

MATRICES (XII, R. S. AGGARWAL)

EXERCISE 5A (Pg. No.: 154)

1. If $A = \begin{bmatrix} 5 & -2 & 6 & 1 \\ 7 & 0 & 8 & -3 \\ \sqrt{2} & 3/5 & 4 & 9 \end{bmatrix}$, then write

- (i) The number of rows in A
(iii) The order of the matrix A
(v) The element $a_{23}, a_{31}, a_{14}, a_{33}, a_{22}$ of A .

- (ii) The number of columns in A
(iv) The number of all entries in A

- Sol.** (i) The number of rows in $A = 3$
(ii) The number of columns in $A = 4$
(iii) The order of the matrix $A = 3 \times 4$
(iv) The number of all entries in $A = 12$
(v) The element $a_{23} = 8, a_{31} = \sqrt{2}, a_{14} = 1, a_{33} = 4, a_{22} = 0$

2. Write the order of each of the following matrices :

(i) $A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & 4/9 \end{bmatrix}$

(ii) $B = \begin{bmatrix} 6 & -5 \\ 1/2 & 3/4 \\ -2 & -1 \end{bmatrix}$

(iii) $C = \begin{bmatrix} 7 & -\sqrt{2} & 5 & 0 \end{bmatrix}$

(iv) $D = \begin{bmatrix} 8 & -3 \end{bmatrix}$

(v) $E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

(vi) $F = \begin{bmatrix} 6 \end{bmatrix}$

Sol. (i) $A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & 4/9 \end{bmatrix}$, order = (2×4) (ii) $B = \begin{bmatrix} 6 & -5 \\ 1/2 & 3/4 \\ -2 & -1 \end{bmatrix}$, order = (3×2)

(iii) $C = \begin{bmatrix} 7 & -\sqrt{2} & 5 & 0 \end{bmatrix}$, order = (1×4) (iv) $D = \begin{bmatrix} 8 & -3 \end{bmatrix}$, order = (1×2)

(v) $E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, order = (3×1)

(vi) $F = \begin{bmatrix} 6 \end{bmatrix}$, order = (1×1)

3. If a matrix has 18 elements, what are the possible orders it can have?

- Sol.** We know that a matrix of order $m \times n$ has mn elements.
Hence, all possible orders of a matrix having 18 element are
 $(18 \times 1), (1 \times 18), (9 \times 2), (2 \times 9), (6 \times 3), (3 \times 6)$

4. Find all possible orders of matrices having 7 elements.

- Sol.** We know that a matrix of order $m \times n$ has mn elements.
Hence, all possible orders of a matrix having 7 element are $(7 \times 1), (1 \times 7)$.

5. Construct a 3×2 matrix whose elements are given by $a_{ij} = (2i - j)$.

- Sol.** A 3×2 matrix has 3 rows and 2 columns.

In general, a 3×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3 \times 2}$,

Thus $a_{ij} = (2i - j)$ for $i = 1, 2, 3$ and $j = 1, 2$

$$\begin{aligned} a_{11} &= 2 - 1 = 1, & a_{12} &= 2(1) - 2 = 2 - 2 = 0, & a_{21} &= 2(2) - 1 = 4 - 1 = 3 \\ a_{22} &= 2(2) - 2 = 4 - 2 = 2, & a_{31} &= 2(3) - 1 = (6 - 1) = 5, & a_{32} &= 2(3) - 2 = 6 - 2 = 4 \end{aligned}$$

Hence, $A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

6. Construct a 4×3 matrix whose elements are given by $a_{ij} = \frac{i}{j}$.

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}_{(4 \times 3)}$. Thus $a_{ij} = \frac{i}{j}$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3$

$$\begin{aligned} a_{11} &= \frac{1}{1} = 1, & a_{12} &= \frac{1}{2}, & a_{13} &= \frac{1}{3}, & a_{21} &= \frac{2}{1} = 2, & a_{22} &= \frac{2}{2} = 1, & a_{23} &= \frac{2}{3}, \\ a_{31} &= \frac{3}{1} = 3, & a_{32} &= \frac{3}{2}, & a_{33} &= \frac{3}{3} = 1, & a_{41} &= \frac{4}{1} = 4, & a_{42} &= \frac{4}{2} = 2, & a_{43} &= \frac{4}{3} \end{aligned}$$

Hence, $A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \\ 4 & 2 & 4/3 \end{bmatrix}_{4 \times 3}$

7. Construct a 2×2 matrix whose elements are $a_{ij} = \frac{(i+2j)^2}{2}$.

Sol. Let, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{(2 \times 2)}$. Thus $a_{ij} = \frac{(i+2j)^2}{2}$ for $i = 1, 2$ and $j = 1, 2$.

$$\begin{aligned} \therefore a_{11} &= \frac{\{1+2(1)\}^2}{2} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}, & a_{12} &= \frac{\{1+2(2)\}^2}{2} = \frac{(1+4)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2} \\ a_{21} &= \frac{\{2+2(1)\}^2}{2} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8, & a_{22} &= \frac{\{2+2(2)\}^2}{2} = \frac{(2+4)^2}{2} = \frac{(6)^2}{2} = \frac{36}{2} = 18 \end{aligned}$$

Hence, $A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}_{(2 \times 2)}$

8. Construct a 2×3 matrix whose elements are $a_{ij} = \frac{(i-2j)^2}{2}$.

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{(2 \times 3)}$. Thus $a_{ij} = \frac{(i-2j)^2}{2}$ for $i = 1, 2$ and $j = 1, 2, 3$

$$\therefore a_{11} = \frac{\{1-2(1)\}^2}{2} = \frac{(1-2)^2}{2} = \frac{(-1)^2}{2} = \frac{1}{2}, \quad a_{12} = \frac{\{1-2(2)\}^2}{2} = \frac{(1-4)^2}{2} = \frac{(-3)^2}{2} = \frac{9}{2}$$

$$a_{13} = \frac{\{1-2(3)\}^2}{2} = \frac{(1-6)^2}{2} = \frac{(-5)^2}{2} = \frac{25}{2}, \quad a_{21} = \frac{\{2-2(1)\}^2}{2} = \frac{(2-2)^2}{2} = 0$$

$$a_{22} = \frac{\{2-2(2)\}^2}{2} = \frac{(2-4)^2}{2} = \frac{(-2)^2}{2} = \frac{4}{2} = 2, \quad a_{23} = \frac{\{2-2(3)\}^2}{2} = \frac{(2-6)^2}{2} = \frac{(-4)^2}{2} = \frac{16}{2} = 8$$

Hence, $A = \begin{bmatrix} 1/2 & 9/2 & 25/2 \\ 0 & 2 & 8 \end{bmatrix}_{(2 \times 3)}$

9. Construct a 3×4 matrix whose elements are given by $a_{ij} = \frac{1}{2}|-3i+j|$.

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{(3 \times 4)}$. Thus $a_{ij} = \frac{1}{2}|-3i+j|$ for $i=1, 2, 3$ and $j=1, 2, 3, 4$

$$\therefore a_{11} = \frac{1}{2}|-3(1)+1| = \frac{1}{2}|-3+1| = \frac{1}{2} \times 2 = 1, \quad a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-3+2| = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$a_{13} = \frac{1}{2}|-3(1)+3| = \frac{1}{2}|-3+3| = 0, \quad a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}|-3+4| = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{1}{2} \times 5 = \frac{5}{2}, \quad a_{22} = \frac{1}{2}|-3(2)+2| = \frac{1}{2}|-6+2| = \frac{1}{2} \times 4 = 2$$

$$a_{23} = \frac{1}{2}|-3(2)+3| = \frac{1}{2}|-6+3| = \frac{1}{2} \times 3 = \frac{3}{2}, \quad a_{24} = \frac{1}{2}|-3(2)+4| = \frac{1}{2}|-6+4| = \frac{1}{2} \times 2 = 1$$

$$a_{31} = \frac{1}{2}|-3(3)+1| = \frac{1}{2}|-9+1| = \frac{1}{2} \times 8 = 4, \quad a_{32} = \frac{1}{2}|-3(3)+2| = \frac{1}{2}|-9+2| = \frac{1}{2} \times 7 = \frac{7}{2}$$

$$a_{33} = \frac{1}{2}|-3(3)+3| = \frac{1}{2}|-9+3| = \frac{1}{2} \times 6 = 3, \quad a_{34} = \frac{1}{2}|-3(3)+4| = \frac{1}{2}|-9+4| = \frac{1}{2} \times 5 = \frac{5}{2}$$

Hence, $A = \begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}_{(3 \times 4)}$

EXERCISE 5B (Pg.No.: 167)

1. If $A = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$, Verify that $(A+B) = (B+A)$.

Sol. LHS = $(A+B) = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 2+3 & -3+2 & 5+(-2) \\ -1+4 & 0+(-3) & 3+1 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix}$

RHS = $(B+A) = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3+2 & 2+(-3) & -2+5 \\ 4+(-1) & -3+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix}$

$\therefore$ LHS = RHS. $\Rightarrow (A+B) = (B+A)$. Hence proved.

