

(i) 15 and 25

15 = 1 × 15

15 = 3 × 5

i.e., the factors of 15 are 1, 3, 5 and 15.

Again, 25 = 1 × 25

25 = 5 × 5 i.e., the factors of 25 are 1, 5 and 25.

Therefore, the common factors of the two numbers are 1 and 5.

(ii) 35 and 50

35 = 1 × 35

35 =5 × 7 i.e., the factors of 35 are 1, 5, 7 and 35.

Again, 50 = 1 × 50

50 = 2 × 25

50 = 5 × 10

i.e., the factors of 50 are 1, 2, 5, 10, 25 and 50.

Therefore, the common factors of the two numbers are 1 and 5.

(iii) 20 and 28

20 = 1 × 20

20 = 2 × 10

20 =4 × 5

i.e., the factors of 20 are 1, 2 , 4, 5, 10 and 20.

Again, 28 = 1 × 28

$28 = 2 \times 14$

$28 = 7 \times 4$

i.e., the factors of 28 are 1, 2, 4, 7, 14 and 28.

Therefore, the common factors of the two numbers are 1, 2 and 4.

Question: 2

Find the common factors of:

Solution:

(i) 5, 15 and 25

Factors of 5 are 1 and 5

Factors of 15 are 1, 3, 5 and 15

Factors of 25 are 1, 5 and 25

Therefore, the common factors of 5, 15, and 25 are 1 and 5.

(ii) 2, 6 and 8

Factors of 2 are 1 and 2

Factors of 6 are 1, 2, 3 and 6

Factors of 8 are 1, 2, 4 and 8

Therefore, the common factors of 2, 6 and 8 are 1 and 2.

Question: 3

Find first three common multiples of 6 and 8

Solution:

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84…

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96…

Therefore, the first three common multiples of 6 and 8 are 24, 48 and 72.

Question: 4

Find first two common multiples of 12 and 18.

Solution:

Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132…

Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198…

Therefore, the first two common multiples of 12 and 18 are 36 and 72.

Question: 5

A number is divisible by both 7 and 16. By which other number will that number be always divisible?

Solution:

Since the number is divisible by 7 and 16, they are the factors of that number.

So, the number will be divisible by the common factor of 7 and 16.

The factors of 7 are 1 and 7.

The factors of 16 are 1, 2, 4, 8, and 16.

Therefore, the common factor of 7 and 16 is 1 and the number is divisible by 1.

Question: 6

A number is divisible by 24. By what other numbers will that number be divisible?

Solution:

Since the number is divisible by 24, it will be divisible by all the factors of 24.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Hence, the number is also divisible by 1, 2, 3, 4, 6, 8 and 12.

Exercise 2.3

Question: 1

What are prime numbers? List all the prime numbers between 1 and 30.

Solution:

Those numbers with only two factors, i.e., 1 and the number itself, are known as prime numbers.

Examples: 2, 3, 5, 7. 11 and 13

The prime numbers between 1 and 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Question: 2

Write all the prime numbers between:

Solution:

(i) 10 and 50

The prime numbers between 10 and 50 are 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

(ii) 70 and 90

The prime numbers between 70 and 90 are 71, 73, 79, 83 and 89.

(iii) 40 and 85

The prime numbers between 40 and 85 are 41, 43, 47, 53, 59, 61, 67, 71, 73, 79 and 83.

(iv) 60 and 100

The prime numbers between 60 and 100 are 61, 67, 71, 73, 79, 83, 89 and 97.

Question: 3

What is the smallest prime number? Is it an even number?

Solution:

The number 2 is the smallest prime number.

It is an even prime number. Except 2, all other even numbers are composite numbers.

Question: 4

What is the smallest odd prime? Is every odd number a prime number? If not, give an example of an odd number which is not prime.

If yes, write the smallest odd composite number.

Solution:

The smallest odd prime number is 3.

No, every odd number is not a prime number. For example, 9 is an odd number but it is not a prime number because its three factors are 1, 3 and 9.

Question: 5

What are composite numbers? Can a composite number be odd?

Solution:

A number which has more than two factors is called a composite number.

For example, the numbers 4, 6, 8, 9 10 and 15 are composite numbers.

Yes, a composite number can be an odd number. The smallest odd number is 9.

Question: 6

What are twin-primes? Write all pairs of twin-primes between 50 and 100.

Solution:

Twin primes: Two prime numbers are said to be twin primes if there is only one composite number between them.

For example, (3, 5) and (5, 7) are twin primes.

Twin primes between 50 and 100 are (59, 61) and (71, 73).

Question: 7

What are co-primes? Give examples of five pairs of co-primes. Are co-primes always prime?

Solution:

Two numbers are said to be co-primes if they do not have any common factors other than 1.

For example, (2, 3), (3, 4), (4, 5), (5, 7) and (13, 17) are co-primes.

Two co-primes numbers need not be both prime numbers.

e.g., (3, 4), (6, 7) and (4, 13).

Question: 8

Which of the following pairs are always co-primes?

Solution:

(i) Two prime numbers

Two prime numbers are always co-primes to each other.

Example: 7 and 11 are co-primes to each other.

(ii) One prime and one composite number

One prime and one composite number are not always co-prime

Example: 3 and 21 are not co-primes to each other.

(iii) Two composite numbers

Two composite numbers are not always co-primes to each other.

Example: 4 and 6 are not co-primes to each other.

Question: 9

Express each of the following numbers as a sum of two or more primes:

Solution:

We can write the given numbers as the sums of the two or more primes as follows:

(i) 13 = 11 + 2

(ii) 130= 59 + 71

(iii) 180 = 139 + 17 + 11 + 13 or 79 + 101

Question: 10

Express each of the following numbers as the sum of two odd primes:

Solution:

We can express the given numbers as the sums of two odd primes as follows:

(i) 36 = 7 + 29 or 17 + 19

(ii) 42 = 5 + 37 or 13 + 29

(iii) 84 = 17 + 67 or 23 + 61

Question: 11

Express each of the following numbers as the sum of three odd prime numbers:

Solution:

We can express the given numbers as the sums of three odd prime numbers as follows:

(i) 31 = 5 + 7+ 9 + 13 or 31 = 11 + 13 + 7

(ii) 35 = 5 + 7 + 23 or 35 = 17 +13 + 5

(iii) 49 = 3 + 5 + 41 or 49 = 7 +11 +31

Question: 12

Express each of the following numbers as the sum of twin primes:

Solution:

We can express the given numbers as the sums of twin primes which are as follows:

(i) 36= 17 +19

(ii) 84 = 41 + 41

(iii) 120 = 59 + 61

Question: 13

Find the possible missing twins for the following numbers so that they become twin primes:

Solution:

(i) The possible missing twins for 29 are 27 and 31. Since 31 is a prime and 37 is not, 31 is the missing twin.

(ii) The possible missing twins for 89 are 87 and 91. Since 87 and 91 are not primes, 89 has no twin.

