Ex 2.1

Q1

By the definition of equality of ordered pairs

$$\left(\frac{a}{3}+1,\ b-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$$

$$\Rightarrow \quad \frac{a}{3}+1=\frac{5}{3} \quad \text{and} \quad b-\frac{2}{3}=\frac{1}{3}$$

$$\Rightarrow \quad \frac{a}{3}=\frac{5}{3}-1 \quad \text{and} \quad b=\frac{1}{3}+\frac{2}{3}$$

$$\Rightarrow \quad \frac{a}{3}=\frac{5-3}{3} \quad \text{and} \quad b=\frac{1+2}{3}$$

$$\Rightarrow \quad \frac{a}{3}=\frac{2}{3} \quad \text{and} \quad b=\frac{3}{3}$$

$$\Rightarrow \quad a=2 \quad \text{and} \quad b=1$$

By the definition of equality of ordered pairs

$$(x+1,1)=(3,y-2)$$

$$\Rightarrow \quad x+1=3 \quad \text{and} \quad 1=y-2$$

$$\Rightarrow \quad x=3-1 \quad \text{and} \quad 1+2=y$$

$$\Rightarrow \quad x=2 \quad \text{and} \quad 3=y$$

$$\Rightarrow \quad x=2 \quad \text{and} \quad y=3$$

Q2

We have,

$$(x,-1)\in\{(a,b):b=2a-3\}$$

and, $(5,y)\in\{(a,b):b=2a-3\}$

$$\Rightarrow \quad -1=2\times x-3 \quad \text{and} \quad y=2\times 5-3$$

$$\Rightarrow \quad -1=2x-3 \quad \text{and} \quad y=10-3$$

$$\Rightarrow \quad 3-1=2x \quad \text{and} \quad y=7$$

$$\Rightarrow \quad 2=2x \quad \text{and} \quad y=7$$

$$\Rightarrow \quad x=1 \quad \text{and} \quad y=7$$

Q3

We have,

$$a + b = 5$$

$$\Rightarrow \quad a = 5 - b$$

$$\therefore \quad b = 0 \Rightarrow a = 5 - 0 = 5,$$

$$b = 3 \Rightarrow a = 5 - 3 = 2,$$

$$b = 6 \Rightarrow a = 5 - 6 = -1,$$

Hence, the required set of ordered pairs (a, b) is $\{(-1,6),(2,3),(5,0)\}$

Q4

We have,

$$a \in \{2, 4, 6, 9\}$$

and, $b \in \{4, 6, 18, 27\}$

Now, a/b stands for 'a divides b'. For the elements of the given sets, we find that $2/4$, $2/6$, $2/18$, $6/18$, $9/18$ and $9/27$

$\therefore$ $\{(2,4), (2,6), (2,18), (6,18), (9,18), (9,27)\}$ are the required set of ordered pairs (a, b).

Q5

We have,

$$A = \{1, 2\} \text{ and } B = \{1, 3\}$$

Now, $A \times B = \{1, 2\} \times \{1, 3\}$

$$= \{(1,1), (1,3), (2,1), (2,3)\}$$

and, $B \times A = \{1, 3\} \times \{1, 2\}$

$$= \{(1,1), (1,2), (3,1), (3,2)\}$$

Q6

We have,

$A = \{1, 2, 3\}$ and $B = \{3, 4\}$

$\therefore \quad A \times B = \{1, 2, 3\} \times \{3, 4\}$

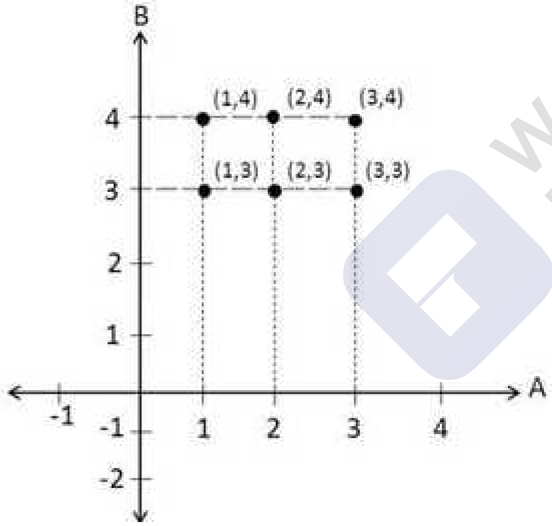
$= \{(1,3),\ (1,4),\ (2,3),\ (2,4),\ (3,3),\ (3,4)\}$

In order to represent $A \times B$ graphically, we follow the following steps:

(a) Draw two mutually perpendicular line one horizontal and other vertical.

(b) On the horizontal line represent the element of set A and on the vertical line represent the elements of set B.

(c) Draw vertical dotted lines through points representing elements of A on horizontal line and horizontal lines through points representing elements of B on the vertical line points of intersection of these lines will represent $A \times B$ graphically.