2. If $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$ verify that $(A+B)+C = A+(B+C)$.



$$\text{Sol. LHS} = (A+B)+C = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + C = \begin{bmatrix} 3+(-1) & 5+(-3) \\ -2+4 & 0+2 \\ 6+(-2) & -1+3 \end{bmatrix} + C$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2+0 & 2+2 \\ 2+3 & 2+(-4) \\ 4+1 & 2+6 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

$$\text{RHS} = A+(B+C) = A + \left(\begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix} \right) = A + \begin{bmatrix} -1+0 & -3+2 \\ 4+3 & 2+(-4) \\ -2+1 & 3+6 \end{bmatrix} = A + \begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} 3+(-1) & 5+(-1) \\ -2+7 & 0+(-2) \\ 6+(-1) & -1+9 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

$\therefore \text{LHS} = \text{RHS} \Rightarrow (A+B)+C = A+(B+C)$. Hence proved.

3. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$ find $(2A-B)$.

$$\text{Sol. } 2A-B = 2 \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-2) & 2-0 & 4-4 \\ 2-5 & 4-(-3) & -6-2 \end{bmatrix} = \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find : (i) $A+2B$ (ii) $B-4C$ (iii) $A-2B+3C$.

$$\text{Sol. (i) } A+2B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+6 \\ 3+(-4) & 2+10 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix}$$

$$\text{(ii) } B-4C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix} = \begin{bmatrix} 1-(-8) & 3-20 \\ -2-12 & 5-16 \end{bmatrix} = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

$$\text{(iii) } A-2B+3C = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2+(-6) & 4-6+15 \\ 3-(-4)+9 & 2-10+12 \end{bmatrix} = \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}$. Compute $5A-3B+4C$.

$$\text{Sol. } 5A-3B+4C = 5 \begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix} - 3 \begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix} + 4 \begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{bmatrix} - \begin{bmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{bmatrix} + \begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3+8 & 5-(-9)+(-20) & -10-(-3)+4 \\ 25-0+(-16) & -5-(-6)+0 & -20-15+24 \end{bmatrix} = \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$$

If $5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$, find A . <https://millionstar.godaddysites.com/>

Sol. $5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix} \Rightarrow A = \frac{1}{5} \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 2 & -3 \\ 2/5 & 3/5 & 4/5 \\ 1/5 & 0 & -1 \end{bmatrix}$

7. Find matrices A and B , if $A+B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix}$ and $A-B = \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$.

Sol. $A+B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix}$... (i) $A-B = \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$... (ii)

Adding equation (i) and (ii), we get

$$(A+B) + (A-B) = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} + \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 1+(-5) & 0+(-4) & 2+8 \\ 5+11 & 4+2 & -6+0 \\ 7+(-1) & 3+7 & 8+4 \end{bmatrix} = \begin{bmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{bmatrix}$$

$$\Rightarrow A = \frac{1}{2} \begin{bmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$$

Putting the value of A in equation (i), we get

$$A+B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} - A \Rightarrow B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} - \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1-(-2) & 0-(-2) & 2-5 \\ 5-8 & 4-3 & -6-(-3) \\ 7-3 & 3-5 & 8-6 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$$

Hence, matrix $A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$

8. Find matrices A and B , if $2A-B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $2B+A = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Sol. $2A-B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$... (i) $\times 2$

$$2B+A = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$
 ... (ii)

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Adding (i) and (ii), we get, $(4A-2B)+(A+2B)=\begin{bmatrix} 12 & 0 & 0 \\ -8 & 4 & 2 \end{bmatrix}+\begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

$$\Rightarrow 5A=\begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix}+\begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \Rightarrow 5A=\begin{bmatrix} 12+3 & -12+2 & 0+5 \\ -8+(-2) & 4+1 & 2+(-7) \end{bmatrix}$$

$$\Rightarrow 5A=\begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix} \Rightarrow A=\frac{1}{5}\begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}=\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

Putting the value of A in equation (i), then $2A-B=\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$

$$\Rightarrow B=2A-\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \Rightarrow B=2\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}-\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{bmatrix}-\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \Rightarrow B=\begin{bmatrix} 6-6 & -4-(-6) & 2-0 \\ -4-(-4) & 2-2 & -2-1 \end{bmatrix}=\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Hence, matrix $A=\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

9. Find matrix X , if $\begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}+X=\begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}$.

Sol. $\begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}+X=\begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix} \Rightarrow X=\begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}-\begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}$

$$\Rightarrow X=\begin{bmatrix} 6-3 & 2-5 & 3-(-9) \\ 4-(-1) & 8-4 & 6-(-7) \end{bmatrix} \Rightarrow X=\begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$$

10. If $A=\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$, find a matrix C such that $A+B-C=O$.

Sol. $A+B-C=O \Rightarrow C=A+B \Rightarrow C=\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix}+\begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$

$$\Rightarrow C=\begin{bmatrix} -2+5 & 3+2 \\ 4+(-7) & 5+3 \\ 1+6 & -6+4 \end{bmatrix} \Rightarrow C=\begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

11. Find the matrix X such that $2A-B+X=O$, where $A=\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$.

Sol. $2A-B+X=O \Rightarrow X=B-2A \Rightarrow X=\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}-2\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

$$\Rightarrow X=\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}-\begin{bmatrix} 6 & 2 \\ 0 & 4 \end{bmatrix}=\begin{bmatrix} -2-6 & 1-2 \\ 0-0 & 3-4 \end{bmatrix} \Rightarrow X=\begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$$

12. If $A=\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, find a matrix C such that $(A+B+C)$ is a zero matrix.

Sol. $A+B+C=O \Rightarrow C=-A-B$

$$\Rightarrow C = - \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow C = \begin{bmatrix} -1-3+1 & 3-2-1 & -2+1 \\ -2-1 & 0-0 & -2+1 \end{bmatrix} = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

13. If $A = \text{diag}[2, -5, 9]$, $B = \text{diag}[-3, 7, 14]$ and $C = \text{diag}[4, -6, 3]$ find

(i) $A+2B$

(ii) $B+C-A$

(iii) $2A+B-5C$

Sol. (i) $A+2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + 2 \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 28 \end{bmatrix}$

$$= \begin{bmatrix} 2-6 & 0 & 0 \\ 0 & -5+14 & 0 \\ 0 & 0 & 9+28 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 37 \end{bmatrix} = \text{diag}[-4, 9, 37]$$

(ii) $B+C-A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$$= \begin{bmatrix} -3+4-2 & 0 & 0 \\ 0 & 7+(-6)-(-5) & 0 \\ 0 & 0 & 14+3-9 \end{bmatrix} = \text{diag}[-1, 6, 8]$$

(iii) $2A+B-5C = 2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - 5 \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 4+(-3)-20 & 0 & 0 \\ 0 & -10+7+30 & 0 \\ 0 & 0 & 18+14-15 \end{bmatrix} = \begin{bmatrix} -19 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

$$= \text{diag}[-19, 27, 17]$$

14. Find the values of x and y , when

(i) $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2x+5 & 7 \\ 6 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$

(iii) $2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 4 \end{bmatrix}$

Sol. (i) $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$x+y=8 \quad \dots(1) \quad x-y=4 \quad \dots(2)$

Adding equation (1) and (2), we get

$(x+y)+(x-y)=8+4 \Rightarrow 2x=12 \Rightarrow x=6$

Putting the value of x in equation (1), we get $x+y=8 \Rightarrow y=8-x=8-6=2$

Hence, $x=6$ and $y=2$.

$$(ii) \begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 1 \\ 0 & -5 \end{bmatrix} \quad \text{https://millionstar.godaddysites.com/}$$

$$\begin{aligned} 2x+5 &= x-3 & \text{and} & \quad 3y-7 = -5 \\ \Rightarrow 2x-x &= -3-5 & \text{and} & \quad 3y = -5+7 \\ \Rightarrow x &= -8 & \text{and} & \quad 3y = 2 \Rightarrow y = \frac{2}{3} \end{aligned}$$

Hence, $x = -8$ and $y = \frac{2}{3}$

$$(iii) 2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 4 \end{bmatrix}$$

$$\begin{aligned} 2x+3 &= 7 & \text{and} & \quad 2y-6+2 = 4 \\ \Rightarrow 2x &= 7-3 & \Rightarrow & \quad 2y = 4-2+6 \\ \Rightarrow 2x &= 4 & \Rightarrow & \quad 2y = 8 \\ \Rightarrow x &= 2 & \Rightarrow & \quad y = 4 \end{aligned}$$

Hence, $x = 2$ and $y = 4$.