(iii) The possible twins for 101 are 99 and 103. Since 103 is a prime and 99 is not , 103 is the missing twin.

Question: 14.

A list consists of the following pairs of numbers:

Solution:

(i) Co-primes: Two natural numbers are said to be co-primes numbers if they have 1 as their only common factor.

Hence, all the given pairs of numbers are co-primes.

(ii) Primes: Natural numbers which have exactly two distinct factors, i.e., 1 and the number itself are called prime numbers.

Hence, (59, 61) and (71, 73) are pairs of prime numbers.

(iii) Composite numbers: Natural numbers which have more than two factors are called composite numbers.

Hence, (55, 57) and (63, 65) are pairs of composite numbers.

Question: 15.

For a number, greater than 10, o be prime what may be the possible digit in the unit’s place?

Solution:

For a number (greater than 10) to be a prime number, the possible digit in the unit’s place may be 1, 3, 7 or 9.

Example: 11, 13, 17 and 19 are prime numbers greater than 10.

Question: 16.

Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Solution:

The required seven consecutive composite numbers are 90, 91, 92, 93, 94, 95 and 96.

Question: 17.

State true (T) and false (F):

(i) The sum of primes cannot be a prime.

(ii) The product of primes cannot be a prime.

(iii) An even number is composite

(iv) Two consecutive numbers cannot be a prime.

(v) Odd numbers cannot be composite.

(vi) Odd numbers cannot be written as sum of primes.

(vii) A number and its successor are always co-primes.

Solution:

(i) False.

2 + 3 = 5 which is a prime number.

(ii) True.

The product of prime number is always a composite number.

(iii) False

The even number 2 is not a composite number.

(iv) False

2 and 3 are consecutive and are also prime numbers.

(v) False.

9 is an odd number but it is composite numbers as its factor are 1, 3 and 9.

(vi) False

9 is an odd number: 9 = 7 +2 where 7 and 2 are prime numbers.

(vii) True

A number and its successor have only one common factor (i.e., 1).

Question: 18.

Fill in the Blank:

Solution:

(i) A number having only two factors is called a prime number.

(ii) A number having more than two factors is called a composite number.

(iii) 1 is neither composite nor prime.

(iv) The smallest prime number is 2.

(v) The smallest composite number is 4.

Exercise 2.4

Question: 1

In which of the following expressions, prime factorization has been done?

Solution:

(i) 24 = 2 x 3 x 4 is not a prime factorization as 4 is not a prime number.

(ii) 56 = 1 x 7 x 2 x2 x 2 is not a prime factorization as 1 is not a prime number.

(iii) 70 = 2 x 5 x 7 is a prime factorization as 2, 5, and 7 are prime numbers.

(iv) 54 = 2 x 3 x 9 is not a prime factorization as 9 is not a prime number.

Question: 2

Determine prime factorization of each of the following numbers:

Solution:

(i) 216

We have:

2	216
2	108
2	54
3	27
3	9
3	3
	1

Therefore, Prime factorization of 216 = 2 x 2 x 2 x 3 x 3

(ii) 420

We have:

2	420

2	210
3	105
5	35
7	7
	1

Therefore, Prime factorization of 420 = 2 x 2 x 3 x 5 x 7

(iii) 468

We have:

2	468
2	234
3	117
3	39
13	13
	1

Therefore, Prime factorization of 468 = 2 x 2 x 3 x 3 x 13

(iv) 945

We have:

3	945
3	315
3	105
5	35
7	7
	1

Therefore, Prime factorization of 945 = 3 x 3 x 3 x 5 x 7

(v) 7325

We have:

5	7325
5	1465
293	293
	1

Therefore, Prime factorization of 732 5= 5 x 5 x 293

(vi) 13915

We have:

5	13915
11	2783
11	253
23	23
	1

Therefore, Prime factorization of 13915 = 5 x 11 x 11 x 23

Question: 3

Write the smallest 4-digit number and express it as a product of primes.

Solution:

The smallest 4-digit number is 1000.

1000 = 2 x 500

= 2 × 2 × 250

= 2 × 2 × 2 × 125

= 2 × 2 × 2 × 5 × 25

= 2 × 2 × 2 × 5 × 5 × 5

Therefore, 1000 = 2 × 2 × 2 × 5 × 5 × 5

Question: 4

Write the largest 4-digit number and express it as product of primes.

Solution:

The largest 4-digit number is 9999.

We have:

3	9999
3	3333
11	1111
101	101
	1

Hence, the largest 4-digit number 9999 can be expressed in the form of its prime factors as 3 x 3 x 11 x 101.

Question: 5

Find the prime factors of 1729. Arrange the factors in ascending order, and find the relation between two consecutive prime factors.

Solution:

The given number is 1729.

We have:

7	1729
13	247
19	19

Thus, the number 1729 can be expressed in the form of its prime factors ass 7 x 13 x19.

Relation between its two consecutive prime factors:

The consecutive prime factors of the given number are 7, 13 and 19.

Clearly, 13 – 7 = 6 and 19 – 13 =6

Here, in two consecutive prime factors, the latter is 6 more than the previous one.

Question: 6

Which factors are not included in the prime factorization of a composite number?

Solution:

1 and the number itself are not included in the prime factorization of a composite number.

Example: 4 is a composite number.

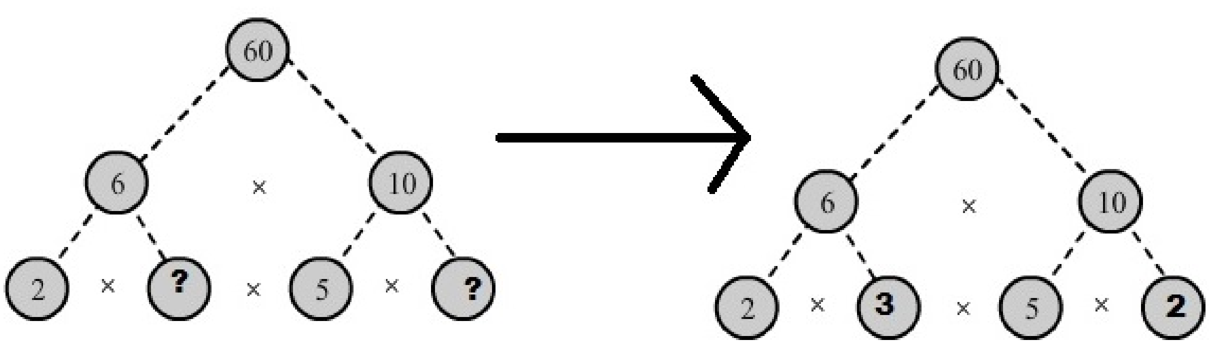
Prime factorization of 4 = 2 x 2.