Q7

We have,

$A = \{1, 2, 3\}$ and $B = \{2, 4\}$

$\therefore \quad A \times B = \{1, 2, 3\} \times \{2, 4\}$

$= \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\},$

$B \times A = \{2, 4\} \times \{1, 2, 3\}$

$= \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\},$

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\},$

$B \times B = \{2, 4\} \times \{2, 4\}$

$= \{(2, 2), (2, 4), (4, 2), (4, 4)\},$

and, $(A \times B) \cap (B \times A)$

$= \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\} \cap \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}$

$= \{(2, 2)\}$

$\Rightarrow \quad (A \times B) \cap (B \times A) = \{(2, 2)\}.$

Q8

We have,

$n(A) = 5$ and $n(B) = 4$

We know that, if A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$

$\therefore \quad n(A \times B) = 5 \times 4 = 20$

Now,

$n[(A \times B) \cap (B \times A)] = 3 \times 3 = 9$ $\quad [\because A$ and B have 3 elements in common$]$

Q9

Let (a,b) be an arbitrary element of $(A\times B)\cap(B\times A)$. Then,

$$(a,b)\in(A\times B)\cap(B\times A)$$
$$\Leftrightarrow\quad (a,b)\in A\times B \quad\text{and}\quad (a,b)\in B\times A$$
$$\Leftrightarrow\quad (a\in A \text{ and } b\in B) \quad\text{and}\quad (a\in B \text{ and } b\in A)$$
$$\Leftrightarrow\quad (a\in A \text{ and } a\in B) \quad\text{and}\quad (b\in A \text{ and } b\in B)$$
$$\Leftrightarrow\quad a\in A\cap B \quad\text{and}\quad b\in A\cap B$$

Hence, the sets $A\times B$ and $B\times A$ have an element in comon iff the sets A and B have an element in common.

Q10

Since $(x,1)$, $(y,2)$, $(z,1)$ are elements of $A\times B$. Therefore, $x,\ y,\ z\in A$ and $1,2\in B$

It is given that $n(A)=3$ and $n(B)=2$

$\therefore\quad x,\ y,\ z\in A$ and $n(A)=3$

$\Rightarrow\quad A=\{x,y,z\}$

$1,2\in B$ and $n(B)=2$

$\Rightarrow\quad B=\{1,2\}$.

Q11

We have,

$A=\{1,2,3,4\}$

and, $R=\{(a,b)=a\in A,\ b\in A,\ a \text{ divides } b\}$

Now,

a/b stands for 'a divides b'. For the elements of the given sets, we find that $1/1,\ 1/2,\ 1/3,\ 1/4,\ 2/2,\ 3/3$ and $4/4$

$\therefore\ R=\{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (2,2),\ (2,4),\ (3,3),\ (4,4)\}$

Q12

We have,

$$A = \{-1, 1\}$$

$$\therefore \quad A \times A = \{-1, 1\} \times \{-1, 1\}$$

$$= \{(-1,-1),\ (-1,1),\ (1,-1),\ (1,1)\}$$

$$\therefore \quad A \times A \times A = \{-1, 1\} \times \{(-1,-1),\ (-1,1),\ (1,-1),\ (1,1)\}$$

$$= \{(-1,-1,-1),\ (-1,-1,1),\ (-1,1,-1),\ (-1,1,1),\ (1,-1,-1),\ (1,-1,1),\ (1,1,-1),\ (1,1,1)\}$$

Q13

(i) False,

If $P = \{m, n\}$ and $Q = \{n, m\}$,

Then,

$$P \times Q = \{(m,n),\ (m,m),\ (n,n),\ (n,m)\}$$

(ii) False,

If A and B are non-empty sets, then AB is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

(iii) True

Q14

We have,

$$A = \{1, 2\}$$

$$\therefore \quad A \times A = \{1, 2\} \times \{1, 2\}$$

$$= \{(1,1),\ (1,2),\ (2,1),\ (2,2)\}$$

$$\therefore \quad A \times A \times A = \{1, 2\} \times \{(1,1),\ (1,2),\ (2,1),\ (2,2)\}$$

$$= \{(1,1,1),\ (1,1,2),\ (1,2,1),\ (1,2,2),\ (2,1,1),\ (2,1,2),\ (2,2,1),\ (2,2,2)\}$$

Q15

We have,

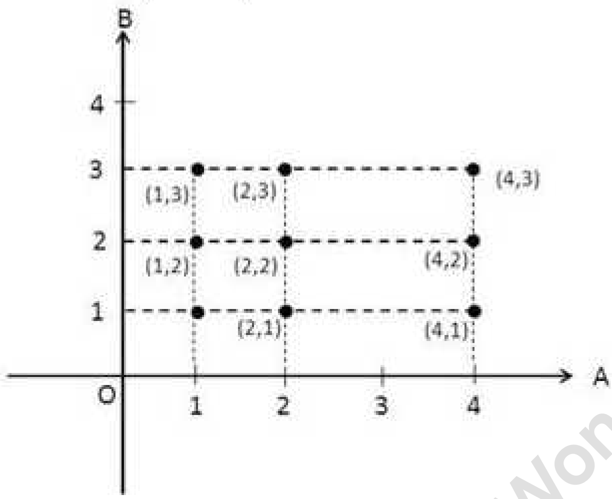
$$A = \{1, 2, 4\} \text{ and } B = \{1, 2, 3\}$$

$$\therefore \quad A \times B = \{1, 2, 4\} \times \{1, 2, 3\}$$

$$= \{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3),\ (4,1),\ (4,2),\ (4,3)\}$$

Hence, we represent A on the horizontal line and B on vertical line.

