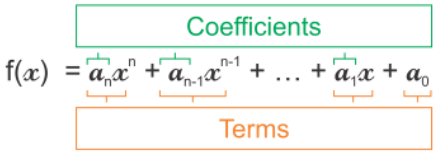
Polynomials

If 'x' is a variable, 'n' is a positive integer and $a_0, a_1, a_2, \ldots, a_n$ are constants, then a polynomial in variable x is $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$



Degree of a Polynomial: The power of the highest degree term

Zero of a Polynomial: A real number α is a zero of a polynomial $f(x)$, iff $f(\alpha) = 0$.
Finding the zero of a polynomial $f(x)$ means solving the polynomial equation $f(x) = 0$

Polynomial Classification

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic

Number of Terms	Name
1	Monomial
2	Binomial
3	Trinomial
4	Polynomial of 4 Terms

Exercise 2A

Question 1:

(i) It is a polynomial, Degree = 5.
(ii) It is polynomial, Degree = 3.
(iii) It is polynomial, Degree = 2.
(iv) It is not a polynomial.
(v) It is not a polynomial.
(vi) It is polynomial, Degree = 108.
(vii) It is not a polynomial.
(viii) It is a polynomial, Degree = 2.
(ix) It is not a polynomial.
(x) It is a polynomial, Degree = 0.
(xi) It is a polynomial, Degree = 0.
(xii) It is a polynomial, Degree = 2.

Question 2:

The degree of a polynomial in one variable is the highest power of the variable.

(i) Degree of $2x - \sqrt{5}$ is 1.
(ii) Degree of $3 - x + x^2 - 6x^3$ is 3.
(iii) Degree of 9 is 0.
(iv) Degree of $8x^4 - 36x + 5x^7$ is 7.
(v) Degree of $x^9 - x^5 + 3x^{10} + 8$ is 10.

(vi) Degree of $2 - 3x^2$ is 2.

Question 3:

(i) Coefficient of x^3 in $2x + x^2 - 5x^3 + x^4$ is -5

(ii) Coefficient of x in $\sqrt{3} - 2\sqrt{2}x + 4x^2$ is $-2\sqrt{2}$

(iii) Coefficient of x^2 in $\frac{\pi}{3}x^2 + 7x - 3$ is $\frac{\pi}{3}$

(iv) Coefficient of x^2 in $3x - 5$ is 0.

Question 4:

(i) $x^{27} - 36$

(ii) y^{16}

(iii) $5x^3 - 8x + 7$

Question 5:

(i) It is a quadratic polynomial.

(ii) It is a cubic polynomial.

(iii) It is a quadratic polynomial.

(iv) It is a linear polynomial.

(v) It is a linear polynomial.

(vi) It is a cubic polynomial.

Exercise 2B

Question 1:

$p(x) = 5 - 4x + 2x^2$

(i) $p(0) = 5 - 4(0) + 2(0)^2 = 5$

(ii) $p(3) = 5 - 4(3) + 2(3)^2$

$= 5 - 12 + 18$

$= 23 - 12 = 11$

(iii) $p(-2) = 5 - 4(-2) + 2(-2)^2$

$= 5 + 8 + 8 = 21$

Question 2:

$p(y) = 4 + 3y - y^2 + 5y^3$

(i) $p(0) = 4 + 3(0) - 0^2 + 5(0)^3$

$= 4 + 0 - 0 + 0 = 4$

(ii) $p(2) = 4 + 3(2) - 2^2 + 5(2)^3$

$= 4 + 6 - 4 + 40$

$= 10 - 4 + 40 = 46$

(iii) $p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^3$

$= 4 - 3 - 1 - 5 = -5$

Question 3:

$f(t) = 4t^2 - 3t + 6$

(i) $f(0) = 4(0)^2 - 3(0) + 6$

$= 0 - 0 + 6 = 6$

(ii) $f(4) = 4(4)^2 - 3(4) + 6$

= 64 – 12 + 6 = 58

(iii) $f(-5) = 4(-5)^2 - 3(-5) + 6$

= 100 + 15 + 6 = 121

Question 4:

(i) p(x) = 0

⇒ x – 5 = 0

⇒ x = 5

⇒ 5 is the zero of the polynomial p(x).

(ii) q(x) = 0

⇒ x + 4 = 0

⇒ x = -4

⇒ -4 is the zero of the polynomial q(x).

(iii) p(t) = 0

⇒ 2t – 3 = 0

⇒ 2t =3

⇒ $t = \frac{3}{2}$

⇒ $t = \frac{3}{2}$ is the zero of the polynomial p(t).

(iv) f(x) = 0

⇒ 3x + 1= 0

⇒ 3x = -1

⇒ $x = \frac{-1}{3}$

⇒ $x = \frac{-1}{3}$ is the zero of the polynomial f(x).

(v) g(x) = 0

⇒ 5 – 4x = 0

⇒ -4x = -5

⇒ $x = \frac{5}{4}$

⇒ $x = \frac{5}{4}$ is the zero of the polynomial g(x).

(vi) h(x) = 0

⇒ 6x – 1 = 0

⇒ 6x = 1

⇒ $x = \frac{1}{6}$

⇒ $x = \frac{1}{6}$ is the zero of the polynomial h(x).

(vii) p(x) = 0

⇒ ax + b = 0

⇒ ax = -b

⇒ $x = \frac{-b}{a}$

⇒ $x = \frac{-b}{a}$ is the zero of the polynomial p(x)

(viii) q(x) = 0

⇒ 4x = 0

⇒ x = 0
⇒ 0 is the zero of the polynomial q(x).

(ix) p(x) = 0
⇒ ax = 0
⇒ x = 0
⇒ 0 is the zero of the polynomial p(x).

Question 5:

(i) p(x) = x – 4
Then, p(4) = 4 – 4 = 0
⇒ 4 is a zero of the polynomial p(x).

(ii) p(x) = x – 3
Then, p(-3) = -3 – 3 = -6
⇒ -3 is not a zero of the polynomial p(x).

(iii) p(y) = 2y + 1
Then, $p\left(-\frac{1}{2}\right)=2\left(\frac{-1}{2}\right)+1=0$
⇒ $\frac{-1}{2}$ is a zero of the polynomial p(y).

(iv) p(x) = 2 – 5x
Then, $p\left(\frac{2}{5}\right)=2-5\left(\frac{2}{5}\right)=2-2=0$
⇒ $\frac{2}{5}$ is a zero of the polynomial p(x).

(v) p(x) = (x – 1) (x – 2)
Then, p(1) = (1 – 1) (1 – 2) = 0 -1 = 0
⇒ 1 is a zero of the polynomial p(x).
Also, p(2) = (2 – 1)(2 – 2) = 1 0 = 0
⇒ 2 is a zero of the polynomial p(x).
Hence, 1 and 2 are the zeroes of the polynomial p(x).

(vi) $p(x) = x^2 – 3x$.
Then, $p(0) = 0^2 – 3(0) = 0$
$p(3) = (3^2) – 3(3) = 9 – 9 = 0$
⇒ 0 and 3 are the zeroes of the polynomial p(x).

(vii) $p(x) = x^2 + x – 6$
Then, $p(2) = 2^2 + 2 – 6$
= 4 + 2 – 6
= 6 – 6 = 0
⇒ 2 is a zero of the polynomial p(x).
Also, $p(-3) = (-3)^2 – 3 – 6$
= 9 – 3 – 6 = 0
⇒ -3 is a zero of the polynomial p(x).
Hence, 2 and -3 are the zeroes of the polynomial p(x).

Exercise 2C

Let p(x) be a polynomial of degree greater than or equal to one and 'a' be a real number. If p(x) is divided by (x - a), then the remainder is equal to p(a).
p(x) = (x - a) q(x) + r(x)

Proof:

Divide p(x) by (x - a), and let q(x) be the quotient and r(x) be the remainder, where r(x) = 0 or degree of r(x) < degree of (x - a).
But degree of (x - a) is 1,
∴ degree of r(x) = 0
Let r(x) = r, then p(x) = (x - a) q(x) + r
Substituting x = a, we have
p(a) = (a - a) q(a) + r
⇒ p(a) = 0 × q(a) + r
⇒ p(a) = 0 + r
⇒ p(a) = r
Thus remainder is p(a) when p(x) is divided by (x - a)

To divide: $(8x^2 + 4x - 2) \div (4x - 2)$

$$\begin{array}{r|l} & 2x + 2 \\ \hline 4x - 2 & 8x^2 + 4x + 2 \\ & 8x^2 - 4x \\ & - \quad + \\ \hline & 8x - 2 \\ & 8x - 4 \\ & - \quad + \\ \hline & 2 \end{array}$$

Using the remainder theorem
p(x) = (x - a) q(x) + r(x),
$p(x) = 8x^2 + 4x - 2$ and $x = \frac{2}{4} = \frac{1}{2}$

$p\left(\frac{1}{2}\right) = 8 \times \left(\frac{1}{2}\right)^2 + 4 \times \frac{1}{2} - 2$

$= 8 \times \frac{1}{4} + 2 - 2$

$= 2 + 0 = 2$

Question 1:

f(x) = $x^3 - 6x^2 + 9x + 3$

Now, x – 1 = 0 ⇒ x = 1

By the remainder theorem, we know that when f(x) is divided by (x – 1) the remainder is f(1).