Question: 7

Here are two different factor trees for 60. Write the missing numbers:

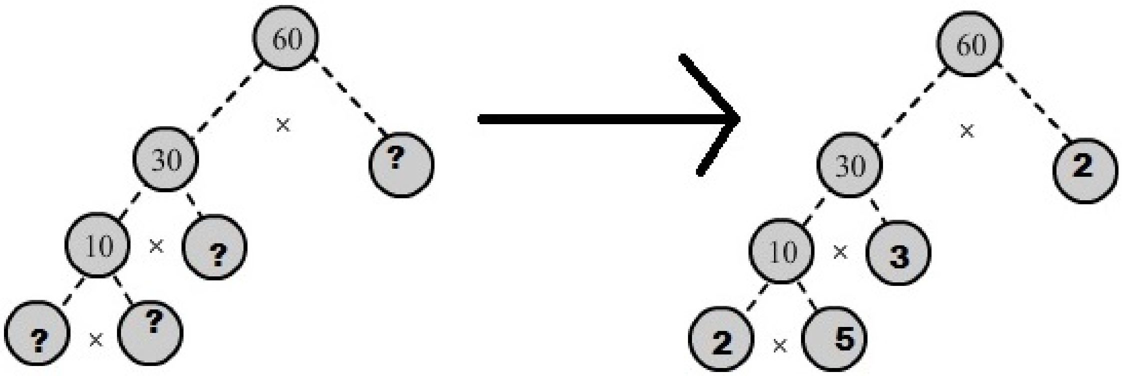
Solution:

(i) Since 6 = 2 x 3 and 10= 5 x 2. We have:



(ii) Since 60 = 30 x 2.

30= 10 x 3 and 10 = 5 x 2 we have:



60
30
×
?
10
×
?
?
×
?
60
30
×
2
10
×
3
2
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5

Exercise 2.5

Question: 1

Test the divisibility of the following numbers by 2:

Solution:

Rule: A natural number is divisible by 2 if its unit digit is 0, 2, 4, 6 or 8.

(i) 6250

Here, the unit’s digit = 0

Thus, the given number is divisible by 2.

(ii) 984325

Here, the unit’s digit = 5

Thus, the given number is not divisible by 2.

(iii) 367314

Here, the unit’s digit = 4

Thus, the given number is divisible by 2.

Question: 2

Test the divisibility of the following numbers by 3:

Solution:

Rule: A number is divisible by 3 if the sum of its digits is divisible by 3.

(i) 70335

Here, the sum of the digits in the given number = 7 + 0 + 3 + 3 + 5 = 18 which is divisible by 3.

Thus, 70,335 is divisible by 3.

(ii) 607439

Here, the sum of the digits in the given number = 6 + 0 + 7 + 4 + 3 + 9 = 29 which is not divisible by 3.

Thus, 6, 07,439 is not divisible by 3.

(iii) 9082746

Here, the sum of the digits in the given number = 9 + 0 + 8 + 2 + 7 + 4 + 6 = 36 which is divisible by 3.

Thus, 90, 82,746 is divisible by 3.

Question: 3

Test the divisibility of the following numbers by 6:

Solution:

Rule: A number is divisible by 6 if it is divisible by 2 as well as 3.

(i) 7020

Here, the units digit = 0

Thus, the given number is divisible by 2.

Also, the sum of the digits = 7 + 0 + 2 + 0 = 9 which is divisible by 3. So, the given number is divisible by 3. Hence, 7,020 is divisible by 6.

(ii) 56423

Here, the units digit = 3 Thus, the given number is not divisible by 2.

Also, the sum of the digits = 5 + 6 + 4 + 2 + 3 = 20 which is not divisible by 3.

So, the given number is not divisible by 3. Since 3,56,423 is neither divisible by 2 nor by 3, it is not divisible by 6.

(iii) 732510

Here, the units digit = 0

Thus, the given number is divisible by 2.

Also, the sum of the digits = 7 + 3 + 2 + 5 + 1 + 0 = 18 which is divisible by 3. So, the given number is divisible by 3.

Hence, 7,32,510 is divisible by 6.

Question: 4

Test the divisibility of the following numbers by 4:

Solution:

Rule: A natural number is divisible by 4 if the number formed by its last two digits is divisible by 4.

(i) 786532

Here, the number formed by the last two digits is 32 which is divisible by 4.
Thus, 7,86,532 is divisible by 4.

(ii) 1020531

Here, the number formed by the last two digits is 31 which is not divisible by 4.
Thus, 10,20,531 is not divisible by 4.

(iii) 9801523

Here, the number formed by the last two digits is 23 which is not divisible by 4.
Thus, 98,01,523 is not divisible by 4.

Question: 5

Test the divisibility of the following numbers by 8:

Solution:

Rule: A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

(i) The given number = 8364

The number formed by its last three digits is 364 which is not divisible by 8.
Therefore, 8,364 is not divisible by 8.

(ii) The given number = 7314

The number formed by its last three digits is 314 which is not divisible by 8.
Therefore, 7,314 is not divisible by 8.

(iii) The given number = 36712

Since the number formed by its last three digit = 712 which is divisible by 8.
Therefore, 36,712 is divisible by 8.

Question: 6

Test the divisibility of the following numbers by 9:

Solution:

Rule: A number is divisible by 9 if the sum of its digits is divisible by 9.

(i) The given number = 187245

The sum of the digits in the given number = 1 + 8 + 7 + 2 + 4 + 5 = 27 which is divisible by 9. Therefore, 1,87,245 is divisible by 9.

(ii) The given number = 3478

The sum of the digits in the given number = 3 + 4 + 7 + 8 = 22 which is not divisible by 9. Therefore, 3,478 is not divisible by 9.

(iii) The given number = 547218

The sum of the digits in the given number = 5 + 4 + 7 + 2 + 1 + 8 = 27 which is divisible by 9. Therefore, 5,47,218 is divisible by 9.

Question: 7

Test the divisibility of the following numbers by 11:

Solution:

(i) The given number is 5,335.

The sum of the digit at the odd places = 5 + 3 = 8

The sum of the digits at the even places = 3 + 5 = 8

Their difference = 8 − 8 = 0

Therefore, 5,335 is divisible by 11.

(ii) The given number is 7,01,69,803.

The sum of the digit at the odd places = 7 + 1 + 9 + 0 = 17

The sum of the digits at the even places = 0 + 6 + 8 + 3 = 17

Their difference = 17 − 17 = 0

Therefore, 7,01,69,803 is divisible by 11.

(iii) The given number is 1,00,00,001.

The sum of the digit at the odd places = 1 + 0 + 0 + 0 = 1

The sum of the digits at the even places = 0 + 0 + 0 + 1 = 1

Their difference = 1 − 1 = 0

Therefore, 1,00,00,001 is divisible by 11.

Question: 8

In each of the following numbers, replace * by the smallest number to make it divisible by 3:

Solution:

We can replace the * by the smallest number to make the given numbers divisible by 3 as follows:

(i) 75*5

75*5 = 7515

As 7 + 5 + 1 + 5 = 18, it is divisible by 3.

