

Exercise 4.1

1. (i) All circles are (congruent, similar).
(ii) All squares are (similar, congruent).
(iii) All triangles are similar (isosceles, equilaterals):
(iv) Two triangles are similar, if their corresponding angles are (proportional, equal)
(v) Two triangles are similar, if their corresponding sides are (proportional, equal)
(vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are and (b) their corresponding sides are (equal, proportional).

Sol:

- (i) All circles are similar
(ii) All squares are similar
(iii) All equilateral triangles are similar
(iv) Two triangles are similar, if their corresponding angles are equal
(v) Two triangles are similar, if their corresponding sides are proportional
(vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.
2. Write the truth value (T/F) of each of the following statements:
(i) Any two similar figures are congruent.
(ii) Any two congruent figures are similar.
(iii) Two polygons are similar, if their corresponding sides are proportional.
(iv) Two polygons are similar if their corresponding angles are proportional.
(v) Two triangles are similar if their corresponding sides are proportional.
(vi) Two triangles are similar if their corresponding angles are proportional.

Sol:

- (i) False
(ii) True
(iii) False
(iv) False
(v) True
(vi) True

Exercise 4.2

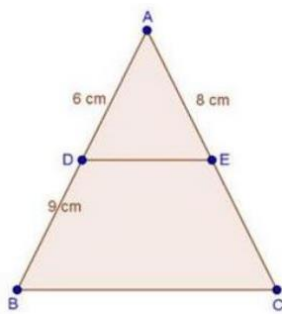
1. In $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$
(i) If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, find AC.
(ii) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm, find AE
(iii) If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18$ cm, find AE



- (iv) If $AD = 4$, $AE = 8$, $DB = x - 4$, and $EC = 3x - 19$, find x .
- (v) If $AD = 8\text{ cm}$, $AB = 12\text{ cm}$ and $AE = 12\text{ cm}$, find CE .
- (vi) If $AD = 4\text{ cm}$, $DB = 4.5\text{ cm}$ and $AE = 8\text{ cm}$, find AC .
- (vii) If $AD = 2\text{ cm}$, $AB = 6\text{ cm}$ and $AC = 9\text{ cm}$, find AE .
- (viii) If $\frac{AD}{BD} = \frac{4}{5}$ and $EC = 2.5\text{ cm}$, find AE .
- (ix) If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x .
- (x) If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = (3x - 1)$, find the value of x .
- (xi) If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$ and $CE = 5x - 3$, find the value of x .
- (xii) If $AD = 2.5\text{ cm}$, $BD = 3.0\text{ cm}$ and $AE = 3.75\text{ cm}$, find the length of AC .

Sol:

(i)



We have,

 $DE \parallel BC$

Therefore, by basic proportionality theorem,

$$\text{We have } \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{6}{9} = \frac{8}{EC}$$

$$\Rightarrow \frac{2}{3} = \frac{8}{EC}$$

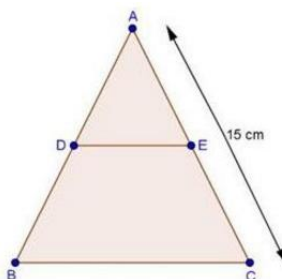
$$\Rightarrow EC = \frac{8 \times 3}{2}$$

$$\Rightarrow EC = 12\text{ cm}$$

$$\Rightarrow \text{Now, } AC = AE + EC = 8 + 12 = 20\text{ cm}$$

$$\therefore AC = 20\text{ cm}$$

(ii)





We have,

$$\frac{AD}{DB} = \frac{3}{4} \text{ and } DE \parallel BC$$

Therefore, by basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Adding 1 on both sides, we get

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\frac{3}{4} + 1 = \frac{AE+EC}{EC}$$

$$\Rightarrow \frac{3+4}{4} = \frac{AC}{EC} \quad [\because AE + EC = AC]$$

$$\Rightarrow \frac{7}{4} = \frac{15}{EC}$$

$$\Rightarrow EC = \frac{15 \times 4}{7}$$

$$\Rightarrow EC = \frac{60}{7}$$

Now, $AE + EC = AC$

$$\Rightarrow AE + \frac{60}{7} = 15$$

$$\Rightarrow AE = 15 - \frac{60}{7}$$

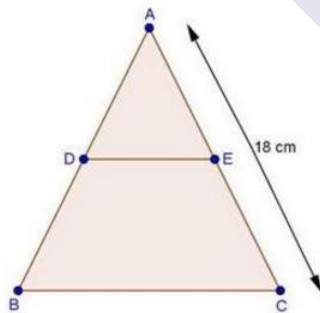
$$= \frac{105-60}{7}$$

$$= \frac{45}{7}$$

$$= 6.43 \text{ cm}$$

$$\therefore AE = 6.43 \text{ cm}$$

(iii)



We have,

$$\frac{AD}{DB} = \frac{2}{3} \text{ and } DE \parallel BC$$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{EC}{AE}$$

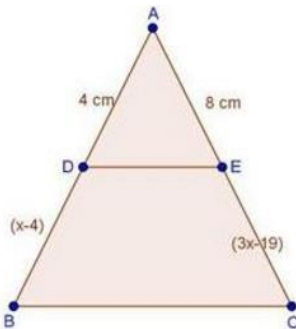
$$\Rightarrow \frac{3}{2} = \frac{EC}{AE}$$

Adding 1 on both sides, we get



$$\begin{aligned}\frac{3}{2} + 1 &= \frac{EC}{AE} + 1 \\ \Rightarrow \frac{3+2}{2} &= \frac{EC+AE}{AE} \\ \Rightarrow \frac{5}{2} &= \frac{AC}{AE} & [\because AE + EC = AC] \\ \Rightarrow \frac{5}{2} &= \frac{18}{AE} & [\because AC = 18] \\ \Rightarrow AE &= \frac{18 \times 2}{5} \\ \Rightarrow AE &= \frac{36}{5} = 7.2 \text{ cm}\end{aligned}$$

(iv)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

$$\Rightarrow 4(3x-19) = 8(x-4)$$

$$\Rightarrow 12x - 76 = 8x - 32$$

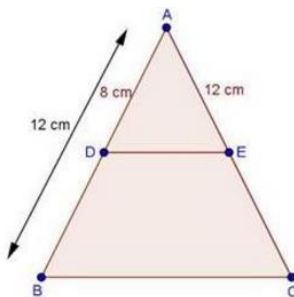
$$\Rightarrow 12x - 8x = -32 + 76$$

$$\Rightarrow 4x = 44$$

$$\Rightarrow x = \frac{44}{4} = 11 \text{ cm}$$

$$\therefore x = 11 \text{ cm}$$

(v)



We have,



$$AD = 8\text{ cm}, AB = 12\text{ cm}$$

$$\therefore BD = AB - AD$$

$$= 12 - 8$$

$$\Rightarrow BD = 4\text{ cm}$$

And, $DE \parallel BC$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{BD} = \frac{AE}{CE}$$

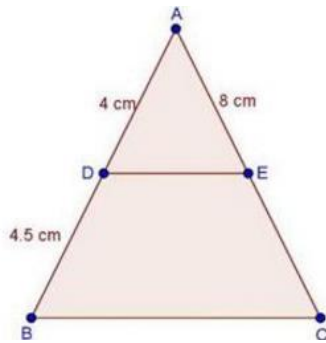
$$\Rightarrow \frac{8}{4} = \frac{12}{CE}$$

$$\Rightarrow CE = \frac{12 \times 4}{8} = \frac{12}{2}$$

$$\Rightarrow CE = 6\text{ cm}$$

$$\therefore CE = 6\text{ cm}$$

(vi)



We have,

$DE \parallel BC$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{4.5} = \frac{8}{EC}$$

$$\Rightarrow EC = \frac{8 \times 4.5}{4}$$

$$\Rightarrow EC = 9\text{ cm}$$

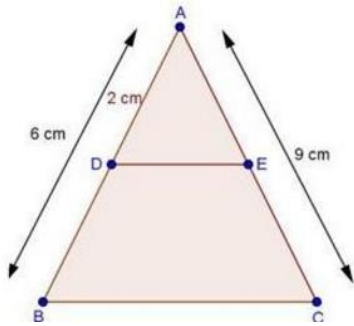
Now, $AC = AE + EC$

$$= 8 + 9$$

$$= 17\text{ cm}$$

$$\therefore AC = 17\text{ cm}$$

(vii)



We have,

$$AD = 2 \text{ cm}, AB = 6 \text{ cm}$$

$$\therefore DB = AB - AD$$

$$= 6 - 2$$

$$\Rightarrow DB = 4 \text{ cm}$$

And, $DE \parallel BC$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Taking reciprocal on both sides, we get,

$$\frac{DB}{AD} = \frac{EC}{AE}$$

$$\frac{4}{2} = \frac{EC}{AE}$$

Adding 1 on both sides, we get

$$\frac{4}{2} + 1 = \frac{EC}{AE} + 1$$

$$\Rightarrow \frac{4+2}{2} = \frac{EC+AE}{AE}$$

$$\Rightarrow \frac{6}{2} = \frac{AC}{AE}$$

$$[\because EC + AE = AC]$$

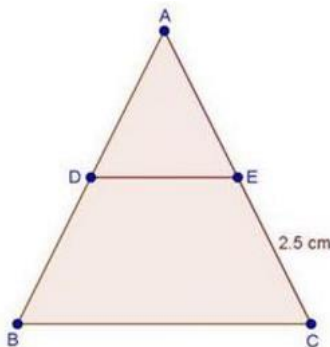
$$\Rightarrow \frac{6}{2} = \frac{9}{AE}$$

$$[\because AC = 9\text{cm}]$$

$$AE = \frac{9 \times 2}{6}$$

$$\Rightarrow AE = 3\text{cm}$$

(viii)



We have, $DE \parallel BC$



Therefore, by basic proportionality theorem,

We have,

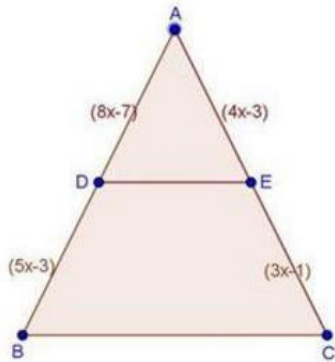
$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{5} = \frac{AE}{2.5}$$

$$\Rightarrow AE = \frac{4 \times 2.5}{5}$$

$$\Rightarrow AE = 2\text{cm}$$

(ix)



We have,

$DE \parallel BC$

Therefore, by basic proportionality theorem,

We have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow x(x-1) = (x+2)(x-2)$$

$$\Rightarrow x^2 - x = x^2 - (2)^2$$

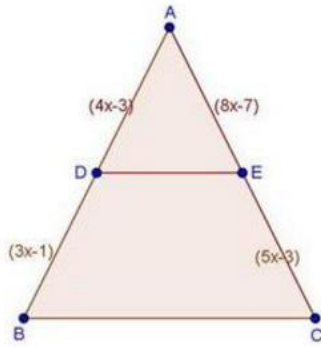
$$[\because (a-b)(a+b) = a^2 - b^2]$$

$$\Rightarrow -x = -4$$

$$\Rightarrow x = 4\text{ cm}$$

$$\therefore x = 4\text{ cm}$$

(x)



We have,

$DE \parallel BC$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (8x-7)(3x-1) = (4x-3)(5x-3)$$

$$\Rightarrow 24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$$

$$\Rightarrow 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$

$$\Rightarrow 2[2x^2 - x - 1] = 0$$

$$\Rightarrow 2x^2 - x - 1 = 0$$

$$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$$

$$\Rightarrow 2x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (2x+1)(x-1) = 0$$

$$\Rightarrow 2x+1 = 0 \text{ or } x-1 = 0$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } x = 1$$

$$x = -\frac{1}{2} \text{ is not possible}$$

$$\therefore x = 1$$

(xi)

We have, $DE \parallel BC$

Therefore, by basic proportionality theorem,

We have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$$

$$\Rightarrow 4x(5x-3) - 3(5x-3) = 8x(3x-1) - 7(3x-1)$$

$$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$



$$\Rightarrow 2(2x^2 - x - 1) = 0$$

$$\Rightarrow 2x^2 - x - 1 = 0$$

$$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$$

$$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$$

$$\Rightarrow (2x + 1)(x - 1) = 0$$

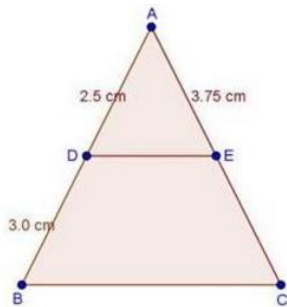
$$\Rightarrow 2x + 1 = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } x = 1$$

$$x = -\frac{1}{2} \text{ is not possible}$$

$$\therefore x = 1$$

(xii)



We have, $DE \parallel BC$

Therefore, by basic proportionality theorem, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2.5}{3.0} = \frac{3.75}{EC}$$

$$\Rightarrow EC = \frac{3.75 \times 3}{2.5} = \frac{375 \times 3}{250}$$

$$\Rightarrow EC = \frac{15 \times 3}{10}$$

$$= \frac{45}{10} = 4.5 \text{ cm}$$

$$\text{Now, } AC = AE + EC = 3.75 + 4.5 = 8.25$$

$$\therefore AC = 8.25 \text{ cm}$$

2. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

(i) $AB = 2\text{cm}$, $AD = 8\text{cm}$, $AE = 12\text{ cm}$ and $AC = 18\text{cm}$.

(ii) $AB = 5.6\text{cm}$, $AD = 1.4\text{cm}$, $AC = 7.2\text{ cm}$ and $AE = 1.8\text{ cm}$.

(iii) $AB = 10.8\text{ cm}$, $BD = 4.5\text{ cm}$, $AC = 4.8\text{ cm}$ and $AE = 2.8\text{ cm}$.

(iv) $AD = 5.7\text{ cm}$, $BD = 9.5\text{ cm}$, $AE = 3.3\text{ cm}$ and $EC = 5.5\text{ cm}$.

Sol:



$AB = 12 \text{ cm}$, $AD = 12 \text{ cm}$ and $AC = 18 \text{ cm}$.

$$DB = AB - AD$$

$$= 12 - 8$$

$$\therefore DB = 4 \text{ cm}$$

And, $EC = AC - AE$

$$= 18 - 12$$

$$\therefore EC = 6 \text{ cm}$$

$$\text{Now, } \frac{AD}{DB} = \frac{12}{4} = 3 \quad \text{[} DB = 4 \text{ cm]}$$

$$\text{And, } \frac{AE}{EC} = \frac{12}{6} = 2 \quad \text{[} EC = 6 \text{ cm]}$$

$$\therefore \frac{AD}{DB} \neq \frac{AE}{EC}$$

7 K X V ' (G L Y L G H V V L G H V \$ % D Q G \$ & R I " \$ % & L Q W K H V D P

Therefore, by the converse of basic proportionality theorem,

(ii)

We have, $DE \parallel BC$

We have,

$AB = 5.6 \text{ cm}$, $AD = 1.4 \text{ cm}$, $AC = 7.2 \text{ cm}$ and $AE = 1.8 \text{ cm}$

$$DB = AB - AD$$

$$= 5.6 - 1.4$$

$$\therefore DB = 4.2 \text{ cm}$$

And, $EC = AC - AE$

$$= 7.2 - 1.8$$

$$\therefore EC = 5.4 \text{ cm}$$



Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ [$\because DB = 4.2$ cm]

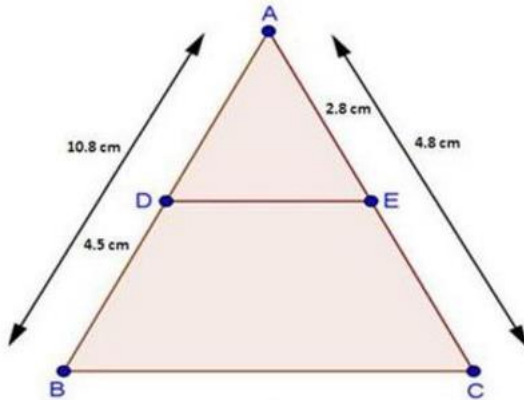
And, $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ [$\because EC = 5.4$ cm]

Thus, DE divides sides AB and AC of $\triangle ABC$ in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(iii)

We have,



We have,

$AB = 10.8$ cm, $BD = 4.5$ cm, $AC = 4.8$ cm and $AE = 2.8$ cm

$\therefore AD = AB - DB = 10.8 - 4.5$

$\Rightarrow AD = 6.3$ cm

And, $EC = AC - AE$

$= 4.8 - 2.8$

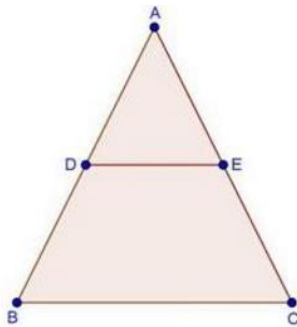
$\Rightarrow EC = 2$ cm

Now, $\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$ [$\because AD = 6.3$ cm]

And, $\frac{AE}{EC} = \frac{2.8}{2} = \frac{28}{20} = \frac{7}{5}$ [$\because EC = 2$ cm]

Thus, DE divides sides AB and AC of $\triangle ABC$ in the same ratio. Therefore, by the converse of basic proportionality theorem.

(iv)



We have,



$DE \parallel BC$

We have, $AD = 5.7$ cm, $BD = 9.5$ cm, $AE = 3.3$ cm and $EC = 5.5$ cm

$$\text{Now } \frac{AD}{BD} = \frac{5.7}{9.5} = \frac{9}{15}$$

$$\frac{AE}{EC} = \frac{3.3}{5.5} = \frac{3}{5}$$

$$\text{And, } \frac{AD}{BD} = \frac{AE}{EC} = \frac{3}{5}$$

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Therefore, by the converse of basic proportionality theorem. We have $DE \parallel BC$

3. In a $\triangle ABC$, P and Q are points on sides AB and AC respectively, such that $PQ \parallel BC$. If $AP = 2.4$ cm, $AQ = 2$ cm, $QC = 3$ cm and $BC = 6$ cm, find AB and PQ.