Now, f(1) = $1^3 - 6 \times 1^2 + 9 \times 1 + 3$

= 1 – 6 + 9 + 3

= 13 – 6 = 7

∴ The required remainder is 7.

Question 2:

f(x) = $(2x^3 - 5x^2 + 9x - 8)$

Now, x – 3 = 0 ⇒ x = 3

By the remainder theorem, we know that when f(x) is divided by (x – 3) the remainder is f(3).

Now, f(3) = $2 \times 3^3 - 5 \times 3^2 + 9 \times 3 - 8$

= 54 – 45 + 27 – 8

= 81 – 53 = 28

∴ The required remainder is 28.

Question 3:

f(x) = $(3x^4 - 6x^2 - 8x + 2)$

Now, x – 2 = 0 ⇒ x = 2

By the remainder theorem, we know that when f(x) is divided by (x – 2) the remainder is f(2).

Now, f(2) = $3 \times 2^4 - 6 \times 2^2 - 8 \times 2 + 2$

= 48 – 24 – 16 + 2

= 50 – 40 = 10

∴ The required remainder is 10.

Question 4:

f(x) = $x^3 - 7x^2 + 6x + 4$

Now, x – 6 = 0 ⇒ x = 6

By the remainder theorem, we know that when f(x) is divide by (x – 6) the remainder is f(6)

Now, f(6) = $6^3 - 7 \times 6^2 + 6 \times 6 + 4$

= 216 – 252 + 36 + 4

= 256 – 252 = 4

∴ The required remainder is 4.

Question 5:

$f(x) = (x^3 - 6x^2 + 13x + 60)$

Now, $x + 2 = 0 \Rightarrow x = -2$

By the remainder the theorem, we know that when f(x) is divide by (x + 2) the remainder is f(-2).

Now, $f(-2) = (-2)^3 - 6(-2)^2 + 13(-2) + 60$

= -8 – 24 – 26 + 60

= -58 + 60 = 2

∴ The required remainder is 2.

Question 6:

$f(x) = (2x^4 + 6x^3 + 2x^2 + x - 8)$

Now, $x + 3 = 0 \Rightarrow x = -3$

By the remainder the theorem, we know that when f(x) is divide by (x + 3) the remainder is f(-3).

$f(-3) = 2(-3)^4 + 6(-3)^3 + 2(-3)^2 - 3 - 8$

= 162 – 162 + 18 – 3 – 8

= 18 – 11 = 7

∴ The required remainder is 7.

Question 7:

$f(x) = (4x^3 - 12x^2 + 11x - 5)$

Now, $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

By the remainder theorem, we know that when f(x) is divided by (2x – 1) the remainder is $f\left(\frac{1}{2}\right)$

Now, $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$

$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + \frac{11}{2} - 5$

$= \frac{1}{2} - 3 + \frac{11}{2} - 5$

$= \frac{1 - 6 + 11 - 10}{2}$

$= \frac{-16 + 12}{2}$

$= \frac{-4}{2} = -2$

∴ The required remainder is -2.

Question 8:

$f(x) = (81x^4 + 54x^3 - 9x^2 - 3x + 2)$

Now, $3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

By the remainder theorem, we know that when f(x) is divided by (3x+ 2) the remainder is $f\left(\frac{-2}{3}\right)$

Now, $f\left(\frac{-2}{3}\right) = 81\left(\frac{-2}{3}\right)^4 + 54\left(\frac{-2}{3}\right)^3 - 9\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) + 2$

$= 81 \times \frac{16}{81} + 54\left(\frac{-8}{27}\right) - 9\left(\frac{4}{9}\right) + 2 + 2$

$= 16 - 16 - 4 + 4 = 0$

∴ The required remainder is 0.

Question 9:

$f(x) = (x^3 - ax^2 + 2x - a)$

Now, $x - a = 0$ x ⇒ $= a$

By the remainder theorem, we know that when f(x) is divided by (x – a) the remainder is f(a)

Now, $f(a) = a^3 - a\,a^2 + 2\,a - a$

$= a^3 - a^3 + 2a - a$

$= a$

∴ The required remainder is a.

Question 10:

Let $f(x) = ax^3 + 3x^2 - 3$

and $g(x) = 2x^3 - 5x + a$

∴ $f(4) = a \times 4^3 + 3 \times 4^2 - 3$

$= 64a + 48 - 3$

$= 64a + 45$

$g(4) = 2 \times 4^3 - 5 \times 4 + a$

$= 128 - 20 + a$

$= 108 + a$

It is given that:

$f(4) = g(4)$

⇒ $64a + 45 = 108 + a$

⇒ $64a - a = 108 - 45$

⇒ $63a = 63$

⇒ $a = \frac{63}{63} = 1$

∴ The value of a is 1.

Question 11:

Let $f(x) = (x^4 - 2x^3 + 3x^2 - ax + b)$

∴ From the given information,

$f(1) = 1^4 - 2(1)^3 + 3(1)^2 - a\,(1\,) + b = 5$

⇒ $1 - 2 + 3 - a + b = 5$

⇒ $2 - a + b = 5$ ....(i)

And,

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

⇒ $1 + 2 + 3 + a + b = 19$

⇒ $6 + a + b = 19$ ....(ii)

Adding (i) and (ii), we get

⇒ $8 + 2b = 24$

⇒ $2b = 24 - 8 = 16$

⇒ $b = \frac{16}{2}$

Substituting the value of b = 8 in (i), we get

$2 - a + 8 = 5$

⇒ $-a + 10 = 5$

⇒ $-a = -10 + 5$

⇒ $-a = -5$

⇒ $a = 5$

∴ a = 5 and b = 8

$f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

$= x^4 - 2x^3 + 3x^2 - 5x + 8$

$\therefore f(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8$

$= 20 - 10 = 10$

$\therefore$ The required remainder is 10.

Exercise 2D

Factor Theorem

Let $p(x)$ be a polynomial of degree greater than or equal to one and 'a' be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.
i.e. $(x - a)$ *is a factor of* $p(x)$, *if* $p(a) = 0$

Proof:

$p(x)$ is a polynomial of degree greater than or equal to one and 'a' is a real number such that $p(a) = 0$.
To prove : $(x - a)$ is a factor of p(x)
Divide p(x) by $(x - a)$, and let q(x) be the quotient.
By Remainder theorem, p(x) when divided by $(x - a)$ gives remainder p(a).

$\therefore\ p(x) = (x - a)\, q(x) + p(a)$

$\Rightarrow p(x) = (x - a)\, q(x)\ [\because p(a) = 0]$

$\Rightarrow (x - a)$ is a factor of p(x)

Find if $(x + 1)$ and $(2x - 4)$ are factors of $2x^3 - 9x^2 + x + 12 = p(x)$

Using Factor theorem:

(i) $p(-1) = 2(-1)^3 - 9(-1)^2 + 1(-1) + 12$
$= -2 - 9 - 1 + 12 = 0$

(ii) $p(\frac{4}{2}) = 2(2)^3 - 9(2)^2 + 1(2) + 12$
$= 16 - 36 + 2 + 12 = -6$

Since (i) = 0, $(x + 1)$ is a factor and (ii) $\neq 0$, $(2x - 4)$ is not a factor of $2x^3 - 9x^2 + x + 12$.

Question 1:

$f(x) = (x^3 - 8)$

By the Factor Theorem, (x – 2) will be a factor of f(x) if f(2) = 0.

Here, $f(2) = (2)^3 - 8$

$= 8 - 8 = 0$

$\therefore (x - 2)$ is a factor of $(x^3 - 8)$.

Question 2:

$f(x) = (2x^3 + 7x^2 - 24x - 45)$

By the Factor Theorem, (x – 3) will be a factor of f(x) if f(3) = 0.

Here, $f(3) = 2 \times 3^3 + 7 \times 3^2 - 24 \times 3 - 45$

$= 54 + 63 - 72 - 45$

$= 117 - 117 = 0$

$\therefore (x - 3)$ is a factor of $(2x^3 + 7x^2 - 24x - 45)$.

Question 3:

$f(x) = (2x^4 + 9x^3 + 6x^2 - 11x - 6)$

By the Factor Theorem, (x – 1) will be a factor of f(x) if f(1) = 0.

Here, $f(1) = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6$

$= 2 + 9 + 6 - 11 - 6$

$= 17 - 17 = 0$

$\therefore (x - 1)$ is factor of $(2x^4 + 9x^3 + 6x^2 - 11x - 6)$.

Question 4:

$f(x) = (x^4 - x^2 - 12)$

By the Factor Theorem, (x + 2) will be a factor of f(x) if f(-2) = 0.

Here, $f(-2) = (-2)^4 - (-2)^2 - 12$

$= 16 - 4 - 12$

$= 16 - 16 = 0$

∴ (x + 2) is a factor of ($x^4 - x^2 - 12$).

Question 5:

$f(x) = 2x^3 + 9x^2 - 11x - 30$

By the Factor Theorem, (x + 5) will be a factor of f(x) if f(-5) = 0.

Here, $f(-5) = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30$

$= -250 + 225 + 55 - 30$

$= -280 + 280 = 0$

∴ (x + 5) is a factor of ($2x^3 + 9x^2 - 11x - 30$).

Question 6:

$f(x) = (2x^4 + x^3 - 8x^2 - x + 6)$

By the Factor Theorem, (x – a) will be a factor of f(x) if f(a) = 0.