(ii) 35*64

35*64 = 35064

As 3 + 5 +6 + 4 = 18, it is divisible by 3.

(iii) 18 * 71

18 * 71= 18171

As 1 + 8 + 1 + 7 + 1 = 18, it is divisible by 3.

Question: 9

In each of the following numbers, replace * by the smallest number to make it divisible by 9:

Solution:

(i) 67 *19

Sum of the given digits = 6 + 7 + 1 + 9 = 23

The multiple of 9 which is greater than 23 is 27.

Therefore, the smallest required number = 27 – 23 = 4

(ii) 66784 *

Sum of the given digits = 6 + 6 + 7 + 8 + 4 = 31

The multiple of 9 which is greater than 31 is 36.

Therefore, the smallest required number = 36 – 31 = 5

(iii) 538 * 8

Sum of the given digits = 5 + 3 + 8 + 8 = 24

The multiple of 9 which is greater than 24 is 27.

Therefore, the smallest required number = 27 – 24 = 3

Question: 10

In each of the following numbers, replace * by the smallest number to make it divisible by 11:

Solution:

Rule: A number is divisible by 11 if the difference of the sums of the alternate digits is either 0 or a multiple of 11.

(i) 86 x 72

Sum of the digits at the odd places = 8 + missing number + 2 = missing number + 10

Sum of the digits at the even places = 6 + 7 = 13

Difference = [missing number + 10] – 13 = Missing number – 3

According to the rule, missing number – 3 = 0 [Because the missing number is a single digit]

Thus, missing number = 3

Hence, the smallest required number is 3.

(ii) 467 x 91

Sum of the digits at the odd places = 4 + 7 + 9 = 20

Sum of the digits at the even places = 6 + missing number + 1 = missing number + 7 Difference = 20 – [missing number + 7] = 13 – missing number

According to rule, 13 – missing number = 11 [Because the missing number is a single digit]

Thus, missing number = 2

Hence, the smallest required number is 2.

(iii) 9 x 8071

Sum of the digits at the odd places = 9 + 8 + 7 = 24

Sum of the digits at the even places = missing number + 0 + 1 = missing number + 1

Difference = 24 – [missing number + 1] = 23 – missing number

According to rule, 23 – missing number = 22 [Because 22 is a multiple of 11 and the missing number is a single digit]

Thus, missing number = 1

Hence, the smallest required number is 1.

Question: 11

Given an example of a number which is divisible by

Solution:

(i) A number which is divisible by 2 but not by 4 is 6.

(ii) A number which is divisible by 3 but not by 6 is 9.

(iii) A number which is divisible by 4 but not by 8 is 28.

(iv) A number which is divisible by 4 and 8 but not by 32 is 48.

Question: 12

Which of the following statement are true?

Solution:

(i) If a number is divisible by 3, it must be divisible by 9.

False. 12 is divisible by 3 but not by 9.

(ii) If a number is divisible by 9, it must be divisible by 3.

True.

(iii) If a number is divisible by 4, it must be divisible by 8.

False. 20 is divisible by 4 but not by 8.

(iv) If a number is divisible by 8, it must be divisible by 4.

True.

(v) A number is divisible by 18, it is divisible by both 3and 6.

False. 12 is divisible by both 3 and 6 but it is not divisible by 18.

(vi) If a number is divisible by both 9 and 10, it must be divisible by 90

True.

(vii) If a number exactly divides three numbers the sum of two numbers, it must exactly divide the numbers separately.

False. 10 divides the sum of 18 and 2 (i.e., 20) but 10 divides neither 18 nor 2.

(viii) If a number divides three numbers exactly, it must divide their sums exactly.

True.

(ix) If two numbers are co-prime, at least one of them must be a co-prime number.

False. 4 and 9 are co-primes and both are composite numbers.

(x) The sum of two consecutive odd numbers is always divisible by 4

True.

Exercise 2.6

Question: 1

Find the H.C. F of the following numbers using prime factors using prime factorization method:

Solution:

(i) 144 and 198

Prime factorization of 144 = 2 × 2 × 2 × 3 × 3

Prime factorization of 198 = 2 × 3 × 3 ×11

Therefore, HCF = 2 × 2 × 3 = 18

(ii) 81 and 117

Prime factorization of 81 = 3 × 3 × 3 × 3

Prime factorization of 117 = 3 × 3 ×13

Therefore, HCF = 3 × 3 = 9

(iii) 84 and 98

Prime factorization of 84 = 2 × 2 × 3 × 7

Prime factorization of 98 = 2 × 7 × 7

Therefore, HCF = 2 × 7 = 14

(iv) 225 and 450

Prime factorization of 225 = 3 × 3 × 5 × 5

Prime factorization of 198 = 2 × 3 × 3 × 5 × 5

Therefore, HCF = 3 × 3 × 5 × 5 = 225

(v) 170 and 238

Prime factorization of 170 = 2 × 5 × 17

Prime factorization of 238 = 2 × 7 × 17

Therefore, HCF = 2 × 17 = 34

(vi) 504 and 980

Prime factorization of 504 = 2 × 2 × 2 × 3 × 3 × 7

Prime factorization of 980 = 2 × 2 × 5 × 7 × 7

Therefore, HCF = 2 × 2 × 7 = 28

(vii) 150, 140 and 210

Prime factorization of 150 = 2 × 3 × 5 × 5

Prime factorization of 140 = 2 × 2 × 5 × 7

Prime factorization of 210 = 2 × 3 × 5 × 7

Therefore, HCF = 2 × 5 = 10

(viii) 84, 120 and 138

Prime factorization of 84 = 2 × 2 × 3 × 7

Prime factorization of 120 = 2 × 2 × 2 × 3 × 5

Prime factorization of 138 = 2 × 3 × 23

Therefore, HCF = 2 × 3 = 6

(i×) 106, 159 and 265

Prime factorization of 106 = 2 × 53

Prime factorization of 159 = 2 × 53

Prime factorization of 265 = 5 × 53

Therefore, HCF = 53

Question: 2

What is the H.C.F of two consecutive?

Solution:

(i) The common factor of two consecutive numbers is always 1.

Therefore, HCF of two consecutive numbers = 1

(ii) The common factors of two consecutive even numbers are 1 and 2.

Therefore, HCF of two consecutive even numbers = 2

(iii) The common factor of two consecutive odd numbers is 1.

Therefore, HCF of two consecutive odd numbers = 1

Question: 3

H.C.F of co-primes numbers 4 and 15 was found as follows:

$4 = 2 \times 2$ and $15 = 3 \times 5$

Since there is no common prime factor. So, H.C.F of 4 and 15 is 0. Is the answer correct? If not what is the correct H.C.F?

Solution:

No, it is not correct.

We know that HCF of two co-prime number is 1.

4 and 15 are co-prime numbers because the only factor common to them is 1.