Graphical representation of $A \times B$ is as shown below:



We have,

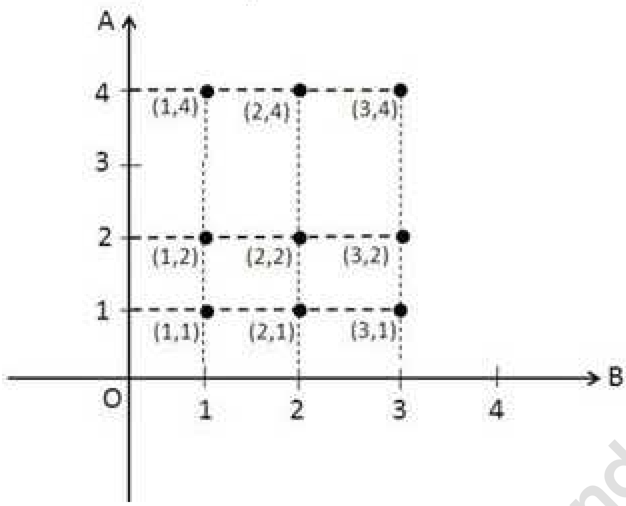
$A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$

$\therefore \quad B \times A = \{1, 2, 3\} \times \{1, 2, 4\}$

$= \{(1,1),\ (1,2),\ (1,4),\ (2,1),\ (2,2),\ (2,4),\ (3,1),\ (3,2),\ (3,4)\}$

Hence, we represent B on the horizontal line and A on vertical line.

Graphical representation of $B \times A$ is as shown below:



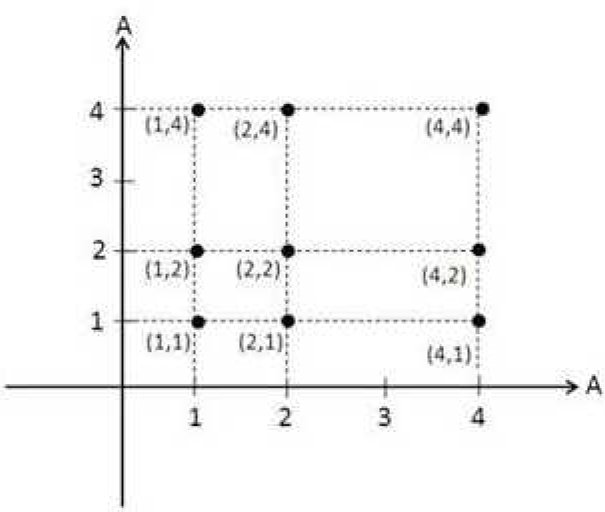
We have,

$A = \{1, 2, 4\}$

$\therefore \quad A \times A = \{1, 2, 4\} \times \{1, 2, 4\}$

$= \{(1,1),\ (1,2),\ (1,4),\ (2,1),\ (2,2),\ (2,4),\ (4,1),\ (4,2),\ (4,4)\}$

Graphical representation of $A \times A$ is shown below:



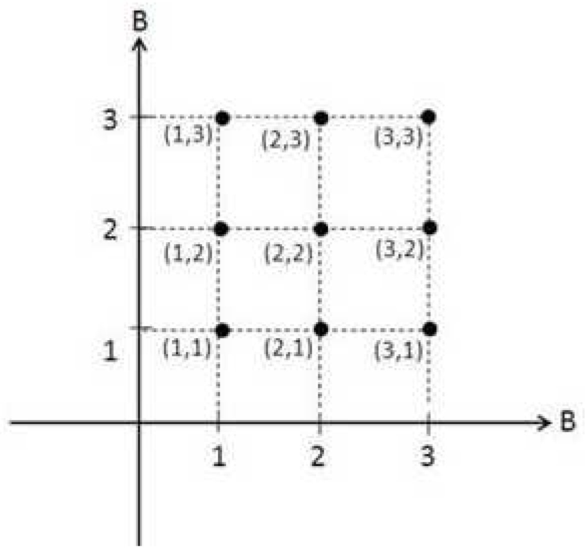
We have,

$$B = \{1,2,3\}$$

$$\therefore \quad B \times B = \{1,2,3\} \times \{1,2,3\}$$

$$= \{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3),\ (3,1),\ (3,2),\ (3,3)\}$$

Graphical representation of $B \times B$ is shown below:



Ex 2.2

Q1

We have,

$A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$

$\therefore$ $A \times B = \{1,2,3\} \times \{3,4\}$

$= \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

and, $B \times C = \{3,4\} \times \{4,5,6\}$

$= \{(3,4), (3,5), (3,6), (4,4), (4,5), (4,6)\}$

$\therefore$ $(A \times B) \cap (B \times C) = \{3,4\}$.

Q2

We have,

$A = \{2,3\}$, $B = \{4,5\}$ and $C = \{5,6\}$

$\therefore$ $B \cup C = \{4,5\} \cup \{5,6\}$

$= \{4,5,6\}$

$\therefore$ $A \times (B \cup C) = \{2,3\} \times \{4,5,6\}$

$= \{(2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Now,

$B \cap C = \{4,5\} \cap \{5,6\} = \{5\}$

$\therefore$ $A \times (B \cap C) = \{2,3\} \times \{5\}$

$= \{(2,5), (3,5)\}$

Now,

$A \times B = \{2,3\} \times \{4,5\}$

$= \{(2,4), (2,5), (3,4), (3,5)\}$

and, $A \times C = \{2,3\} \times \{5,6\}$

$= \{(2,5), (2,6), (3,5), (3,6)\}$

$\therefore$ $(A \times B) \cup (A \times C) = \{(2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Q3

We have,

$A = \{1,2,3\}$, $B = \{4\}$ and $C = \{5\}$

$\therefore \quad B \cup C = \{4\} \cup \{5\} = \{4,5\}$

$\therefore \quad A \times (B \cup C) = \{1,2,3\} \times \{4,5\}$

$\Rightarrow \quad A \times (B \cup C) = \{(1,4),\ (1,5),\ (2,4),\ (2,5),\ (3,4),\ (3,5)\}$ ---(i)

Now,

$A \times B = \{1,2,3\} \times \{4\}$

$= \{(1,4),\ (2,4),\ (3,4)\}$

and, $A \times C = \{1,2,3\} \times \{5\}$

$= \{(1,5),\ (2,5),\ (3,5)\}$

$\therefore \quad (A \times B) \cup (A \times C) = \{(1,4),\ (2,4),\ (3,4)\} \cup \{(1,5),\ (2,5),\ (3,5)\}$

$\Rightarrow \quad (A \times B) \cup (A \times C) = \{(1,4),\ (1,5),\ (2,4),\ (2,5),\ (3,4),\ (3,5)\}$ ---(ii)

From equation(i) and(ii), we get

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence verified.