Here, $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

$$\therefore f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$$

$$= 2 \times \frac{81}{16} + \frac{27}{8} - 8 \times \frac{9}{4} - \frac{3}{2} + 6$$

$$= \frac{81}{8} + \frac{27}{8} - 18 - \frac{3}{2} + 6$$

$$= \frac{81 + 27 - 144 - 12 + 48}{8}$$

$$= \frac{156 - 156}{8} = 0$$

∴ (2x – 3) is a factor of ($2x^4 + x^3 - 8x^2 - x + 6$).

Question 7:

$f(x) = (7x^2 - 4\sqrt{2}x - 6 = 0)$

By the Factor Theorem, (x – a) will be a factor of f(x) if f(a) = 0.

Here, $f(\sqrt{2}) = 7(\sqrt{2})^2 - 4\sqrt{2} \times \sqrt{2} - 6$

$= 14 - 8 - 6$

$= 14 - 14 = 0$

∴ $(x - \sqrt{2})$ is a factor of ($7 - 4\sqrt{2}x - 6 = 0$).

Question 8:

$f(x) = (4\sqrt{2}x^2 + 5x + \sqrt{2} = 0)$

By the Factor Theorem, (x – a) will be a factor of f(x) if f(a) = 0.

Here, $f(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$

$= 2\sqrt{2} \times 2 - 5\sqrt{2} + \sqrt{2}$

$= 4\sqrt{2} - 5\sqrt{2} + \sqrt{2}$

$= 5\sqrt{2} - 5\sqrt{2} = 0.$

∴ $(x + \sqrt{2})$ is a factor of ($4\sqrt{2}x^2 + 5x + \sqrt{2} = 0$).

Question 9:

$f(x) = (2x^3 + 9x^2 + x + k)$

$x - 1 = 0 \Rightarrow x = 1$

$\therefore f(1) = 2 \times 1^3 + 9 \times 1^2 + 1 + k$

$= 2 + 9 + 1 + k$

$= 12 + k$

Given that (x – 1) is a factor of f(x).

By the Factor Theorem, (x – a) will be a factor of f(x) if f(a) = 0 and therefore f(1) = 0.

$\Rightarrow f(1) = 12 + k = 0$

$\Rightarrow k = -12.$

Question 10:

$f(x) = (2x^3 - 3x^2 - 18x + a)$

$x - 4 = 0 \Rightarrow x = 4$

$\therefore f(4) = 2(4)^3 - 3(4)^2 - 18 \times 4 + a$

$= 128 - 48 - 72 + a$

$= 128 - 120 + a$

$= 8 + a$

Given that $(x - 4)$ is a factor of $f(x)$.

By the Factor Theorem, $(x - a)$ will be a factor of $f(x)$ if $f(a) = 0$ and therefore $f(4) = 0$.

$\Rightarrow f(4) = 8 + a = 0$

$\Rightarrow a = -8$

Question 11:

$f(x) = x^4 - x^3 - 11x^2 - x + a$

$x + 3 = 0 \Rightarrow x = -3$

$\therefore f(-3) = (-3)^4 - (-3)^3 - 11(-3)^2 - (-3) + a$

$= 81 + 27 - 11 \times 9 + 3 + a$

$= 81 + 27 - 99 + 3 + a$

$= 111 - 99 + a$

$= 12 + a$

Given that $f(x)$ is divisible by $(x + 3)$, that is $(x+3)$ is a factor of $f(x)$.

By the Factor Theorem, $(x - a)$ will be a factor of $f(x)$ if $f(a) = 0$ and therefore $f(-3) = 0$.

$\Rightarrow f(-3) = 12 + a = 0$

$\Rightarrow a = -12$.

Question 12:

$f(x) = (2x^3 + ax^2 + 11x + a + 3)$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

Given that $f(x)$ is exactly divisible by $(2x - 1)$, that is $(2x - 1)$ is a factor of $f(x)$.

By the Factor Theorem, $(x - a)$ will be a factor of $f(x)$ if $f(a) = 0$

and therefore $f\left(\frac{1}{2}\right) = 0$.

Therefore, we have

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} + a + 3 = 0$$

$$\Rightarrow 2 \times \frac{1}{8} + a \times \frac{1}{4} + \frac{11}{2} + a + 3 = 0$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4}a + \frac{11}{2} + a + 3 = 0$$

$$\Rightarrow \frac{1 + a + 22 + 4a + 12}{4} = 0$$

$$\Rightarrow \frac{5a + 35}{4} = 0$$

$$\Rightarrow 5a + 35 = 0$$

$$\Rightarrow 5a = -35$$

$$\Rightarrow a = \frac{-35}{5} = -7$$

$\therefore$ The value of $a = -7$.

Question 13:

Let $f(x) = (x^3 - 10x^2 + ax + b)$, then by factor theorem

$(x - 1)$ and $(x - 2)$ will be factors of $f(x)$ if $f(1) = 0$ and $f(2) = 0$.

$f(1) = 1^3 - 10 \_\_ 1^2 + a \_\_ 1 + b = 0$

$\Rightarrow 1 - 10 + a + b = 0$

$\Rightarrow a + b = 9$ ....(i)

And $f(2) = 2^3 - 10 \_\_ 2^2 + a \_\_ 2 + b = 0$

$\Rightarrow 8 - 40 + 2a + b = 0$

$\Rightarrow 2a + b = 32$ ....(ii)

Subtracting (i) from (ii), we get

$a = 23$

Substituting the value of $a = 23$ in (i), we get

$\Rightarrow 23 + b = 9$

$\Rightarrow b = 9 - 23$

$\Rightarrow b = -14$

$\therefore a = 23$ and $b = -14$.

Question 14:

Let $f(x) = (x^4 + ax^3 - 7x^2 - 8x + b)$

Now, $x + 2 = 0$ $x = -2$ and $x + 3 = 0$ $x = -3$

By factor theorem, $(x + 2)$ and $(x + 3)$ will be factors of $f(x)$ if $f(-2) = 0$ and $f(-3) = 0$

$\therefore f(-2) = (-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b = 0$

$\Rightarrow 16 - 8a - 28 + 16 + b = 0$

$\Rightarrow -8a + b = -4$

$\Rightarrow 8a - b = 4$ ....(i)

And, $f(-3) = (-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b = 0$

$\Rightarrow 81 - 27a - 63 + 24 + b = 0$

$\Rightarrow -27a + b = -42$

$\Rightarrow 27a - b = 42$ ....(ii)

Subtracting (i) from (ii), we get,

$19a = 38$

So, $a = 2$

Substituting the value of $a = 2$ in (i), we get

$8(2) - b = 4$

$\Rightarrow 16 - b = 4$

$\Rightarrow -b = -16 + 4$

$\Rightarrow -b = -12$

$\Rightarrow b = 12$

$\therefore a = 2$ and $b = 12$.

Question 15:

Let $f(x) = x^3 - 3x^2 - 13x + 15$

Now, $x^2 + 2x - 3 = x^2 + 3x - x - 3$

$= x(x + 3) - 1(x + 3)$

$= (x + 3)(x - 1)$

Thus, $f(x)$ will be exactly divisible by $x^2 + 2x - 3 = (x + 3)(x - 1)$ if $(x + 3)$ and $(x - 1)$ are both factors of $f(x)$, so by factor theorem, we should have $f(-3) = 0$ and $f(1) = 0$.

Now, $f(-3) = (-3)^3 - 3(-3)^2 - 13(-3) + 15$

$= -27 - 3 \times 9 + 39 + 15$

$= -27 - 27 + 39 + 15$

$= -54 + 54 = 0$

And, $f(1) = 1^3 - 3 \times 1^2 - 13 \times 1 + 15$

$= 1 - 3 - 13 + 15$

$= 16 - 16 = 0$

$\therefore f(-3) = 0$ and $f(1) = 0$

So, $x^2 + 2x - 3$ divides $f(x)$ exactly.

Question 16:

Let $f(x) = (x^3 + ax^2 + bx + 6)$

Now, by remainder theorem, f(x) when divided by (x – 3) will leave a remainder as f(3).

So, $f(3) = 3^3 + a \times 3^2 + b \times 3 + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\Rightarrow 9a + 3b + 33 = 3$

$\Rightarrow 9a + 3b = 3 - 33$

$\Rightarrow 9a + 3b = -30$

$\Rightarrow 3a + b = -10$ ....(i)

Given that (x – 2) is a factor of f(x).

By the Factor Theorem, (x – a) will be a factor of f(x) if f(a) = 0 and therefore f(2) = 0.

$f(2) = 2^3 + a \times 2^2 + b \times 2 + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\Rightarrow 4a + 2b = -14$

$\Rightarrow 2a + b = -7$ ....(ii)

Subtracting (ii) from (i), we get,

$\Rightarrow a = -3$

Substituting the value of a = -3 in (i), we get,

$\Rightarrow 3(-3) + b = -10$

$\Rightarrow -9 + b = -10$

$\Rightarrow b = -10 + 9$

$\Rightarrow b = -1$

$\therefore a = -3$ and $b = -1$.