Thus, HCF of 4 and 15 is 1.

Exercise 2.7

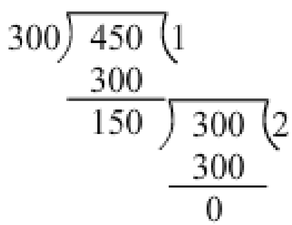
Question: 1

Determine the H.C.F of the following numbers by using Euclid’s algorithm (I – x):

Solution:

(i) 300 and 450

Dividend = 450 and divisor = 300

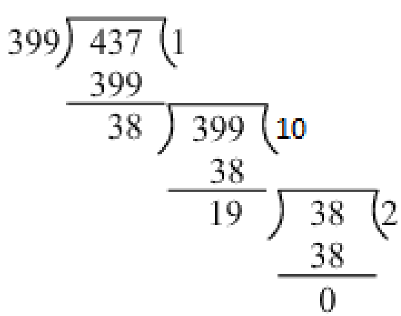


Clearly, the last divisor is 150.

Hence, HCF of the given number is 150.

(ii) 399 and 437

We have dividend = 399 and divisor= 437

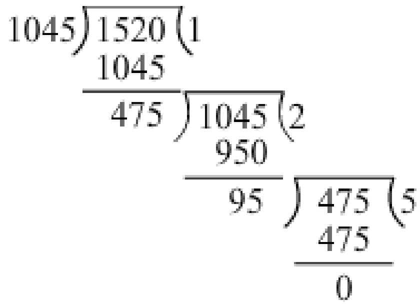


Clearly, the last divisor is 19.

Hence, HCF of the given number is 19

(iii) 1045 and 1520

We have dividend = 1045 and divisor = 1520



Clearly, the last divisor is 95.

Hence, HCF of given numbers is 95.

Question: 2

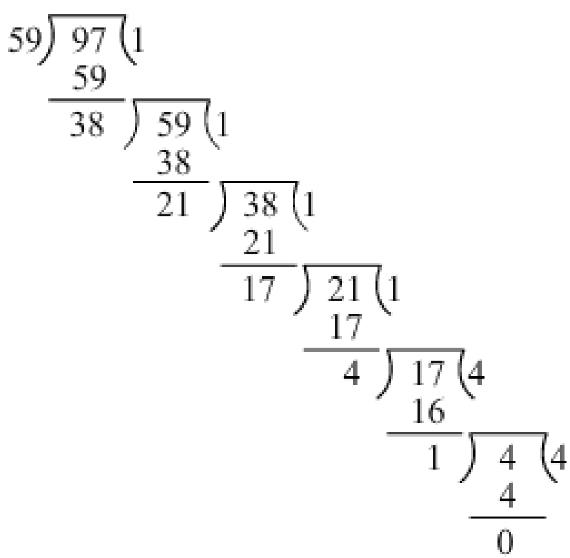
Show that the following pairs are co-prime:

Solution:

We know that two numbers are co-primes if their HCF is 1.

(i) 59 and 97

Here, dividend = 97 and divisor = 59

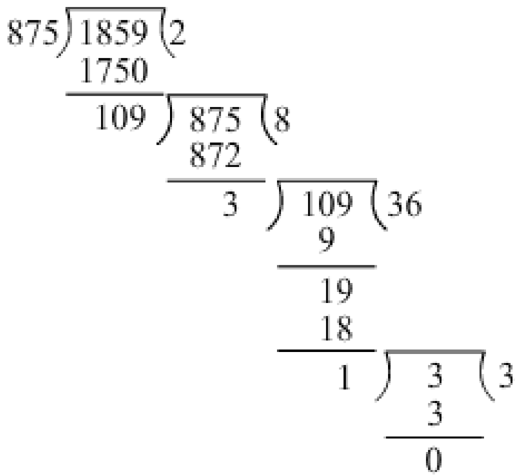


Clearly, the last divisor is 1.

Hence, the given numbers are co-primes.

(ii) 875 and 1859

Here, dividend = 1859 and divisor = 875

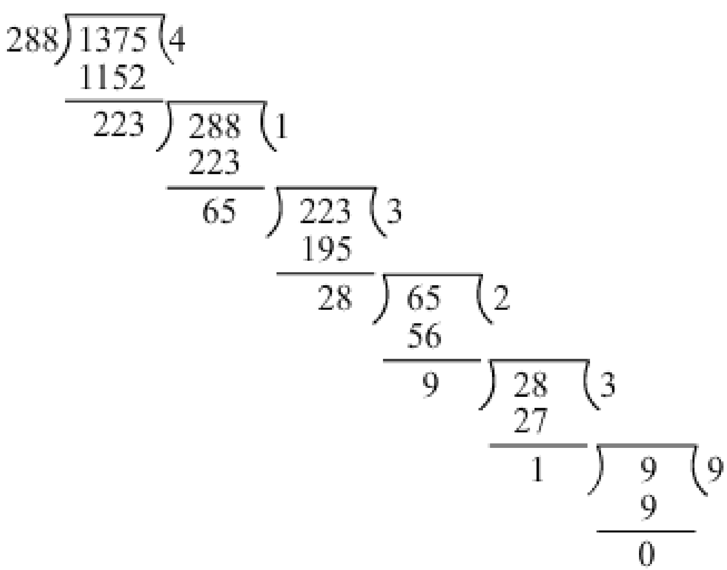


Clearly, the last divisor is 1.

Hence, the given numbers are co-prime.

(iii) 288 and 1375

Here, dividend = 288 and divisor = 1375



Clearly, the last divisor is 1.

Hence, the given numbers are co-prime.

Question: 3

What is the H.C.F of two consecutive numbers?

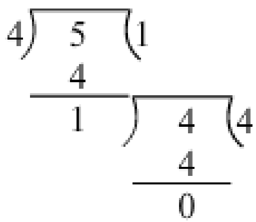
Solution:

The HCF of two consecutive numbers is 1.

Example:

D = 4 and d= 5 are two consecutive numbers.

Here, we have dividend = 5 and divisor = 4



Clearly, the last divisor is 1.

Hence, HCF of 4 and 5 is 1.

Question: 4

Write true (T) or false (F) for each of the following statements:

Solution:

(i) The H.CF of two distinct prime numbers is 1

True.

(ii) The H.CF of two co-prime number is 1.

True.

(iii) The H.CF of an even and an odd number is 1.

False. HCF of 6 and 9 is 3 not 1.

(iv) The H.C.F of two consecutive even numbers is 2.

True.

(v) The H.C.F of two consecutive odd numbers is 2.

False.

HCF of two consecutive odd numbers is 1.

Example: HCF of 25 and 27 is 1.

Exercise 2.8

Question: 1

Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Solution:

We have to find the largest number which divides (615 – 6) and (963 – 6) exactly.