We have,

$A = \{1,2,3\}$, $B = \{4\}$ and $C = \{5\}$

$\therefore \quad B \cap C = \{4\} \cap \{5\} = \phi$

$\therefore \quad A \times (B \cap C) = \{1,2,3\} \times \phi$

$\Rightarrow \quad A \times (B \cap C) = \phi$ ---(i)

Now,

$A \times B = \{1,2,3\} \times \{4\}$

$= \{(1,4),\ (2,4),\ (3,4)\}$

and, $A \times C = \{1,2,3\} \times \{5\}$

$= \{(1,5),\ (2,5),\ (3,5)\}$

$\therefore \quad (A \times B) \cap (A \times C) = \{(1,4),\ (2,4),\ (3,4)\} \cap \{(1,5),\ (2,5),\ (3,5)\}$

$\Rightarrow \quad (A \times B) \cap (A \times C) = \phi$ ---(ii)

From equation(i) and equation(ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence verified.

We have,

$$A = \{1,2,3\},\ B = \{4\} \text{ and } C = \{5\}$$

$$\therefore\quad B - C = \{4\}$$

$$\therefore\quad A \times (B - C) = \{1,2,3\} \times \{4\}$$

$$\Rightarrow\quad A \times (B - C) = \{(1,4),\ (2,4),\ (3,4)\} \qquad \text{---(i)}$$

Now,

$$A \times B = \{1,2,3\} \times \{4\}$$
$$= \{(1,4),\ (2,4),\ (3,4)\}$$

and,
$$A \times C = \{1,2,3\} \times \{5\}$$
$$= \{(1,5),\ (2,5),\ (3,5)\}$$

$$\therefore\quad (A \times B) - (A \times C) = \{(1,4),\ (2,4),\ (3,4)\} \qquad \text{---(ii)}$$

From equation (i) and equation (ii), we get

$$A \times (B - C) = (A \times B) - (A \times C)$$

Hence verified.

Q4

We have,

$$A = \{1,2\},\ B = \{1,2,3,4\},\ C = \{5,6\} \text{ and } D = \{5,6,7,8\}$$

$$\therefore\quad B \times D = \{1,2,3,4\} \times \{5,6,7,8\}$$

$$= \begin{bmatrix} (1,5), & (1,6), & (1,7), & (1,8), & (2,5), & (2,6), & (2,7), & (2,8), \\ (3,5), & (3,6), & (3,7), & (3,8), & (4,5), & (4,6), & (4,7), & (4,8) \end{bmatrix} \qquad \text{---(i)}$$

and,
$$A \times C = (1,2) \times (5,6)$$
$$= \{(1,5),\ (1,6),\ (2,5),\ (2,6)\} \qquad \text{---(ii)}$$

Clearly from equation (i) and equation (ii), we get

$$A \times C \subset B \times D$$

Hence verified.

We have,

$A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5,6,7,8\}$

$\therefore \quad B \cap \cap C = \{1,2,3,4\} \cap \{5,6\} = \phi$

$A \times (B \cap C) = \{1,2\} \times \phi = \phi$ --- (i)

Now,

$A \times B = \{1,2\} \times \{1,2,3,4\}$

$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

and, $A \times C = \{1,2\} \times \{5,6\}$

$= \{(1,5), (1,6), (2,5), (2,6)\}$

$\therefore \quad (A \times B) \cap (A \times C) = \theta$ --- (ii)

From equation (i) and equation (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence verified.

Q5

Given:

A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

(i) A × (B ∩ C)

Now,

(B ∩ C) = {4}

∴ A × (B ∩ C) = {(1, 4), (2, 4), (3, 4)}

(ii) (A × B) ∩ (A × C)

Now,

(A × B) = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

And,

(A × C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

∴ (A × B) ∩ (A × C) = {(1, 4), (2, 4), (3, 4)}

(iii) A × (B ∪ C)

Now,

(B ∪ C) = {3, 4, 5, 6}

∴ A × (B ∪ C) = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

(iv) (A × B) ∪ (A × C)

Now,

(A × B) = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

And,

(A × C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

∴ (A × B) ∪ (A × C) = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

Q6

Let (a,b) be an arbitrary element of $(A \cup B) \times C$. Then,

$(a,b) \in (A \cup B) \times C$

$\Rightarrow \quad a \in A \cup B$ and $b \in C$ [By defination]

$\Rightarrow \quad (a \in A$ or $a \in B)$ and $b \in C$ [By defination]

$\Rightarrow \quad (a \in A$ and $b \in C)$ or $(a \in B$ and $b \in C)$

$\Rightarrow \quad (a,b) \in A \times C$ or $(a,b) \in B \times C$

$\Rightarrow \quad (a,b) \in (A \times C) \cup (B \times C)$

$\Rightarrow \quad (a,b) \in (A \cup B) \times C$

$\Rightarrow \quad (a,b) \in (A \times C) \cup (B \times C)$

$\Rightarrow \quad (A \cup B) \times C \subseteq (A \times C) \cup (B \times C)$ ---(i)