15. Find the value of $(x+y)$ from the following equation

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Sol. Given $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\begin{aligned} \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \\ \Rightarrow 2+y &= 5 \text{ and } 2x+2 = 8 \Rightarrow y = 5-2 \text{ and } 2x = 8-2 \\ \Rightarrow y &= 3 \text{ and } x = 3 \quad \therefore x = 3 \text{ and } y = 3 \end{aligned}$$

16. If $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$ then write the value of $(x+y+z)$

Sol. Given $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$

$$\begin{aligned} \Rightarrow x-y &= 1, 2y = 4, 2y+z = 9 \text{ and } x+y = 5 \\ \text{Now, } 2y &= 4 \Rightarrow y = 2 \\ x-y &= 1 \Rightarrow x-2 = 1 \Rightarrow x = 3 \\ 2y+z &= 9 \Rightarrow 4+z = 9 \Rightarrow z = 5 \\ \therefore x+y+z &= 3+2+5 = 10 \end{aligned}$$

EXERCISE 5C (Pg.No.: 186)

1. Compute AB and BA , whichever exists when

$$(i) A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$(iv) A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$(v) A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

Sol. (i) $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(-2) - 1(0) & 2(3) - 1(4) \\ 3(-2) + 0(0) & 3(3) + 0(4) \\ -1(-2) + 4(0) & -1(3) + 4(4) \end{bmatrix} = \begin{bmatrix} -4 - 0 & 6 - 4 \\ -6 + 0 & 9 + 0 \\ 2 + 0 & -3 + 16 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$$

B is 2×2 , A is 3×2 column of B not equal to row of A

$\therefore BA$ does not exist.

$$(ii) A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

$\therefore AB$ does not exist.

$$BA = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(-2) + 1(-3) & 3(1) - 2(2) + 1(3) \\ 0(-1) + 1(-2) + 2(-3) & 0(1) + 1(2) + 2(3) \\ -3(-1) + 4(-2) - 5(-3) & -3(1) + 4(2) - 5(3) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 - 3 & 3 - 4 + 3 \\ 0 - 2 - 6 & 0 + 2 + 6 \\ 3 - 8 + 15 & -3 + 8 - 15 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0(1) + 1(-1) - 5(0) & 0(3) + 1(0) - 5(5) \\ 2(1) + 4(-1) + 0(0) & 2(3) + 4(0) + 0(5) \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 1 - 0 & 0 + 0 - 25 \\ 2 - 4 + 0 & 6 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+3(2) & 1(1)+3(4) & 1(-5)+3(0) \\ -1(0)+0(2) & -1(1)+0(4) & -1(-5)+0(0) \\ 0(0)+5(2) & 0(1)+5(4) & 0(-5)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+6 & 1+12 & -5+0 \\ 0+0 & -1+0 & 5+0 \\ 0+10 & 0+20 & 0+0 \end{bmatrix} = \begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$$

(iv) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3)+4(4) \end{bmatrix} = \begin{bmatrix} 1+4+9+16 \end{bmatrix} = \begin{bmatrix} 30 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(1) & 1(2) & 1(3) & 1(4) \\ 2(1) & 2(2) & 2(3) & 2(4) \\ 3(1) & 3(2) & 3(3) & 3(4) \\ 4(1) & 4(2) & 4(3) & 4(4) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

(v) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1(2)+0(3)+1(-1) & 1(1)+0(2)+1(1) \\ -1(2)+2(3)+1(-1) & -1(1)+2(2)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0-1 & 1+0+1 \\ -2+6-1 & -1+4+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2. Show that $AB \neq BA$ in each of the following cases:

(i) $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Sol. (i) $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2)+(-1)3 & 5(1)+(-1)4 \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix} = \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix} = \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Hence, $AB \neq BA$.

$$(ii) A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & 1(3)+1(0)+0(0) \\ 0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & 3+0+0 \\ 0+0+1 & 0-1+1 & 0-0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

Hence, $AB \neq BA$.

3. Show that $AB = BA$ in each of the following cases.

$$(i) A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

Sol. (i) $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$

$$AB = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix} \Rightarrow AB = \begin{bmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) \\ \sin(\theta + \phi) & \cos(\theta + \phi) \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} \cos \phi \cos \theta - \sin \phi \sin \theta & \cos \phi(-\sin \theta) - (-\sin \phi) \cos \theta \\ \sin \phi \cos \theta + \cos \phi \sin \theta & \sin \phi(-\sin \theta) + \cos \phi \cos \theta \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) \\ \sin(\theta + \phi) & \cos(\theta + \phi) \end{bmatrix}. \text{ Hence, } AB = BA.$$

$$(ii) A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(10)+2(-11)+1(9) & 1(-4)+2(5)+1(-5) & 1(-1)+2(0)+1(1) \\ 3(10)+4(-11)+2(9) & 3(-4)+4(5)+2(-5) & 3(-1)+4(0)+2(1) \\ 1(10)+3(-11)+2(9) & 1(-4)+3(5)+2(-5) & 1(-1)+3(0)+2(1) \end{bmatrix} \\ &= \begin{bmatrix} 10-22+9 & -4+10-5 & -1+0+1 \\ 30-44+18 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \\ BA &= \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 10(1)+(-4)(3)-1(1) & 10(2)-4(4)-1(3) & 10(1)-4(2)-1(2) \\ -11(1)+5(3)+0(1) & -11(2)+5(4)+0(3) & -11(1)+5(2)+0(2) \\ 9(1)-5(3)+1(1) & 9(2)-5(4)+1(3) & 9(1)-5(2)+1(2) \end{bmatrix} \\ &= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \end{aligned}$$

Hence, $AB = BA$.

$$\begin{aligned} \text{(iii)} \quad A &= \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \\ AB &= \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1(-2)+3(-1)-1(-6) & 1(3)+3(2)-1(9) & 1(-1)+3(-1)-1(-4) \\ 2(-2)+2(-1)-1(-6) & 2(3)+2(2)-1(9) & 2(-1)+2(-1)-1(-4) \\ 3(-2)+0(-1)+(-1)(-6) & 3(3)+0(2)-1(9) & 3(-1)+0(-1)-1(-4) \end{bmatrix} \\ &= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \\ BA &= \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1) \\ -1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1) \\ -6(1)+9(2)-4(3) & -6(3)+9(2)+(-4)(0) & -6(-1)+9(-1)-4(-1) \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} -2+6-3 & -6+6-0 & -2+9+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18-0 & 6-9+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I. \quad \text{Hence, } AB = BA.$$

4. If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $AB = A$ and $BA = B$.

Sol. $AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 2(2)-3(-1)-5(1) & 2(-2)-3(3)-5(-2) & 2(-4)-3(4)-5(-3) \\ -1(2)+4(-1)+5(1) & -1(-2)+4(3)+5(-2) & -1(-4)+4(4)+5(-3) \\ 1(2)-3(-1)-4(1) & 1(-2)-3(3)-4(-2) & 1(-4)-3(4)-4(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A. \quad \text{Hence } AB = A.$$

and $BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-3)-2(4)-4(-3) & 2(-5)-2(5)-4(-4) \\ -1(2)+3(-1)+4(1) & -1(-3)+3(4)+4(-3) & -1(-5)+3(5)+4(-4) \\ 1(2)-2(-1)-3(1) & 1(-3)-2(4)-3(-3) & 1(-5)-2(5)-3(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B. \quad \text{Hence } BA = B.$$

5. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that AB is a zero matrix.

Sol. $AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$= \begin{bmatrix} 0(a^2)+c(ab)-b(ac) & a(ab)+c(b^2)-b(bc) & 0(ac)+c(bc)-b(c^2) \\ -c(a^2)+0(ab)+a(ac) & -c(ab)+0(b^2)+a(bc) & c(ac)+0(bc)+a(c^2) \\ b(a^2)-a(ab)+0(ac) & b(ab)-a(b^2)+0(bc) & b(ac)-a(bc)+0(c^2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+abc-abc & a^2b+cb^2-b^2c & 0+bc^2-bc^2 \\ -a^2c^2+0+a^2c & -abc+0+abc & -c^2a+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, $AB = O$.

For the following matrices, verify that $A(BC) = (AB)C$.

(i) $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 \end{bmatrix}$

Sol. (i) $LHS = A(BC) = A \left(\begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \right) = A \begin{bmatrix} 2(1)+3(4)+0(5) \\ 1(1)+0(4)+4(5) \\ 1(1)-1(4)+2(5) \end{bmatrix} = A \begin{bmatrix} 2+12+0 \\ 1+0+20 \\ 1-4+10 \end{bmatrix}$
 $= \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(14)+2(21)+5(7) \\ 0(14)+1(21)+3(7) \end{bmatrix} = \begin{bmatrix} 14+42+35 \\ 0+21+21 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$

$RHS = (AB)C = \left(\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \right) C$
 $= \begin{bmatrix} 1(2)+2(1)+5(1) & 1(3)+2(0)+5(-1) & 1(0)+2(4)+5(2) \\ 0(2)+1(1)+3(1) & 0(3)+1(0)+3(-1) & 0(0)+1(4)+3(2) \end{bmatrix}$
 $= \begin{bmatrix} 2+2+5 & 3+0-5 & 0+8+10 \\ 0+1+3 & 0+0-3 & 0+4+6 \end{bmatrix} C = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$
 $= \begin{bmatrix} 9(1)-2(4)+18(5) \\ 4(1)-3(4)+10(5) \end{bmatrix} = \begin{bmatrix} 9-8+90 \\ 4-12+50 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Hence, $A(BC) = (AB)C$.