Therefore, the required number = HCF of 609 and 957

Resolving 609 and 957 into prime factors, we have:

$609 = 3 \times 7 \times 29$

$957 = 3 \times 11 \times 29$

Therefore, HCF of 609 and 957 = $29 \times 3 = 87$

Hence, the required largest number is 87.

Question: 2

Find the largest number that divides 285 and 1249 leaving remainders 9 and 7 respectively.

Solution:

We have to find the greatest number which divides (285 – 9) and (1,249 – 7) exactly.

The required number will be given by the HCF of 276 and 1242.

Resolving 276 and 1242 into prime factors, we have:

$276 = 2 \times 2 \times 3 \times 23$

$1242 = 2 \times 3 \times 3 \times 3 \times 23$

HCF of 276 and 1242 is $2 \times 3 \times 23 = 138$.

Question: 3

What is the largest number that divides 626, 3127 and 15628 leaving remainders 1, 2 and 3 respectively.

Solution:

We have to find the largest number which divides (626 – 1), (3,127 – 2), and (15,628 – 3) exactly.

The required number will be given by the HCF of 625, 3,125 and 15,625.

Resolving 625, 3125, and 15625 into prime factors, we have:

625 = 5 × 5 × 5 × 5 3,

125 = 5 × 5 × 5 × 5 × 5 15,

625 = 5 × 5 × 5 × 5 × 5 × 5

Therefore, HCF of 625, 3125 and 15625 = 5 × 5 × 5 × 5 = 625 Hence, the required largest number is 625.

Question: 4

The length, breadth and height of the room are 8cm 25cm, 6m 75 cm and 4m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Solution:

Given:

Length of the room = 8 m 25 cm = 825 cm

Breadth of the room = 6 m 75 cm = 675 cm

Height of the room = 4 m 50 cm = 450 cm

The longest rod will be given by the HCF of 825, 675 and 450.

Prime factorization of 825 = 3 × 5 × 5 × 11

Prime factorization of 675 = 3 × 3 × 3 × 5 × 5

Prime factorization of 450 = 2 × 3 × 3 × 5 × 5 Therefore, HCF of 825, 675 and 450 = 3 × 5 × 5 = 75

Thus, the required length of the longest rod is 75 cm.

Question: 5

A rectangular courtyard is 20 m 16 cm long and 15m 60 cm broad. It is to be paved with square roots of the same size. Find the least possible number of such stones.

Solution:

Length of the rectangular courtyard = 20 m 16 cm = 2,016 cm

Breadth of the rectangular courtyard = 15 m 60 cm = 1,560 cm

Least possible side of the square stones used to pave the rectangular courtyard = HCF of (2,016 and 1,560)

Prime factorization of 2,016 = 2 × 2 × 2 × 2 × 2 × 3 × 3 × 7

Prime factorization of 1,560 = 2 × 2 × 2 × 3 × 5 × 13 HCF of (2,016, 1,560) = 2 × 2 × 2 × 3 = 24

Least possible side of square stones used to pave the rectangular courtyard is 24 cm. Number of square stones used to pave the rectangular courtyard

= Area of rectangular courtyard Area of square stone = 2016 cm × 1560 cm (24 cm) 2 = 5460 Thus, the least number of square stones used to pave the rectangular courtyard is 5,460.

Question: 6

Determine the longest tape which can be used to measure exactly the lengths 7m, 3m 85 cm and 12 m 95 cm?

Solution:

Given: Length of the first tape = 7 m = 700 cm

Length of the second tape = 3 m 85 cm = 385 cm

Length of the third tape = 12 m 95 cm = 1,295 cm

The length of the longest tape will be the HCF of 700, 385, and 1,295.

Prime factorization of 700 = 2 × 2 × 5 × 5 × 7

Prime factorization of 385 = 5 × 7 × 11

Prime factorization of 1,295 = 5 × 7 × 37 I-ICF of 700, 385, and 1,295 = 5 × 7 = 35

Required length of the longest tape = 35 cm

Question: 7

105 goats, 140 donkeys and 175 cows have to b taken across a river .Three is only one boat which will have to make many trips in order to do so .The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same

kind .He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Solution:

We have to find the largest possible number of animals. Thus, we will have to find the HCF of 105, 140, and 175.

Prime factorization of 105 = 3 × 5 × 7

Prime factorization of 140 = 2 × 2 × 5 × 7

Prime factorization of 175 = 5 × 5 × 7

Required HCF = 5 × 7 = 35 Hence, 35 animals went in each trip.

Question: 8

Two brands of chocolates are available in packs of 24 and 15 respectively. If in need to but an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Solution:

Let the brand 'A' contain 24 chocolates in one packet and brand 'B' contain 14 chocolates in one packet.

Equal number of chocolates of each kind can be found out by taking LCM of the number of chocolates in each packet.

Therefore, LCM of 15 and 24 is:

2	15,24
2	15,12
2	15,6
3	15,3
5	5,1
	1,1

Required LCM = 2 × 2 × 2 × 3 × 5 = 120

Therefore, minimum 12o chocolates of each kind should be purchased.

Number of boxes of brand 'A' which needs to be purchased = 120 ÷ 24 = 5

Number of boxes of brand ‘B’ which needs to be purchased = $120 \div 15 = 8$

Question: 9

During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would need to buy?

Solution:

To find the required number of pencils and crayons, we need to find the LCM of 24 and 32.

Prime factorization of 24 = $2 \times 2 \times 2 \times 3$

Prime factorization of 32 = $2 \times 2 \times 2 \times 2 \times 2$

Required LCM of 24 and 32 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

Thus, number of pencils and crayons needed to be bought is 96 each, i.e. 96•24 = 4 packs of color

pencils and $96 \div 32 = 3$ packs of crayons.

Question: 10

Reduce each of the following fractions to the lowest terms:

Solution:

(i) 161/207

For reducing the given fraction to the lowest terms, we have to divide its numerator and denominator by their HCF.

Now, we have to find the HCF of 161 and 207.

Prime factorization of 161 = 7×23

Prime factorization of 207 = $3 \times 3 \times 23$

Therefore, HCF of 161 and 207 = 23

Now, 161+23207+ 23 = 79

Hence, 79 is the required fraction.

(ii) 296/491<

For reducing the given fraction to the lowest terms, we have to divide its numerator and denominator by their HCF.

Now, we have to find the HCF of 296 and 481.

Prime factorization of 296 = 2 × 2 × 2 × 37

Prime factorization of 481 = 13 × 37

Therefore, HCF of 296 and 481 = 37

Now, 296 + 37481 + 37 = 813

Hence, 813 is the required fraction.

Question: 11

A merchant has 120 liters of oil of one kind, 180 liters of another kind and 340 liters of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Solution:

The maximum capacity of the required tin is the HCF of the three quantities of oil.