Sol:

We have $PQ \parallel BC$

Therefore, by BPT

We have,

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

$$\frac{2.4}{PB} = \frac{2}{3}$$

$$\Rightarrow PB = \frac{2.4 \times 3}{2} = \frac{7.2}{2} = 3.6$$

$$\Rightarrow PB = 3.6 \text{ cm}$$

Now, $AB = AP + PB$

$$= 2.4 + 3.6 = 6 \text{ cm}$$

$\triangle APQ \sim \triangle ABC$ [common]

$$\angle APQ = \angle ABC$$

[common]

$$\angle AQP = \angle ACB$$

[$PQ \parallel BC$ Corresponding angles are equal]

$\therefore \triangle APQ \sim \triangle ABC$ [By AA criteria]

$$\frac{AP}{AB} = \frac{AQ}{AC}$$

[corresponding sides of similar triangles are proportional]

$$\Rightarrow PQ = \frac{2.4 \times 6}{6} = 2.4$$

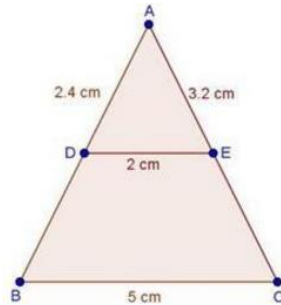
$$\Rightarrow PQ = 2.4 \text{ cm}$$



Hence, $AB = 6$ cm and $PO = 2.4$ cm

4. In a $\triangle ABC$, D and E are points on AB and AC respectively such that $DE \parallel BC$. If $AD = 2.4$ cm, $AE = 3.2$ cm, $DE = 2$ cm and $BC = 5$ cm, find BD and CE.

Sol:



We have,

$DE \parallel BC$

Now, In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{common}]$$

$$\angle ADE = \angle ABC \quad [\because DE \parallel BC \Rightarrow \text{Corresponding angles are equal}]$$

$$\Rightarrow \triangle ADE \sim \triangle ABC \quad [\text{By AA criteria}]$$

$$\Rightarrow \frac{AB}{BC} = \frac{AD}{DE} \quad [\text{corresponding sides of similar triangles are proportional}]$$

$$\Rightarrow AB = \frac{2.4 \times 5}{2}$$

$$\Rightarrow AB = 1.2 \times 5 = 6.0 \text{ cm}$$

$$\Rightarrow AB = 6 \text{ cm}$$

$$\therefore BD = 6 \text{ cm}$$

$$BD = AB - AD$$

$$= 6 - 2.4 = 3.6 \text{ cm}$$

$$\Rightarrow DB = 3.6 \text{ cm}$$

Now,

$$\frac{AC}{BC} = \frac{AE}{DE} \quad [\because \text{Corresponding sides of similar triangles are equal}]$$

$$\Rightarrow \frac{AC}{5} = \frac{3.2}{2}$$

$$\Rightarrow AC = \frac{3.2 \times 5}{2} = 1.6 \times 5 = 8.0 \text{ cm}$$

$$\Rightarrow AC = 8 \text{ cm}$$

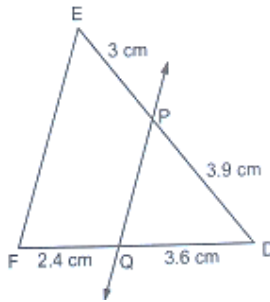
$$\therefore CE = AC - AE$$

$$= 8 - 3.2 = 4.8 \text{ cm}$$

Hence, $BD = 3.6$ cm and $CE = 4.8$ cm



5. In below Fig., state if $PQ \parallel EF$.



Sol:

We have,

$DP = 3.9$ cm, $PE = 3$ cm, $DQ = 3.6$ cm and $QF = 2.4$ cm

$$\text{Now, } \frac{DP}{PE} = \frac{3.9}{3} = \frac{1.3}{1} = \frac{13}{10}$$

$$\text{And, } \frac{DQ}{QF} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2}$$

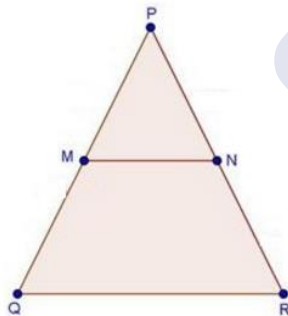
$$\Rightarrow \frac{DP}{PE} \neq \frac{DQ}{QF}$$

So, PQ is not parallel to EF

6. M and N are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $MN \parallel QR$

- (i) $PM = 4$ cm, $QM = 4.5$ cm, $PN = 4$ cm and $NR = 4.5$ cm

Sol:



- (i) We have, $PM = 4$ cm, $QM = 4.5$ cm, $PN = 4$ cm and $NR = 4.5$ cm

$$\text{Hence, } \frac{PM}{QM} = \frac{4}{4.5} = \frac{8}{9}$$

$$\text{Also, } \frac{PN}{NR} = \frac{4}{4.5} = \frac{8}{9}$$

$$\text{Hence, } \frac{PM}{QM} = \frac{PN}{NR}$$

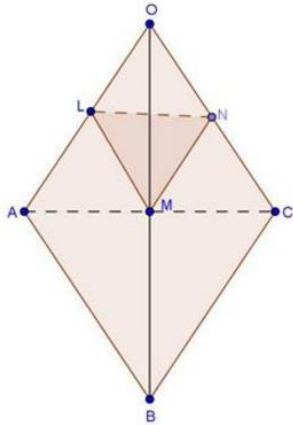
By converse of proportionality theorem

$MN \parallel QR$



7. In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that LM \parallel AB and MN \parallel BC but neither of L, M, N nor of A, B, C are collinear. Show that LN \parallel AC.

Sol:



We have,

LM \parallel AB and MN \parallel BC

Therefore, by basic proportionality theorem,

We have,

$$\frac{OL}{LA} = \frac{OM}{MB} \quad \dots(i)$$

$$\text{and, } \frac{ON}{NC} = \frac{OM}{MB} \quad \dots(ii)$$

Comparing equation (i) and equation (ii), we get,

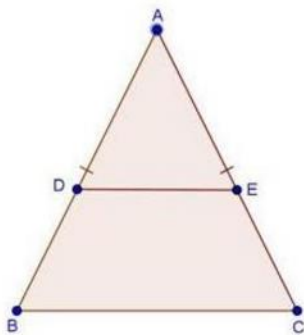
$$\frac{OL}{LA} = \frac{ON}{NC}$$

Thus, LN divides sides OA and OC of $\triangle OAC$ in the same ratio. Therefore, by the converse of basic proportionality theorem,

we have, LN \parallel AC

8. If D and E are points on sides AB and AC respectively of a $\triangle ABC$ such that DE \parallel BC and BD = CE. Prove that $\triangle ABC$ is isosceles.

Sol:





We have, $DE \parallel BC$

Therefore, by BPT, we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{DB} \quad [\because BD = CE]$$

$$\Rightarrow AD = AE$$

Adding DB on both sides

$$\Rightarrow AD + DB = AE + DB$$

$$\Rightarrow AD + DB = AE + EC \quad [\because BD = CE]$$

$$\Rightarrow AB = AC$$

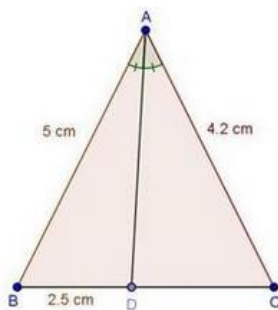
$$\Rightarrow \Delta ABC \text{ is isosceles}$$

Exercise 4.3

1. In a ΔABC , AD is the bisector of $\angle A$, meeting side BC at D.
 - (i) If $BD = 2.5\text{cm}$, $AB = 5\text{cm}$ and $AC = 4.2\text{cm}$, find DC.
 - (ii) If $BD = 2\text{cm}$, $AB = 5\text{cm}$ and $DC = 3\text{cm}$, find AC.
 - (iii) If $AB = 3.5\text{ cm}$, $AC = 4.2\text{ cm}$ and $DC = 2.8\text{ cm}$, find BD.
 - (iv) If $AB = 10\text{ cm}$, $AC = 14\text{ cm}$ and $BC = 6\text{ cm}$, find BD and DC.
 - (v) If $AC = 4.2\text{ cm}$, $DC = 6\text{ cm}$ and 10 cm , find AB
 - (vi) If $AB = 5.6\text{ cm}$, $AC = 6\text{cm}$ and $DC = 3\text{cm}$, find BC.
 - (vii) If $AD = 5.6\text{ cm}$, $BC = 6\text{cm}$ and $BD = 3.2\text{ cm}$, find AC.
 - (viii) If $AB = 10\text{cm}$, $AC = 6\text{ cm}$ and $BC = 12\text{ cm}$, find BD and DC.

Sol:

(i)



We have,

$$\angle BAD = \angle CAD$$

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{2.5}{DC} = \frac{5}{4.2}$$

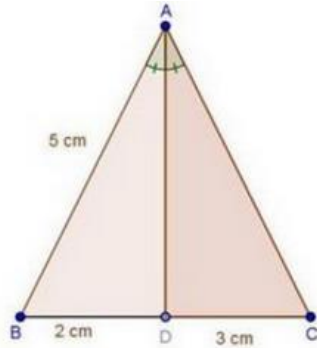


$$\Rightarrow DC = \frac{2.5 \times 4.2}{5}$$

$$= \frac{25 \times 42}{5 \times 100} = \frac{5 \times 42}{100} = \frac{210}{100} = 2.1 \text{ cm}$$

$$\therefore DC = 2.1 \text{ cm}$$

(ii)



We have,

AD is the bisector of $\angle A$

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

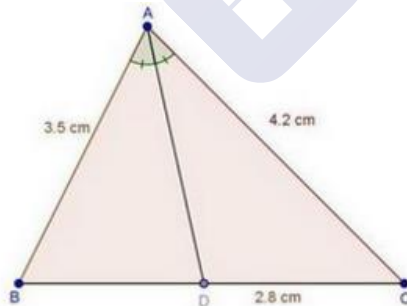
$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{2}{3} = \frac{5}{AC}$$

$$\Rightarrow AC = \frac{5 \times 3}{2} = \frac{15}{2}$$

$$\Rightarrow AC = 7.5 \text{ cm}$$

(iii)

In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{BD}{2.8} = \frac{3.5}{4.2}$$

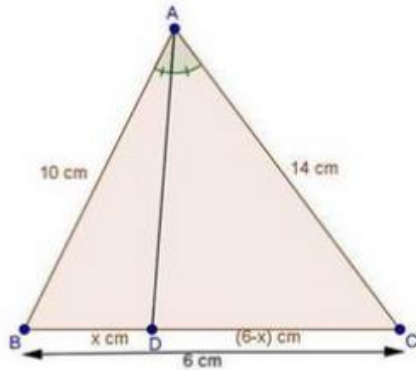
$$= \frac{3.5 \times 2}{3}$$



$$= \frac{7}{3} = 2.33 \text{ cm}$$

$$\therefore BD = 2.3 \text{ cm}$$

(iv)



In $\triangle ABC$, AD is the bisector of $\angle A$

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$$

$$\Rightarrow 14x = 10(6-x)$$

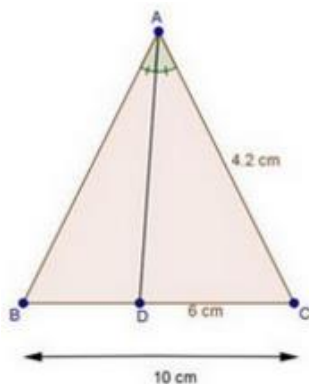
$$\Rightarrow 24x = 60$$

$$\Rightarrow x = \frac{60}{24} = \frac{5}{2} = 2.5 \text{ cm}$$

Since, $DC = 6 - x = 6 - 2.5 = 3.5 \text{ cm}$

Hence, $BD = 2.5 \text{ cm}$, and $DC = 3.5 \text{ cm}$

(v)



We have,

$BC = 10 \text{ cm}$, $DC = 6 \text{ cm}$ and $AC = 4.2 \text{ cm}$

$$\therefore BD = BC - DC = 10 - 6 = 4 \text{ cm}$$

$$\Rightarrow BD = 4 \text{ cm}$$

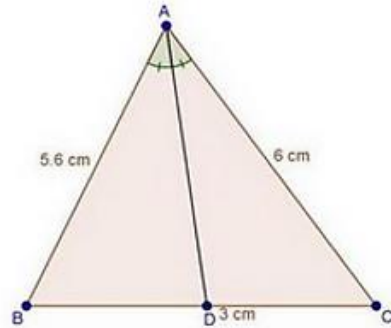


In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\begin{aligned}\therefore \frac{BD}{DC} &= \frac{AB}{AC} \\ \Rightarrow \frac{4}{6} &= \frac{AB}{4.2} \quad [\because BD = 4 \text{ cm}] \\ \Rightarrow AB &= 2.8 \text{ cm}\end{aligned}$$

(vi)

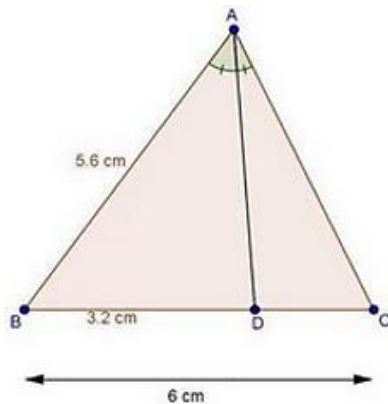


We have, In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\begin{aligned}\therefore \frac{BD}{DC} &= \frac{AB}{AC} \\ \Rightarrow \frac{BD}{3} &= \frac{5.6}{6} \\ \Rightarrow BD &= \frac{5.6 \times 3}{6} = \frac{5.6}{2} = 2.8 \text{ cm} \\ \Rightarrow BD &= 2.8 \text{ cm} \\ \text{Since, } BC &= BD + DC \\ &= 2.8 + 3 \\ &= 5.8 \text{ cm} \\ \therefore BC &= 5.8 \text{ cm}\end{aligned}$$

(vii)



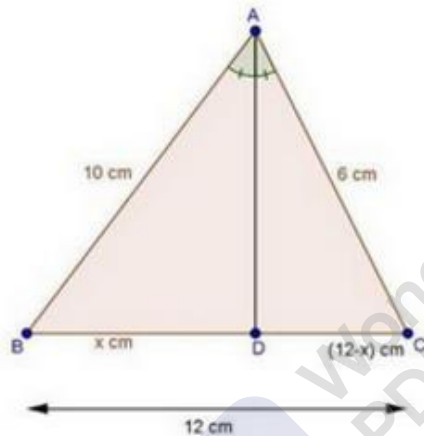
We have,

In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the containing the angle.

$$\begin{aligned}\therefore \frac{AB}{AC} &= \frac{BD}{DC} \\ \frac{5.6}{AC} &= \frac{3.2}{6-3.2} \quad [\because DC = BC - BD] \\ \Rightarrow \frac{5.6}{AC} &= \frac{3.2}{2.8} \\ \Rightarrow AC &= \frac{5.6 \times 2.8}{3.2} \\ &= \frac{5.6 \times 7}{8} = 0.7 \times 7 \\ &= 4.9 \text{ cm}\end{aligned}$$

(viii)



In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\begin{aligned}\therefore \frac{BD}{DC} &= \frac{AB}{AC} \\ \Rightarrow \frac{x}{12-x} &= \frac{10}{6} \\ \Rightarrow 6x &= 10(12-x) \\ \Rightarrow 6x &= 120 \\ \Rightarrow x &= \frac{120}{6} = 7.5 \text{ cm} \\ \therefore BD &= 7.5 \text{ cm and } DC = 12 - x = 12 - 7.5 = 4.5 \text{ cm} \\ \text{Hence, } BD &= 7.5 \text{ cm and } DC = 4.5 \text{ cm}\end{aligned}$$

2. In Fig. 4.57, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If AB = 10cm, AC = 6cm and BC = 12 cm, find CE.

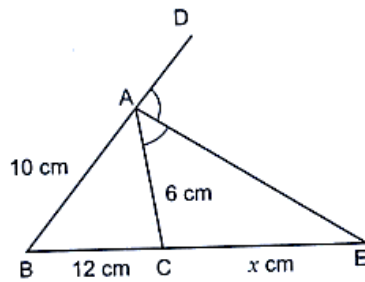


Fig. 4.57

Sol:

In $\triangle ABC$, AD is the bisector of $\angle A$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{12-x} = \frac{10}{6}$$

$$\Rightarrow 6(12 + x) = 10x$$

$$\Rightarrow 72 + 6x = 10x$$

$$\Rightarrow 4x = 72$$

$$\Rightarrow x = \frac{72}{4} = 18 \text{ cm}$$

$$\therefore CE = 18 \text{ cm}$$

3. In Fig. 4.58, $\triangle ABC$ is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70^\circ$, $\angle C = 50^\circ$. Find $\angle BAD$.