(ii) $LHS = A(BC) = A \left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} \right) = A \begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix} = A \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = A \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$
 $= \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1)+3(1)-1(2) & 2(-2)+3(-2)-1(-4) \\ 3(1)+0(1)+2(2) & 3(-2)+0(-2)+2(-4) \end{bmatrix}$
 $= \begin{bmatrix} 2+3-2 & -4-6+4 \\ 3+0+4 & -6+0-8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

$RHS = (AB)C = \left(\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right) C = \begin{bmatrix} 2(1)+3(1)-1(2) \\ 3(1)+0(1)+2(2) \end{bmatrix} C = \begin{bmatrix} 2+3-2 \\ 3+0+4 \end{bmatrix} C$
 $= \begin{bmatrix} 3 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-2) \\ 7(1) & 7(-2) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Hence, $A(BC) = (AB)C$ <https://millionstar.godaddysites.com/>

Verify that $A(B+C) = (AB+AC)$, when

$$(i) A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Sol. (i) $LHS = A(B+C) = A \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + A \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2+1 & 0-1 \\ 1+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 1(3)+2(1) & 1(-1)+2(-2) \\ 3(3)+4(1) & 3(-1)+4(-2) \end{bmatrix} = \begin{bmatrix} 3+2 & -1-4 \\ 9+4 & -3-8 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$RHS = AB + AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+2(1) & 1(0)+2(-3) \\ 3(2)+4(1) & 3(0)+4(-3) \end{bmatrix} + \begin{bmatrix} 1(1)+2(0) & 1(-1)+2(1) \\ 3(1)+4(0) & 3(-1)+4(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 0-6 \\ 6+4 & 0-12 \end{bmatrix} + \begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

Hence, $A(B+C) = (AB+AC)$

(ii) $LHS = A(B+C) = A \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + A \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = A \begin{bmatrix} 6+(-1) & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = A \begin{bmatrix} 5 & -1 \\ 5 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 2(4)+3(5) & 2(-1)+3(5) \\ -1(4)+4(5) & -1(-1)+4(5) \\ 0(4)+1(5) & 0(-1)+1(5) \end{bmatrix} = \begin{bmatrix} 8+15 & -2+15 \\ -4+20 & 1+20 \\ 0+5 & 0+5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$RHS = AB + BC = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2(5)+3(2) & 2(3)+3(1) \\ -1(5)+4(2) & -1(-3)+4(1) \\ 0(5)+1(2) & 0(-3)+1(1) \end{bmatrix} + \begin{bmatrix} 2(-1)+3(3) & 2(2)+3(4) \\ -1(-1)+4(3) & -1(2)+4(4) \\ 0(-1)+1(3) & 0(2)+1(4) \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -6+3 \\ -5+8 & 3+4 \\ 0+2 & 0+1 \end{bmatrix} + \begin{bmatrix} -2+9 & 4+12 \\ 1+12 & -2+16 \\ 0+3 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 16+7 & -3+16 \\ 3+13 & 7+14 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

Hence $A(B+C) = AB + AC$.

If $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$; verify that $A(B-C) = (AB-AC)$.

Sol. LHS = $A(B-C) = A \left(\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right) = A \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+0(-1)-2(-6) & 1(0)+0(0)-2(3) & 1(-6)+0(3)-2(1) \\ 3(-1)-1(-1)+0(-1) & 3(0)-1(0)+0(1) & 3(-6)-1(3)+0(1) \\ -2(-1)+1(-1)+1(-1) & -2(0)+1(0)+1(1) & -2(-6)+1(3)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} -1-0+2 & 0+0-2 & -6+0-2 \\ -3+1-0 & 0-0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

RHS = $AB-AC = \left(\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \right) - \left(\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right)$

$$= \begin{bmatrix} 1(0)+0(-2)-2(-1) & 1(5)+0(1)-2(0) & 1(-4)+0(3)-2(3) \\ 3(0)-1(-2)+0(-1) & 3(5)-1(1)+0(0) & 3(-4)-1(3)+0(2) \\ -2(0)+1(-2)+1(-1) & -2(5)+1(1)+1(0) & -2(-4)+1(3)+1(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(-1)-2(0) & 1(5)+0(1)-2(-1) & 1(2)+0(0)+(-2)(1) \\ 3(1)-1(-1)+0(0) & 3(5)-1(1)+0(-1) & 3(2)-1(0)+0(1) \\ -2(1)+1(-1)+1(0) & -2(5)+1(1)+1(-1) & -2(2)+1(0)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 0-0+2 & 5+0-0 & -4+0-4 \\ 0+2-0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} = \begin{bmatrix} 1-0-0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6-0+0 \\ -2-1+0 & -10+1-1 & -4+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix} = \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -15-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

∴ LHS = RHS. Hence, $A(B-C) = AB-AC$.

9. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$.

Sol. $A^2 = A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} ab(ab)+b^2(-a^2) & ab(b^2)+b^2(-ab) \\ -a^2(ab)-ab(-a^2) & -a^2(b^2)-ab(-ab) \end{bmatrix}$

$$= \begin{bmatrix} a^2b^2-a^2b^2 & ab^3-ab^3 \\ -a^3b+a^3b & -a^2b^2+a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Hence, $A^2 = O$.

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10. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$.

Sol. $A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 2(2) - 2(-1) - 4(1) & 2(-2) - 2(3) - 4(-2) & 2(-4) - 2(4) - 4(-3) \\ -1(2) + 3(-1) + 4(1) & -1(-2) + 3(3) + 4(-2) & -1(-4) + 3(4) + 4(-3) \\ 1(2) - 2(-1) - 3(1) & 1(-2) - 2(3) - 3(-2) & 1(-4) - 2(4) - 3(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 2 - 4 & -4 - 6 + 8 & -8 - 8 + 12 \\ -2 - 3 + 4 & 2 + 9 - 8 & 4 + 12 - 12 \\ 2 + 2 - 3 & -2 - 6 + 6 & -4 - 8 + 9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Hence, $A^2 = A$.

11. If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I$.

Sol. $A^2 = A.A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 4(4) - 1(3) - 4(3) & 4(-1) - 1(0) - 4(-1) & 4(-4) - 1(-4) - 4(-3) \\ 3(4) + 0(3) - 4(3) & 3(-1) + 0(0) - 4(-1) & 3(-4) + 0(-4) - 4(-3) \\ 3(4) - 1(3) - 3(3) & 3(-1) - 1(0) - 3(-1) & 3(-4) - 1(-4) - 3(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 3 - 12 & -4 - 0 + 4 & -16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 - 0 + 12 \\ 12 - 3 - 9 & -3 - 0 + 3 & -12 + 4 + 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, $A^2 = I$.

12. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

Sol. $3A^2 - 2B + I = 3 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= 3 \begin{bmatrix} 2(2) - 1(3) & 2(-1) - 1(2) \\ 3(2) + 2(3) & 3(-1) + 2(2) \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 4 - 3 & -2 - 2 \\ 6 + 6 & -3 + 4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -20 \\ 38 & -11 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -20 \\ 38 & -10 \end{bmatrix}$$

13. If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$ then find $(-A^2 + 6A)$.

$$\begin{aligned} \text{Sol. } -A^2 + 6A &= -\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} + 6 \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \\ &= -\begin{bmatrix} 2(2) - 2(-3) & 2(-2) - 2(4) \\ -3(2) + 4(-3) & -3(-2) + 4(4) \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} \\ &= \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} -10+12 & 12+(-12) \\ 18+(-18) & -22+24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

14. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

$$\begin{aligned} \text{Sol. LHS} &= A^2 - 5A + 7I \\ &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3(3) + 1(-1) & 3(1) + 1(2) \\ -1(3) + 2(-1) & -1(1) + 2(2) \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \end{aligned}$$

Hence, $A^2 - 5A + 7I = O$.

15. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = O$.

$$\begin{aligned} \text{Sol. LHS} &= A^3 - 4A^2 + A \\ &= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2(2) + 3(1) & 2(3) + 3(2) \\ 1(2) + 2(1) & 1(3) + 2(2) \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2(2) + 3(1) & 2(3) + 3(2) \\ 1(2) + 2(1) & 1(3) + 2(2) \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 7(2) + 12(1) & 7(3) + 12(2) \\ 4(2) + 7(1) & 4(3) + 7(2) \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0. \text{ Hence, } A^3 - 4A^2 + A = O.$$

16. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Sol. $A^2 = kA - 2I \Rightarrow kA = A^2 + 2I$

$$\Rightarrow kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 1+2 & -2+0 \\ 4+0 & -4+2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$3k = 3, -2k = -2, 4k = 4, -2k = -2. \text{ Hence, } k = 1.$$

17. If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 2x + 3$.

Sol. $f(x) = x^2 - 2x + 3 \Rightarrow f(A) = A^2 - 2A + 3I$

$$\Rightarrow f(A) = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} - 2 \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -1(-1)+2(3) & -1(2)+2(1) \\ 3(-1)+1(3) & 3(2)+1(1) \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 1+6 & -2+2 \\ -3+3 & 6+1 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow f(A) = \begin{bmatrix} 7+2+3 & 0-4+0 \\ 0-6+0 & 7-2+3 \end{bmatrix} = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$$

18. If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 + 4x + 5$, find $f(A)$.

Sol. $f(x) = 2x^3 + 4x + 5 \Rightarrow f(A) = 2A^3 + 4A + 5I$

$$\Rightarrow f(A) = 2 \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4 \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = 2 \begin{bmatrix} 1(1)+2(4) & 1(2)+2(-3) \\ 4(1)-3(4) & 4(2)+(-3)(-3) \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow f(A) = 2 \begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4+5 & 8+0 \\ 16+0 & -12+5 \end{bmatrix}$$

$$\Rightarrow f(A) = 2 \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow f(A) &= 2 \begin{bmatrix} 9(1)-4(4) & 9(2)-4(-3) \\ -8(1)+17(4) & -8(2)+17(-3) \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \\ \Rightarrow f(A) &= 2 \begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \Rightarrow f(A) = 2 \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \\ \Rightarrow f(A) &= \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \Rightarrow f(A) = \begin{bmatrix} -14+9 & 60+8 \\ 120+16 & -134-7 \end{bmatrix} = \begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix} \end{aligned}$$