Prime factorization of 120 = 2 × 2 × 2 × 3 × 5

Prime factorization of 180 = 2 × 2 × 3 × 3 × 5

Prime factorization of 240 =2 × 2 × 2 × 2 × 3 × 5

Therefore, HCF of 120, 180, and 240 = 2 × 2 × 3 × 5 = 60

Hence, the required greatest capacity of the tin must be 60 liters.

Exercise 2.9

Question: 1

Determine the L.C.M of the numbers given below:

Solution:

(i) 48, 60

Prime factorization of 48 = $2 \times 2 \times 2 \times 2 \times 3$

Prime factorization of 60 = $2 \times 2 \times 3 \times 5$

Therefore, Required LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$

(ii) 42, 63

Prime factorization of 42 = $2 \times 3 \times 7$

Prime factorization of 63 = $3 \times 3 \times 7$

Therefore, Required LCM = $2 \times 3 \times 3 \times 7 = 126$

(iii) 18, 17

Prime factorization of 18 = $2 \times 3 \times 3$

Prime factorization of 17 = 17

Therefore, Required LCM = $2 \times 3 \times 3 \times 17 = 306$

(iv) 15, 30, 90

Prime factorization of 15 = 3×5

Prime factorization of 30 = $2 \times 3 \times 5$

Prime factorization of 90 = $2 \times 3 \times 3 \times 5$

Therefore, Required LCM = $2 \times 3 \times 3 \times 5 = 90$

(v) 56, 65, 85

Prime factorization of 56 = $2 \times 2 \times 2 \times 7$

Prime factorization of 65 = 5×13

Prime factorization of 85 = 5×17

Therefore, Required LCM = $2 \times 2 \times 2 \times 5 \times 7 \times 13 \times 17 = 61,880$

(vi) 180, 384, 144

Prime factorization of 180 = 2 × 2 × 3 × 3 × 5

Prime factorization of 384 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

Prime factorization of 144 = 2 × 2 × 2 × 2 × 3 × 3

Therefore,

Therefore, Required LCM = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 5 = 5,760

(vii) 108, 135, 162

Prime factorization of 108 = 2 × 2 × 3 × 3 × 3

Prime factorization of 135 = 3 × 3 × 3 × 5

Prime factorization of 162 = 2 × 3 × 3 × 3 × 3

Therefore, Required LCM = 2 × 2 × 3 × 3 × 3 × 3 × 5 = 1,620

(viii) 28, 36, 45, 60

Prime factorization of 28 = 2 × 2 × 7

Prime factorization of 36 = 2 × 2 × 3 × 3

Prime factorization of 45 = 3 × 3 × 5

Prime factorization of 60 = 2 × 2 × 3 × 5

Therefore, Required LCM =2 × 2 × 3 × 3 × 5 × 7= 1,260

Exercise 2.10

Question: 1

What is the smallest number which when divided by 24, 36 and 54 gives a remainder of 5 each time?

Solution:

We have to find prime factorization of 24, 36, and 54.

Prime factorization of 24 = $2 \times 2 \times 2 \times 3$

Prime factorization of 36 = $2 \times 2 \times 3 \times 3$

Prime factorization of 54 = $2 \times 3 \times 3 \times 3$

Therefore, Required LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$

Thus, 216 is the smallest number exactly divisible by 24, 36, and 54.

To get the remainder as 5:

Smallest number = $216 + 5 = 221$

Thus, the required number is 221.

Question: 2

What is the smallest number that both 33 and 39divide leaving remainders of 5?

Solution:

We have to find prime factorization of 33 and 39.

Prime factorization of 33 = 3×11

Prime factorization of 39 = 3×13

Therefore, Required LCM = $3 \times 11 \times 13 = 429$

Thus, 429 is the smallest number exactly divisible by 33 and 39.

To get the remainder as 5: Smallest number = $429 + 5 = 434$

Thus, the required number is 434.

Question: 3

Find the least number that is divisible ny all the numbers between 1 and 10 (both inclusive)

Solution:

To find the required least number, we have to find the LCM of the numbers from 1 to 10. We know that 2, 3, 5, and 7 are prime number.

Prime factorization of 4 = 2×2

Prime factorization of 6 = 2×3

Prime factorization of 8 = $2 \times 2 \times 2$

Prime factorization of 9 = 3×3

Prime factorization of 10 = 2×5

Therefore, Required least number = $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2,520$

Question: 4

What is the smallest number that, when divided by 35, 56 and 91 leaves remainder of 7 in each case?

Solution:

We have to find the prime factorization of 35, 56, and 91.

Prime factorization of 35 = 5×7

Prime factorization of 56 = $2 \times 2 \times 2 \times 7$

Prime factorization of 91 = 7×13

Therefore, Required LCM = $2 \times 2 \times 2 \times 5 \times 7 \times 13 = 3,640$

Thus, 3,640 is the smallest number exactly divisible by 35, 56, and 91.

To get the remainder as 7:

Smallest number = $3,640 + 7 = 3,647$

Thus, the required number is 3,647.

Question: 5

In school there are two sections- section A and section B of class VI. There are 32 students in section- A and 36 in section B. determine the minimum number of books required for their class library so that they can be distributed equally among students of section A and section B

Solution:

We have to find the LCM of 32 and 36.

Prime factorization of 32 = 2 × 2 × 2 × 2 × 2

Prime factorization of 36 = 2 × 2 × 3 × 3

Required LCM = 2 × 2 × 2 × 2 × 2 × 3 × 3 =288

Therefore, Minimum number of books required = LCM of 32 and 36 = 288 books

Question: 6

In a morning walk three persons step off together. Their steps measure 80 cm. 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Solution:

We have to find the LCM of 80 cm, 85 cm, and 90 cm.

Prime factorization of 80 = 2 × 2 × 2 × 2 × 5

Prime factorization of 85 = 5 × 17

Prime factorization of 90 = 2 × 3 × 3 × 5

Therefore, Required LCM=2 × 2 × 2 × 2 × 3 × 3 × 5 × 17 = 12,240

Therefore, Required minimum distance = LCM of 80 cm, 85 cm, and 90 cm

= 12,240 cm

= 122 m 40 cm (since 1 m =100 cm)

Question: 7

Determine the number nearest to 10000 but greater than 10000 which is exactly divisible by each of 8, 15 and 21.

Solution:

First, we have to find the L.C.M of 8, 15 and 21.

Prime factorization of 8 = 2 × 2 × 2

Prime factorization of 15 = 3 × 5

Prime factorization of 21 = 3 × 7

Therefore, required LCM= 2 × 2 × 2 × 3 × 5 × 7 = 840

The number nearest to 1, 00,000 and exactly divisible by each 8, 15 and 21 should also be divisible by their LCM (i.e. 840)

We have to divide 1, 00,000 by 840.

```
840) 100000 (119
     840
     ----
     1600
      840
     ----
     7600
     7560
     ----
       40
```

Remainder = 40

Therefore, Number greater than 1,00,000 and exactly divisible by 840 =
1,00,000 + (840 – 40) = 1,00,000 + 800 = 1,00,800

Therefore, Required number = 1, 00, 800.