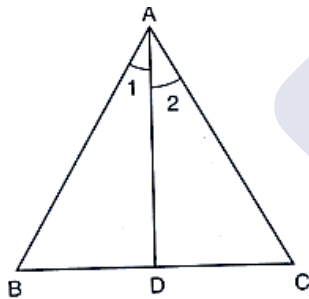


Fig. 4.58

Sol:

We have, if a line through one vertex of a triangle divides the opposite side in the ratio of the other two sides, then the line bisects the angle at the vertex.

$$\therefore \angle 1 = \angle 2$$

In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 70^\circ + 50^\circ = 180^\circ \quad [\because \angle B = 70^\circ \text{ and } \angle C = 50^\circ]$$

$$\Rightarrow \angle A = 180^\circ - 120^\circ = 60^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 60^\circ$$

$$\Rightarrow \angle 1 + \angle 1 = 60^\circ \quad [\because \angle 1 = \angle 2]$$



$$\Rightarrow 2\angle 1 = 60^\circ$$

$$\Rightarrow \angle 1 = 30^\circ$$

$$\therefore \angle BAD = 30^\circ$$

4. In $\triangle ABC$ (Fig., 4.59), if $\angle 1 = \angle 2$, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

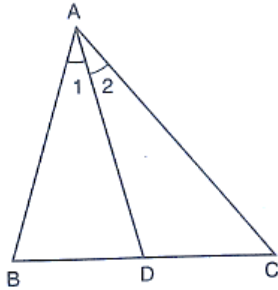
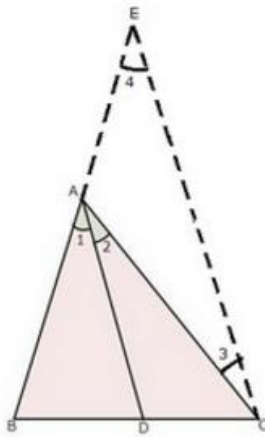


Fig. 4.59

Sol:



Given: A $\triangle ABC$ in which $\angle 1 = \angle 2$

To prove: $\frac{AB}{AC} = \frac{BD}{DC}$

Construction: Draw $CE \parallel DA$ to meet BA produced in E .

Proof: since, $CE \parallel DA$ and AC cuts them.

$$\therefore \angle 2 = \angle 3 \quad \dots (i) \quad [\text{Alternate angles}]$$

$$\text{And, } \angle 1 = \angle 4 \quad \dots (ii) \quad [\text{Corresponding angles}]$$

$$\text{But, } \angle 1 = \angle 2 \quad [\text{Given}]$$

From (i) and (ii), we get

$$\angle 3 = \angle 4$$

Thus, in $\triangle ACE$, we have

$$\angle 3 = \angle 4$$

$$\Rightarrow AE = AC \quad \dots (iii) \quad [\text{Sides opposite to equal angles are equal}]$$

Now, In $\triangle BCE$, we have

$$DA \parallel CE$$



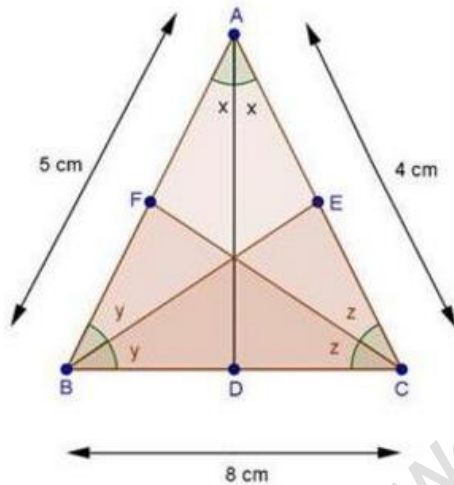
$$\Rightarrow \frac{BD}{DC} = \frac{BA}{AE} \quad [\text{Using basic proportionality theorem}]$$

$$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC} \quad [\because BA = AB \text{ and } AE = AC \text{ from (iii)}]$$

$$\text{Hence, } \frac{AB}{AC} = \frac{BD}{DC}$$

5. D, E and F are the points on sides BC, CA and AB respectively of $\triangle ABC$ such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If $AB = 5$ cm, $BC = 8$ cm and $CA = 4$ cm, determine AP, CE and BD.

Sol:



In $\triangle ABC$, CF bisects $\angle C$.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \frac{AF}{FB} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8} \quad [\because FB = AB - AF = 5 - AF]$$

$$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$$

$$\Rightarrow 2AF = 5 - AF$$

$$\Rightarrow 2AF + AF = 5$$

$$\Rightarrow 3AF = 5$$

$$\Rightarrow AF = \frac{5}{3} \text{ cm}$$

Again, In $\triangle ABC$, BE bisects $\angle B$.

$$\therefore \frac{AE}{EC} = \frac{AB}{BC}$$

$$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8} \quad [\because AE = AC - CE = 4 - CE]$$

$$\Rightarrow 8(4 - CE) = 5 \times CE$$

$$\Rightarrow 32 - 8CE = 5CE$$

$$\Rightarrow 32 = 13CE$$

$$\Rightarrow CE = \frac{32}{13} \text{ cm}$$

Similarly,

$$\frac{BD}{DC} = \frac{AD}{AC}$$

$$\Rightarrow \frac{BD}{8-BD} = \frac{5}{4} \quad [\because DC = BC - BD = 8 - BD]$$

$$\Rightarrow 4BD = 40 - 5BD$$

$$\Rightarrow 9BD = 40$$

$$\Rightarrow BD = \frac{40}{9} \text{ cm}$$

$$\text{Hence, } AF = \frac{5}{3} \text{ cm, } CE = \frac{32}{13} \text{ cm and } BD = \frac{40}{9} \text{ cm.}$$

6. In fig., 4.60, check whether AD is the bisector of $\angle A$ of $\triangle ABC$ in each of the following:

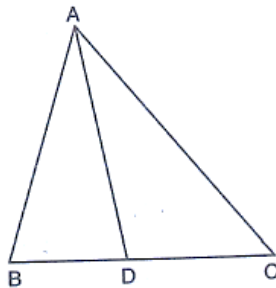


Fig. 4.60

- (i) $AB = 5\text{ cm}$, $AC = 10\text{ cm}$, $BD = 1.5\text{ cm}$ and $CD = 3.5\text{ cm}$
- (ii) $AB = 4\text{ cm}$, $AC = 6\text{ cm}$, $BD = 1.6\text{ cm}$ and $CD = 2.4\text{ cm}$
- (iii) $AB = 8\text{ cm}$, $AC = 24\text{ cm}$, $BD = 6\text{ cm}$ and $BC = 24\text{ cm}$
- (iv) $AB = 6\text{ cm}$, $AC = 8\text{ cm}$, $BD = 1.5\text{ cm}$ and $CD = 2\text{ cm}$.
- (v) $AB = 5\text{ cm}$, $AC = 12\text{ cm}$, $BD = 2.5\text{ cm}$ and $BC = 9\text{ cm}$

Sol:

Now,

$$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$$

$$\text{And, } \frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \frac{BD}{CD} \neq \frac{AB}{AC}$$

\Rightarrow AD is not the bisector of $\angle A$.

Now,

$$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$$

$$\text{And, } \frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$$

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$$

\Rightarrow AD is the bisector of $\angle A$.

$$\text{Now, } \frac{AB}{AC} = \frac{8}{24} = \frac{1}{3}$$



$$\text{And, } \frac{BD}{CD} = \frac{BD}{BC-BD} \quad [\because CD = BC - BD]$$

$$= \frac{BD}{24-6}$$

$$= \frac{6}{18}$$

$$= \frac{1}{3}$$

$$\therefore \frac{AB}{AC} = \frac{BD}{CD}$$

$\therefore AD$ is the bisector of $\angle A$ of $\triangle ABC$.

$$\frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$$

$$\text{And, } \frac{BD}{CD} = \frac{2.5}{BC-BD} \quad [\because CD = BC - BD]$$

$$= \frac{2.5}{9-2.5}$$

$$= \frac{2.5}{6.5}$$

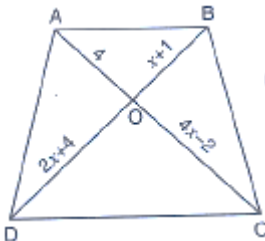
$$= \frac{1}{3}$$

$$\therefore \frac{AB}{AC} \neq \frac{BD}{CD}$$

$\therefore AD$ is not the bisector of $\angle A$ of $\triangle ABC$.

Exercise 4.4

1. (i) In below fig., If $AB \parallel CD$, find the value of x .



Sol:

Since diagonals of a trapezium divide each other proportionally.

$$\therefore \frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{4}{4x-2} = \frac{x+1}{2x+4}$$

$$\Rightarrow 4(2x+4) = (x+1)(4x-2)$$

$$\Rightarrow 8x+16 = x(4x-2) + 1(4x-2)$$

$$\Rightarrow 8x+16 = 4x^2+2x-2$$

$$\Rightarrow 4x^2+2x-8x-2-16=0$$

$$\Rightarrow 4x^2-6x-18=0$$

$$\Rightarrow 2[2x^2-3x-9]=0$$

$$\Rightarrow 2x^2-3x-9=0$$



$$\Rightarrow 2x(x - 3) + 3(x - 3) = 0$$

$$\Rightarrow (x - 3)(2x + 3) = 0$$

$$\Rightarrow x - 3 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow x = 3 \text{ or } x = -\frac{3}{2}$$

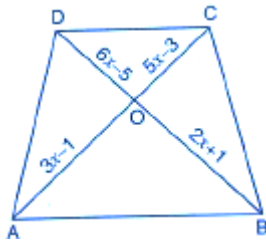
$$\Rightarrow x = 3 \text{ or } x = -\frac{3}{2}$$

$$x = -\frac{3}{2} \text{ is not possible, because } OB = x + 1 = -\frac{3}{2} + 1 = -\frac{1}{2}$$

Length cannot be negative

$$\therefore \frac{AO}{OC} = \frac{BO}{OD}$$

(ii) In the below fig., If $AB \parallel CD$, find the value of x .



$$\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$$

$$\Rightarrow (3x - 1)(6x - 5) = (2x + 1)(5x - 3)$$

$$\Rightarrow 3x(6x - 5) - 1(6x - 5) = 2x(5x - 3) + 1(5x - 3)$$

$$\Rightarrow 18x^2 - 15x - 6x + 5 = 10x^2 - 6x + 5x - 3$$

$$\Rightarrow 8x^2 - 20x + 8 = 0$$

$$\Rightarrow 4(2x^2 - 5x + 2) = 0$$

$$\Rightarrow 2x^2 - 4x - 1x + 2 = 0$$

$$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (2x - 1)(x - 2) = 0$$

$$\Rightarrow 2x - 1 = 0 \text{ or } x - 2 = 0$$

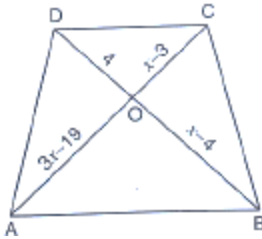
$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2$$

$$x = \frac{1}{2} \text{ is not possible, because, } OC = 5x - 3$$

$$= 5\left(\frac{1}{2}\right) - 3$$

$$= \frac{5-6}{2} = -\frac{1}{2}$$

(iii) In below fig., $AB \parallel CD$. If $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$, find x .



Since diagonals of a trapezium divide each other proportionally.

$$\therefore \frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{3x-19}{x-3} = \frac{x-4}{4}$$

$$\Rightarrow 4(3x-19) = (x-4)(x-3)$$

$$\Rightarrow 12x-76 = x(x-3)-4(x-3)$$

$$\Rightarrow 12x-76 = x^2-3x-4x+12$$

$$\Rightarrow x^2-7x-12x+12+76=0$$

$$\Rightarrow x^2-19x+88=0$$

$$\Rightarrow x^2-11x-8x+88=0$$

$$\Rightarrow x(x-11)-8(x-11)=0$$

$$\Rightarrow (x-11)(x-8)=0$$

$$\Rightarrow x-11=0 \text{ or } x-8=0$$

$$\Rightarrow x=11 \text{ or } x=8$$

Exercise 4.5

1. In fig. 4.136, $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, find CA and AQ .

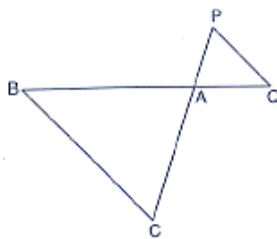


Fig. 4.136

Sol:

Given $\triangle ACB \sim \triangle APQ$

$$\text{Then, } \frac{AC}{AP} = \frac{BC}{PQ} = \frac{AB}{AQ}$$

[corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} \text{ and } \frac{8}{4} = \frac{6.5}{AQ}$$

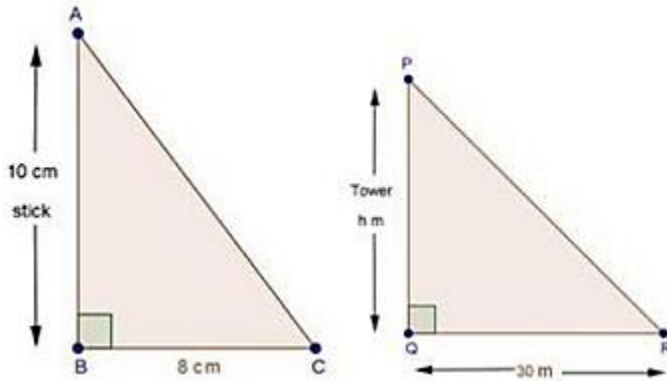
$$\Rightarrow AC = \frac{8}{4} \times 2.8 \text{ and } AQ = 6.5 \times \frac{4}{8}$$



$$\Rightarrow AC = 5.6 \text{ cm and } AQ = 3.25 \text{ cm}$$

2. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a shadow 30 m long. Determine the height of the tower.

Sol:



Length of stick = 10 cm

Length of shadow of stick = 8 cm

Length of shadow of tower = h cm

In $\triangle ABC$ and $\triangle PQR$

$$\angle B = \angle Q = 90^\circ$$

And, $\angle C = \angle R$ [Angular elevation of sun]

Then, $\triangle ABC \sim \triangle PQR$ [By AA similarity]

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\Rightarrow \frac{10 \text{ cm}}{8 \text{ cm}} = \frac{h \text{ cm}}{3000}$$

$$\Rightarrow h = \frac{10}{8} \times 3000 = 3750 \text{ cm} = 37.5 \text{ m}$$

3. In Fig. 4.137, $AB \parallel QR$. Find the length of PB.

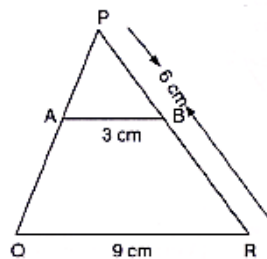


Fig. 4.137

Sol:

We have, $\triangle PAB$ and $\triangle PQR$

$$\angle P = \angle P \quad \text{[common]}$$

$$\angle PAB = \angle PQR \quad \text{[corresponding angles]}$$



Then, $\Delta PAB \sim \Delta PQR$

[By AA similarity]

$$\therefore \frac{PB}{PR} = \frac{AB}{QR}$$

[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{PB}{6} = \frac{3}{9}$$

$$\Rightarrow PB = \frac{3}{9} \times 6 = 2 \text{ cm}$$

4. In fig. 4.138, $XY \parallel BC$. Find the length of XY

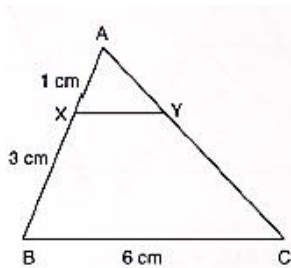


Fig. 4.138

Sol:

We have, $XY \parallel BC$

In ΔAXY and ΔABC

$$\angle A = \angle A$$

[common]

$$\angle AXY = \angle ABC$$

[corresponding angles]

Then, $\Delta AXY \sim \Delta ABC$

[By AA similarity]

$$\therefore \frac{AX}{AB} = \frac{XY}{BC}$$

[Corresponding parts of similar Δ are proportional]

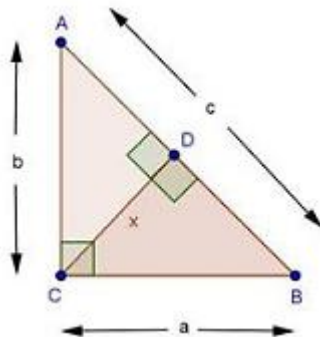
$$\Rightarrow \frac{1}{4} = \frac{XY}{6}$$

$$\Rightarrow XY = \frac{6}{4} = 1.5 \text{ cm}$$

5. In a right angled triangle with sides a and b and hypotenuse c , the altitude drawn on the hypotenuse is x . Prove that $ab = cx$.