Again, let (x,y) be an arbitrary element of $(A \times C) \cup (B \times C)$. Then,

$(x,y) \in (A \times C) \cup (B \times C)$

$\Rightarrow \quad (x,y) \in A \times C$ or $(x,y) \in B \times C$

$\Rightarrow \quad x \in A$ and $y \in C$ or $x \in B$ and $y \in C$

$\Rightarrow \quad (x \in A$ or $x \in B)$ and $y \in C$

$\Rightarrow \quad x \in A \cup B$ and $y \in C$

$\Rightarrow \quad (x,y) \in (A \cup B) \times C$

$\Rightarrow \quad (x,y) \in (A \times C) \cup (B \times C)$

$\Rightarrow \quad (x,y) \in (A \cup B) \times C$

$\Rightarrow \quad (A \times C) \cup (B \times C) \subseteq (A \cup B) \times C$ ---(ii)

Using equation (i) and equation (ii), we get

$(A \cup B) \times C = (A \times C) \cup (B \times C)$

Hence proved.

Let (a,b) be an arbitrary element of $(A \cap B)\times C$. Then,

$$(a,b) \in (A \cap B)\times C$$
$\Rightarrow$ $a \in A \cap B$ and $b \in C$
$\Rightarrow$ $(a \in A$ and $a \in B)$ and $b \in C$ [By defination]
$\Rightarrow$ $(a \in A$ and $b \in C)$ and $(a \in B$ and $b \in C)$
$\Rightarrow$ $(a,b) \in A \times C$ and $(a,b) \in B \times C$
$\Rightarrow$ $(a,b) \in (A \times C) \cap (B \times C)$
$\Rightarrow$ $(a,b) \in (A \cap B)\times C$
$\Rightarrow$ $(a,b) \in (A \times C) \cap (B \times C)$
$\Rightarrow$ $(A \cap B)\times C \subseteq (A \times C) \cap (B \times C)$ ---(i)

Let (x,y) be an arbitrary element of $(A \times C) \cap (B \times C)$. Then,
$$(x,y) \in (A \times C) \cap (B \times C)$$
$\Rightarrow$ $(x,y) \in A \times C$ and $(x,y) \in B \times C$ [By defination]
$\Rightarrow$ $(x \in A$ and $y \in C)$ and $(x \in B$ and $y \in C)$
$\Rightarrow$ $(x \in A$ and $x \in B)$ and $y \in C$
$\Rightarrow$ $x \in A \cap B$ and $y \in C$
$\Rightarrow$ $(x,y) \in (A \cap B)\times C$
$\Rightarrow$ $(x,y) \in (A \times C) \cap (B \times C)$
$\Rightarrow$ $(x,y) \in (A \cap B)\times C$
$\Rightarrow$ $(A \times C) \cap (B \times C) \subseteq (A \cap B)\times C$ ---(ii)

Using equation (i) and equation (ii), we get
$$(A \cap B)\times C = (A \times C) \cap (B \times C)$$

Q7

Let (a, b) be an arbitrary element of $A \times B$. then,

$(a, b) \in A \times B$

$\Rightarrow \quad a \in A$ and $b \in B$ ---(i)

Now,

$(a, b) \in A \times B$

$\Rightarrow \quad (a, b) \in C \times D$ $[\because A \times B \subseteq C \times D]$

$\Rightarrow \quad a \in C$ and $b \in D$ ---(ii)

$\therefore \quad a \in A \Rightarrow a \in C$ [Using (i) and (ii)]

$\Rightarrow \quad A \subseteq C$

and,

$b \in B \Rightarrow b \in D$

$\Rightarrow \quad B \subseteq D$

Hence, proved

Ex 2.3

Q1

(i) We have,

$A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$\{(1, 6), (3, 4), (5, 2)\}$ is not a relation from A to B as it is not a subset of $A \times B$.

(ii) We have,

$A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$\{(1, 5), (2, 6), (3, 4), (3, 6)\}$ is a subset of $A \times B$, so it is a relation from A to B.

(iii) We have,

$A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$\{(4, 2), (4, 3), (5, 1)\}$ is not a relation from A to B as it is not a subset of $A \times B$.

(iv) We have,

$A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$A \times B$ is a relation from A to B.

Q2

We have,

$A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$

It is given that $(x, y) \in R \Leftrightarrow x$ is relatively prime to y

$\therefore \quad (2, 3) \in R, (2, 7) \in R, (3, 7) \in R, (3, 10) \in R, (4, 3) \in R, (4, 7) \in R, (5, 3) \in R,$ and $(5, 7) \in R$

Thus,

$R = \{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 7)\}$

Clearly, Domain $(R) = \{2, 3, 4, 5\}$ and Range $= \{3, 7, 10\}$.