Question: 8

A school bus picking up children in a colony of flats stops at every sixth block of flats. Another school bus starting from the same place stops at every eight blocks of flats. Which is the first bus stop at which both of them will stop?

Solution:

First bus stop at which both the buses will stop together = LCM of 6th block and 8th block

Prime factorization of 6 = 2 × 3

Prime factorization of 8 = 2 × 2 × 2

Therefore, Required LCM = 2 × 2 × 2 × 3 = 24

Hence, the first bus stop at which both the buses will stop together will be at the 24th block.

Question: 9

Telegraph pole occur at equal distances of 220 m along a road and heaps of stones are put at equal distances of 300 m along the same road. The first heap is at the foot of the first pole. How far from it along the road is the next heap which lies at the foot of a pole?

Solution:

We have to find the LCM of 220 m and 300 m.

Prime factorization of 220 = 2 × 2 × 5 × 11

Prime factorization of 300 = 2 × 2 × 3 × 5 × 5

Therefore, Required LCM = 2 × 2 × 3 × 5 × 5 × 11 = 3,300

Hence, 3,300 m far is the next heap that lies at the foot of a pole.

Question: 10

Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

Solution:

First, we have to find the LCM of 28 and 32.

Prime factorization of 28 = 2 × 2 × 7

Prime factorization of 32 = 2 × 2 × 2 × 2 × 2

Therefore, Required LCM = 2 × 2 × 2 × 2 × 2 × 7 = 224

It is given that when we divide the number by 28, the remainder is 8 and when we divide the number by 32, the remainder is 12.

We observe:

28 – 8 = 20

32 – 12 = 20

Therefore, Required number = 224 – 20 = 204

Exercise 2.11

Question: 1

For each of the following pairs of numbers, verify the property:

Product of the number = Product of their H.C.F and L.C.M

Solution:

(i) Given numbers are 25 and 65

Prime factorization of 25 = 5 × 5

Prime factorization of 65 = 5 × 13

HCF of 25 and 65 = 5

LCM of 25 and 65 = 5 × 5 × 13 = 325

Product of the given numbers = 25 × 65 = 1, 625

Product of their HCF and LCM = 5 × 325 = 1, 625

Therefore, Product of the number = Product of their HCF and LCM (Verified)

(ii) Given numbers are 117 and 221.

Prime factorization of 117 = 3 × 3 × 13

Prime factorization of 221 = 13 × 17

HCF of 117 and 221 = 13

LCM of 117 and 221 = 3 × 3 × 13 × 17 = 1, 989

Product of the given number = 117 × 221 = 12, 857

Product of their HCF and LCM = 13 × 1, 989 = 12,857

Therefore, Product of the number = Product of their HCF and LCM (verified)

(iii) Given numbers are 35 and 40.

Prime factorization of 35 = 5 × 7

Prime factorization of 40 = 2 × 2 × 2 × 5

HCF of 35 and 40 = 5

LCM of 35 and 40 = 2 × 2 × 2 × 5 × 7 = 280

Product of the given number = 35 × 40 = 1400

Product of their HCF and LCM = 5 × 280 = 1400

Therefore, Product of the number = Product of their HCF and LCM (Verified)

(iv) Given numbers are 87 and 145.

Prime factorization of 87= 3 × 29

Prime factorization of 145= 5 × 29

HCF of 87 and 145 = 29

LCM of 87 and 145 = 3 × 5 × 29 = 435

Product of the given number = 87 × 145 = 12615

Product of their HCF and LCM = 29 × 435 = 12615

Therefore, Product of the number = Product of their HCF and LCM (Verified)

(v) Given numbers are 490 and 1155.

Prime factorization of 490= 2 × 5 × 7 × 7

Prime factorization of 1155= 3 × 5 × 7 × 11

HCF of 490 and 1155 = 35

LCM of 490 and 1155 = 2 × 3 × 3 × 5 × 7 × 7 × 11 = 16710

Product of the given number = 490 × 1155 = 5, 65,950

Product of their HCF and LCM = 35 × 16, 170 = 5, 65,950

Therefore, Product of the number = Product of their HCF and LCM (Verified)

Question: 2

Find the H.C. F and L.C.M of the following pairs and numbers:

Solution:

(i) 1174 and 221

Prime factorization of 117 = 3 × 3 × 13

Prime factorization of 221 = 13 × 17

Therefore, Required HCF of 117 and 221 = 13

Therefore, Required LCM of 117 and 221 = 3 × 3 × 13 × 17 = 1989

(ii) 234 and 572.

Prime factorization of 234 = 2 × 3 × 3 × 13

Prime factorization of 572 = 2 × 2 × 11 × 13

Therefore, Required HCF of 234 and 572 = 226

Therefore, Required LCM of 117 and 221 = 2 × 2 × 3 × 3 × 11 × 13 = 5148

(iii) 145 and 232

Prime factorization of 145 = 5 × 29

Prime factorization of 232 = 2 × 2 × 2 × 29

Therefore, Required HCF of 145 and 232 = 289

Therefore, Required LCM of 145 and 232 = 2 × 2 × 2 × 5 × 29 = 1160

(v) 861 and 1353

Prime factorization of 861 = 3 × 7 × 41

Prime factorization of 1353 = 3 × 11 × 41

Therefore, Required HCF of 861 and 1353 = 123

Therefore, Required LCM of 861 and 1353 = 3 × 7 × 11 × 41 = 9471

Question: 3

The L.C.M and H.C.F of two numbers are 180 and 6 respectively. If one of the number is 30, find the other number.

Solution:

Given: HCF of two numbers = 6

LCM of two numbers = 180

One of the given number = 30

Product of the two numbers = Product of their HCF and LCM

Therefore, 30 × other number = 6 × 180

Other number = 6 × 18030 = 36

Thus, the required number is 36.

Question: 4

The H.C.F of two numbers is 16, and their product is 3072. Find their L.C.M

Solution:

Given: HCF of two numbers = 16

Product of these two numbers = 3,072

Product of the two numbers = Product of their HCF and LCM

Therefore, 3,072 = 16 × LCM

LCM = 307216 = 192

Thus, the required LCM is 192.

Question: 5

The H.C.F of two numbers is 145, their L.C.M is 2175. If one number is 725, find the other.

Solution:

HCF of two numbers = 145

LCM of two numbers = 2,175

One of the given numbers = 725

Product of the given two numbers = Product of their LCM and HCF

Therefore, 725 × other number = 145 × 2,175

Other number = 145 × 2175725 = 435

Thus, the required number is 435.

Question: 6

Can two numbers have 16 as their HCF and 380 as their L.C.M? Give reasons.

Solution:

No. We know that HCF of the given two numbers must exactly divide their LCM.

But 16 does not divide 380 exactly.

Hence, there can be no two numbers with 16 as their HCF and 380 as their LCM.