Sol:

We have: $\angle C = 90^\circ$ and $CD \perp AB$



In ΔACB and ΔCDB



$$\begin{aligned}
 \angle B &= \angle B && \text{[common]} \\
 \angle ACB &= \angle CDB && \text{[Each } 90^\circ\text{]} \\
 \text{Then, } \triangle ACB &\sim \triangle CDB && \text{[By AA similarity]} \\
 \therefore \frac{AC}{CD} &= \frac{AB}{CB} && \text{[Corresponding parts of similar } \Delta \text{ are proportional]} \\
 \Rightarrow \frac{b}{x} &= \frac{c}{a} \\
 \Rightarrow ab &= cx
 \end{aligned}$$

6. In Fig. 4.139, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, find CD .

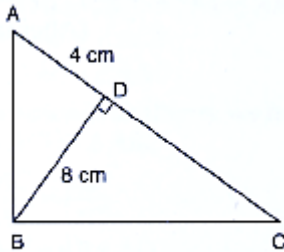


Fig. 4.139

Sol:

We have, $\angle ABC = 90^\circ$ and $BD \perp AC$

Now, $\angle ABD + \angle DBC = 90^\circ$... (i) [$\because \angle ABC = 90^\circ$]

And, $\angle C + \angle DBC = 90^\circ$... (ii) [By angle sum prop. in $\triangle BCD$]

Compare equations (i) & (ii)

$\angle ABD = \angle C$... (iii)

In $\triangle ABD$ and $\triangle BCD$

$\angle ABD = \angle C$ [From (iii)]

$\angle ADB = \angle BDC$ [Each 90°]

Then, $\triangle ABD \sim \triangle BCD$ [By AA similarity]

$\therefore \frac{BD}{CD} = \frac{AD}{BD}$ [Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{8}{CD} = \frac{4}{8}$$

$$\Rightarrow CD = \frac{8 \times 8}{4} = 16 \text{ cm}$$

7. In Fig. 4.14, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC .

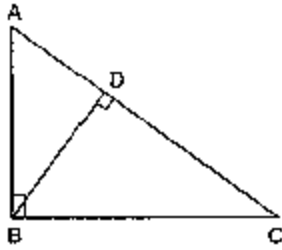


Fig. 4.140

Sol:

We have, $\angle ABC = 90^\circ$ and $BD \perp AC$

In $\triangle ABC$ and $\triangle BDC$

$$\angle ABC = \angle BDC \quad [\text{Each } 90^\circ]$$

$$\angle C = \angle C \quad [\text{Common}]$$

Then, $\triangle ABC \sim \triangle BDC$ [By AA similarity]

$$\therefore \frac{AB}{BD} = \frac{BC}{DC} \quad [\text{Corresponding parts of similar } \triangle \text{ are proportional}]$$

$$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$$

$$\Rightarrow BC = \frac{5.7}{3.8} \times 8.1 \text{ cm}$$

8. In Fig. 4.141, $DE \parallel BC$ such that $AE = \frac{1}{4} AC$. If $AB = 6$ cm, find AD .

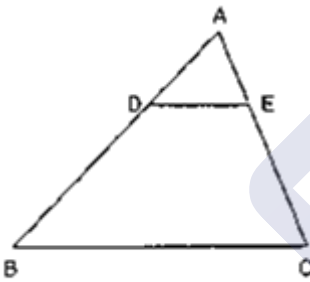


Fig. 4.141

Sol:

We have, $DE \parallel BC$, $AB = 6$ cm and $AE = \frac{1}{4} AC$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle ABC \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \quad [\text{Corresponding parts of similar } \triangle \text{ are proportional}]$$

$$\Rightarrow \frac{AD}{6} = \frac{\frac{1}{4}AC}{AC} \quad [\because AE = \frac{1}{4} AC \text{ given}]$$

$$\Rightarrow \frac{AD}{6} = \frac{1}{4}$$

$$\Rightarrow AD = \frac{6}{4} = 1.5 \text{ cm}$$

9. In fig., 4.142, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

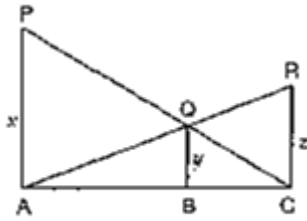


Fig. 4.142

Sol:

We have, $PA \perp AC$, $QB \perp AC$ and $RC \perp AC$

Let, $AB = a$ and $BC = b$

In ΔCQB and ΔCPA

$$\angle QCB = \angle PCA$$

[Common]

$$\angle QBC = \angle PAC$$

[Each 90°]

Then, $\Delta CQB \sim \Delta CPA$

[By AA similarity]

$$\therefore \frac{QB}{PA} = \frac{CB}{CA}$$

[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{y}{x} = \frac{b}{a+b}$$

....(i)

In ΔAQB and ΔARC

$$\angle QAB = \angle RAC$$

[common]

$$\angle ABQ = \angle ACR$$

[Each 90°]

Then, $\Delta AQB \sim \Delta ARC$

[By AA similarity]

$$\therefore \frac{QB}{RC} = \frac{AB}{AC}$$

[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{y}{z} = \frac{a}{a+b}$$

....(ii)

Adding equations (i) & (ii)

$$\frac{y}{x} + \frac{y}{z} = \frac{b}{a+b} + \frac{a}{a+b}$$

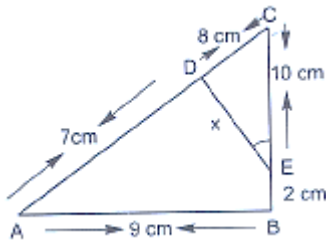
$$\Rightarrow y \left(\frac{1}{x} + \frac{1}{z} \right) = \frac{b+a}{a+b}$$

$$\Rightarrow y \left(\frac{1}{x} + \frac{1}{z} \right) = 1$$

$$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

10. In below fig., $\angle A = \angle CED$, Prove that $\Delta CAB \sim \Delta CED$. Also, find the value of x.





Sol:

We have, $\angle A = \angle CED$

In $\triangle CAB$ and $\triangle CED$

$$\angle C = \angle C$$

[Common]

$$\angle A = \angle CED$$

[Given]

Then, $\triangle CAB \sim \triangle CED$

[By AA similarity]

$$\therefore \frac{CA}{CE} = \frac{AB}{ED}$$

[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{15}{10} = \frac{9}{x}$$

$$\Rightarrow x = \frac{10 \times 9}{15} = 6 \text{ cm}$$

11. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Sol:

Assume ABC and PQR to be 2 triangles

We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Perimeter of } \triangle ABC = 25 \text{ cm}$$

$$\text{Perimeter of } \triangle PQR = 15 \text{ cm}$$

$$AB = 9 \text{ cm}$$

$$PQ = ?$$

$$\text{Since, } \triangle ABC \sim \triangle PQR$$

Then, ratio of perimeter of triangles = ratio of corresponding sides

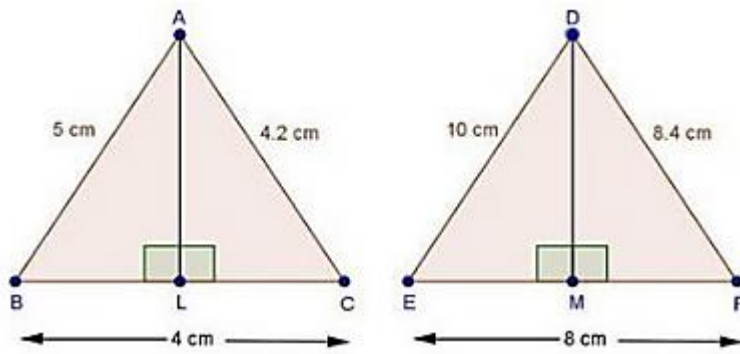
$$\Rightarrow \frac{25}{15} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{25}{15} = \frac{9}{PQ}$$

$$\Rightarrow PQ = \frac{15 \times 9}{25} = 5.4 \text{ cm}$$

12. In $\triangle ABC$ and $\triangle DEF$, it is being given that: $AB = 5 \text{ cm}$, $BC = 4 \text{ cm}$ and $CA = 4.2 \text{ cm}$; $DE = 10 \text{ cm}$, $EF = 8 \text{ cm}$ and $FD = 8.4 \text{ cm}$. If $AL \perp BC$ and $DM \perp EF$, find AL , DM .

Sol:



$$\text{Since, } \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$$

Then, $\triangle ABC \sim \triangle DEF$

[By SSS similarity]

Now, In $\triangle ABL \sim \triangle DEM$

$$\angle B = \angle E$$

[$\triangle ABC \sim \triangle DEF$]

$$\angle ALB = \angle DME$$

[Each 90°]

Then, $\triangle ABL \sim \triangle DEM$

[By AA similarity]

$$\therefore \frac{AB}{DE} = \frac{AL}{DM}$$

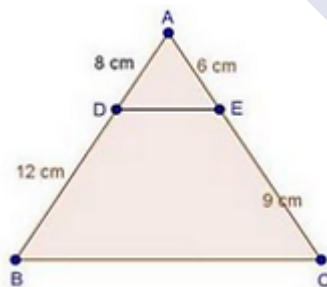
[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{5}{10} = \frac{AL}{DM}$$

$$\Rightarrow \frac{1}{2} = \frac{AL}{DM}$$

13. D and E are the points on the sides AB and AC respectively of a $\triangle ABC$ such that: AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that $BC = \frac{5}{2} DE$.

Sol:



We have,

$$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$$

$$\text{And, } \frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$$

$$\text{Since, } \frac{AD}{DB} = \frac{AE}{EC}$$

Then, by converse of basic proportionality theorem

$DE \parallel BC$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle B \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

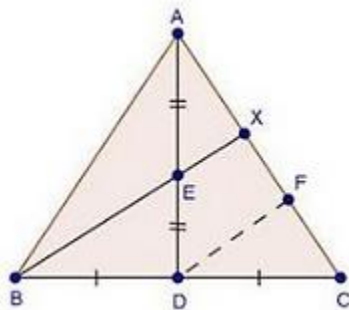
$$\Rightarrow \frac{8}{20} = \frac{DE}{BC}$$

$$\Rightarrow \frac{2}{5} = \frac{DE}{BC}$$

$$\Rightarrow BC = \frac{5}{2} DE$$

14. D is the mid-point of side BC of a $\triangle ABC$. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that $BE : EX = 3 : 1$

Sol:



Given: In $\triangle ABC$, D is the mid-point of BC and E is the mid-point of AD.

To prove: $BE : EX = 3 : 1$

Const: Through D, draw $DF \parallel BX$

Proof: In $\triangle EAX$ and $\triangle ADF$

$$\angle EAX = \angle ADF \quad [\text{Common}]$$

$$\angle AXE = \angle DAF \quad [\text{Corresponding angles}]$$

Then, $\triangle AEX \sim \triangle ADF$ [By AA similarity]

$$\therefore \frac{EX}{DF} = \frac{AE}{AD} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

$$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE} \quad [AE = ED \text{ given}]$$

$$\Rightarrow DF = 2EX \quad \dots (i)$$

In $\triangle CDF$ and $\triangle CBX$ [By AA similarity]

$$\therefore \frac{CD}{CB} = \frac{DF}{BX} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

$$\Rightarrow \frac{1}{2} = \frac{DF}{BE + EX} \quad [BD = DC \text{ given}]$$

$$\Rightarrow BE + EX = 2DF$$

$$\Rightarrow BE + EX = 4EX$$

$$\Rightarrow BE = 4EX - EX$$

$$\Rightarrow BE = 4EX - EX$$

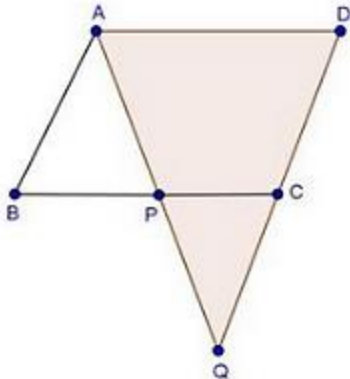
[By using (i)]



$$\Rightarrow \frac{BE}{EX} = \frac{3}{1}$$

15. ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the AB and BC.

Sol:



Given: ABCD is a parallelogram

To prove: $BP \times DQ = AB \times BC$

Proof: In $\triangle ABP$ and $\triangle QDA$

$$\angle B = \angle D$$

[Opposite angles of parallelogram]

$$\angle BAP = \angle AQD$$

[Alternate interior angles]

Then, $\triangle ABP \sim \triangle QDA$

[By AA similarity]

$$\therefore \frac{AB}{QD} = \frac{BP}{DA}$$

[Corresponding parts of similar Δ are proportional]

$$\text{But, } DA = BC$$

[Opposite sides of parallelogram]

$$\text{Then, } \frac{AB}{QD} = \frac{BP}{BC}$$

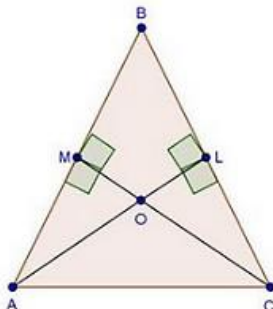
$$\Rightarrow AB \times BC = QD \times BP$$

16. In $\triangle ABC$, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O, prove that:

(i) $\triangle OMA$ and $\triangle OLC$

$$(ii) \frac{OA}{OC} = \frac{OM}{OL}$$

Sol:





We have,

$AL \perp BC$ and $CM \perp AB$

In $\triangle OMA$ and $\triangle OLC$

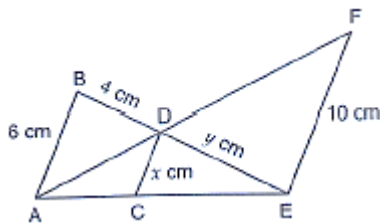
$\angle MOA = \angle LOC$ [Vertically opposite angles]

$\angle AMO = \angle CLO$ [Each 90°]

Then, $\triangle OMA \sim \triangle OLC$ [By AA similarity]

$\therefore \frac{OA}{OC} = \frac{OM}{OL}$ [Corresponding parts of similar \triangle are proportional]

17. In Fig below we have $AB \parallel CD \parallel EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm, calculate the values of x and y .



Sol:

We have $AB \parallel CD \parallel EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm

In $\triangle ECD$ and $\triangle EAB$

$\angle CED = \angle AEB$ [common]

$\angle ECD = \angle EAB$ [corresponding angles]

Then, $\triangle ECD \sim \triangle EAB$ (i) [By AA similarity]

$\therefore \frac{EC}{EA} = \frac{CD}{AB}$ [Corresponding parts of similar \triangle are proportional]

$\Rightarrow \frac{EC}{EA} = \frac{x}{6}$ (ii)

In $\triangle ACD$ and $\triangle AEF$

$\angle CAD = \angle EAF$ [common]

$\angle ACD = \angle AEF$ [corresponding angles]

Then, $\triangle ACD \sim \triangle AEF$ [By AA similarity]

$\therefore \frac{AC}{AE} = \frac{CD}{EF}$

$\Rightarrow \frac{AC}{AE} = \frac{x}{10}$ (iii)

Add equations (iii) & (ii)

$\therefore \frac{EC}{EA} + \frac{AC}{AE} = \frac{x}{6} + \frac{x}{10}$

$\Rightarrow \frac{AE}{AE} = \frac{5x+3x}{30}$

$\Rightarrow 1 = \frac{8x}{30}$

$\Rightarrow x = \frac{30}{8} = 3.75$ cm

From (i) $\frac{DC}{AB} = \frac{ED}{BE}$



$$\Rightarrow \frac{3.75}{6} = \frac{y}{y+4}$$

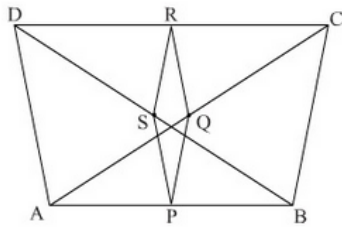
$$\Rightarrow 6y = 3.75y + 15$$

$$\Rightarrow 2.25y = 15$$

$$\Rightarrow y = \frac{15}{2.25} = 6.67 \text{ cm}$$

- 18.** ABCD is a quadrilateral in which $AD = BC$. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.

Sol:



$AD = BC$ and P, Q, R and S are the mid-points of sides AB, AC, CD and BD respectively, show that PQRS is a rhombus.

In $\triangle BAD$, by mid-point theorem

$$PS \parallel AD \text{ and } PS = \frac{1}{2} AD \quad \dots(i)$$

In $\triangle CAD$, by mid-point theorem

$$QR \parallel AD \text{ and } QR = \frac{1}{2} AD \quad \dots(ii)$$

Compare (i) and (ii)

$$PS \parallel QR \text{ and } PS = QR$$

Since one pair of opposite sides is equal as well as parallel then

$$PQRS \text{ is a parallelogram} \quad \dots(iii)$$

Now, In $\triangle ABC$, by mid-point theorem

$$PQ \parallel BC \text{ and } PQ = \frac{1}{2} BC \quad \dots(iv)$$

$$\text{And, } AD = BC \quad \dots(v) \text{ [given]}$$

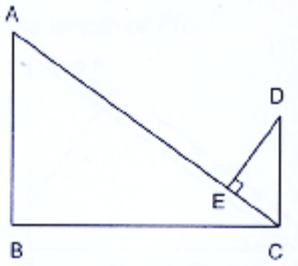
Compare equations (i) (iv) and (v)

$$PS = PQ \quad \dots(vi)$$

From (iii) and (vi)

Since, PQRS is a parallelogram with $PS = PQ$ then PQRS is a rhombus

- 19.** In Fig. below, if $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$, Prove that $\triangle CED \sim \triangle ABC$.



Sol:

Given: $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$

To prove: $\Delta CED \sim \Delta ABC$

Proof:

$$\angle BAC + \angle BCA = 90^\circ \quad \dots(i) \quad [\text{By angle sum property}]$$

$$\text{And, } \angle BCA + \angle ECD = 90^\circ \quad \dots(ii) \quad [DC \perp BC \text{ given}]$$

Compare equation (i) and (ii)

$$\angle BAC = \angle ECD \quad \dots(iii)$$

In ΔCED and ΔABC

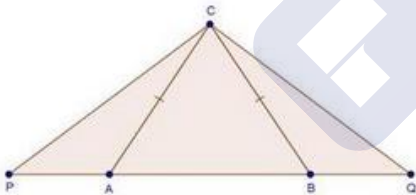
$$\angle CED = \angle ABC \quad [\text{Each } 90^\circ]$$

$$\angle ECD = \angle BAC \quad [\text{From (iii)}]$$

$$\text{Then, } \Delta CED \sim \Delta ABC \quad [\text{By AA similarity}]$$

20. In an isosceles ΔABC , the base AB is produced both the ways to P and Q such that $AP \times BQ = AC^2$. Prove that $\Delta APC \sim \Delta BCQ$.