19. Find the value of x and y when $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Sol. $\begin{bmatrix} 2(x)-3(y) \\ 1(x)+1(y) \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-3y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$2x-3y=1 \quad \dots(1)$$

and $x+y=3 \quad \dots(2) \times 3$

$$\Rightarrow 5x=10 \Rightarrow x=2$$

Putting the value of x in equation (2), we get $2x-3y=1 \Rightarrow 2(2)-3y=1$

$$\Rightarrow 3y=4-1 \Rightarrow 3y=3 \Rightarrow y=1 \quad \text{Hence, } x=2 \text{ and } y=1.$$

20. Solve for x and y when $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$.

Sol. $\begin{bmatrix} 3(x)-4(y) \\ 1(x)+2(y) \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 3x-4y \\ x+2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$$3x-4y=3 \quad \dots(1)$$

$$x+2y=11 \quad \dots(2) \times 2$$

Solving equations (1) and (2), we get $5x=25 \Rightarrow x=5$

Putting the value of x in equation (1), we get $3x-4y=3 \Rightarrow 4y=3x-3$

$$\Rightarrow 4y=3(5)-3 \Rightarrow 4y=15-3 \Rightarrow 4y=12 \Rightarrow y=3. \quad \text{Hence, } x=5 \text{ and } y=3.$$

21. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$.

Sol. $A^2 + xI = yA$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} + x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3(3)+1(7) & 3(1)+1(5) \\ 7(3)+5(7) & 7(1)+5(5) \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$16+x=3y \quad \dots(1)$$

$$8+0=y \quad \dots(2)$$

$\Rightarrow y=8$, putting the value of y in equation (1), we get, $16+x=3y$

$$\Rightarrow 16+x=3(8) \Rightarrow x=24-16 \Rightarrow x=8. \text{ Hence, } x=8 \text{ and } y=8.$$

If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the values of a and b such that $A^2 + aA + bI = O$.

Sol. $A^2 + aA + bI = O \Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} 3(3)+2(1) & 3(2)+2(1) \\ 1(3)+1(1) & 1(2)+1(1) \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O \Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$$

$$11+3a+b=0 \quad \dots(1)$$

$$4+a+0=0 \Rightarrow a=-4$$

Putting the value of a in equation (1), we get $11+3(-4)+b=0 \Rightarrow b=12-11=1$

Hence, $a=-4$ and $b=1$.

23. Find the matrix A such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

Sol. Let matrix $A = \begin{bmatrix} a & b \\ x & y \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ x & y \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 5a-7x & 5b-7y \\ -2a+3x & -2b+3y \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$5a-7x=-16 \quad \dots(1) \times 3$$

$$-2a+3x=7 \quad \dots(2) \times 7$$

$$5b-7y=-6 \quad \dots(3) \times 3$$

$$-2b+3y=2 \quad \dots(4) \times 7$$

Solving equations (1) and (2), we get $a=1$

Putting the value of a in equation (1), we get

$$5a-7x=-16 \Rightarrow 5(1)-7x=-16 \Rightarrow 5-7x=-16 \Rightarrow 7x=5+16$$

$$\Rightarrow 7x=21 \Rightarrow x=3$$

Now, solving the equation (3) and (4), we get $b=-4$

Putting the value of b in equation (3), we get $5b-7y=-6 \Rightarrow 7y=5b+6$

$$\Rightarrow 7y=5(-4)+6 \Rightarrow 7y=-20+6 \Rightarrow 7y=-14 \Rightarrow y=-2$$

$\therefore a=1, b=-4, x=3$ and $y=-2$. Hence, matrix $A = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$.

24. Find the matrix A such that $A \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} a & x \\ b & y \end{bmatrix} \Rightarrow \begin{bmatrix} a & x \\ b & y \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2a+4x & 3a+5x \\ 2b+4y & 3b+5y \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

$$2a+4x=0 \quad \dots(1) \times 5$$

$$3a+5x=-4 \quad \dots(2) \times 4$$

$$2b+4y=10 \quad \dots(3) \times 5$$

$$3b+5y=3 \quad \dots(4) \times 4$$

Solving the equation (1) and (2), we get $a = -8$

Putting the value of a in equation (1), we get

$$2a + 4x = 0 \Rightarrow 2(-8) + 4x = 0 \Rightarrow 4x = 16 \Rightarrow x = 4$$

Now solve the equation (3) and (4), we get $-2b = 38 \Rightarrow b = -19$

Putting the value of b in equation (iii), we get $2b + 4y = 10$

$$\Rightarrow 2(-19) + 4y = 10 \Rightarrow 4y = 10 + 38 \Rightarrow 4y = 48 \Rightarrow y = 12$$

$$\therefore a = -8, b = -19, x = 4 \text{ and } y = 12. \text{ Hence, } A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}.$$

Or alternatively : $A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

25. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = (A^2 + B^2)$ then find the values of a and b .

Sol. Let $(A+B)^2 = A^2 + B^2 = (A+B)(A+B) = A^2 + B^2$

$$\Rightarrow A^2 + AB + BA + B^2 = A^2 + B^2 \Rightarrow AB + BA = 0$$

$$\Rightarrow AB = -BA \text{ and try it yourself, then find } a = 1, b = 4$$

26. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x) \cdot F(y) = F(x+y)$

Sol. Let $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{aligned} \therefore F(x) \cdot F(y) &= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) \end{aligned}$$

Hence, $F(x) \cdot F(y) = F(x+y)$

27. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cdot \cos \alpha + \sin \alpha (-\sin \alpha) & \cos \alpha \cdot \sin \alpha + \sin \alpha \cdot \cos \alpha \\ -\sin \alpha \cdot \cos \alpha + \cos \alpha (-\sin \alpha) & -\sin \alpha \cdot \sin \alpha + \cos \alpha \cdot \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 2 \sin \alpha \cos \alpha \\ -2 \sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}. \text{ Hence proved.}$$

28. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$, find x .

Sol. $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

$$\Rightarrow [1(1) + x(4) + 1(3) \quad 1(2) + x(5) + 1(2) \quad 1(3) + x(6) + 1(5)] \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow [1 + 4x + 3 \quad 2 + 5x + 2 \quad 3 + 6x + 5] \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow [4 + 4x \quad 5x + 4 \quad 6x + 8] \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0 \Rightarrow [(4x + 4)(1) + (5x + 4)(-2) + (6x + 8)3] = 0$$

$$\Rightarrow [4x + 4 - 10x - 8 + 18x + 24] = 0 \Rightarrow 12x + 20 = 0 \Rightarrow 12x = -20 \Rightarrow x = \frac{-20}{12} = \frac{-5}{3}$$

29. If $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = O$, find x .

Sol. $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$

$$\Rightarrow [x(2) + 4(1) + 1(0) \quad x(1) + 4(0) + 1(2) \quad x(2) + 4(2) + 1(-4)] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [2x + 4 + 0 \quad x + 0 + 2 \quad 2x + 8 - 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0 \Rightarrow [(2x + 4)x + (x + 2)4 + (2x + 4)(-1)] = 0$$

$$\Rightarrow [2x^2 + 4x + 4x + 8 - 2x - 4] = 0 \Rightarrow 2x^2 + 6x + 4 = 0 \Rightarrow 2x^2 + 4x + 2x + 4 = 0$$

$$\Rightarrow 2x(x + 2) + 2(x + 2) = 0 \Rightarrow (x + 2)(2x + 2) = 0 \Rightarrow x + 2 = 0 \text{ or } 2x + 2 = 0$$

$$\therefore x = -2 \text{ or } x = \frac{-2}{2} = -1. \text{ Hence, } x = -2 \text{ or } -1.$$

30. Find the values of a and b for which $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

Sol. $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} a(2)+b(-1) \\ -a(2)+2b(-1) \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$$2a - b = 5 \quad \dots(1)$$

$$-2a - 2b = 4 \quad \dots(2)$$

Solving equations (1) and (2), we get $-3b = 9 \Rightarrow b = -3$

Putting the value of b in equation (1), we get $2a - b = 5 \Rightarrow 2a = 5 + b$

$$\Rightarrow 2a = 5 + (-3) \Rightarrow 2a = 2 \Rightarrow a = 1. \text{ Hence, } a = 1 \text{ and } b = -3.$$

31. If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$ find $f(A)$, where $f(x) = x^2 - 5x + 7$.

Sol. $f(x) = x^2 - 5x + 7 \Rightarrow f(A) = A^2 - 5A + 7I$

$$\Rightarrow f(A) = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 3(3)+4(-4) & 3(4)+4(-3) \\ -4(3)-3(-4) & -4(4)-3(-3) \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -7-15+7 & 0-20+0 \\ 0+20+0 & -7+15+7 \end{bmatrix} \Rightarrow f(A) = \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$$

32. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all $n \in N$.

Sol. We shall prove the result by using the principle of mathematical induction.