Q3

We have,

$A = \{1,2,3,4,5\}$ [$\because A$ is the set of first five natural number]

It is given that R be a relation on A defined as $(x,y) \in R \Leftrightarrow x \le y$

For the elements of the given sets A and A, we find that

$1 = 1,\ 1 < 2,\ 1 < 3,\ 1 < 4,\ 1 < 5,\ 2 = 2,\ 2 < 3,\ 2 < 4,\ 2 < 5,\ 3 = 3,\ 3 < 4,\ 3 < 5,\ 4 = 4,\ 4 < 5$ and $5 = 5$

$\therefore$ $(1,1) \in R,\ (1,2) \in R,\ (1,3) \in R,\ (1,4) \in R,\ (1,5) \in R,\ (2,2) \in R,\ (2,3) \in R,\ (2,4) \in R,\ (2,5) \in R,$
$(3,3) \in R,\ (3,4) \in R,\ (3,5) \in R,\ (4,4) \in R,\ (4,5) \in R$ and $(5,5) \in R$

Thus,

$$R = \left\{\begin{matrix}(1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),\ (2,2),\ (2,3),\ (2,4),\ (2,5),\ (3,3),\ (3,4),\ (3,5),\ (4,4), \\ (4,5),\ (5,5)\end{matrix}\right\}$$

Also,

$$R^{-1} = \left\{\begin{matrix}(1,1),\ (2,1),\ (3,1),\ (4,1),\ (5,1),\ (2,2),\ (3,2),\ (4,2),\ (5,2),\ (3,3),\ (4,3),\ (5,3),\ (4,4), \\ (5,4),\ (5,5)\end{matrix}\right\}$$

(i) Domain$(R^{-1}) = \{1,2,3,4,5\}$

(ii) Range$(R) = \{1,2,3,4,5\}$

Q4

(i) We have,

$R = \{(1,2), (1,3), (2,3), (3,2), (5,6)\}$

$\Rightarrow \quad R^{-1} = \{(2,1), (3,1), (3,2), (2,3), (6,5)\}$

(ii) We have,

$R = \{(x,y) : x, y \in N, x + 2y = 8\}$

Now,

$x + 2y = 8$

$\Rightarrow \quad x = 8 - 2y$

Putting $y = 1,2,3$ we get $x = 6,4,2$ respectively.

For $y = 4$, we get $x = 0 \notin N$. Also for $y > 4$, $x \notin N$

$\therefore \quad R = \{(6,1), (4,2), (2,3)\}$

Thus,

$R^{-1} = \{(1,6), (2,4), (3,2)\}$

$\Rightarrow \quad R^{-1} = \{(3,2), (2,4), (1,6)\}$

(iii) We have,

R is a relation from $\{11,12,13,\}$ to $\{8,10,12,\}$ defined by $y = x - 3$

Now,

$y = x - 3$

Putting $x = 11,12,13$ we get $y = 8,9,10$ respectively

$\Rightarrow \quad (11,8) \in R, \; (12,9) \notin R$ and $(13,10) \in R$

Thus,

$R = \{(11,8), (13,10)\}$

$\Rightarrow \quad R^{-1} = \{(8,11), (10,13)\}$

Q5

() We have,

$x = 2y$

Putting $y = 1, 2, 3$ we get $x = 2, 4, 6$ respectively.

$\therefore \quad R = \{(2,1), (4,2), (6,3)\}$

(i) We have,

It is given that relation R on the set $\{1,2,3,4,5,6,7\}$ defined by $(x,y) \in R \Leftrightarrow x$ is relatively prime to y.

$\therefore$ $(2,3) \in R$, $(2,5) \in R$, $(2,7) \in R$, $(3,2) \in R$, $(3,4) \in R$, $(3,5) \in R$, $(3,7) \in R$, $(4,3) \in R$, $(4,5) \in R$, $(4,7) \in R$, $(5,2) \in R$, $(5,3) \in R$, $(5,4) \in R$, $(5,6) \in R$, $(5,7) \in R$, $(6,5) \in R$, $(6,7) \in R$, $(7,2) \in R$, $(7,3) \in R$, $(7,4) \in R$, $(7,5) \in R$ and $(7,6) \in R$.

Thus,

$$R = \begin{Bmatrix} (2,3), (2,5), (2,7), (3,2), (3,4), (3,5), (3,7), (4,3), (4,5), (4,7), (5,2), \\ (5,3), (5,4), (5,6), (5,7), (6,5), (6,7), (7,2), (7,3), (7,4), (7,5), (7,6) \end{Bmatrix}$$

(ii) We have,

$2x + 3y = 12$

$\Rightarrow \quad 2x = 12 - 3y$

$\Rightarrow \quad x = \dfrac{12 - 3y}{2}$

Putting $y = 0, 2, 4$ we get $x = 6, 3, 0$ respectively.

For $y = 1, 3, 5, 6, 7, 8, 9, 10$, $x \notin$ given set

$\therefore \quad R = \{(6,0), (3,2), (0,4)\}$

$= \{(0,4), (3,2), (6,0)\}$

(iv) We have,

$A = (5,6,7,8)$ and $B = (10,12,15,16,18)$

Now,

a/b stands for 'a divides b'. For the elements of the given set A and B, we find that $5/10$, $5/15$, $6/12$, $6/18$ and $8/16$

$\therefore \quad (5,10) \in R$, $(5,15) \in R$, $(6,12) \in R$, $(6,18) \in R$, and $(8,16) \in R$

Thus,

$R = \{(5,10), (5,15), (6,12), (6,18), (8,16)\}$

Q6

We have,

$$(x, y) \in R \Leftrightarrow x + 2y = 8$$

Now,

$$x + 2y = 8$$

$$\Rightarrow \quad x = 8 - 2y$$

Putting $y = 1, 2, 3,$ we get $x = 6, 4, 2$ respectively

For $y = 4,$ we get $x = 0 \notin N$

Also, for $y > 4,$ $x \notin N$

$$\therefore \quad R = \{(6,1), (4,2), (2,3)\}$$

Thus,

$$R^{-1} = \{(1,6), (2,4), (3,2)\}$$

$$\Rightarrow \quad R^{-1} = \{(3,2), (2,4), (1,6)\}$$

Q7

We have,

$$A = \{3,5\}, \quad B = \{7,11\}$$

and, $R = \{(a,b) : a \in A, b \in B, a - b \text{ is odd}\}$

For the elements of the given sets A and B, we find that

$$3 - 7 = -4, \; 3 - 11 = -8, \; 5 - 7 = -2 \text{ and } 5 - 11 = -6$$

$$\therefore \quad (3,7) \notin R, \; (3,11) \notin R, \; (5,7) \notin R \text{ and } (5,11) \notin R,$$

Thus, R is an empty relation from A into B.