Sol:



Given: In ΔABC , $CA = CB$ and $AP \times BQ = AC^2$

To prove: $\Delta APC \sim \Delta BCQ$

Proof:

$$AP \times BQ = AC^2 \quad [\text{Given}]$$

$$\Rightarrow AP \times BQ = AC \times AC$$

$$\Rightarrow AP \times BQ = AC \times BC \quad [AC = BC \text{ given}]$$

$$\Rightarrow \frac{AP}{BC} = \frac{AC}{BQ} \quad \dots(i)$$

$$\text{Since, } CA = CB \quad [\text{Given}]$$

$$\text{Then, } \angle CAB = \angle CBA \quad \dots(ii) \quad [\text{Opposite angles to equal sides}]$$

$$\text{Now, } \angle CAB + \angle CAP = 180^\circ \quad \dots(iii) \quad [\text{Linear pair of angles}]$$

$$\text{And, } \angle CBA + \angle CBQ = 180^\circ \quad \dots(iv) \quad [\text{Linear pair of angles}]$$

Compare equation (ii) (iii) & (iv)

$$\angle CAP = \angle CBQ \quad \dots(v)$$

In $\triangle APC$ and $\triangle BCQ$

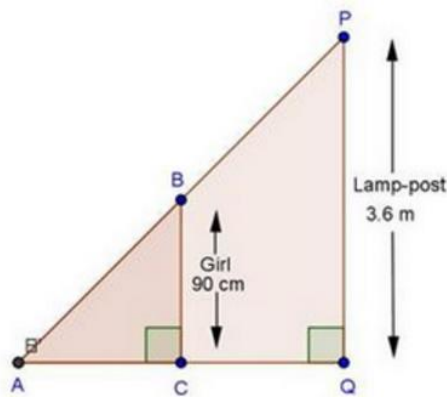
$$\angle CAP = \angle CBQ \quad [\text{From (v)}]$$

$$\frac{AP}{BC} = \frac{AC}{BQ} \quad [\text{From (i)}]$$

$$\text{Then, } \triangle APC \sim \triangle BCQ \quad [\text{By SAS similarity}]$$

21. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Sol:



We have,

Height of girl = 90 cm = 0.9 m

Height of lamp-post = 3.6 m

Speed of girl = 1.2 m/sec

\therefore Distance moved by girl (CQ) = Speed \times Time

$$= 1.2 \times 4 = 4.8\text{m}$$

Let length of shadow (AC) = x cm

In $\triangle ABC$ and $\triangle APQ$

$$\angle ACB = \angle AQP \quad [\text{Each } 90^\circ]$$

$$\angle BAC = \angle PAQ \quad [\text{Common}]$$

$$\text{Then, } \triangle ABC \sim \triangle APQ \quad [\text{By AA similarity}]$$

$$\therefore \frac{AC}{AQ} = \frac{BC}{PQ} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

$$\Rightarrow \frac{x}{x+4.8} = \frac{0.9}{3.6}$$

$$\Rightarrow \frac{x}{x+4.8} = \frac{1}{4}$$

$$\Rightarrow 4x = x + 4.8$$

$$\Rightarrow 4x - x = 4.8$$

$$\Rightarrow 3x = 4.8$$

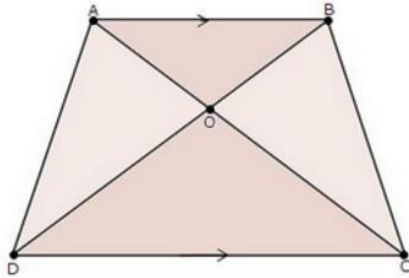
$$\Rightarrow x = \frac{4.8}{3} = 1.6\text{ m}$$



\therefore Length of shadow = 1.6m

22. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol:



We have,

ABCD is a trapezium with $AB \parallel DC$

In $\triangle AOB$ and $\triangle COD$

$$\angle AOB = \angle COD$$

[Vertically opposite angles]

$$\angle OAB = \angle OCD$$

[Alternate interior angles]

Then, $\triangle AOB \sim \triangle COD$

[By AA similarity]

$$\therefore \frac{OA}{OC} = \frac{OB}{OD}$$

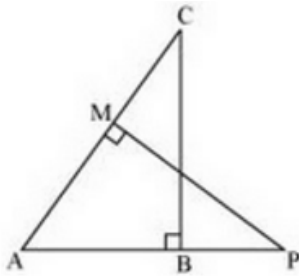
[Corresponding parts of similar Δ are proportional]

23. If $\triangle ABC$ and $\triangle AMP$ are two right triangles, right angled at B and M respectively such that $\angle MAP = \angle BAC$. Prove that

(i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:



We have,

$$\angle B = \angle M = 90^\circ$$

$$\text{And, } \angle BAC = \angle MAP$$

In $\triangle ABC$ and $\triangle AMP$

$$\angle B = \angle M$$

[Each 90°]

$$\angle BAC = \angle MAP$$

[Given]

Then, $\triangle ABC \sim \triangle AMP$

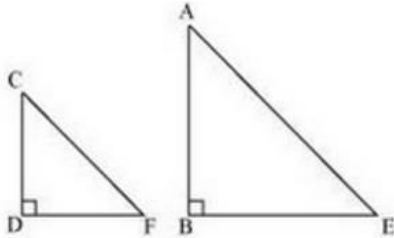
[By AA similarity]

$$\therefore \frac{CA}{PA} = \frac{BC}{MP}$$

[Corresponding parts of similar Δ are proportional]

24. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol:



Let AB be a tower

CD be a stick, $CD = 6\text{m}$

Shadow of AB is $BE = 28\text{m}$

Shadow of CD is $DF = 4\text{m}$

At same time light rays from sun will fall on tower and stick at same angle.

So, $\angle DCF = \angle BAE$

And $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$

(tower and stick are vertical to ground)

Therefore $\Delta ABE \sim \Delta CDF$ (By AA similarity)

So,

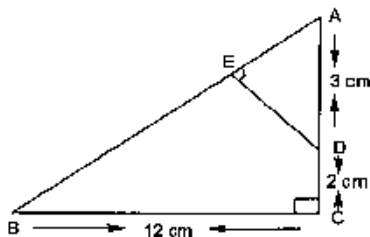
$$\frac{AB}{CD} = \frac{BE}{DF}$$

$$\frac{AB}{6} = \frac{28}{4}$$

$$AB = 28 \times \frac{6}{4} = 42\text{m}$$

So, height of tower will be 42 metres.

25. In below Fig., ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and Hence find the lengths of AE and DE.



Sol:

In ΔACB , by Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = (5)^2 + (12)^2$$



$$\Rightarrow AB^2 = 25 + 144 = 169$$

$$\Rightarrow AB = \sqrt{169} = 13 \text{ cm}$$

In $\triangle AED$ and $\triangle ACB$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle AED = \angle ACB \quad [\text{Each } 90^\circ]$$

Then, $\triangle AED \sim \triangle ACB$ [By AA similarity]

$$\therefore \frac{AE}{AC} = \frac{DE}{CB} = \frac{AD}{AB} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

$$\Rightarrow \frac{AE}{5} = \frac{DE}{12} = \frac{3}{13}$$

$$\Rightarrow \frac{AE}{5} = \frac{3}{13} \text{ and } \frac{DE}{12} = \frac{3}{13}$$

$$\Rightarrow AE = \frac{15}{13} \text{ cm and } DE = \frac{36}{13} \text{ cm}$$

Exercise 4.6

1. Triangles ABC and DEF are similar

(i) If area $(\triangle ABC) = 16 \text{ cm}^2$, area $(\triangle DEF) = 25 \text{ cm}^2$ and $BC = 2.3 \text{ cm}$, find EF .

(ii) If area $(\triangle ABC) = 9 \text{ cm}^2$, area $(\triangle DEF) = 64 \text{ cm}^2$ and $DE = 5.1 \text{ cm}$, find AB .

(iii) If $AC = 19 \text{ cm}$ and $DF = 8 \text{ cm}$, find the ratio of the area of two triangles.

(iv) If area $(\triangle ABC) = 36 \text{ cm}^2$, area $(\triangle DEF) = 64 \text{ cm}^2$ and $DE = 6.2 \text{ cm}$, find AB .

(v) If $AB = 1.2 \text{ cm}$ and $DE = 1.4 \text{ cm}$, find the ratio of the areas of $\triangle ABC$ and $\triangle DEF$.

Sol:

(i)

We have,

$$\triangle ABC \sim \triangle DEF$$

$$\text{Area } (\triangle ABC) = 16 \text{ cm}^2,$$

$$\text{Area } (\triangle DEF) = 25 \text{ cm}^2$$

$$\text{And } BC = 2.3 \text{ cm}$$

$$\text{Since, } \triangle ABC \sim \triangle DEF$$

$$\text{Then, } \frac{\text{Area } (\triangle ABC)}{\text{Area } (\triangle DEF)} = \frac{BC^2}{EF^2}$$

[By area of similar triangle theorem]

$$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$$

$$\Rightarrow \frac{4}{5} = \frac{2.3}{EF}$$

[By taking square root]

$$\Rightarrow EF = \frac{11.5}{4} = 2.875 \text{ cm}$$

(ii)

We have,

$$\triangle ABC \sim \triangle DEF$$

$$\text{Area } (\triangle ABC) = 9 \text{ cm}^2$$

$$\text{Area } (\triangle DEF) = 64 \text{ cm}^2$$



And $DE = 5.1$ cm

Since, $\triangle ABC \sim \triangle DEF$

Then, $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AB^2}{DE^2}$ [By area of similar triangle theorem]

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$$

$$\Rightarrow \frac{3}{8} = \frac{AB}{5.1} \quad \text{[By taking square root]}$$

$$\Rightarrow AB = \frac{3 \times 5.1}{8} = 1.9125 \text{ cm}$$

(iii)

We have,

$\triangle ABC \sim \triangle DEF$

$AC = 19$ cm and $DF = 8$ cm

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$$

We have,

$\triangle ABC \sim \triangle DEF$

$AC = 19$ cm and $DF = 8$ cm

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$$

(iv)

We have, $\text{Area}(\triangle ABC) = 36 \text{ cm}^2$

$\text{Area}(\triangle DEF) = 64 \text{ cm}^2$

$DE = 6.2$ cm

And, $\triangle ABC \sim \triangle DEF$

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2} \quad \text{[By taking square root]}$$

$$\Rightarrow AB = \frac{6 \times 6.2}{8} = 4.65 \text{ cm}$$

(v)

We have,

$\triangle ABC \sim \triangle DEF$

$AB = 1.2$ cm and $DF = 1.4$ cm

By area of similar triangle theorem

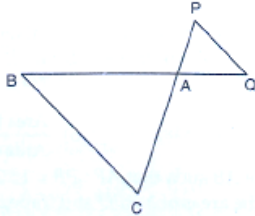
$$\begin{aligned} \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} &= \frac{AB^2}{DE^2} \\ &= \frac{(1.2)^2}{(1.4)^2} \end{aligned}$$



$$= \frac{1.44}{1.96}$$

$$= \frac{36}{49}$$

2. In fig. below $\triangle ACB \sim \triangle APQ$. If $BC = 10$ cm, $PQ = 5$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, find CA and AQ . Also, find the area $(\triangle ACB)$: $area(\triangle APQ)$



Sol:

We have,

$$\triangle ACB \sim \triangle APQ$$

Then, $\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$ [Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} = \frac{6.5}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} \text{ and } \frac{10}{5} = \frac{6.5}{AQ}$$

$$\Rightarrow AC = \frac{10}{5} \times 2.8 \text{ and } AQ = 6.5 \times \frac{5}{10}$$

$$\Rightarrow AC = 5.6 \text{ cm and } AQ = 3.25 \text{ cm}$$

By area of similar triangle theorem

$$\frac{Area(\triangle ACB)}{Area(\triangle APQ)} = \frac{BC^2}{PQ^2}$$

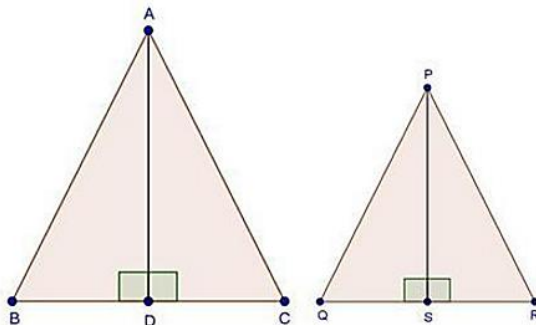
$$= \frac{(10)^2}{(5)^2}$$

$$= \frac{100}{25}$$

$$= \frac{4}{1}$$

3. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

Sol:





We have,

$$\Delta ABC \sim \Delta PQR$$

$$\text{Area}(\Delta ABC) = 81 \text{ cm}^2,$$

$$\text{Area}(\Delta PQR) = 49 \text{ cm}^2$$

And AD and PS are the altitudes

By area of similar triangle theorem

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{81}{49} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{9}{7} = \frac{AB}{PQ} \quad \dots(i) \quad [\text{Taking square root}]$$

In ΔABD and ΔPQS

$$\angle B = \angle Q \quad [\Delta ABC \sim \Delta PQR]$$

$$\angle ADB = \angle PSQ \quad [\text{Each } 90^\circ]$$

$$\text{Then, } \Delta ABD \sim \Delta PQS \quad [\text{By AA similarity}]$$

$$\therefore \frac{AB}{PQ} = \frac{AD}{PS} \quad \dots(ii) \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

Compare (1) and (2)

$$\frac{AD}{PS} = \frac{9}{7}$$

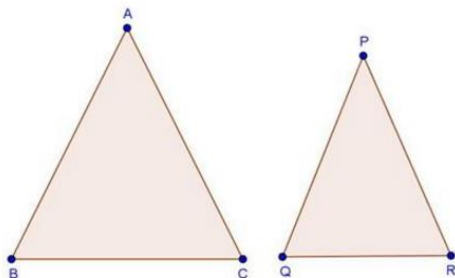
$$\therefore \text{Ratio of altitudes} = \frac{9}{7}$$

Since, the ratio of the area of two similar triangles is equal to the ratio of the squares of the squares of their corresponding altitudes and is also equal to the squares of their corresponding medians.

Hence, ratio of altitudes = Ratio of medians = 9 : 7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Sol:



We have,

$$\Delta ABC \sim \Delta PQR$$

$$\text{Area}(\Delta ABC) = 169 \text{ cm}^2$$

$$\text{Area}(\Delta PQR) = 121 \text{ cm}^2$$

$$\text{And } AB = 26 \text{ cm}$$



By area of similar triangle theorem

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

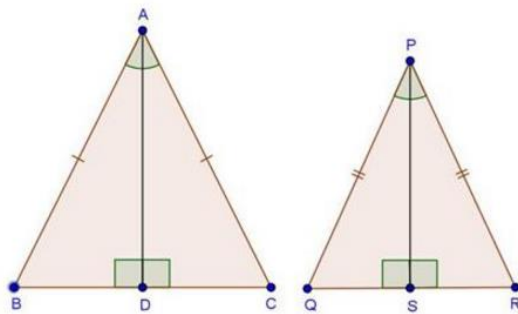
$$\Rightarrow \frac{169}{121} = \frac{(26)^2}{PQ^2}$$

$$\Rightarrow \frac{13}{11} = \frac{26}{PQ} \quad [\text{Taking square root}]$$

$$\Rightarrow PQ = \frac{11}{13} \times 26 = 22 \text{ cm}$$

5. Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Sol:



Given: $AB = AC$, $PQ = PR$ and $\angle A = \angle P$

And, AD and PS are altitudes

$$\text{And, } \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{36}{25} \quad \dots(i)$$

To find: $\frac{AD}{PS}$

Proof: Since, $AB = AC$ and $PQ = PR$

$$\text{Then, } \frac{AB}{AC} = 1 \text{ and } \frac{PQ}{PR} = 1$$

$$\therefore \frac{AB}{AC} = \frac{PQ}{PR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR} \quad \dots(ii)$$

In ΔABC and ΔPQR

$$\angle A = \angle P \quad [\text{Given}]$$

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad [\text{From (2)}]$$

$$\text{Then, } \Delta ABC \sim \Delta PQR \quad [\text{By SAS similarity}]$$

$$\therefore \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \dots(iii) \quad [\text{By area of similar triangle theorem}]$$

Compare equation (i) and (iii)

$$\frac{AB^2}{PQ^2} = \frac{36}{25}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{6}{5} \quad \dots(iv)$$



In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q$$

$$[\triangle ABC \sim \triangle PQR]$$

$$\angle ADB = \angle PSQ$$

$$[\text{Each } 90^\circ]$$

Then, $\triangle ABD \sim \triangle PQS$

[By AA similarity]

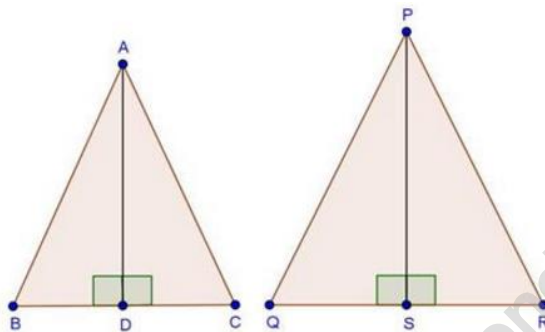
$$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$$

$$\Rightarrow \frac{6}{5} = \frac{AD}{PS}$$

[From (iv)]

6. The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm , find the corresponding altitude of the other.

Sol:



We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Area } (\triangle ABC) = 25 \text{ cm}^2$$

$$\text{Area } (\triangle PQR) = 36 \text{ cm}^2$$

$$AD = 2.4 \text{ cm}$$

And AD and PS are the altitudes

To find: PS

Proof: Since, $\triangle ABC \sim \triangle PQR$

Then, by area of similar triangle theorem

$$\frac{\text{Area } (\triangle ABC)}{\text{Area } (\triangle PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{25}{36} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{5}{6} = \frac{AB}{PQ} \quad \dots(i)$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q$$

$$[\triangle ABC \sim \triangle PQR]$$

$$\angle ADB = \angle PSQ$$

$$[\text{Each } 90^\circ]$$

Then, $\triangle ABD \sim \triangle PQS$

[By AA similarity]

$$\therefore \frac{AB}{PS} = \frac{AD}{PS}$$

....(ii) [Corresponding parts of similar Δ are proportional]

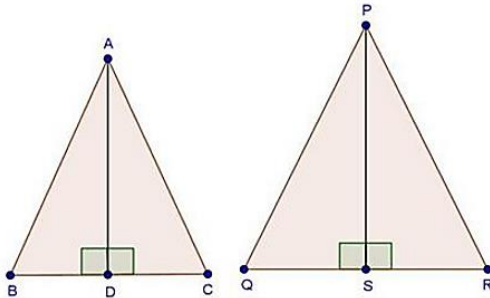
Compare (i) and (ii)



$$\begin{aligned}\frac{AD}{PS} &= \frac{5}{6} \\ \Rightarrow \frac{2.4}{PS} &= \frac{5}{6} \\ \Rightarrow PS &= \frac{2.4 \times 6}{5} = 2.88 \text{ cm}\end{aligned}$$

7. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Sol:



We have,

$$\Delta ABC \sim \Delta PQR$$

$$AD = 6 \text{ cm}$$

$$\text{And, } PS = 9 \text{ cm}$$

By area of similar triangle theorem

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \dots(i)$$

In ΔABD and ΔPQS

$$\angle B = \angle Q \quad [\Delta ABC \sim \Delta PQR]$$

$$\angle ADB = \angle PSQ \quad [\text{Each } 90^\circ]$$

Then, $\Delta ABD \sim \Delta PQS$ [By AA similarity]

$$\therefore \frac{AB}{PQ} = \frac{AD}{PS} \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

$$\Rightarrow \frac{AB}{PQ} = \frac{6}{9}$$

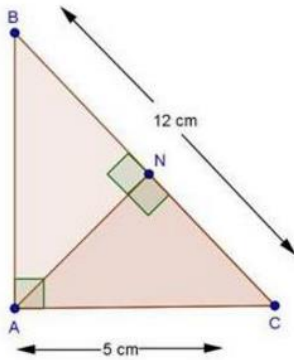
$$\Rightarrow \frac{AB}{PQ} = \frac{2}{3} \quad \dots(ii)$$

Compare equations (i) and (ii)

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

8. ABC is a triangle in which $\angle A = 90^\circ$, $AN \perp BC$, $BC = 12 \text{ cm}$ and $AC = 5 \text{ cm}$. Find the ratio of the areas of ΔANC and ΔABC .