When $n = 1$, we have $A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Thus, the result is true for $n = 1$.

Let the result be true for $n = k$. then $A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

$$\therefore A^{k+1} = A.A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{k+1} = \begin{bmatrix} 1(1)+k(0) & 1(1)+k(1) \\ 0(1)+1(0) & 0(1)+1(1) \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1+0 & 1+k \\ 0 & 0+1 \end{bmatrix} \Rightarrow A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

Thus, the result is true for $n = (k+1)$, whenever it is true for $n = k$.

So, the result is true for $n \in N$. Hence, $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all values of $n \in N$.



33. Give an example of two matrices A and B such that $A \neq B$, $B \neq O$, $AB = O$ and $BA \neq O$.

Sol. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$;

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AB = O$$

$$\text{Again let } BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow BA = \begin{bmatrix} 0+0 & 0+0 \\ 1+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow BA \neq O$$

34. Give an example of three matrices A, B, C such that $AB = AC$ but $B \neq C$.

Sol. Let the three matrices be $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AC = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $AB = AC$

35. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

$$\begin{aligned} \text{Sol. } 3A^2 - 2B + I &\Rightarrow 3 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 3 \begin{bmatrix} 1(1)+0(-1) & 1(0)+0(7) \\ -1(1)+7(-1) & -1(0)+7(7) \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 3 \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 3 \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix} \end{aligned}$$

36. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find the value of x

$$\begin{aligned} \text{Sol. Given } \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} &= \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} &= \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & -6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow x = 13 \end{aligned}$$

EXERCISE 5D (Pg.No.: 201) <https://millionstar.godaddysites.com/>

1. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$, verify that $(A')' = A$.

Sol. $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{bmatrix} \Rightarrow (A')' = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix} \therefore (A')' = A$. Hence proved.

2. If $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$, verify that $(2A)' = 2A'$.

Sol. $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$.

$$\text{LHS} = 2A = 2 \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{bmatrix} \Rightarrow (2A)' = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$$

$$\text{RHS} = 2A' = 2 \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix} = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix} \therefore (2A)' = 2A' \text{. Hence proved.}$$

3. If $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$, verify that $(A+B)' = (A'+B')$.

Sol. $A' = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$

$$\Rightarrow (A+B)' = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}' + \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}' = \begin{bmatrix} 3-4 & 2-5 & -1-2 \\ -5+3 & 0+1 & -6+8 \end{bmatrix}' = \begin{bmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{bmatrix}'$$

$$\text{LHS} = (A+B)' = \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\text{RHS} = (A'+B') = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix} = \begin{bmatrix} 3-4 & -5+3 \\ 2-5 & 0+1 \\ -1-2 & -6+8 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

$\therefore \text{LHS} = \text{RHS}$. Hence proved that $(A+B)' = (A'+B')$.

4. If $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$, verify that $(P+Q)' = (P'+Q')$.

Sol. $\text{LHS} = (P+Q)' = \left(\begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix} \right)' = \begin{bmatrix} 3+7 & 4-5 \\ 2-4 & -1+0 \\ 0+2 & 5+6 \end{bmatrix}' = \begin{bmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{bmatrix}'$

$$\Rightarrow (P+Q)' = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix} \quad \text{https://millionstar.godaddysites.com/}$$

$$\text{RHS, } P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix}, Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow (P'+Q') = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 3+7 & 2-4 & 0+2 \\ 4-5 & -1+0 & 5+6 \end{bmatrix} = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

$\therefore \text{LHS} = \text{RHS}$. Hence proved that $(P+Q)' = (P'+Q')$.

5. If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that $(A+A')$ is symmetric.

$$\text{Sol. } A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

$$(A+A') = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 4+4 & 1+5 \\ 5+1 & 8+8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

$$\Rightarrow (A+A')' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix} \therefore (A+A') = (A+A')'. \text{ Hence, } (A+A') \text{ is a symmetric.}$$

6. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $(A-A')$ is skew-symmetric.

$$\text{Sol. } A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$(A-A') = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow (A-A')' = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$\therefore (A-A') = -(A-A')'$. Hence, $(A-A')$ is a skew symmetric.

7. Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew symmetric.

$$\text{Sol. } A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \Rightarrow A' = -\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$\Rightarrow A' = -A$. Hence, A is a skew symmetric.

8. Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, as the sum of a symmetric matrix and a skew-symmetric matrix.

$$\text{Sol. } A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$(A+A') = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2+2 & 3-1 \\ -1+3 & 4+4 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2-2 & 3-3 \\ -1-3 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

∴ Sum of the symmetric and skew symmetric matrix

$$= \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

9. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

$$(A + A') = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

∴ Sum of the symmetric and a skew-symmetric matrix,

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -3/2 \\ -3/2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$$

10. Express the matrix $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Sol. $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}$

$$(A + A') = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} -1-1 & 5+2 & 1+7 \\ 2+5 & 3+3 & 4+0 \\ 7+1 & 0+4 & 9+9 \end{bmatrix} = \begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} -1+1 & 5-2 & 1-7 \\ 2-5 & 3-3 & 4-0 \\ 7-1 & 0-4 & 9-9 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

∴ Sum of a symmetric and a skew-symmetric matrix

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 7/2 & 4 \\ 7/2 & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 3/2 & -3 \\ -3/2 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

11. Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

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$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

$$(A + A') = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 3+3 & -1+2 & 0+1 \\ 2-1 & 0+0 & 3-1 \\ 1+0 & -1+3 & 2+2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 3-3 & -1-2 & 0-1 \\ 2+1 & 0-0 & 3+1 \\ 1-0 & -1-3 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

∴ Sum of a symmetric and a skew-symmetric matrix, $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$$= \frac{1}{2} \begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1/2 & 1/2 \\ 1/2 & 0 & 1 \\ 1/2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3/2 & -1/2 \\ 3/2 & 0 & 2 \\ 1/2 & -2 & 0 \end{bmatrix}$$

12. Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is

skew-symmetric.

Sol. $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$

$$(A + A') = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 3+3 & 2+4 & 5+0 \\ 4+2 & 1+1 & 3+6 \\ 0+5 & 6+3 & 7+7 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 3-3 & 2-4 & 5-0 \\ 4-2 & 1-1 & 3-6 \\ 0-5 & 6-3 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

∴ Sum of a symmetric and skew-symmetric matrix $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$$= \frac{1}{2} \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 1 & 9/2 \\ 5/2 & 9/2 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 5/2 \\ 1 & 0 & -3/2 \\ -5/2 & 3/2 & 0 \end{bmatrix}$$

13. For each of the following pairs of matrices A and B , verify that $(AB)' = B'A'$.

(i) $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

(iii) $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$.

(iv) $A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$

(i) $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$ <https://millionstar.godaddysites.com/>

$$\text{LHS} = (AB)'$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1(1)+3(2) & 1(4)+3(5) \\ 2(1)+4(2) & 2(4)+4(5) \end{bmatrix} = \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix} = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$$

$$\Rightarrow (AB)' = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\text{RHS} = B'A', \quad B' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \text{ and } A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(1)+2(3) & 1(2)+2(4) \\ 4(1)+5(3) & 4(2)+5(4) \end{bmatrix} = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$\therefore \text{LHS} = \text{RHS}$. Hence proved that $(AB)' = B'A'$.

(ii) $A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

$$\text{LHS} = (AB)'$$

$$\Rightarrow AB = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3(1)-1(2) & 3(-3)-1(-1) \\ 2(1)-2(2) & 2(-3)-2(-1) \end{bmatrix} = \begin{bmatrix} 3-2 & -9+1 \\ 2-4 & -6+2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$$

$$\Rightarrow (AB)' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

$$\text{RHS} = B'A', \quad B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \text{ and } A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 1(3)+2(-1) & 1(2)+2(-2) \\ -3(3)-1(-1) & -3(2)-1(-2) \end{bmatrix} = \begin{bmatrix} 3-2 & 2-4 \\ -9+1 & -6+2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

$\therefore \text{LHS} = \text{RHS}$. Hence proved that $(AB)' = (B'A')$.

(iii) $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$

$$\text{LHS} = (AB)'$$

$$\Rightarrow AB = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \end{bmatrix} = \begin{bmatrix} -1(-2) & -1(-1) & -1(-4) \\ 2(-2) & 2(-1) & 2(-4) \\ 3(-2) & 3(-1) & 3(-4) \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

$$\Rightarrow (AB)' = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

$$\text{RHS} = B'A', \quad B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \text{ and } A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -2(-1) & -2(2) & -2(3) \\ -1(-1) & -1(2) & -1(3) \\ -4(-1) & -4(2) & -4(3) \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

$\therefore$ LHS = RHS. Hence proved that $(AB)' = B'A'$

$$\text{(iv)} \quad A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\text{LHS} = (AB)'$$

$$\Rightarrow AB = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 5 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1(3) + 2(2) - 3(-1) & -1(-4) + 2(1) - 3(0) \\ 4(3) - 5(2) + 5(-1) & 4(-4) - 5(1) + 5(0) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 + 3 & 4 + 2 - 0 \\ 12 - 10 - 5 & -16 - 5 + 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix} \Rightarrow (AB)' = \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

$$\text{RHS} = B'A', \quad B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \text{ and } A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 3(-1) + 2(2) - 1(-3) & 3(4) + 2(-5) - 1(6) \\ -4(-1) + 1(2) + 0(-3) & -4(4) + 1(-5) + 0(6) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 + 3 & 12 - 10 - 6 \\ 4 + 2 + 0 & -16 - 5 + 0 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

$\therefore$ LHS = RHS. Hence, proved that $(AB)' = B'A'$.

14. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, show that $A'A = I$.

Sol. $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$$A'A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

Hence, proved that $A'A = I$.

15. If matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, write AA' .

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$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 \\ 3 \end{bmatrix}; \quad AA' = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(2) + 3(3) \end{bmatrix} = \begin{bmatrix} 1 + 4 + 9 \end{bmatrix} = \begin{bmatrix} 14 \end{bmatrix}$$

EXERCISE 5E (Pg.No.: 211)

Using elementary row transformations, find the inverse of each of the following matrices.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 3R_1$, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + 2R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} A$. Hence, $A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow 5R_1 + 2R_2$, $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow -R_2$, $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A \Rightarrow 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$. Hence $A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$

3. $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow 2R_2 + 3R_1$, $\begin{bmatrix} 2 & 5 \\ 0 & 17 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} A$

Applying $R_1 \rightarrow 17R_1 - 3R_2$, $\begin{bmatrix} 34 & 0 \\ 0 & 17 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ 3 & 2 \end{bmatrix} A$

Applying $R_1 \rightarrow \frac{1}{34}R_1$ and $R_2 \rightarrow \frac{1}{17}R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{bmatrix} A$. Hence, $A^{-1} = \begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{bmatrix}$

Sol.
$$\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$,
$$\begin{bmatrix} 2 & -3 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow 11R_1 + 3R_2$,
$$\begin{bmatrix} 22 & 0 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{22}R_1$ and $R_2 \rightarrow \frac{1}{11}R_2$,
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix} A \quad \text{Hence, } A^{-1} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$$

5.
$$\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$$

Sol. We have
$$\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 2R_2 - R_1$,
$$\begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{4}R_1$ and $R_2 \rightarrow \frac{1}{10}R_2$,
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix} A \quad \text{Hence } A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix}$$

6.
$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

Sol. We have,
$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 6R_2 - 8R_1$,
$$\begin{bmatrix} 6 & 7 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 6 \end{bmatrix} A$$

Applying $R_1 \rightarrow 2R_1 + 7R_2$,
$$\begin{bmatrix} 12 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -54 & 42 \\ -8 & 6 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{12}R_1$ and $R_2 \rightarrow -\frac{1}{2}R_2$,
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ -4 & 3 \end{bmatrix} A \quad \text{Hence, } A^{-1} = \begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ -4 & 3 \end{bmatrix}$$

7.
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Sol. We have,
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \rightarrow R_3 - 3R_1$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$

Applying $R_3 \rightarrow R_3 + 5R_2$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow \frac{1}{2}R_3$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + R_3$, $R_2 \rightarrow R_2 - 2R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \leftrightarrow R_3$, $\begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - R_2$, $\begin{bmatrix} 1 & -4 & -1 \\ 2 & 2 & 3 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & -1 & 0 \end{bmatrix} A$

Applying $R_2 \leftrightarrow R_3$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & -5 & 0 \\ 0 & 10 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & 0 \\ 0 & 3 & -2 \end{bmatrix} A$

Applying $R_3 \rightarrow R_3 + 2R_2$, $R_2 \rightarrow -\frac{1}{5}R_2$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + 4R_2$, $R_3 \rightarrow \frac{1}{5}R_3$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Hence, $A^{-1} = \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{5} \begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \rightarrow R_3 - 2R_1$, $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow 3R_2 - R_1$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 15 & 25 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow \frac{1}{5}R_2$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

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Applying $R_3 \rightarrow 3R_3 - 4R_2$,
$$\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & -11 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & \frac{3}{5} & 0 \\ -\frac{26}{5} & -\frac{12}{5} & 3 \end{bmatrix} A$$

Applying $R_3 \rightarrow -\frac{1}{11}R_3$,
$$\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_3$ and $R_1 \rightarrow R_1 - 2R_3$,
$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{55} & -\frac{25}{55} & \frac{6}{11} \\ -\frac{141}{55} & -\frac{27}{55} & \frac{15}{11} \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$, $R_2 \rightarrow \frac{1}{3}R_2$,
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{55} & -\frac{8}{55} & \frac{2}{11} \\ -\frac{47}{55} & \frac{9}{55} & \frac{5}{11} \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$$

Hence, $A^{-1} = -\frac{1}{55} \begin{bmatrix} -1 & 8 & -10 \\ 47 & 9 & -25 \\ -26 & -12 & 15 \end{bmatrix}$

10.
$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

Sol. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$,
$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 8 \\ 0 & -9 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 2R_2$, $R_3 \rightarrow R_3 - 9R_2$,
$$\begin{bmatrix} 1 & 0 & 13 \\ 0 & -1 & 8 \\ 0 & 0 & -67 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -2 & 1 & 0 \\ 15 & -9 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow -\frac{1}{67}R_3$,
$$\begin{bmatrix} 1 & 0 & 13 \\ 0 & -1 & 8 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -2 & 1 & 0 \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} A$$

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Applying $R_1 \rightarrow R_1 - 13R_3$, $R_2 \rightarrow R_2 - 8R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ -\frac{14}{67} & -\frac{5}{67} & \frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} A$

Applying $R_2 \rightarrow (-1)R_2$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} A$ Hence, $A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

11. $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Sol. We have $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$

Applying $R_1 \rightarrow R_1 - R_2$, $\begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$, $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ -3 & 3 & 1 \end{bmatrix} . A$

Applying $R_2 \rightarrow \frac{1}{2}R_2$, $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & \frac{3}{2} & 0 \\ -3 & 3 & 1 \end{bmatrix} . A$

Applying $R_1 \rightarrow R_1 + R_2$, $R_3 \rightarrow R_3 + 2R_2$, $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & 0 \\ -5 & 6 & 1 \end{bmatrix} . A$

Applying $R_3 + \frac{1}{4}R_3$, $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & 0 \\ -\frac{5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix} . A$

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Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, $R_2 \rightarrow R_2 + \frac{1}{2}R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{5}{8} & \frac{5}{4} & \frac{1}{8} \\ -\frac{3}{8} & \frac{3}{4} & -\frac{1}{8} \\ -\frac{5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix} \cdot A$

$$\therefore A^{-1} = -\frac{1}{8} \begin{bmatrix} -5 & 5(2) & 1 \\ -3 & 3(2) & -1 \\ -5 & 3(4) & 1(2) \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{8} \begin{bmatrix} 5 & -10 & -1 \\ 3 & -6 & 1 \\ 10 & -12 & -2 \end{bmatrix}$$

12. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{cases} R_2 \rightarrow R_2 + 3R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix} A \begin{cases} R_2 \rightarrow R_2 + 2R_3 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ -2 & 0 & 1 \end{bmatrix} A \begin{cases} R_2 \rightarrow -R_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & -1 & -2 \\ 3 & -5 & -9 \end{bmatrix} A \begin{cases} R_1 \rightarrow R_1 - 3R_2 \\ R_3 \rightarrow R_3 + 5R_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & -1 & -2 \\ -3 & 5 & 9 \end{bmatrix} A \begin{cases} R_3 \rightarrow -R_3 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A \begin{cases} R_1 \rightarrow R_1 - R_3 \\ R_2 \rightarrow R_2 + R_3 \end{cases}$$

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$$\Rightarrow I = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} \cdot A \quad \therefore A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$$

13. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 + 2R_1$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 - 2R_2$, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Sol. We have $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & -3 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 - R_2$, $\begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1$, $\begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 1 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + 3R_2$, $R_3 \rightarrow R_3 - 4R_2$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 8 & -6 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A$

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Applying $R_1 \rightarrow R_1 + R_3$, $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \cdot A$. Hence, $A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$.

15. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now, $A = I \cdot A$

$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \{R_1 \leftrightarrow R_2\}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A \begin{cases} R_2 \rightarrow R_2 + R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{cases}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \cdot A \{R_3 \rightarrow R_3 + 2R_2\}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot A \{R_2 \leftrightarrow R_3\}$

$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} \cdot A \begin{cases} R_1 + R_1 - 2R_2 \\ R_3 + R_3 - 3R_2 \end{cases}$

$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & -1 & 1 \\ 5 & -4 & 3 \end{bmatrix} \cdot A \{R_3 \leftrightarrow -R_3\}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -7 & 3 \end{bmatrix} \cdot A \begin{cases} R_1 \rightarrow R_1 + R_3 \\ R_2 + R_2 - 2R_3 \end{cases}$

$\Rightarrow I = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$ Hence $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$



EXERCISE 5F (Pg.No.: 213)

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Solve the following Short Answer Questions.

1. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}(i-2j)^2$.