Exercise 2E

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Question 1:

$9x^2 + 12xy = 3x(3x + 4y)$

Question 2:

$18x^2y - 24xyz = 6xy(3x - 4z)$

Question 3:

$27a^3b^3 - 45a^4b^2 = 9a^3b^2(3b - 5a)$

Question 4:

$2a(x+y) - 3b(x+y) = (x+y)(2a-3b)$

Question 5:

$2x(p^2+q^2) + 4y(p^2+q^2)$

$= (2x+4y)(p^2+q^2)$

$= 2(x+2y)(p^2+q^2)$

Question 6:

$x(a-5) + y(5-a)$

$= x(a-5) + y(-1)(a-5)$

$= (x-y)(a-5)$

Question 7:

$4(a+b) - 6(a+b)^2$

$= (a+b)[4 - 6(a+b)]$

$= 2(a+b)(2-3a-3b)$

$= 2(a+b)(2-3a-3b)$

Question 8:

$8(3a-2b)^2 - 10(3a-2b)$

$= (3a-2b)[8(3a-2b) - 10]$

$= (3a-2b)\,2[4(3a-2b) - 5]$

$= 2(3a-2b)(12a - 8b - 5)$

Question 9:

$x(x+y)^3 - 3x^2y(x+y)$

$= x(x+y)[(x+y)^2 - 3xy]$

$= x(x+y)(x^2+y^2+2xy-3xy)$

$= x(x+y)(x^2+y^2-xy)$

Question 10:

$x^3 + 2x^2 + 5x + 10$

$= x^2(x+2) + 5(x+2)$

$= (x^2+5)(x+2)$

Question 11:

$x^2 + xy - 2xz - 2yz$

$= x(x+y) - 2z(x+y)$

$= (x+y)(x-2z)$

Question 12:

$a^3b - a^2b + 5ab - 5b$

$= a^2b(a-1) + 5b(a-1)$

$= (a-1)(a^2b+5b)$

$= (a-1)\,b(a^2+5)$

$= b(a-1)(a^2+5)$

Question 13:

$8 - 4a - 2a^3 + a^4$

$= 4(2-a) - a^3(2-a)$

$= (2-a)(4-a^3)$

Question 14:

$x^3 - 2x^2y + 3xy^2 - 6y^3$

$= x^2 (x - 2y) + 3y^2 (x - 2y)$

$= (x - 2y) (x^2 + 3y^2)$

Question 15:

$px + pq - 5q - 5x$

$= p(x + q) - 5 (q + x)$

$= (x + q) (p - 5)$

Question 16:

$x^2 - xy + y - x$

$= x (x - y) - 1 (x - y)$

$= (x - y) (x - 1)$

Question 17:

$(3a - 1)^2 - 6a + 2$

$= (3a - 1)^2 - 2 (3a - 1)$

$= (3a - 1) [(3a - 1) - 2]$

$= (3a - 1) (3a - 3)$

$= 3(3a - 1) (a - 1)$

Question 18:

$(2x - 3)^2 - 8x + 12$

$= (2x - 3)^2 - 4 (2x - 3)$

$= (2x - 3) (2x - 3 - 4)$

$= (2x - 3) (2x - 7)$

Question 19:

$a^3 + a - 3a^2 - 3$

$= a(a^2 + 1) - 3 (a^2 + 1)$

$= (a - 3) (a^2 + 1)$

Question 20:

$3ax - 6ay - 8by + 4bx$

$= 3a (x - 2y) + 4b (x - 2y)$

$= (x - 2y) (3a + 4b)$

Question 21:

$abx^2 + a^2x + b^2x + ab$

$= ax (bx + a) + b (bx + a)$

$= (bx + a) (ax + b)$

Question 22:

$x^3 - x^2 + ax + x - a - 1$

$= x^3 - x^2 + ax - a + x - 1$

$= x^2 (x - 1) + a (x - 1) + 1 (x - 1)$

$= (x - 1) (x^2 + a + 1)$

Question 23:

$2x + 4y - 8xy - 1$

$= 2x - 1 - 8xy + 4y$

= (2x – 1) – 4y (2x – 1)
= (2x – 1) (1 – 4y)

Question 24:

$ab(x^2 + y^2) - xy(a^2 + b^2)$
$= abx^2 + aby^2 - a^2xy - b^2xy$
$= abx^2 - a^2xy + aby^2 - b^2xy$
$= ax(bx - ay) + by(ay - bx)$
$= (bx - ay)(ax - by)$

Question 25:

$a^2 + ab(b + 1) + b^3$
$= a^2 + ab^2 + ab + b^3$
$= a^2 + ab + ab^2 + b^3$
$= a(a + b) + b^2(a + b)$
$= (a + b)(a + b^2)$

Question 26:

$a^3 + ab(1 - 2a) - 2b^2$
$= a^3 + ab - 2a^2b - 2b^2$
$= a(a^2 + b) - 2b(a^2 + b)$
$= (a^2 + b)(a - 2b)$

Question 27:

$2a^2 + bc - 2ab - ac$
$= 2a^2 - 2ab - ac + bc$
$= 2a(a - b) - c(a - b)$
$= (a - b)(2a - c)$

Question 28:

$(ax + by)^2 + (bx - ay)^2$
$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$
$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
$= a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2$
$= x^2(a^2 + b^2) + y^2(a^2 + b^2)$
$= (a^2 + b^2)(x^2 + y^2)$

Question 29:

$a(a + b - c) - bc$
$= a^2 + ab - ac - bc$
$= a(a + b) - c(a + b)$
$= (a - c)(a + b)$

Question 30:

$a(a - 2b - c) + 2bc$
$= a^2 - 2ab - ac + 2bc$
$= a(a - 2b) - c(a - 2b)$
$= (a - 2b)(a - c)$

Question 31:

$a^2x^2 + (ax^2 + 1)x + a$
$= a^2x^2 + ax^3 + x + a$

$= ax^2 (a + x) + 1 (x + a)$

$= (ax^2 + 1) (a + x)$

Question 32:

$ab (x^2 + 1) + x (a^2 + b^2)$

$= abx^2 + ab + a^2x + b^2x$

$= abx^2 + a^2x + ab + b^2x$

$= ax (bx + a) + b (bx + a)$

$= (bx + a) (ax + b)$

Question 33:

$x^2 - (a + b) x + ab$

$= x^2 - ax - bx + ab$

$= x (x - a) - b(x - a)$

$= (x - a) (x - b)$

Question 34:

$$x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$$

$$= \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$$

Exercise 2F

Question 1:

$25x^2 - 64y^2$

$= (5x)^2 - (8y)^2$

$= (5x + 8y) (5x - 8y)$

$\left[\because a^2 - b^2 = (a + b) (a - b)\right]$

Question 2:

$100 - 9x^2$

$= (10)^2 - (3x)^2$

$= (10 + 3x) (10 - 3x)$

$\left[\because a^2 - b^2 = (a + b) (a - b)\right]$

Question 3:

$5x^2 - 7y^2$

$= \left(\sqrt{5}x\right)^2 - \left(\sqrt{7}y\right)^2$

$= \left(\sqrt{5}x + \sqrt{7}y\right)\left(\sqrt{5}x - \sqrt{7}y\right) \qquad \left[\because a^2 - b^2 = (a + b) (a - b)\right]$

Question 4:

$(3x + 5y)^2 - 4z^2$

$= (3x + 5y)^2 - (2z)^2$

$= (3x + 5y + 2z) (3x + 5y - 2z)$

$\left[\because a^2 - b^2 = (a + b) (a - b)\right]$

Question 5:

$150 - 6x^2$

$= 6(25 - x^2)$

$= 6(5^2 - x^2)$

$= 6(5 + x)(5 - x)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 6:

$20x^2 - 45$

$= 5(4x^2 - 9)$

$= 5[(2x)^2 - (3)^2]$

$= 5(2x + 3)(2x - 3)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 7:

$3x^3 - 48x$

$= 3x(x^2 - 16)$

$= 3x[(x)^2 - (4)^2]$

$= 3x(x + 4)(x - 4)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 8:

$2 - 50x^2$

$= 2(1 - 25x^2)$

$= 2[(1)^2 - (5x)^2]$

$= 2(1 + 5x)(1 - 5x)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 9:

$27a^2 - 48b^2$

$= 3(9a^2 - 16b^2)$

$= 3[(3a)^2 - (4b)^2]$

$= 3(3a + 4b)(3a - 4b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 10:

$x - 64x^3$

$= x(1 - 64x^2)$

$= x[(1)^2 - (8x)^2]$

$= x(1 + 8x)(1 - 8x)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 11:

$8ab^2 - 18a^3$

$= 2a(4b^2 - 9a^2)$

$= 2a[(2b)^2 - (3a)^2]$

$= 2a(2b + 3a)(2b - 3a)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 12:

$3a^3b - 243ab^3$

$= 3ab\,(a^2 - 81\,b^2)$

$= 3ab\,[(a)^2 - (9b)^2]$

$= 3ab\,(a + 9b)\,(a - 9b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 13:

$(a + b)^3 - a - b$

$= (a + b)^3 - (a + b)$

$= (a + b)\,[(a + b)^2 - 1^2]$

$= (a + b)\,(a + b + 1)\,(a + b - 1)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 14:

$108a^2 - 3(b - c)^2$

$= 3\,[(36a^2 - (b - c)^2]$

$= 3\,[(6a)^2 - (b - c)^2]$

$= 3\,(6a + b - c)\,(6a - b + c)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 15:

$x^3 - 5x^2 - x + 5$

$= x^2\,(x - 5) - 1\,(x - 5)$

$= (x - 5)\,(x^2 - 1)$

$= (x - 5)\,(x + 1)\,(x - 1)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 16:

$a^2 + 2ab + b^2 - 9c^2$

$= (a + b)^2 - (3c)^2$

$= (a + b + 3c)\,(a + b - 3c)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 17:

$9 - a^2 + 2ab - b^2$

$= 9 - (a^2 - 2ab + b^2)$

$= 3^2 - (a - b)^2$

$= (3 + a - b)\,(3 - a + b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 18:

$a^2 - 4ac + 4c^2 - b^2$

$= a^2 - 4ac + 4c^2 - b^2$

$= a^2 - 2\,a\,2c + (2c)^2 - b^2$

$= (a - 2c)^2 - b^2$

$= (a - 2c + b)\,(a - 2c - b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 19:

$9a^2 + 3a - 8b - 64b^2$

$= 9a^2 - 64b^2 + 3a - 8b$

$= (3a)^2 - (8b)^2 + (3a - 8b)$

$= (3a + 8b)(3a - 8b) + (3a - 8b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (3a - 8b)(3a + 8b + 1)$

Question 20:

$x^2 - y^2 + 6y - 9$

$= x^2 - (y^2 - 6y + 9)$

$= x^2 - (y^2 - 2 \text{ y } 3 + 3^2)$

$= x^2 - (y - 3)^2$

$= [x + (y - 3)][x - (y - 3)]$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (x + y - 3)(x - y + 3)$

Question 21:

$4x^2 - 9y^2 - 2x - 3y$

$= (2x)^2 - (3y)^2 - (2x + 3y)$

$= (2x + 3y)(2x - 3y) - (2x + 3y)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (2x + 3y)(2x - 3y - 1)$

Question 22:

$x^4 - 1$

$= (x^2)^2 - 1^2$

$= (x^2 + 1)(x^2 - 1)$ $\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (x^2 + 1)(x + 1)(x - 1)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Question 23:

$a - b - a^2 + b^2$

$= (a - b) - (a^2 - b^2)$

$= (a - b) - (a - b)(a + b)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (a - b)(1 - a - b)$

Question 24:

$x^4 - 625$

$= (x^2)^2 - (25)^2$

$= (x^2 + 25)(x^2 - 25)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$= (x^2 + 25)(x^2 - 5^2)$

$= (x^2 + 25)(x + 5)(x - 5)$

$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$

Exercise 2G

Question 1:

$x^2 + 11x + 30$

$= x^2 + 6x + 5x + 30$

$= x(x + 6) + 5(x + 6)$

$= (x + 6)(x + 5)$.

Question 2:

$x^2 + 18x + 32$

$= x^2 + 16x + 2x + 32$

$= x(x + 16) + 2(x + 16)$

$= (x + 16)(x + 2)$.

Question 3:

$x^2 + 7x - 18$

$= x^2 + 9x - 2x - 18$

$= x(x + 9) - 2(x + 9)$

$= (x + 9)(x - 2)$.

Question 4:

$x^2 + 5x - 6$

$= x^2 + 6x - x - 6$

$= x(x + 6) - 1(x + 6)$

$= (x + 6)(x - 1)$.

Question 5:

$y^2 - 4y + 3$

$= y^2 - 3y - y + 3$

$= y(y - 3) - 1(y - 3)$

$= (y - 3)(y - 1)$.

Question 6:

$x^2 - 21x + 108$

$= x^2 - 12x - 9x + 108$

$= x(x - 12) - 9(x - 12)$

$= (x - 12)(x - 9)$.

Question 7:

$x^2 - 11x - 80$

$= x^2 - 16x + 5x - 80$

$= x(x - 16) + 5(x - 16)$

$= (x - 16)(x + 5)$.

Question 8:

$x^2 - x - 156$

$= x^2 - 13x + 12x - 156$

$= x(x - 13) + 12(x - 13)$

$= (x - 13)(x + 12)$.

Question 9:

$z^2 - 32z - 105$

$= z^2 - 35z + 3z - 105$
$= z(z - 35) + 3(z - 35)$
$= (z - 35)(z + 3)$

Question 10:

$40 + 3x - x^2$
$= 40 + 8x - 5x - x^2$
$= 8(5 + x) - x(5 + x)$
$= (5 + x)(8 - x)$.

Question 11:

$6 - x - x^2$
$= 6 + 2x - 3x - x^2$
$= 2(3 + x) - x(3 + x)$
$= (3 + x)(2 - x)$.

Question 12:

$7x^2 + 49x + 84$
$= 7(x^2 + 7x + 12)$
$= 7[x^2 + 4x + 3x + 12]$
$= 7[x(x + 4) + 3(x + 4)]$
$= 7(x + 4)(x + 3)$.

Question 13:

$m^2 + 17mn - 84n^2$
$= m^2 + 21mn - 4mn - 84n^2$
$= m(m + 21n) - 4n(m + 21n)$
$= (m + 21n)(m - 4n)$.

Question 14:

$5x^2 + 16x + 3$
$= 5x^2 + 15x + x + 3$
$= 5x(x + 3) + 1(x + 3)$
$= (5x + 1)(x + 3)$.

Question 15:

$6x^2 + 17x + 12$
$= 6x^2 + 9x + 8x + 12$
$= 3x(2x + 3) + 4(2x + 3)$
$= (2x + 3)(3x + 4)$.

Question 16:

$9x^2 + 18x + 8$
$= 9x^2 + 12x + 6x + 8$
$= 3x(3x + 4) + 2(3x + 4)$
$= (3x + 4)(3x + 2)$.

Question 17:

$14x^2 + 9x + 1$
$= 14x^2 + 7x + 2x + 1$
$= 7x(2x + 1) + (2x + 1)$
$= (7x + 1)(2x + 1)$.

Question 18:

$2x^2 + 3x - 90$

$= 2x^2 - 12x + 15x - 90$

$= 2x(x - 6) + 15(x - 6)$

$= (x - 6)(2x + 15)$.

Question 19:

$2x^2 + 11x - 21$

$= 2x^2 + 14x - 3x - 21$

$= 2x(x + 7) - 3(x + 7)$

$= (x + 7)(2x - 3)$.

Question 20:

$3x^2 - 14x + 8$

$= 3x^2 - 12x - 2x + 8$

$= 3x(x - 4) - 2(x - 4)$

$= (x - 4)(3x - 2)$.

Question 21:

$18x^2 + 3x - 10$

$= 18x^2 - 12x + 15x - 10$

$= 6x(3x - 2) + 5(3x - 2)$

$= (6x + 5)(3x - 2)$.

Question 22:

$15x^2 + 2x - 8$

$= 15x^2 - 10x + 12x - 8$

$= 5x(3x - 2) + 4(3x - 2)$

$= (3x - 2)(5x + 4)$.

Question 23:

$6x^2 + 11x - 10$

$= 6x^2 + 15x - 4x - 10$

$= 3x(2x + 5) - 2(2x + 5)$

$= (2x + 5)(3x - 2)$.

Question 24:

$30x^2 + 7x - 15$

$= 30x^2 - 18x + 25x - 15$

$= 6x(5x - 3) + 5(5x - 3)$

$= (5x - 3)(6x + 5)$.

Question 25:

$24x^2 - 41x + 12$

$= 24x^2 - 32x - 9x + 12$

$= 8x(3x - 4) - 3(3x - 4)$

$= (3x - 4)(8x - 3)$.

Question 26:

$2x^2 - 7x - 15$

$= 2x^2 - 10x + 3x - 15$

$= 2x(x - 5) + 3(x - 5)$

= (x – 5) (2x + 3).

Question 27:

$6x^2 – 5x – 21$

$= 6x^2 + 9x – 14x – 21$

= 3x (2x + 3) – 7 (2x + 3)

= (3x – 7) (2x + 3).

Question 28:

$10x^2 – 9x – 7$

$= 10x^2 + 5x – 14x – 7$

= 5x (2x + 1) – 7 (2x+ 1)

= (2x + 1) (5x – 7).

Question 29:

$5x^2 – 16x – 21$

$= 5x^2 + 5x – 21x – 21$

= 5x (x + 1) -21 (x + 1)

= (x + 1) (5x – 21).

Question 30:

$2x^2 – x – 21$

$= 2x^2 + 6x – 7x – 21$

= 2x (x + 3) – 7 (x + 3)

= (x + 3) (2x – 7).

Question 31:

$15x^2 – x – 28$

$= 15x^2 + 20x – 21x – 28$

= 5x (3x + 4) – 7 (3x + 4)

= (3x + 4) (5x – 7).

Question 32:

$8a^2 – 27ab + 9b^2$

$= 8a^2 – 24ab – 3ab + 9b^2$

= 8a (a – 3b) – 3b (a – 3b)

= (a – 3b) (8a – 3b).

Question 33:

$5x^2 + 33xy – 14y^2$

$= 5x^2 + 35xy – 2xy – 14y^2$

= 5x (x + 7y) – 2y (x + 7y)

= (x + 7y) (5x – 2y).

Question 34:

$3x^3 – x^2 – 10x$

$= x (3x^2 – x – 10)$

$= x [3x^2 – 6x + 5x – 10]$

= x [3x (x – 2) + 5 (x – 2)]

= x (x – 2) (3x + 5).