Sol:



In $\triangle ANC$ and $\triangle ABC$

$$\angle C = \angle C \quad [\text{Common}]$$

$$\angle ANC = \angle BAC \quad [\text{Each } 90^\circ]$$

Then, $\triangle ANC \sim \triangle BAC$ [By AA similarity]

By area of similarity triangle theorem

$$\begin{aligned} \frac{\text{Area}(\triangle ANC)}{\text{Area}(\triangle BAC)} &= \frac{AN^2}{BC^2} \\ &= \frac{12^2}{5^2} \\ &= \frac{144}{25} \end{aligned}$$

9. In Fig. 4.178, $DE \parallel BC$

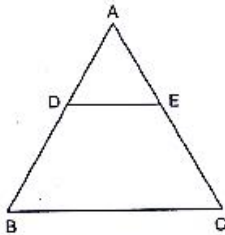


Fig. 4.178

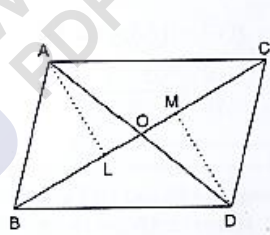


Fig. 4.179

- (i) If $DE = 4$ cm, $BC = 6$ cm and $\text{Area}(\triangle ADE) = 16$ cm², find the area of $\triangle ABC$.
- (ii) If $DE = 4$ cm, $BC = 8$ cm and $\text{Area}(\triangle ADE) = 25$ cm², find the area of $\triangle ABC$.
- (iii) If $DE : BC = 3 : 5$. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium BCED.

Sol:

We have, $DE \parallel BC$, $DE = 4$ cm, $BC = 6$ cm and $\text{area}(\triangle ADE) = 16$ cm²

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle ABC \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

\therefore By area of similar triangle theorem

$$\begin{aligned} \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} &= \frac{DE^2}{BC^2} \\ \Rightarrow \frac{16}{\text{Area}(\triangle ABC)} &= \frac{4^2}{6^2} \end{aligned}$$



$$\Rightarrow \text{Area}(\triangle ABC) = \frac{16 \times 36}{16} = 36 \text{ cm}^2$$

we have, $DE \parallel BC$, $DE = 4 \text{ cm}$, $BC = 8 \text{ cm}$ and $\text{area}(\triangle ADE) = 25 \text{ cm}^2$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle ABC \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{16}{\text{Area}(\triangle ABC)} = \frac{4^2}{8^2}$$

$$\Rightarrow \text{Area}(\triangle ABC) = \frac{16 \times 36}{16} = 36 \text{ cm}^2$$

We have, $DE \parallel BC$, $DE = 4 \text{ cm}$, $BC = 8 \text{ cm}$ and $\text{area}(\triangle ADE) = 25 \text{ cm}^2$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle ABC \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\frac{25}{\text{Area}(\triangle ABC)} = \frac{4^2}{8^2}$$

$$\Rightarrow \text{Area}(\triangle ABC) = \frac{25 \times 64}{16} = 100 \text{ cm}^2$$

We have, $DE \parallel BC$, and $\frac{DE}{BC} = \frac{3}{5} \dots (i)$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle B \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ADE) + \text{ar}(\text{trap. DECB})} = \frac{3^2}{5^2} \quad [\text{From (i)}]$$

$$\Rightarrow 25 \text{ar}(\triangle ADE) = 9 \text{ar}(\triangle ADE) + 9 \text{ar}(\text{trap. DECB})$$

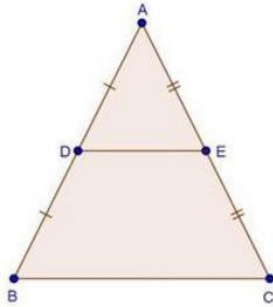
$$\Rightarrow 25 \text{ar}(\triangle ADE) - 9 \text{ar}(\triangle ADE) = 9 \text{ar}(\text{trap. DECB})$$

$$\Rightarrow 16 \text{ar}(\triangle ADE) = 9 \text{ar}(\text{trap. DECB})$$

$$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\text{trap. DECB})} = \frac{9}{16}$$

10. In $\triangle ABC$, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of $\triangle ADE$ and $\triangle ABC$

Sol:



We have, D and E as the mid-points of AB and AC

So, according to the mid-point theorem

$$DE \parallel BC \text{ and } DE = \frac{1}{2} BC \quad \dots(i)$$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle ADE = \angle B \quad [\text{Corresponding angles}]$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2} \quad [\text{From (i)}]$$

$$= \frac{\frac{1}{4}BC^2}{BC^2}$$

$$= \frac{1}{4}$$

11. In Fig., 4.179, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD and BC intersect at O, prove that $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$

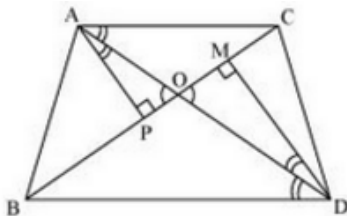
Sol:

We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{height}$

Since $\triangle ABC$ and $\triangle DBC$ are on the same base,

Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC.





In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each is 90°)

$\angle AOP = \angle DOM$ (vertically opposite angles)

$\angle OAP = \angle ODM$ (remaining angle)

Therefore $\triangle APO \sim \triangle DMO$ (By AAA rule)

Therefore $\frac{AP}{DM} = \frac{AO}{DO}$

Therefore $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$

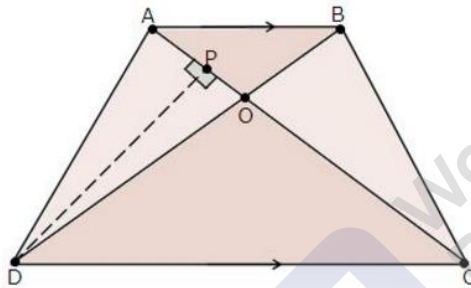
12. ABCD is a trapezium in which $AB \parallel CD$. The diagonals AC and BD intersect at O. Prove that: (i) $\triangle AOB$ and $\triangle COD$ (ii) If $OA = 6$ cm, $OC = 8$ cm,

Find:

(a) $\frac{\text{area}(\triangle AOB)}{\text{area}(\triangle COD)}$

(b) $\frac{\text{area}(\triangle AOD)}{\text{area}(\triangle COD)}$

Sol:



We have,

$AB \parallel DC$

In $\triangle AOB$ and $\triangle COD$

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate interior angles]

Then, $\triangle AOB \sim \triangle COD$ [By AA similarity]

(a) By area of similar triangle theorem

$$\frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{OA^2}{OC^2} = \frac{6^2}{8^2} = \frac{36}{64} = \frac{9}{16}$$

(b) Draw $DP \perp AC$

$$\therefore \frac{\text{area}(\triangle AOD)}{\text{area}(\triangle COD)} = \frac{\frac{1}{2} \times AO \times DP}{\frac{1}{2} \times CO \times DP}$$

$$= \frac{AO}{CO}$$

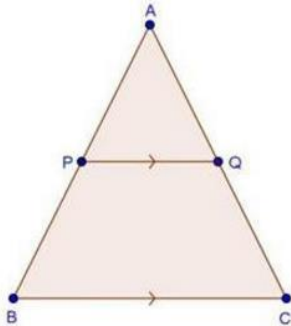
$$= \frac{6}{8}$$

$$= \frac{3}{4}$$



13. In $\triangle ABC$, P divides the side AB such that $AP : PB = 1 : 2$. Q is a point in AC such that $PQ \parallel BC$. Find the ratio of the areas of $\triangle APQ$ and trapezium BPQC.

Sol:



We have,

$PQ \parallel BC$

$$\text{And } \frac{AP}{PB} = \frac{1}{2}$$

In $\triangle APQ$ and $\triangle ABC$

$\angle A = \angle A$ [Common]

$\angle APQ = \angle B$ [Corresponding angles]

Then, $\triangle APQ \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{AP^2}{AB^2}$$

$$\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle APQ) + \text{ar}(\text{trap. BPQC})} = \frac{1^2}{3^2} \left[\frac{AP}{PB} = \frac{1}{2} \right]$$

$$\Rightarrow 9\text{ar}(\triangle APQ) = \text{ar}(\triangle APQ) + \text{ar}(\text{trap. BPQC})$$

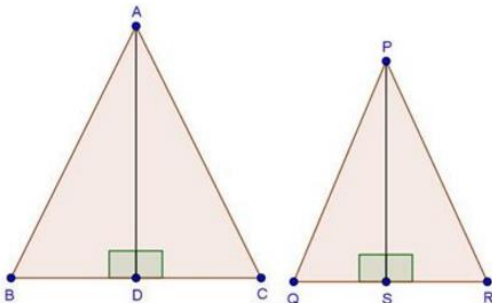
$$\Rightarrow 9\text{ar}(\triangle APQ) - \text{ar}(\triangle APQ) = \text{ar}(\text{trap. BPQC})$$

$$\Rightarrow 8\text{ar}(\triangle APQ) = \text{ar}(\text{trap. BPQC})$$

$$\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\text{trap. BPQC})} = \frac{1}{8}$$

14. The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude the bigger triangle is 5 cm, find the corresponding altitude of the other.

Sol:



We have, $\triangle ABC \sim \triangle PQR$



$$\text{Area}(\Delta ABC) = 100 \text{ cm}^2,$$

$$\text{Area}(\Delta PQR) = 49 \text{ cm}^2$$

$$AD = 5 \text{ cm}$$

And AD and PS are the altitudes

By area of similar triangle theorem

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{100}{49} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{10}{7} = \frac{AB}{PQ} \quad \dots(i)$$

In ΔABD and ΔPQS

$$\angle B = \angle Q \quad [\Delta ABC \sim \Delta PQR]$$

$$\angle ADB = \angle PSQ \quad [\text{Each } 90^\circ]$$

Then, $\Delta ABD \sim \Delta PQS$ [By AA similarity]

$$\therefore \frac{AB}{PQ} = \frac{AD}{PS} \quad \dots(ii) \quad [\text{Corresponding parts of similar } \Delta \text{ are proportional}]$$

Compare (i) and (ii)

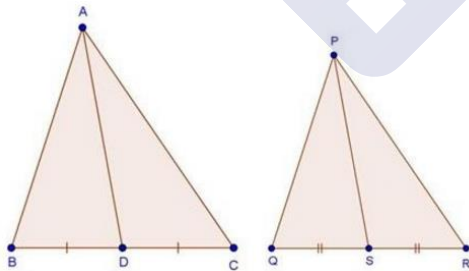
$$\frac{AD}{PS} = \frac{10}{7}$$

$$\Rightarrow \frac{5}{PS} = \frac{10}{7}$$

$$\Rightarrow PS = \frac{5 \times 7}{10} = 3.5 \text{ cm}$$

15. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm , find the corresponding median of the other.

Sol:



We have,

$$\Delta ABC \sim \Delta PQR$$

$$\text{Area}(\Delta ABC) = 121 \text{ cm}^2,$$

$$\text{Area}(\Delta PQR) = 64 \text{ cm}^2$$

$$AD = 12.1 \text{ cm}$$

And AD and PS are the medians

By area of similar triangle theorem

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$



$$\Rightarrow \frac{121}{64} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{11}{8} = \frac{AB}{PQ} \quad \dots(i)$$

Since, $\triangle ABC \sim \triangle PQR$

Then, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QS} \quad \text{[AD and PS are medians]}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS} \quad \dots(ii)$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q \quad [\triangle ABC \sim \triangle PQS]$$

$$\frac{AB}{PQ} = \frac{BD}{QS} \quad \text{[From (ii)]}$$

Then, $\triangle ABD \sim \triangle PQS$ [By SAS similarity]

$$\therefore \frac{AB}{PQ} = \frac{AD}{PS} \quad \dots(iii) \quad \text{[Corresponding parts of similar } \Delta \text{ are proportional]}$$

Compare (i) and (iii)

$$\frac{11}{8} = \frac{AD}{PS}$$

$$\Rightarrow \frac{11}{8} = \frac{12.1}{PS}$$

$$\Rightarrow PS = \frac{8 \times 12.1}{11}$$

$$\Rightarrow PS = \frac{8 \times 12.1}{11} = 8.8 \text{ cm}$$

16. If $\triangle ABC \sim \triangle DEF$ such that $AB = 5 \text{ cm}$, $\text{area}(\triangle ABC) = 20 \text{ cm}^2$ and $\text{area}(\triangle DEF) = 45 \text{ cm}^2$, determine DE .

Sol:

We have,

$\triangle ABC \sim \triangle DEF$ such that $AB = 5 \text{ cm}$,

$\text{Area}(\triangle ABC) = 20 \text{ cm}^2$ and $\text{area}(\triangle DEF) = 45 \text{ cm}^2$

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{20}{45} = \frac{5^2}{DE^2}$$

$$\Rightarrow \frac{4}{9} = \frac{5^2}{DE^2}$$

$$\Rightarrow \frac{2}{3} = \frac{5}{DE}$$

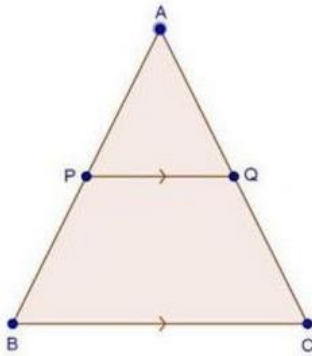
[Taking square root]

$$\Rightarrow DE = \frac{3 \times 5}{2} = 7.5 \text{ cm}$$

17. In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$ and PQ divides $\triangle ABC$ into two parts equal in area. Find $\frac{BP}{AB}$



Sol:



We have,

$PQ \parallel BC$

And $\text{ar}(\Delta APQ) = \text{ar}(\text{trap. PQCB})$

$\Rightarrow \text{ar}(\Delta APQ) = \text{ar}(\Delta ABC) - \text{ar}(\Delta APQ)$

$\Rightarrow 2\text{ar}(\Delta APQ) = \text{ar}(\Delta ABC) \quad \dots(i)$

In ΔAPQ and ΔABC

$\angle A = \angle A$ [common]

$\angle APQ = \angle B$ [corresponding angles]

Then, $\Delta APQ \sim \Delta ABC$ [By AA similarity]

\therefore By area of similar triangle theorem

$$\frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$\Rightarrow \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta APQ)} = \frac{AP^2}{AB^2} \quad [\text{By using (i)}]$$

$$\Rightarrow \frac{1}{2} = \frac{AP^2}{AB^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB}$$

[Taking square root]

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB - BP}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = 1 - \frac{BP}{AB}$$

$$= \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

18. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If $BC = 4.5$ cm, find the length of QR.

Sol:

We have,





$$\Delta ABC \sim \Delta PQR$$

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

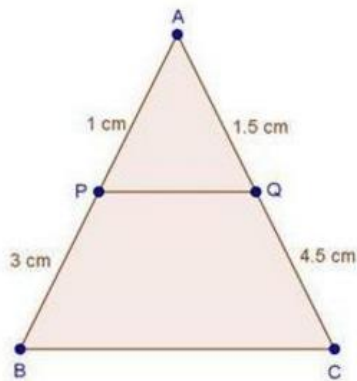
$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$\Rightarrow \frac{3}{4} = \frac{4.5}{QR} \quad [\text{Taking square root}]$$

$$\Rightarrow QR = \frac{4 \times 4.5}{3} = 6\text{ cm}$$

19. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQ is one- sixteenth of the area of ABC.

Sol:



We have,

AP = 1 cm, PB = 3 cm, AQ = 1.5 cm and QC = 4.5 m

In ΔAPQ and ΔABC

$$\angle A = \angle A \quad [\text{Common}]$$

$$\frac{AP}{AB} = \frac{AQ}{AC} \quad [\text{Each equal to } \frac{1}{4}]$$

Then, $\Delta APQ \sim \Delta ABC$ [By SAS similarity]

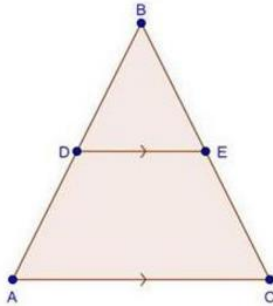
By area of similar triangle theorem

$$\frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{1^2}{4^2}$$

$$\Rightarrow \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{1}{16} \times \text{ar}(\Delta ABC)$$

20. If D is a point on the side AB of ΔABC such that AD : DB = 3:2 and E is a Point on BC such that DE \parallel AC. Find the ratio of areas of ΔABC and ΔBDE .