Q8

We have,

$A = \{1,2\}$ and $B = \{3,4\}$

$\therefore$ $n(A) = 2$ and $n(B) = 2$

$\Rightarrow$ $n(A) \times n(B) = 2 \times 2 = 4$

$\Rightarrow$ $n(A \times B) = 4$ $\quad [\because n(A \times B) = n(A) \times n(B)]$

So, there are $2^4 = 16$ relations from A to B. $\quad \left[\begin{array}{l}\because n(x) = a, n(y) = b \\ \Rightarrow \text{Total number of relations} = 2^{ab}\end{array}\right]$

Q9

(i) We have,

$R = \{(x, x+5) : x \in \{0,1,2,3,4,5\}\}$

For the elements of the given sets, we find that

$R = \{(0,5),\ (1,6),\ (2,7),\ (3,8),\ (4,9),\ (5,10)\}$

Clearly, Domain$(R) = \{0,1,2,3,4,5\}$ and Range$(R) = \{5,6,7,8,9,10\}$

(ii) We have,

$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

For the elements of the given sets, we find that

$x = 2, 3, 5, 7$

$\therefore$ $(2,8) \in R,\ (3,27) \in R,\ (5,125) \in R$ and $(7,343) \in R$

$\Rightarrow$ $R = \{(2,8),\ (3,27),\ (5,125),\ (7,343)\}$

Clearly, Domain$(R) = \{2,3,5,7\}$ and Range$(R) = \{8,27,125,343\}$

Q10

(i) We have,

$$R = \{(a,b) : a \in N,\ a < 5,\ b = 4\}$$

$\Rightarrow$ $a = 1,2,3,4$ and $b = 4$

Thus, $R = \{(1,4),\ (2,4),\ (3,4),\ (4,4)\}$

Clearly, Domain$(R) = \{1,2,3,4\}$ and Range$(R) = \{4\}$

(ii) We have,

$$S = \{(a,b) : b = |a-1|,\ a \in z \text{ and } |a| \le 3\}$$

$\Rightarrow$ $a = -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3$

For $a = -3,-2,-1,0,1,2,3$ we get
$b = 4,3,2,1,0,1,2$ respectively

Thus, $S = \{(-3,4),\ (-2,3),\ (-1,2),\ (0,1),\ (2,1),\ (3,2)\}$

Domain$(S) = \{-3,-2,-1,0,1,2,3\}$ and
Range$(R) = \{0,1,2,3,4\}$

Q11

Here, $A = \{a,b\}$
we know that,
Number of relations $= 2^{m.n}$
$= 2^{2.2}$
$= 24$
$= 16$
Number of relations on $A = 16$

Relations on A are given by

$R = \{a,a\},\{a,b\},\{b,a\},\{b,b\}$
$\{(a,a),(a,b)\},\{(a,a),(b,a)\},\{(a,a),(b,b)\},$
$\{(a,b),(b,a)\},\{(a,b),(b,b)\},\{(b,a),(b,b)\},$
$\{(a,a),(a,b),(b,a)\},\{(a,b),(b,a),(b,b)\},$
$\{(b,a),(b,b),(a,a)\},\{(b,b),(a,a),(a,b)\},$
$\{(a,a),(b,a),(b,b)\},\{(a,a),(b,a),(b,b)\}$

Q12

We have,

$A = \{x, y, z\}$ and $B = \{a, b\}$

$\Rightarrow \quad n(A) = 3$ and $n(B) = 2$

$\Rightarrow \quad n(A) \times n(B) = 3 \times 2 = 6$

$\Rightarrow \quad n(A \times B) = 6 \qquad [\because n(A \times B) = n(A) \times n(B)]$

So, there are $2^6 = 64$ relations from A to B. $\left[\begin{array}{l}\because n(x) = a, n(y) = b \\ \Rightarrow \text{Total number of relations} = 2^{ab}\end{array}\right]$

Q13

We have,

$R = \{(a, b) : a,\ b \in N \text{ and } a = b^2\}$

(i) This statement is not true because $(5, 5) \notin R$.

(ii) This statement is not true because $(25, 5) \in R$ but $(5, 25) \notin R$.

(iii) This statement is not true because $(36, 6) \in R$ and $(25, 5) \in R$ but $(36, 5) \notin R$.

Q14

We have,

$$3x - y = 0$$
$$\Rightarrow \quad 3x = y$$
$$\Rightarrow \quad y = 3x$$

Putting $x = 1, 2, 3, 4$ we get, $y = 3, 6, 9, 12$ respectively

For $x > 4$, we get $y > 14$ which does not belong to set A.