Sol. Let matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3 \times 2}$. Thus $a_{ij} = \frac{1}{2}(i-2j)^2$ for $i=1, 2$ for $j=1, 2, 3$.

$$\therefore a_{11} = \frac{1}{2}\{1-2(1)\}^2 = \frac{1}{2}(1-2)^2 = \frac{1}{2}(-1)^2 = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$a_{12} = \frac{1}{2}\{1-2(2)\}^2 = \frac{1}{2}(1-4)^2 = \frac{1}{2}(-3)^2 = \frac{1}{2} \times 9 = \frac{9}{2}, \quad a_{21} = \frac{1}{2}\{2-2(1)\}^2 = \frac{1}{2}(2-2)^2 = \frac{1}{2} \times 0 = 0$$

$$a_{22} = \frac{1}{2}\{2-2(2)\}^2 = \frac{1}{2}(2-4)^2 = \frac{1}{2}(-2)^2 = \frac{1}{2} \times 4 = 2, \quad a_{31} = \frac{1}{2}\{3-2(1)\}^2 = \frac{1}{2}(3-2)^2 = \frac{1}{2}(1)^2 = \frac{1}{2}$$

$$a_{32} = \frac{1}{2}\{3-2(2)\}^2 = \frac{1}{2}(3-4)^2 = \frac{1}{2}(-1)^2 = \frac{1}{2}$$

Hence, $A = \begin{bmatrix} 1/2 & 9/2 \\ 0 & 2 \\ 1/2 & 1/2 \end{bmatrix}$

2. Construct a 2×3 matrix whose elements are given by $a_{ij} = \frac{1}{2}|-3i+j|$.

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, Thus $a_{ij} = \frac{1}{2}|-3i+j|$ for $i=1, 2$ and $j=1, 2, 3$

$$\therefore a_{11} = \frac{1}{2}|-3(1)+1| = \frac{1}{2}|-3+1| = \frac{1}{2} \times 2 = 1, \quad a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-3+2| = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$a_{13} = \frac{1}{2}|-3(1)+3| = \frac{1}{2}|-3+3| = \frac{1}{2} \times 0 = 0, \quad a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{1}{2} \times 5 = \frac{5}{2}$$

$$a_{22} = \frac{1}{2}|-3(2)+2| = \frac{1}{2}|-6+2| = \frac{1}{2} \times 4 = 2, \quad a_{23} = \frac{1}{2}|-3(2)+3| = \frac{1}{2}|-6+3| = \frac{1}{2} \times 3 = \frac{3}{2}$$

Hence, $A = \begin{bmatrix} 1 & 1/2 & 0 \\ 5/2 & 2 & 3/2 \end{bmatrix}$

3. If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y .

Sol. $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$

$$x+2y = -4 \quad \dots(1)$$

$$3x = 6 \Rightarrow x = 2$$

Putting the value of x in equation (1), we get $x+2y = -4 \Rightarrow 2+2y = -4$

$$\Rightarrow 2y = -4-2 \Rightarrow 2y = -6 \Rightarrow y = -3. \text{ Hence, } x = 2 \text{ and } y = -3.$$

4. Find the values of x and y , if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

$$\text{Sol. } 2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$2+y=5 \quad \text{and} \quad 2x+2=8$$

$$\Rightarrow y=5-2 \quad \text{and} \quad \Rightarrow 2x=8-2$$

$$\Rightarrow y=3 \quad \text{and} \quad \Rightarrow x=3 \quad \text{Hence, } x=3 \text{ and } y=3.$$

5. If $x \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y .

Sol. $\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$2x-y=10 \quad \dots(1)$$

$$3x+y=5 \quad \dots(2)$$

Solving the equations (1) and (2), we get $5x=15 \Rightarrow x=3$

Putting the value of x in equation (1), we get $2x-y=10 \Rightarrow 2(3)-y=10$

$$\Rightarrow y=6-10 \Rightarrow y=-4. \text{ Hence, } x=3 \text{ and } y=-4.$$

6. If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find the values of x, y, z, w .

Sol. $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

$$x=3 \quad \dots(1)$$

$$\text{and } 2x+z=4 \quad \dots(2)$$

$$\Rightarrow z=4-2x \Rightarrow z=4-2(3) \Rightarrow z=4-6=-2$$

Now, $3y-w=7 \Rightarrow w=3y-7 \Rightarrow w=3(7)-7=21-7=14$. Hence, $x=3, y=7, z=-2, w=14$

7. If $\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3 \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, find the values of x, y, z, w .

Sol. $\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$

$$x+4=3x \Rightarrow 4=2x \Rightarrow x=2$$

$$6+x+y=3y \Rightarrow 6+x=2y \Rightarrow 6+2=2y \Rightarrow y=4$$

$$\therefore 2w+3=3w \Rightarrow w=3$$

$$-1+z+w=3z \Rightarrow -1+w=2z \Rightarrow -1+3=2z \Rightarrow 2=2z \Rightarrow z=1$$

Hence, $x=2, y=4, z=1, w=3$.

8. If $A = \text{diag}(3 \ -2 \ 5)$ and $B = \text{diag}(1 \ 3 \ -4)$, find $(A+B)$

Sol. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

$$\Rightarrow (A+B) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \Rightarrow (A+B) = \begin{bmatrix} 3+1 & 0 & 0 \\ 0 & -2+3 & 0 \\ 0 & 0 & 5+(-4) \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow (A+B) = \text{diag} \begin{pmatrix} 4 & 1 \end{pmatrix} \quad \text{https://millionstar.godaddysites.com/}$$

9. Show that $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} = I$.

Sol. LHS = $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 \theta + \cos^2 \theta & \sin \theta \cos \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = \text{RHS.} \quad \text{Hence proved.}$$

10. If $A = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$, find the matrix C such that $A+B+C$ is a zero matrix.

Sol. $A+B+C=O \Rightarrow C=-A-B \Rightarrow C = -\begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$

$$\Rightarrow C = \begin{bmatrix} -1-3 & 5-1 \\ 3-2 & -2+1 \\ -4+2 & 2-3 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 1 & -1 \\ -2 & -1 \end{bmatrix}$$

11. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then find the least value of α for which $A+A'=I$.

Sol. Let $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$A+A'=I \Rightarrow \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2\cos \alpha = 1 \Rightarrow \cos \alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

12. Find the values of x and y for which $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Sol. $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-3y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$2x-3y=1 \quad \dots(1)$$

$$\text{and } x+y=3 \quad \dots(2) \times 3$$

$$\text{Solving equations (1) and (2), we get } 5x=10 \Rightarrow x=2$$

$$\text{Putting the value of } x \text{ in equation (1), we get } 2x-3y=1 \Rightarrow 2(2)-3y=1$$

$$\Rightarrow 4-1=3y \Rightarrow 3y=3 \Rightarrow y=1. \text{ Hence, } x=2 \text{ and } y=1.$$

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Find the values of x and y for which $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

Sol. $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x+2y \\ 3y+2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$x+2y=3 \quad \dots(1) \times 2$$

$$\text{and } 2x+3y=5 \quad \dots(2)$$

Solving equations (1) and (2), we get $y=1$

Putting the value of y in equation (1), we get $x+2y=3 \Rightarrow x=3-2y$

$\Rightarrow x=3-2(1)=3-2=1$. Hence, $x=1$ and $y=1$.

14. If $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$, show that $(A+A')$ is symmetric.

Sol. $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

$$\Rightarrow (A+A') = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 4+4 & 5+1 \\ 1+5 & 8+8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix} \Rightarrow (A+A')' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

$\therefore (A+A') = (A+A')'$. Hence, proved that $(A+A')$ is a symmetric matrix.

15. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, show that $(A-A')$ is skew-symmetric.

Sol. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

$$(A-A') = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Since, in resultant matrix diagonal values are just opposite. Hence prove that $(A-A')$ is a skew-symmetric.

16. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find a matrix X such that $A+2B+X=O$.

Sol. $A+2B+X=O \Rightarrow X=-A-2B \Rightarrow X = -\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow X = \begin{bmatrix} -2 & 3 \\ -4 & -5 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 0 & 6 \end{bmatrix} \Rightarrow X = \begin{bmatrix} -2+2 & 3-4 \\ -4-0 & -5-6 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$$

17. If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that $3A-2B+X=O$.

Sol. $3A-2B+X=O \Rightarrow X=2B-3A \Rightarrow X = 2\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$$\Rightarrow X = \begin{bmatrix} -4 & 2 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} -4-12 & 2-6 \\ 6-3 & 4-9 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$$

18. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, show that $A'A=I$.

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I. \text{ Hence proved.}$$

19. If A and B are symmetric matrices of the same order, show that $(AB - BA)$ is a skew-symmetric matrix.

Sol. If A and B are symmetry matrix, then $A' = A, B' = B$

$$\text{Then, } (AB - BA)' = (AB)' - (BA)' = B'A' - A'B' = BA - AB$$

$$\therefore A' = A, B' = B = -AB + BA \Rightarrow (AB - BA)' = -(AB - BA)$$

Hence, $AB - BA$ is skew-symmetry matrix.

20. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 1$, find $f(A)$.

Sol. Given, $f(x) = x^2 - 4x + 1 \Rightarrow f(A) = A^2 - 4A + I$

$$A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$\therefore f(A) = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

21. If the matrix A is both symmetric and skew-symmetric, show that A is a zero matrix.

Sol. Given A is symmetric matrix. Hence, $A' = A$... (1)

Also, A is skew-symmetric. Hence, $A' = -A$... (2)

Subtracting (1) and (2), we have, $0 = 2A \Rightarrow A = 0$

Hence, A is zero matrix.