Question 35:

$\frac{1}{3}x^2 - 2x - 9$

$= \frac{1}{3}x^2 - 3x + x - 9$
$= x\left(\frac{x}{3} - 3\right) + (x - 9)$
$= \frac{x}{3}(x - 9) + (x - 9)$
$= (x - 9)\left(\frac{x}{3} + 1\right)$
$= (x - 9)\frac{(x + 3)}{3} = \frac{1}{3}(x - 9)(x + 3).$

Question 36:

$x^2 - 2x + \frac{7}{16}$

$= \frac{1}{16}\left(16x^2 - 32x + 7\right)$
$= \frac{1}{16}\left(16x^2 - 4x - 28x + 7\right)$
$= \frac{1}{16}\left[4x(4x - 1) - 7(4x - 1)\right]$
$= \frac{1}{16}(4x - 1)(4x - 7).$

Question 37:

$\sqrt{2}x^2 + 3x + \sqrt{2}$

$= \sqrt{2}x^2 + x + 2x + \sqrt{2}$
$= x\left(\sqrt{2}x + 1\right) + \sqrt{2}\left(\sqrt{2}x + 1\right)$
$= \left(\sqrt{2}x + 1\right)(x + \sqrt{2}).$

Question 38:

$\sqrt{5}x^2 + 2x - 3\sqrt{5}$

$= \sqrt{5}x^2 + 5x - 3x - 3\sqrt{5}$
$= \sqrt{5}x(x + \sqrt{5}) - 3(x + \sqrt{5})$
$= (\sqrt{5}x - 3)(x + \sqrt{5}).$

Question 39:

$2x^2 + 3\sqrt{3}x + 3$

$= 2x^2 + 2\sqrt{3}x + \sqrt{3}x + 3$
$= 2x(x + \sqrt{3}) + \sqrt{3}(x + \sqrt{3})$
$= (x + \sqrt{3})(2x + \sqrt{3}).$

Question 40:

$2\sqrt{3}x^2 + x - 5\sqrt{3}$

$= 2\sqrt{3}x^2 + 6x - 5x - 5\sqrt{3}$
$= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3})$
$= (x + \sqrt{3})(2\sqrt{3}x - 5).$

Question 41:

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$
$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$
$= (\sqrt{5}x + 3)(5x + \sqrt{5}).$

Question 42:

$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

$= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2}$
$= 7\sqrt{2}x(x - \sqrt{2}) + 4(x - \sqrt{2})$
$= (x - \sqrt{2})(7\sqrt{2}x + 4)$.

Question 43:

$6\sqrt{3}x^2 - 47x + 5\sqrt{3}$

$= 6\sqrt{3}x^2 - 45x - 2x + 5\sqrt{3}$
$= 3\sqrt{3}x(2x - 5\sqrt{3}) - 1(2x - 5\sqrt{3})$
$= (2x - 5\sqrt{3})(3\sqrt{3}x - 1)$.

Question 44:

$7x^2 + 2\sqrt{14}x + 2$

$= 7x^2 + \sqrt{2}\sqrt{7}x + \sqrt{2}\sqrt{7}x + 2$
$= \sqrt{7}x(\sqrt{7}x + \sqrt{2}) + \sqrt{2}(\sqrt{7}x + \sqrt{2})$
$= (\sqrt{7}x + \sqrt{2})(\sqrt{7}x + \sqrt{2}) = (\sqrt{7}x + \sqrt{2})^2$.

Question 45:

Let $x + y = z$

Then, $2(x + y)^2 - 9(x + y) - 5$

$= 2z^2 - 9z - 5$
$= 2z^2 - 10z + z - 5$
$= 2z(z - 5) + 1(z - 5)$
$= (z - 5)(2z + 1)$

Now, replacing z by $(x + y)$, we get

$2(x + y)^2 - 9(x + y) - 5$

$= [(x + y) - 5][(2(x + y) + 1)]$
$= (x + y - 5)(2x + 2y + 1)$.

Question 46:

Let $2a - b = c$

Then, $9(2a - b)^2 - 4(2a - b) - 13$

$= 9c^2 - 4c - 13$
$= 9c^2 - 13c + 9c - 13$
$= c(9c - 13) + 1(9c - 13)$
$= (c + 1)(9c - 13)$

Now, replacing c by $(2a - b)$, we get

$9(2a - b)^2 - 4(2a - b) - 13$

$= (2a - b + 1)[9(2a - b) - 13]$
$= (2a - b + 1)(18a - 9b - 13)$

Question 47:

Let $x - 2y = z$

Then, $7(x - 2y)^2 - 25(x - 2y) + 12$

$= 7z^2 - 25z + 12$
$= 7z^2 - 21z - 4z + 12$
$= 7z(z - 3) - 4(z - 3)$
$= (z - 3)(7z - 4)$

Now replace z by $(x - 2y)$, we get

$7(x - 2y)^2 - 25(x - 2y) + 12$

$= (x - 2y - 3)[7 (x - 2y) - 4]$
$= (x - 2y - 3)(7x - 14y - 4).$

Question 48:

Let $x^2 = y$

Then, $4x^4 + 7x^2 - 2$

$= 4y^2 + 7y - 2$
$= 4y^2 + 8y - y - 2$
$= 4y(y + 2) - 1(y + 2)$
$= (y + 2)(4y - 1)$

Now replacing y by x^2, we get

$4x^4 + 7x^2 - 2$

$= (x^2 + 2)(4x^2 - 1)$ $[\because a^2 - b^2 = (a - b)(a + b)]$
$= (x^2 + 2)(2x + 1)(2x - 1).$

Exercise 2H

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Question 1:

We know:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(a + 2b + 5c)^2$

$= (a)^2 + (2b)^2 + (5c)^2 + 2(a)(2b) + 2(2b)(5c) + 2(5c)(a)$

$= a^2 + 4b^2 + 25c^2 + 4ab + 20bc + 10ac$

(ii) $(2a - b + c)^2$

$= (2a)^2 + (-b)^2 + (c)^2 + 2(2a)(-b) + 2(-b)(c) + 2(c)(2a)$

$= 4a^2 + b^2 + c^2 - 4ab - 2bc + 4ac.$

(iii) $(a - 2b - 3c)^2$

$= (a)^2 + (-2b)^2 + (-3c)^2 + 2(a)(-2b) + 2(-2b)(-3c) + 2(-3c)(a)$

$= a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ac.$

Question 2:

We know:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(2a - 5b - 7c)^2$

$= (2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(-7c) + 2(-7c)(2a)$

$= 4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ac.$

(ii) $(-3a + 4b - 5c)^2$

$= (-3a)^2 + (4b)^2 + (-5c)^2 + 2(-3a)(4b) + 2(4b)(-5c) + 2(-5c)(-3a)$

$= 9a^2 + 16b^2 + 25c^2 - 24ab - 40bc + 30ac.$

(iii) $\left(\frac{1}{2}a - \frac{1}{4}b + 2\right)^2$

$= \left(\frac{1}{2}a\right)^2 + \left(-\frac{1}{4}b\right)^2 + (2)^2 + 2\left(\frac{1}{2}a\right)\left(-\frac{1}{4}b\right) + 2\left(-\frac{1}{4}b\right)(2) + 2(2)\left(\frac{1}{2}a\right) = \frac{a^2}{4} + \frac{b^2}{16} + 4 - \frac{ab}{4} - b + 2a$

Question 3:

$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

Question 4:

$9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$

$= (-3x)^2 + (4y)^2 + (2z)^2 + 2(-3x)(4y) + 2(4y)(2z) + 2(2z)(-3x)$

$= (-3x + 4y + 2z)^2.$

Question 5:

$25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$

$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$

$= (5x - 2y + 3z)^2$

Question 6:

(i) $(99)^2$

$= (100 - 1)^2$

$\left[\because (a - b)^2 = a^2 - 2ab + b^2\right]$

$= (100)^2 - 2(100)(1) + (1)^2$

$= 10000 - 200 + 1$

$= 9801.$

(ii) $(998)^2$

$= (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$

$= 1000000 - 4000 + 4$

$= 996004.$

Exercise 2I

1. $(a+b)^2 = a^2 + 2ab + b^2 = (-a-b)^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a-b)(a+b) = a^2 - b^2$
4. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a+b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
10. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
11. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
    $= (a+b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
    $= (a-b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Question 1:

(i) $(3x+2)^3$

$= (3x)^3 + (2)^3 + 3 \times 3x \times 2(3x+2)$

$\left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$

$= 27x^3 + 8 + 18x(3x+2)$

$= 27x^3 + 8 + 54x^2 + 36x.$

(ii) $(3a-2b)^3$

$= (3a)^3 - (2b)^3 - 3 \times 3a \times 2b(3a-2b)$

$\left[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right]$

$= 27a^3 - 8b^3 - 18ab(3a-2b)$

$= 27a^3 - 8b^3 - 54a^2b + 36ab^2.$

(iii) $\left(\frac{2}{3}x + 1\right)^3$

$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3 \times \frac{2}{3}x \times 1\left(\frac{2}{3}x + 1\right)$

$\left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$

$= \frac{8}{27}x^3 + 1 + 2x\left(\frac{2}{3}x + 1\right)$

$= \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x.$

Question 2:

(i)

$$\left(2x - \frac{2}{x}\right)^3$$

$$= (2x)^3 - \left(\frac{2}{x}\right)^3 - 3 \times 2x \times \frac{2}{x}\left(2x - \frac{2}{x}\right)$$

$$\left[\because (a-b)^3 = a^3 - b^3 - 3ab\,(a-b)\right]$$

$$= 8x^3 - \frac{8}{x^3} - 12\left(2x - \frac{2}{x}\right)$$

$$= 8x^3 - \frac{8}{x^3} - 24x + \frac{24}{x}.$$

(ii)

$$\left(3a + \frac{1}{4b}\right)^3$$

$$= (3a)^3 + \left(\frac{1}{4b}\right)^3 + 3 \times 3a \times \frac{1}{4b}\left(3a + \frac{1}{4b}\right)$$

$$\left[\because (a+b)^3 = a^3 + b^3 + 3ab\,(a+b)\right]$$

$$= 27a^3 + \frac{1}{64b^3} + \frac{9a}{4b}\left(3a + \frac{1}{4b}\right)$$

$$= 27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}.$$

(iii)

$$\left(\frac{4}{5}x - 2\right)^3$$

$$= \left(\frac{4}{5}x\right)^3 - (2)^3 - 3 \times \frac{4}{5}x \times 2\left(\frac{4}{5}x - 2\right)$$

$$\left[\because (a-b)^3 = a^3 - b^3 - 3ab\,(a-b)\right]$$

$$= \frac{64}{125}x^3 - 8 - \frac{24}{5}x\left(\frac{4}{5}x - 2\right)$$

$$= \frac{64}{125}x^3 - 8 - \frac{96}{25}x^2 + \frac{48}{5}x.$$

Question 3:

(i) $(95)^3$

$= (100 - 5)^3$

$= (100)^3 - (5)^3 - 3 \times 100 \times 5\,(100 - 5)$

$= 1000000 - 125 - (1500\,95)$

$= 857375.$

(ii) $(999)^3$

$= (1000 - 1)^3$

$= (1000)^3 - (1)^3 - 3 \times 1000 \times 1\,(1000 - 1)$

$= 1000000000 - 1 - 3000\,(1000 - 1)$

$= 1000000000 - 1 - (3000\,999)$

$= 997002999.$

Exercise 2J

1. $(a+b)^2 = a^2 + 2ab + b^2 = (-a-b)^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a-b)(a+b) = a^2 - b^2$
4. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a+b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
10. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
11. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
    $= (a+b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
    $= (a-b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Question 1:

$x^3 + 27$

$= x^3 + 3^3$

$= (x+3)(x^2 - 3x + 9)$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

Question 2:

$8x^3 + 27y^3$

$= (2x)^3 + (3y)^3$

$= (2x+3y)[(2x)^2 - (2x)(3y) + (3y)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= (2x+3y)(4x^2 - 6xy + 9y^2)$.

Question 3:

$343 + 125\,b^3$

$= (7)^3 + (5b)^3$

$= (7+5b)[(7)^2 - (7)(5b) + (5b)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= (7+5b)(49 - 35b + 25b^2)$

Question 4:

$1 + 64x^3$

$= (1)^3 + (4x)^3$

$= (1+4x)[(1)^2 - 1(4x) + (4x)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= (1+4x)(1 - 4x + 16x^2)$.

Question 5:

$125a^3 + \frac{1}{8}$

We know that

$a^3+b^3=(a+b)(a^2-a\times b+b^2)$

Let us rewrite

$125a^3+\frac{1}{8}$

$$= (5a)^3 + \left(\frac{1}{2}\right)^3$$
$$= \left(5a + \frac{1}{2}\right)\left[(5a)^2 - 5a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right]$$
$$= \left(5a + \frac{1}{2}\right)\left(25a^2 - \frac{5a}{2} + \frac{1}{4}\right).$$

Question 6:

$216x^3+\frac{1}{125}$

We know that

$a^3+b^3=(a+b)(a^2-a\times b+b^2)$

Let us rewrite

$216x^3+\frac{1}{125}$

$$= (6x)^3 + \left(\frac{1}{5}\right)^3$$
$$= \left(6x + \frac{1}{5}\right)\left[(6x)^2 - 6x \times \frac{1}{5} + \left(\frac{1}{5}\right)^2\right]$$
$$= \left(6x + \frac{1}{5}\right)\left(36x^2 - \frac{6x}{5} + \frac{1}{25}\right).$$

Question 7:

$16x^4 + 54x$

$= 2x (8x^3 + 27)$

$= 2x [(2x)^3 + (3)^3]$

$= 2x (2x + 3) [(2x)^2 - 2x(3) + 3^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$=2x(2x+3)(4x^2 -6x +9)$

Question 8:

$7a^3 + 56b^3$

$= 7(a^3 + 8b^3)$

$= 7 [(a)^3 + (2b)^3]$

$= 7 (a + 2b) [a^2 - a\ 2b + (2b)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= 7 (a + 2b) (a^2 - 2ab + 4b^2).$

Question 9:

$x^5 + x^2$

$= x^2(x^3 + 1)$

$= x^2 (x + 1) [(x)^2 - x (1) + (1)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= x^2 (x + 1) (x^2 - x + 1).$

Question 10:

$a^3 + 0.008$

$= (a)^3 + (0.2)^3$

$= (a + 0.2) [(a)^2 - a(0.2) + (0.2)^2]$

Since $a^3+b^3=(a+b)(a^2-a\times b+b^2)$

$= (a + 0.2) (a^2 - 0.2a + 0.04).$

Question 11:

$x^6 + y^6$

$= (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)\,[(x^2)^2 - x^2\,(y^2) + (y^2)^2]$

Since $a^3 + b^3 = (a+b)(a^2 - a \times b + b^2)$

$= (x^2 + y^2)\,(x^4 - x^2y^2 + y^4)$.

Question 12:

$2a^3 + 16b^3 - 5a - 10b$

$= 2\,(a^3 + 8b^3) - 5\,(a + 2b)$

$= 2\,[(a)^3 + (2b)^3] - 5\,(a + 2b)$

$= 2\,(a + 2b)\,[(a)^2 - a\,(2b) + (2b)^2] - 5\,(a + 2b)$

Since $a^3 + b^3 = (a+b)(a^2 - a \times b + b^2)$

$= (a + 2b)\,[2(a^2 - 2ab + 4b^2) - 5]$

Question 13:

$x^3 - 512$

$= (x)^3 - (8)^3$

$= (x - 8)\,[(x)^2 + x\,(8) + (8)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (x - 8)\,(x^2 + 8x + 64)$.

Question 14:

$64x^3 - 343$

$= (4x)^3 - (7)^3$

$= (4x - 7)\,[(4x)^2 + 4x\,(7) + (7)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (4x - 7)\,(16x^2 + 28x + 49)$.

Question 15:

$1 - 27x^3$

$= (1)^3 - (3x)^3$

$= (1 - 3x)\,[(1)^2 + 1\,(3x) + (3x)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (1 - 3x)\,(1 + 3x + 9x^2)$.

Question 16:

$1 - 27x^3$

$= (1)^3 - (3x)^3$

$= (1 - 3x)\,[(1)^2 + 1\,(3x) + (3x)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (1 - 3x)\,(1 + 3x + 9x^2)$.

Question 17:

We know that

$a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

Let us rewrite

$8x^3 - \frac{1}{27y^3}$

$= (2x)^3 - \left(\frac{1}{3y}\right)^3$

$= \left(2x - \frac{1}{3y}\right)\left[(2x)^2 + 2x \times \frac{1}{3y} + \left(\frac{1}{3y}\right)^2\right]$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$.

Question 18:

$a^3 - 0.064$

$= (a)^3 - (0.4)^3$

$= (a - 0.4)[(a)^2 + a(0.4) + (0.4)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (a - 0.4)(a^2 + 0.4a + 0.16)$.

Question 19:

$(a + b)^3 - 8$

$= (a + b)^3 - (2)^3$

$= (a + b - 2)[(a + b)^2 + (a + b)2 + (2)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (a + b - 2)[a^2 + b^2 + 2ab + 2(a + b) + 4]$.

Question 20:

$x^6 - 729$

$= (x^2)^3 - (9)^3$

$= (x^2 - 9)[(x^2)^2 + x^2 9 + (9)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (x^2 - 9)(x^4 + 9x^2 + 81)$

$= (x + 3)(x - 3)[(x^2 + 9)^2 - (3x)^2]$

$= (x + 3)(x - 3)(x^2 + 3x + 9)(x^2 - 3x + 9)$.

Question 21:

We know that,

$a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

Therefore,

$(a + b)^3 - (a - b)^3$

$= [a + b - (a - b)][(a + b)^2 + (a + b)(a - b) + (a - b)^2]$

$= (a + b - a + b)[a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab]$

$= 2b(3a^2 + b^2)$.

Question 22:

$x - 8xy^3$

$= x(1 - 8y^3)$

$= x[(1)^3 - (2y)^3]$

$= x(1 - 2y)[(1)^2 + 1(2y) + (2y)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= x(1 - 2y)(1 + 2y + 4y^2)$.

Question 23:

$32x^4 - 500x$

$= 4x(8x^3 - 125)$

$= 4x[(2x)^3 - (5)^3]$

$= 4x[(2x - 5)[(2x)^2 + 2x(5) + (5)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= 4x(2x - 5)(4x^2 + 10x + 25)$.