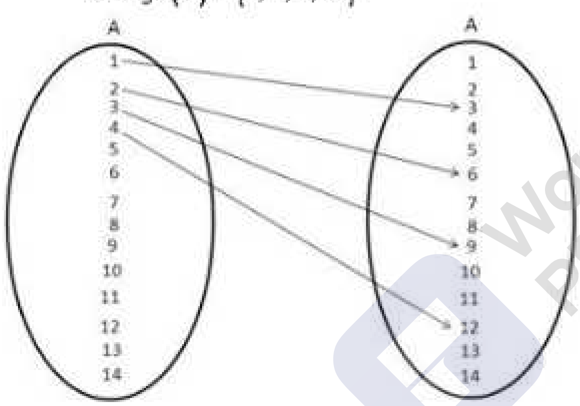
$$\therefore \quad R = \{(1,3),\ (2,6),\ (3,9),\ (4,12)\}$$

The arrow diagram representing R is as follows:

Clearly, Domain$(R) = \{1, 2, 3, 4\}$,

Co-domain$(R) = \{1, 2, 3, 4 \ldots, 14\}$ and

Range$(R) = \{3, 6, 9, 12\}$



Q15

We have,

$R = \{(x, y) : y = x + 5,\ x \text{ is a natural number less than } 4,\ x, y \in N\}$

(i) Putting $x = 1, 2, 3$ we get, $y = 6, 7, 8$ respectively

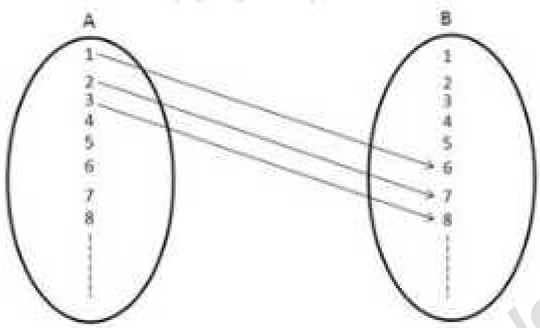
$\therefore$ Relation R in roster form is

$R = \{(1,6),\ (2,7),\ (3,8)\}$

(ii) The arrow diagram representing R is as follows:

Clearly, Domain$(R) = \{1,2,3\}$ and

Range$(R) = \{6,7,8\}$



Q16

We have,

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$

It is given that,

$R = \{(x, y) :$ the difference between x and y is odd, $x \in A,\ y \in B\}$

For the elements of the given sets A and B, we find that

$(1, 4) \in R,\ (1, 6) \in R,\ (2, 9) \in R,\ (3, 4) \in R,\ (3, 6) \in R,\ (5, 4) \in R$ and $(5, 6) \in R$

$\therefore\ R = \{(1, 4),\ (1,6),\ (2,9),\ (3,4),\ (3,6),\ (5,4),\ (5,6)\}$

Hence, relation R in roster form is $\{(1,4),\ (1,6),\ (2,9),\ (3,4),\ (3,6),\ (5,4),\ (5,6)\}$

Q17

We have,

$$R=\{(x,x^3): x \text{ is a prime number less than } 10\}$$

For the elements of the given sets, we find that

$$x = 2, 3, 5, 7$$

$\therefore \quad (2,8)\in R,\ (3,27)\in R,\ (5,125)\in R$ and $(7,343)\in R$

$\therefore$ Relation R in roster form is $=\{(2,8),\ (3,27),\ (5,125),\ (7,343)\}$

Q18

We have,

$$A=\{1,2,3,4,5,6\}$$

and, $R=\{(a,b)\ a,\ b\in A,\ b \text{ is exactly divisible by } a\}$

(i) Now, a/b stands for 'a divides b'. For the elements of the given sets A and A, we find that

$$1/1,\ 1/2,\ 1/3,\ 1/4,\ 1/5,\ 1/6,\ 2/2,\ 2/4,\ 2/6,\ 3/3,\ 3/6,\ 4/4,\ 5/5,\ 6/6$$

$\therefore$ Relation R in roster form is

$R=\{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),\ (1,6),\ (2,2),\ (2,4),\ (2,6),\ (3,3),\ (3,6),\ (4,4),\ (5,5),\ (6,6)\}$

(i) Domain$(R)=\{1,2,3,4,5,6\}$

(ii) Range$(R)=\{1,2,3,4,5,6\}$

Q19

(i) Set builder form of the relation from P to Q is

$$R=\{(x,y): y=x-2,\ x\in P,\ y\in Q\}$$

(ii) Roster form of the relation from P to Q is

$$R=\{(5,3),\ (6,4),\ (7,5)\}$$

Domain$(R)=\{5,6,7\}$
Range$(R)=\{3,4,5\}$

Q20

We have,

$R = \{(a,b) : a,\ b \in Z,\ a - b \text{ is an integer}\}$

Clearly, Domain$(R) = z$,

Range$(R) = z$.

Q21

Let $\left(1, \frac{-1}{2}\right) \in R_1$ and $\left(\frac{-1}{2}, -4\right) \in R_1$

$\Rightarrow \quad 1 + 1 \times \frac{-1}{2} > 0$ and $1 + \left(\frac{-1}{2}\right) - 4 > 0$

But, $1 + 1 \times (-4) = 1 - 4$

$= -3 < 0$

So, $(1, -4) \notin R_1$

Q22

We have,

$(a,b) R (c,d) \Leftrightarrow a + d = b + c$ for all (a,b), $(c,d) \in N \times N$

(i) We have,

$a + b = b + a$ for all $a, b \in N$

$\therefore \quad (a,b) R (a,b)$ for all, $a, b \in N$

(ii) Now,

$(a,b) R (c,d)$

$\Rightarrow \quad a + d = b + c$

$\Rightarrow \quad c + b = d + a$

$\Rightarrow \quad (c,d) R (a,b)$

(iii) Now,

$(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow \quad a + d = b + c$ and $c + f = d + e$

$\Rightarrow \quad a + d + c + f = b + c + d + e \qquad$ [Adding]

$\Rightarrow \quad a + f = b + e$

$\Rightarrow \quad (a,b) R (e,f)$