Sol:



We have,

$$\frac{AD}{DB} = \frac{3}{2}$$

$$\Rightarrow \frac{DB}{AD} = \frac{2}{3}$$

In $\triangle BDE$ and $\triangle BAC$

$$\angle B = \angle B$$

[common]

$$\angle BDE = \angle BAC$$

[corresponding angles]

Then, $\triangle BDE \sim \triangle BAC$

[By AA similarity]

By area of similar triangle theorem

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle BDE)} = \frac{AB^2}{BD^2}$$

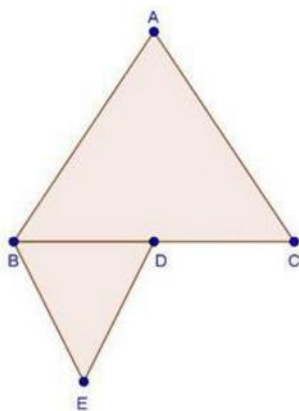
$$= \frac{5^2}{2^2}$$

$$= \frac{25}{4}$$

$$\left[\frac{AD}{DB} = \frac{3}{2} \right]$$

21. If $\triangle ABC$ and $\triangle BDE$ are equilateral triangles, where D is the mid-point of BC, find the ratio of areas of $\triangle ABC$ and $\triangle BDE$.

Sol:



We have,

$\triangle ABC$ and $\triangle BDE$ are equilateral triangles then both triangles are equiangular

$\therefore \triangle ABC \sim \triangle BDE$ [By AAA similarity]

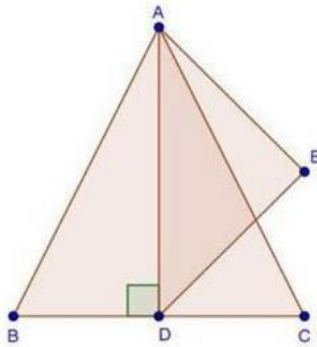
By area of similar triangle theorem



$$\begin{aligned}
 \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle BDE)} &= \frac{BC^2}{BD^2} \\
 &= \frac{2(BD)^2}{BD^2} && \text{[D is the mid-point of BC]} \\
 &= \frac{4BD^2}{BD^2} \\
 &= \frac{4}{1}
 \end{aligned}$$

22. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area ($\triangle ADE$): Area ($\triangle ABC$) = 3: 4

Sol:



We have,

$\triangle ABC$ is an equilateral triangle

Then, $AB = BC = AC$

Let, $AB = BC = AC = 2x$

Since, $AD \perp BC$ then $BD = DC = x$

In $\triangle ADB$, by Pythagoras theorem

$$AB^2 = (2x)^2 - (x)^2$$

$$\Rightarrow AD^2 = 4x^2 - x^2 = 3x^2$$

$$\Rightarrow AD = \sqrt{3}x \text{ cm}$$

Since, $\triangle ABC$ and $\triangle ADE$ both are equilateral triangles then they are equiangular

$\therefore \triangle ABC \sim \triangle ADE$ [By AA similarity]

By area of similar triangle theorem

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$= \frac{(\sqrt{3}x)^2}{(2x)^2}$$

$$= \frac{3x^2}{4x^2}$$

$$= \frac{3}{4}$$

**Exercise 4.7**

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.

Sol:

We have,

Sides of triangle

$$AB = 3 \text{ cm}$$

$$BC = 4 \text{ cm}$$

$$AC = 6 \text{ cm}$$

$$\therefore AB^2 = 3^2 = 9$$

$$BC^2 = 4^2 = 16$$

$$AC^2 = 6^2 = 36$$

$$\text{Since, } AB^2 + BC^2 \neq AC^2$$

Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2. The sides of certain triangles are given below. Determine which of them right triangles are.

(i) $a = 7 \text{ cm}$, $b = 24 \text{ cm}$ and $c = 25 \text{ cm}$

(ii) $a = 9 \text{ cm}$, $b = 16 \text{ cm}$ and $c = 18 \text{ cm}$

(iii) $a = 1.6 \text{ cm}$, $b = 3.8 \text{ cm}$ and $c = 4 \text{ cm}$

(iv) $a = 8 \text{ cm}$, $b = 10 \text{ cm}$ and $c = 6 \text{ cm}$

Sol:

We have,

$$a = 7 \text{ cm}, b = 24 \text{ cm} \text{ and } c = 25 \text{ cm}$$

$$\therefore a^2 = 49, b^2 = 576 \text{ and } c^2 = 625$$

$$\text{Since, } a^2 + b^2 = 49 + 576$$

$$= 625$$

$$= c^2$$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

We have,

$$a = 9 \text{ cm}, b = 16 \text{ cm} \text{ and } c = 18 \text{ cm}$$

$$\therefore a^2 = 81, b^2 = 256 \text{ and } c^2 = 324$$

$$\text{Since, } a^2 + b^2 = 81 + 256 = 337$$

$$\neq c^2$$

Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,

$$a = 1.6 \text{ cm}, b = 3.8 \text{ cm} \text{ and } C = 4 \text{ cm}$$

$$\therefore a^2 = 64, b^2 = 100 \text{ and } c^2 = 36$$

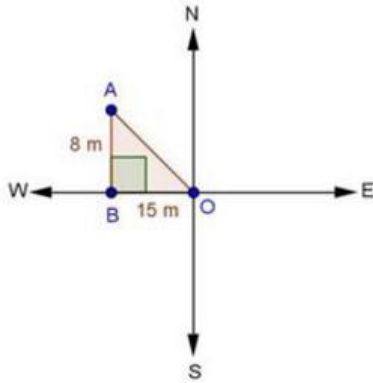
$$\text{Since, } a^2 + c^2 = 64 + 36 = 100 = b^2$$



Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Sol:



Let the starting point of the man be O and final point be A.

∴ In $\triangle ABO$, by Pythagoras theorem $AO^2 = AB^2 + BO^2$

$$\Rightarrow AO^2 = 8^2 + 15^2$$

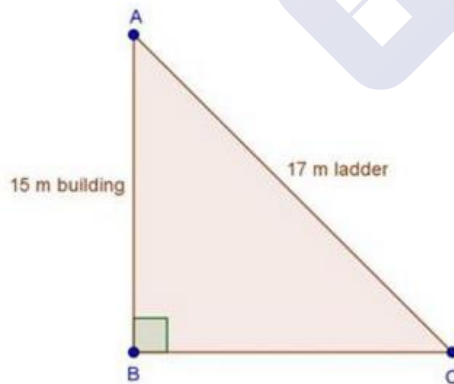
$$\Rightarrow AO^2 = 64 + 225 = 289$$

$$\Rightarrow AO = \sqrt{289} = 17m$$

∴ He is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Sol:



In $\triangle ABC$, by Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow 15^2 + BC^2 = 17^2$$

$$\Rightarrow 225 + BC^2 = 17^2$$

$$\Rightarrow BC^2 = 289 - 225$$

$$\Rightarrow BC^2 = 64$$

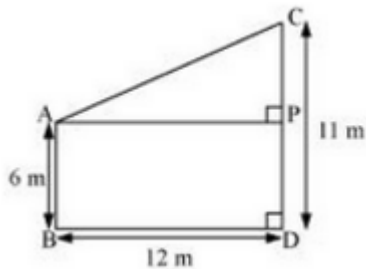


$$\Rightarrow BC = 8 \text{ m}$$

\therefore Distance of the foot of the ladder from building = 8 m

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



Let CD and AB be the poles of height 11 and 6 m.

Therefore $CP = 11 - 6 = 5 \text{ m}$

From the figure we may observe that $AP = 12 \text{ m}$

In triangle APC, by applying Pythagoras theorem

$$AP^2 + PC^2 = AC^2$$

$$12^2 + 5^2 = AC^2$$

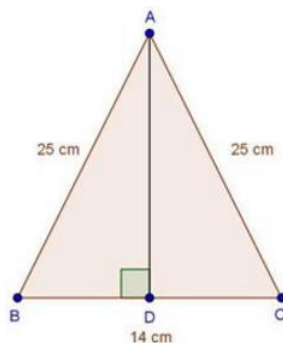
$$AC^2 = 144 + 25 = 169$$

$$AC = 13$$

Therefore distance between their tops = 13m.

6. In an isosceles triangle ABC, $AB = AC = 25 \text{ cm}$, $BC = 14 \text{ cm}$. Calculate the altitude from A on BC.

Sol:



We have

$AB = AC = 25 \text{ cm}$ and $BC = 14 \text{ cm}$

In $\triangle ABD$ and $\triangle ACD$

$$\angle ADB = \angle ADC \quad [\text{Each } 90^\circ]$$

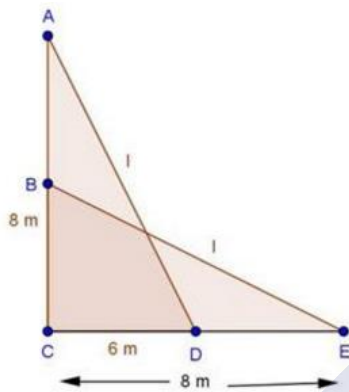
$$AB = AC \quad [\text{Each } 25 \text{ cm}]$$



$AD = AD$ [Common]
 Then, $\triangle ABD \cong \triangle ACD$ [By RHS condition]
 $\therefore BD = CD = 7 \text{ cm}$ [By c.p.c.t]
 In $\triangle ADB$, by Pythagoras theorem
 $AD^2 + BD^2 = AB^2$
 $\Rightarrow AD^2 + 7^2 = 25^2$
 $\Rightarrow AD^2 = 625 - 49 = 576$
 $\Rightarrow AD = \sqrt{576} = 24 \text{ cm}$

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Sol:



Let, length of ladder be $AD = BE = l \text{ m}$

In $\triangle ACD$, by Pythagoras theorem

$$AD^2 = AC^2 + CD^2$$

$$\Rightarrow l^2 = 8^2 + 6^2 \quad \dots(i)$$

In $\triangle BCE$, by pythagoras theorem

$$BE^2 = BC^2 + CE^2$$

$$\Rightarrow l^2 = BC^2 + 8^2 \quad \dots(ii)$$

Compare (i) and (ii)

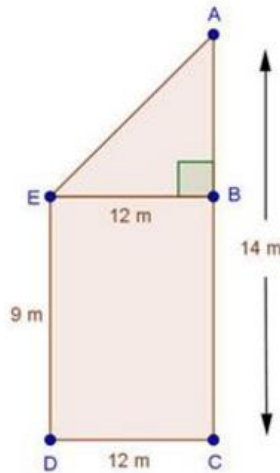
$$BC^2 + 8^2 = 8^2 + 6^2$$

$$\Rightarrow BC^2 = 6^2$$

$$\Rightarrow BC = 6 \text{ m}$$

8. Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



We have,

$AC = 14$ m, $DC = 12$ m and $ED = BC = 9$ m

Construction: Draw $EB \perp AC$

$\therefore AB = AC - BC = 14 - 9 = 5$ m

And, $EB = DC = 12$ m

In $\triangle ABE$, by Pythagoras theorem,

$$AE^2 = AB^2 + BE^2$$

$$\Rightarrow AE^2 = 5^2 + 12^2$$

$$\Rightarrow AE^2 = 25 + 144 = 169$$

$$\Rightarrow AE = \sqrt{169} = 13 \text{ m}$$

\therefore Distance between their tops = 13 m

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219

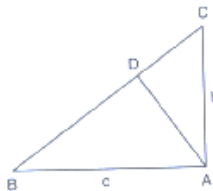


Fig. 4.219

Sol:

We have,

In $\triangle BAC$, by Pythagoras theorem

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC^2 = c^2 + b^2$$

$$\Rightarrow BC = \sqrt{c^2 + b^2} \quad \dots(i)$$

In $\triangle ABD$ and $\triangle CBA$

$$\angle B = \angle B$$

[Common]



$$\angle ADB = \angle BAC$$

[Each 90°]

$$\text{Then, } \triangle ABD \sim \triangle CBA$$

[By AA similarity]

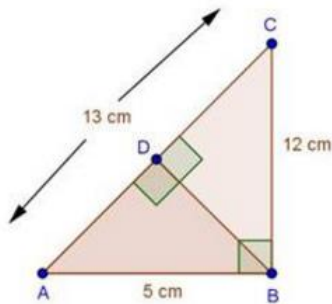
$$\therefore \frac{AB}{CB} = \frac{AD}{CA}$$

[Corresponding parts of similar Δ are proportional]

$$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$$

$$\Rightarrow AD = \frac{bc}{\sqrt{c^2+b^2}}$$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol:

Let, $AB = 5\text{ cm}$, $BC = 12\text{ cm}$ and $AC = 13\text{ cm}$. Then, $AC^2 = AB^2 + BC^2$. This proves that $\triangle ABC$ is a right triangle, right angles at B. Let BD be the length of perpendicular from B on AC.

$$\text{Now, Area } \triangle ABC = \frac{1}{2}(BC \times BA)$$

$$= \frac{1}{2}(12 \times 5)$$

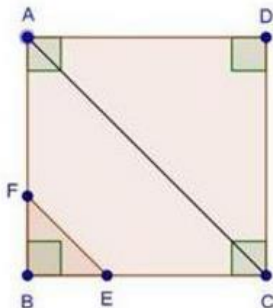
$$= 30\text{ cm}^2$$

$$\text{Also, Area of } \triangle ABC = \frac{1}{2}AC \times BD = \frac{1}{2}(13 \times BD)$$

$$\Rightarrow (13 \times BD) = 30 \times 2$$

$$\Rightarrow BD = \frac{60}{13}\text{ cm}$$

11. ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of $\triangle FBE = 108\text{ cm}^2$, find the length of AC.

Sol:



Since, ABCD is a square

Then, $AB = BC = CD = DA = x$ cm

Since, F is the mid-point of AB

Then, $AF = FB = \frac{x}{2}$ cm

Since, BE is one third of BC

Then, $BE = \frac{x}{3}$ cm

We have, area of $\triangle FBE = 108$ cm²

$$\Rightarrow \frac{1}{2} \times BE \times FB = 108$$

$$\Rightarrow \frac{1}{2} \times \frac{x}{3} \times \frac{x}{2} = 108$$

$$\Rightarrow x^2 = 108 \times 2 \times 3 \times 2$$

$$\Rightarrow x^2 = 1296$$

$$\Rightarrow x = \sqrt{1296} = 36 \text{ cm}$$

In $\triangle ABC$, by pythagoras theorem $AC^2 = AB^2 + BC^2$

$$\Rightarrow AC^2 = x^2 + x^2$$

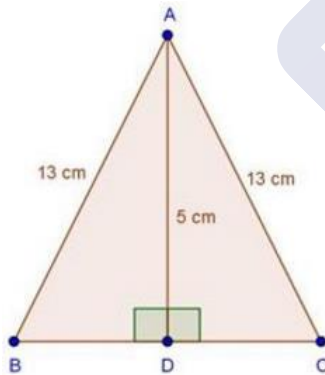
$$\Rightarrow AC^2 = 2x^2$$

$$\Rightarrow AC^2 = 2 \times (36)^2$$

$$\Rightarrow AC = 36\sqrt{2} = 36 \times 1.414 = 50.904 \text{ cm}$$

12. In an isosceles triangle ABC, if $AB = AC = 13$ cm and the altitude from A on BC is 5 cm, find BC.

Sol:



In $\triangle ADB$, by Pythagoras theorem

$$AD^2 + BD^2 = 13^2$$

$$\Rightarrow 25 + BD^2 = 169$$

$$\Rightarrow BD^2 = 169 - 25 = 144$$

$$\Rightarrow BD = \sqrt{144} = 12 \text{ cm}$$

In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC \quad [\text{Each } 90^\circ]$$

$$AB = AC \quad [\text{Each } 13 \text{ cm}]$$

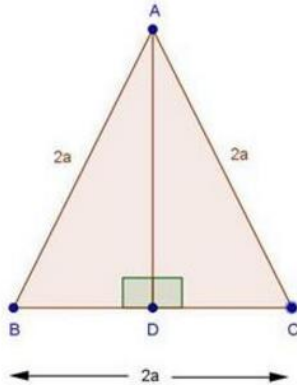


$AD = AD$ [Common]
 Then, $\triangle ADB \cong \triangle ADC$ [By RHS condition]
 $\therefore BD = CD = 12 \text{ cm}$ [By c.p.c.t]
 Hence, $BC = 12 + 12 = 24 \text{ cm}$

13. In a $\triangle ABC$, $AB = BC = CA = 2a$ and $AD \perp BC$. Prove that

(i) $AD = a\sqrt{3}$ (ii) $\text{Area}(\triangle ABC) = \sqrt{3} a^2$

Sol:

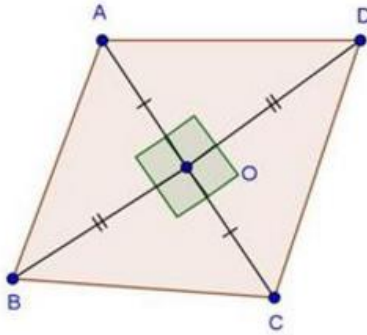


- (i) In $\triangle ABD$ and $\triangle ACD$
 $\angle ADB = \angle ADC$ [Each 90°]
 $AB = AC$ [Given]
 $AD = AD$ [Common]
 Then, $\triangle ABD \cong \triangle ACD$ [By RHS condition]
 $\therefore BD = CD = a$ [By c.p.c.t]
 In $\triangle ADB$, by Pythagoras theorem
 $AD^2 + BD^2 = AB^2$
 $\Rightarrow AD^2 + (a)^2 = (2a)^2$
 $\Rightarrow AD^2 + a^2 = 4a^2$
 $\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$
 $\Rightarrow AD = a\sqrt{3}$
- (ii) $\text{Area of } \triangle ABC = \frac{1}{2} \times BC \times AD$
 $= \frac{1}{2} \times 2a \times a\sqrt{3}$
 $= \sqrt{3}a^2$

14. The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.

Sol:





We have,

ABCD is a rhombus with diagonals $AC = 10$ cm and $BD = 24$ cm

We know that diagonal of a rhombus bisect each other at 90°

$\therefore AO = OC = 5$ cm and $BO = OD = 12$ cm

In $\triangle AOB$, by Pythagoras theorem

$$AB^2 = AO^2 + BO^2$$

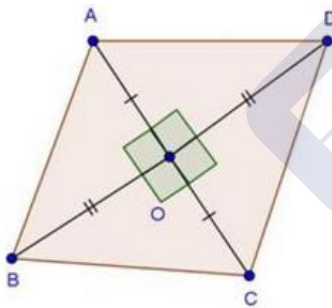
$$\Rightarrow AB^2 = 5^2 + 12^2$$

$$\Rightarrow AB^2 = 25 + 144 = 169$$

$$\Rightarrow AB = \sqrt{169} = 13 \text{ cm}$$

15. Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

Sol:



We have,

ABCD is a rhombus with side 10 cm and diagonal $BD = 16$ cm

We know that diagonals of a rhombus bisect each other at 90°

$\therefore BO = OD = 8$ cm

In $\triangle AOB$, by pythagoras theorem

$$AO^2 + BO^2 = AB^2$$

$$\Rightarrow AO^2 + 8^2 = 10^2$$

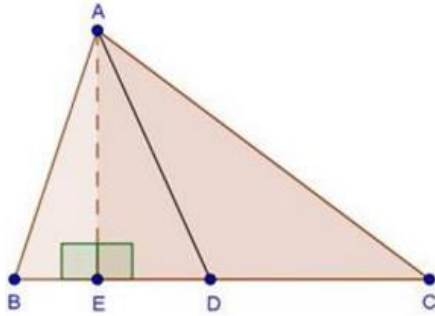
$$\Rightarrow AO^2 = 100 - 64 = 36$$

$$\Rightarrow AO = \sqrt{36} = 6 \text{ cm} \quad [\text{By above property}]$$

$$\text{hence, } AC = 6 + 6 = 12 \text{ cm}$$

16. In an acute-angled triangle, express a median in terms of its sides.

Sol:



We have,

In $\triangle ABC$, AD is a median.