Question 24:

$3a^7b - 81a^4b^4$

$= 3a^4b(a^3 - 27b^3)$

$= 3a^4b[(a)^3 - (3b)^3]$

$= 3a^4b(a - 3b)[(a)^2 + a(3b) + (3b)^2]$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= 3a^4b(a - 3b)(a^2 + 3ab + 9b^2)$.

Question 25:

We know that

$a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$$

$$= a^3 - \frac{1}{a^3} - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + a \times \frac{1}{a} + \frac{1}{a^2}\right) - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - 2\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right).$$

Question 26:

$8a^3 - b^3 - 4ax + 2bx$

$= 8a^3 - b^3 - 2x(2a - b)$

$= (2a)^3 - (b)^3 - 2x(2a - b)$

$= (2a - b)[(2a)^2 + 2a(b) + (b)^2] - 2x(2a - b)$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b)$

$= (2a - b)(4a^2 + 2ab + b^2 - 2x)$.

Question 27:

$8a^3 - b^3 - 4ax + 2bx$

$= 8a^3 - b^3 - 2x(2a - b)$

$= (2a)^3 - (b)^3 - 2x(2a - b)$

$= (2a - b)[(2a)^2 + 2a(b) + (b)^2] - 2x(2a - b)$

Since $a^3 - b^3 = (a-b)(a^2 + a \times b + b^2)$

$= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b)$

$= (2a - b)(4a^2 + 2ab + b^2 - 2x)$.

Exercise 2K

1. $(a+b)^2 = a^2 + 2ab + b^2 = (-a-b)^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a-b)(a+b) = a^2 - b^2$
4. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a+b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
10. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
11. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
    $= (a+b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
    $= (a-b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Question 1:

$125a^3 + b^3 + 64c^3 - 60abc$

$= (5a)^3 + (b)^3 + (4c)^3 - 3(5a)(b)(4c)$

$= (5a + b + 4c)[(5a)^2 + b^2 + (4c)^2 - (5a)(b) - (b)(4c) - (5a)(4c)]$

[$\because a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$]

$= (5a + b + 4c)(25a^2 + b^2 + 16c^2 - 5ab - 4bc - 20ac)$.

Question 2:

$a^3 + 8b^3 + 64c^3 - 24abc$

$= (a)^3 + (2b)^3 + (4c)^3 - 3\,a\,2b\,4c$

$= (a + 2b + 4c)[a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$.

Question 3:

$1 + b^3 + 8c^3 - 6bc$

$= 1 + (b)^3 + (2c)^3 - 3(b)(2c)$

$= (1 + b + 2c)[1 + b^2 + (2c)^2 - b - b\,2c - 2c]$

$= (1 + b + 2c)(1 + b^2 + 4c^2 - b - 2bc - 2c)$.

Question 4:

$216 + 27b^3 + 8c^3 - 108bc$

$= (6)^3 + (3b)^3 + (2c)^2 - 3\,6\,3b\,2c$

$= (6 + 3b + 2c)[(6)^2 + (3b)^2 + (2c)^2 - 6\,3b - 3b\,2c - 2c\,6]$

$= (6 + 3b + 2c)(36 + 9b^2 + 4c^2 - 18b - 6bc - 12c)$.

Question 5:

$27a^3 - b^3 + 8c^3 + 18abc$

$= (3a)^3 + (-b)^3 + (2c)^3 + 3(3a)(-b)(2c)$

$= [3a + (-b) + 2c][(3a)^2 + (-b)^2 + (2c)^2 - 3a(-b) - (-b)(2c) - (2c)(3a)]$

$= (3a - b + 2c)(9a^2 + b^2 + 4c^2 + 3ab + 2bc - 6ca)$.

Question 6:

$8a^3 + 125b^3 - 64c^3 + 120abc$

$= (2a)^3 + (5b)^3 + (-4c)^3 - 3(2a)(5b)(-4c)$

$= (2a + 5b - 4c)[(2a)^2 + (5b)^2 + (-4c)^2 - (2a)(5b) - (5b)(-4c) - (-4c)(2a)]$

= (2a + 5b – 4c) ($4a^2 + 25b^2 + 16c^2 – 10ab + 20bc + 8ca$).

Question 7:

$8 – 27b^3 – 343c^3 – 126bc$

$= (2)^3 + (-3b)^3 + (-7c)^3 – 3(2)(-3b)(-7c)$

$= (2 – 3b – 7c)[(2)^2 + (-3b)^2 + (-7c)^2 – (2)(-3b) – (-3b)(-7c) – (-7c)(2)]$

$= (2 – 3b – 7c)(4 + 9b^2 + 49c^2 + 6b – 21bc + 14c)$.

Question 8:

$125 – 8x^3 – 27y^3 – 90xy$

$= (5)^3 + (-2x)^3 + (-3y)^3 – 3(5)(-2x)(-3y)$

$= (5 – 2x – 3y)[(5)^2 + (-2x)^2 + (-3y)^2 – (5)(-2x) – (-2x)(-3y) – (-3y)(5)]$

$= (5 – 2x – 3y)(25 + 4x^2 + 9y^2 + 10x – 6xy + 15y)$.

Question 9:

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 – 12abc$

$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + (c)^3 – 3(\sqrt{2}a)(2\sqrt{2}b)(c)$

$= (\sqrt{2}a + 2\sqrt{2}b + c)$
$\left[(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 – (\sqrt{2}a)(2\sqrt{2}b) – (2\sqrt{2}b)(c) – (c)(\sqrt{2}a)\right]$

$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 – 4ab – 2\sqrt{2}bc – \sqrt{2}ac)$.

Question 10:

$x^3 + y^3 – 12xy + 64$

$= x^3 + y^3 + 64 – 12xy$

$= (x)^3 + (y)^3 + (4)^3 – 3(x)(y)(4)$

$= (x + y + 4)[(x)^2 + (y)^2 + (4)^2 – x \times y – y \times 4 – 4 \times x]$

$= (x + y + 4)(x^2 + y^2 + 16 – xy – 4y – 4x)$.

Question 11:

Putting (a – b) = x, (b – c) = y and (c – a) = z, we get,

$(a – b)^3 + (b – c)^3 + (c – a)^3$

$= x^3 + y^3 + z^3$, where (x + y + z) = (a – b) + (b – c) + (c – a) = 0

$= 3xyz$ [$\because (x + y + z) = 0 \Rightarrow (x^3 + y^3 + z^3) = 3xyz$]

= 3(a – b) (b – c) (c – a).

Question 12:

We have:

(3a – 2b) + (2b – 5c) + (5c – 3a) = 0

So, $(3a – 2b)^3 + (2b – 5c)^3 + (5c – 3a)^3$

= 3(3a – 2b) (2b – 5c) (5c – 3a).

Question 13:

$a^3 (b – c)^3 + b^3 (c – a)^3 + c^3 (a – b)^3$

$= [a (b – c)]^3 + [b (c – a)]^3 + [c (a – b)]^3$

Now, since, a (b – c) + b (c -a) + c (a – b)

= ab – ac + bc – ba + ca – bc = 0

So, $a^3 (b – c)^3 + b^3 (c – a)^3 + c^3 (a – b)^3$

= 3a (b – c) b (c – a) c (a – b)

= 3abc (a – b) (b – c) (c – a).

Question 14:

$(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3$

Since, (5a – 7b) + (9c – 5a) + (7b – 9c)
= 5a – 7b + 9c – 5a + 7b – 9c = 0
So, $(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3$
= 3(5a – 7b) (9c – 5a) (7b – 9c).

Question 15:

$(x + y - z)(x^2 + y^2 + z^2 - xy + yz + zx)$
$= [x + y + (-z)][(x)^2 + (y)^2 + (-z)^2 - (x)(y) - (y)(-z) - (-z)(x)]$
$= x^3 + y^3 - z^3 + 3xyz.$

Question 16:

$(x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9)$
$= [x + (-2y) + 3][(x)^2 + (-2y)^2 + (3) - (x)(-2y) - (-2y)(3) - (3)(x)]$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= a^3 + b^3 + c^3 - 3abc$
Where, x = a, (-2y) = b and 3 = c
$(x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9)$
$= (x)^3 + (-2y)^3 + (3)^2 - 3(x)(-2y)(3)$
$= x^3 - 8y^3 + 27 + 18xy.$

Question 17:

$(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx - 2yz)$
$= [x + (-2y) + (-z)][(x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (-2y)(-z) - (-z)(x)]$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= a^3 + b^3 + c^3 - 3abc$
Where x = a, (-2y) = b and (-z) = c
$(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx - 2yz)$
$= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z)$
$= x^3 - 8y^3 - z^3 - 6xyz.$

Question 18:

Given, x + y + 4 = 0
We have $(x^3 + y^3 - 12xy + 64)$
$= (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4)$
= 0.
Since, we know $a + b + c = 0 \Rightarrow (a^3 + b^3 + c^3) = 3abc$

Question 19:

Given x = 2y + 6
Or, x – 2y – 6 = 0
We have, $(x^3 - 8y^3 - 36xy - 216)$
$= (x^3 - 8y^3 - 216 - 36xy)$
$= (x)^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6)$
$= (x - 2y - 6)[(x)^2 + (-2y)^2 + (-6)^2 - (x)(-2y) - (-2y)(-6) - (-6)(x)]$
$= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$
$= 0\,(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$
= 0.