Draw $AE \perp BC$

In $\triangle AEB$, by pythagoras theorem

$$AB^2 = AE^2 + BE^2$$

$$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2 \quad [\text{By Pythagoras theorem}]$$

$$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \times DE$$

$$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE \quad \dots(i) \quad [BC = 2BD \text{ given}]$$

Again, In $\triangle AEC$, by pythagoras theorem

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + CD)^2 \quad [\text{By Pythagoras theorem}]$$

$$\Rightarrow AC^2 = AD^2 + CD^2 + 2CD \times DE$$

$$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE \quad \dots(ii) \quad [BC = 2CD \text{ given}]$$

Add equations (i) and (ii)

$$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$$

$$\Rightarrow 2AB^2 + 2AC^2 = 4AD^2 + BC^2 \quad [\text{Multiply by 2}]$$

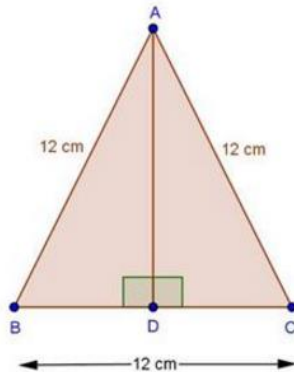
$$\Rightarrow 4AD^2 = 2AB^2 + 2AC^2 - BC^2$$

$$\Rightarrow AD^2 = \frac{2AB^2 + 2AC^2 - BC^2}{4}$$



17. Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Sol:



We have,

$\triangle ABC$ is an equilateral \triangle with side 12 cm.

Draw $AE \perp BC$

In $\triangle ABD$ and $\triangle ACD$

$\angle ADB = \angle ADC$

[Each 90°]

$AB = AC$

[Each 12 cm]

$AD = AD$

[Common]

Then, $\triangle ABD \cong \triangle ACD$

[By RHS condition]

$$\therefore AD^2 + BD^2 = AB^2$$

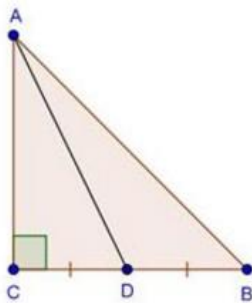
$$\Rightarrow AD^2 + 6^2 = 12^2$$

$$\Rightarrow AD^2 = 144 - 36 = 108$$

$$\Rightarrow AD = \sqrt{108} = 10.39 \text{ cm}$$

18. In right-angled triangle ABC in which $\angle C = 90^\circ$, if D is the mid-point of BC , prove that $AB^2 = 4 AD^2 - 3 AC^2$.

Sol:



We have,

$\angle C = 90^\circ$ and D is the mid-point of BC

In $\triangle ACB$, by Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 + (2CD)^2$$

[D is the mid-point of BC]

$$\begin{aligned}
 AB^2 &= AC^2 + 4CD^2 \\
 \Rightarrow AB^2 &= AC^2 + 4(AD^2 - AC^2) && [\text{In } \triangle ACD, \text{ by Pythagoras theorem}] \\
 \Rightarrow AB^2 &= AC^2 + 4AD^2 - 4AC^2 \\
 \Rightarrow AB^2 &= 4AD^2 - 3AC^2
 \end{aligned}$$

19. In Fig. 4.220, D is the mid-point of side BC and $AE \perp BC$. If $BC = a$, $AC = b$, $AB = c$, $ED = x$, $AD = p$ and $AE = h$, prove that:

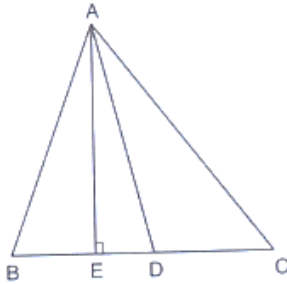


Fig. 4.220

- (i) $b^2 = p^2 + ax + \frac{a^2}{4}$
- (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$
- (iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Sol:

We have, D as the mid-point of BC

- (i) $AC^2 = AE^2 + EC^2$
 $b^2 = AE^2 + (ED + DC)^2$ [By pythagoras theorem]
 $b^2 = AD^2 + DC^2 + 2DC \times ED$
 $b^2 = p^2 + \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right) \times x$ [BC = 2CD given]
 $\Rightarrow b^2 = p^2 + \frac{a^2}{4} + ax$... (i)
- (ii) In $\triangle AEB$, by pythagoras theorem
 $AB^2 = AE^2 + BE^2$
 $\Rightarrow c^2 = AD^2 - ED^2 + (BD - ED)^2$ [By pythagoras theorem]
 $\Rightarrow c^2 = p^2 - ED^2 + BD^2 + ED^2 - 2BD \times ED$
 $\Rightarrow c^2 = p^2 + \left(\frac{a}{2}\right)^2 - 2\left(\frac{a}{2}\right) \times x$... (ii)
- (iii) Add equations (i) and (ii)
 $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

20. In Fig., 4.221, $\angle B < 90^\circ$ and segment $AD \perp BC$, show that

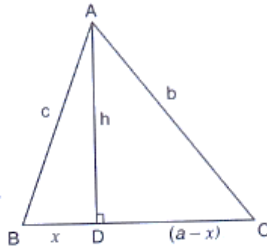


Fig. 4.221

$$(i) \quad b^2 = h^2 + a^2 + x^2 - 2ax$$

$$(ii) \quad b^2 = a^2 + c^2 - 2ax$$

Sol:

In $\triangle ADC$, by pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow b^2 = h^2 + (a - x)^2$$

$$\Rightarrow b^2 = h^2 + a^2 + x^2 - 2ax$$

$$\Rightarrow b^2 = a^2 + (h^2 + x^2) - 2ax$$

$$\Rightarrow b^2 = a^2 + c^2 - 2ax$$

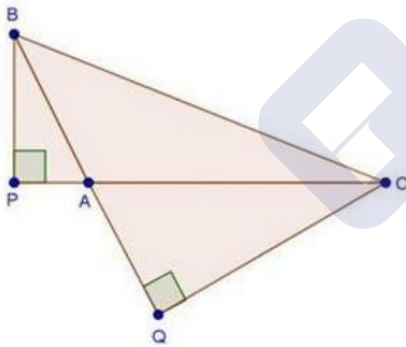
by Pythagoras theorem

21. In $\triangle ABC$, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that:

$$(i) \quad AB \times AQ = AC \times AP$$

$$(ii) \quad BC^2 = (AC \times CP + AB \times BQ)$$

Sol:



Then, $\triangle APB \sim \triangle AQC$

[By AA similarity]

$$\therefore \frac{AP}{AQ} = \frac{AB}{AC}$$

[Corresponding parts of similar \triangle are proportional]

$$\Rightarrow AP \times AC = AQ \times AB$$

...(i)

(ii) In $\triangle BPC$, by pythagoras theorem

$$BC^2 = BP^2 + PC^2$$

$$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2 \quad [\text{By pythagoras theorem}]$$

$$\Rightarrow BC^2 = AB^2 + AC^2 + 2AP \times AC \quad \dots(ii)$$

In $\triangle BQC$, by pythagoras theorem,

$$BC^2 = CQ^2 + BQ^2$$

$$\Rightarrow BC^2 = AC^2 - AQ^2 + (AB + AQ)^2 \quad [\text{By pythagoras theorem}]$$



$$\Rightarrow BC^2 = AC^2 - AQ^2 + AB^2 + AQ^2 + 2AB \times AQ$$

$$\Rightarrow BC^2 = AC^2 + AB^2 + 2AB \times AQ \quad \dots(iii)$$

Add equations (ii) & (iii)

$$2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$$

$$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$$

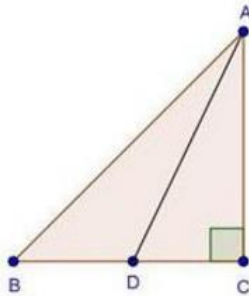
$$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$$

$$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$$

$$\Rightarrow BC^2 = AC \times PC + AB \times BQ \quad [\text{Divide by 2}]$$

22. In a right $\triangle ABC$ right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$

Sol:



To prove: $BC^2 = 4[AD^2 - AC^2]$

We have, $\angle C = 90^\circ$ and D is the mid-point of BC.

$$\text{LHS} = BC^2$$

$$= (2CD)^2$$

[D is the mid-point of BC]

$$= 4CD^2$$

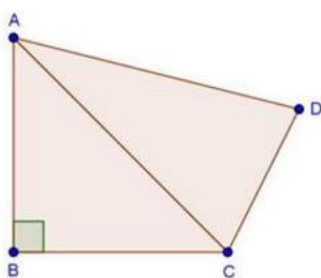
$$= 4[AD^2 - AC^2]$$

[In $\triangle ACD$, by pythagoras theorem]

$$= \text{RHS}$$

23. In a quadrilateral ABCD, $\angle B = 90^\circ$, $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.

Sol:



We have, $\angle B = 90^\circ$ and $AD^2 = AB^2 + BC^2 + CD^2$

$$\therefore AD^2 = AB^2 + BC^2 + CD^2 \quad [\text{Given}]$$

$$\text{But } AB^2 + BC^2 = AC^2$$

[By pythagoras theorem]



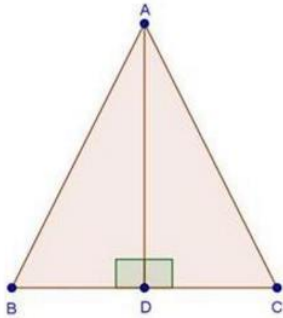
Then, $AD^2 = AC^2 + CD^2$

By converse of by pythagoras theorem

$$\angle ACD = 90^\circ$$

24. In an equilateral $\triangle ABC$, $AD \perp BC$, prove that $AD^2 = 3BD^2$.

Sol:



We have, $\triangle ABC$ is an equilateral \triangle and $AD \perp BC$

In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC \quad [\text{Each } 90^\circ]$$

$$AB = AC \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common}]$$

Then, $\triangle ADB \cong \triangle ADC$ [By RHS condition]

$$\therefore BD = CD = \frac{BC}{2} \dots (i) \quad [\text{corresponding parts of similar } \triangle \text{ are proportional}]$$

In, $\triangle ABD$, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow BC^2 = AD^2 + BD^2 \quad [AB = BC \text{ given}]$$

$$\Rightarrow [2BD]^2 = AD^2 + BD^2 \quad [\text{From (i)}]$$

$$\Rightarrow 4BD^2 - BD^2 = AD^2$$

$$\Rightarrow 3BD^2 = AD^2$$

25. $\triangle ABD$ is a right triangle right angled at A and $AC \perp BD$. Show that:

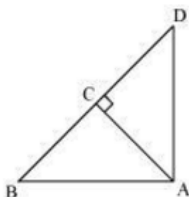
$$(i) \quad AB^2 = CB \times BD$$

$$(ii) \quad AC^2 = DC \times BC$$

$$(iii) \quad AD^2 = BD \times CD$$

$$(iv) \quad \frac{AB^2}{AC^2} = \frac{BD}{DC}$$

Sol:



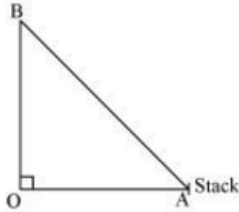


- (i) In $\triangle ADB$ and $\triangle CAB$
 $\angle DAB = \angle ACB = 90^\circ$
 $\angle ABD = \angle CBA$ (common angle)
 $\angle ADB = \angle CAB$ (remaining angle)
 So, $\triangle ADB \sim \triangle CAB$ (by AAA similarity)
 Therefore $\frac{AB}{CB} = \frac{BD}{AB}$
 $\Rightarrow AB^2 = CB \times BD$
- (ii) Let $\angle CAB = x$
 In $\triangle CBA$
 $\angle CBA = 180^\circ - 90^\circ - x$
 $\angle CBA = 90^\circ - x$
 Similarly in $\triangle CAD$
 $\angle CAD = 90^\circ - \angle CAB = 90^\circ - x$
 $\angle CDA = 90^\circ - \angle CAB$
 $= 90^\circ - x$
 $\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$
 $\angle CDA = x$
 Now in $\triangle CBA$ and $\triangle CAD$ we may observe that
 $\angle CBA = \angle CAD$
 $\angle CAB = \angle CDA$
 $\angle ACB = \angle DCA = 90^\circ$
 Therefore $\triangle CBA \sim \triangle CAD$ (by AAA rule)
 Therefore $\frac{AC}{DC} = \frac{BC}{AC}$
 $\Rightarrow AC^2 = DC \times BC$
- (iii) In $\triangle DCA$ & $\triangle DAB$
 $\angle DCA = \angle DAB$ (both are equal to 90°)
 $\angle CDA = \angle ADB$ (common angle)
 $\angle DAC = \angle DBA$ (remaining angle)
 $\triangle DCA \sim \triangle DAB$ (AAA property)
 Therefore $\frac{DC}{DA} = \frac{DA}{DB}$
 $\Rightarrow AD^2 = BD \times CD$
- (iv) From part (i) $AB^2 = CB \times BD$
 From part (ii) $AC^2 = DC \times BC$
 Hence $\frac{AB^2}{AC^2} = \frac{CB \times BD}{DC \times BC}$
 $\frac{AB^2}{AC^2} = \frac{BD}{DC}$
 Hence proved



26. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol:



Let OB be the pole and AB be the wire. Therefore by pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$24^2 = 18^2 + OA^2$$

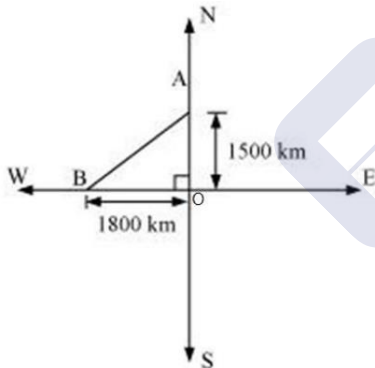
$$OA^2 = 576 - 324$$

$$OA = \sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$$

Therefore distance from base = $6\sqrt{7} \text{ m}$

27. An aeroplane leaves an airport and flies due north at a speed of 1000km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after 1 hours?

Sol:



Distance traveled by the plane flying towards north in $1\frac{1}{2}$ hrs

$$= 1000 \times 1\frac{1}{2} = 1500 \text{ km}$$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hrs

$$= 1200 \times 1\frac{1}{2} = 1800 \text{ km}$$

Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

$$\text{Distance between these planes after } 1\frac{1}{2} \text{ hrs } AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$$



$$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$$

So, distance between these planes will be $300\sqrt{61}$ km, after $1\frac{1}{2}$ hrs

28. Determine whether the triangle having sides $(a - 1)$ cm, $2\sqrt{a}$ cm and $(a + 1)$ cm is a right-angled triangle.

Sol:

Let ABC be the Δ with

$$AB = (a - 1) \text{ cm } BC = 2\sqrt{a} \text{ cm, } CA = (a + 1) \text{ cm}$$

$$\text{Hence, } AB^2 = (a - 1)^2 = a^2 + 1 - 2a$$

$$BC^2 = (2\sqrt{a})^2 = 4a$$

$$CA^2 = (a + 1)^2 = a^2 + 1 + 2a$$

$$\text{Hence } AB^2 + BC^2 = AC^2$$

So ΔABC is right angled Δ at B.

