

ADJOINT AND INVERSE OF A MATRIX (XII, R. S. AGGARWAL)

EXERCISE 7 (Pg.No.: 293)

Find the adjoint of the given matrix and verify in each case that $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$.

1. $\begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(9) = 9$; $C_{12} = (-1)^{1+2}(5) = -5$

$C_{21} = (-1)^{2+1}(3) = -3$; $C_{22} = (-1)^{2+2}(2) = 2$

$\therefore \text{adj } A = \begin{bmatrix} 9 & -5 \\ -3 & 2 \end{bmatrix}' = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix} \Rightarrow |A| = [2(9) - 5(3)] \Rightarrow |A| = (18 - 15) \Rightarrow |A| = 3$

M.H.S. $= (\text{adj } A) \cdot A = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} 18-15 & 27-27 \\ -10+10 & -15+18 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

L.H.S. $= A(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 18-15 & -6+6 \\ 45-45 & -15+18 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

and R.H.S. $= |A| \cdot I = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. Hence $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$

2. $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(2) = 2$; $C_{12} = (-1)^{1+2}(-1) = 1$

$C_{21} = (-1)^{2+1}(-5) = 5$; $C_{22} = (-1)^{2+2}(3) = 3$

$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}' = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} \Rightarrow |A| = [3(2) - (-1)(-5)] \Rightarrow |A| = (6 - 5) \Rightarrow |A| = 1$

Now, M.H.S. $= (\text{adj } A) \cdot A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, L.H.S. = $A \cdot (\text{adj } A) = \begin{bmatrix} 5 & 15 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5-15 & 15-15 \\ -2+2 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, R.H.S. = $|A| \cdot I = 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Hence, $A(\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$

3. $\begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(\cos \alpha) = \cos \alpha$; $C_{12} = (-1)^{1+2}(\sin \alpha) = -\sin \alpha$

$C_{21} = (-1)^{2+1}(\sin \alpha) = -\sin \alpha$; $C_{22} = (-1)^{2+2}(\cos \alpha) = \cos \alpha$

$\therefore \text{adj } A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix}$

$\Rightarrow |A| = [\cos \alpha \cdot \cos \alpha - \sin \alpha \cdot \sin \alpha] \Rightarrow |A| = (\cos^2 \alpha - \sin^2 \alpha) \Rightarrow |A| = \cos 2\alpha$

Now, L.H.S. = $A \cdot (\text{adj } A) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$
 $= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Now, M.H.S. = $(\text{adj } A) \cdot A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$
 $= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Now, R.H.S. = $|A| \cdot I = \cos 2\alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$. Hence, $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$

4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = [1(3) - 0(-2)] = (3 - 0) = 3$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -[3(3) - (-2)(1)] = -(9 + 2) = -11$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = [3(0) - 1(1)] = (0 - 1) = -1$

$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -[(-1)(3) - 0(2)] = -(-3 - 0) = 3$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = [1(3) - 1(2)] = (3-2) = 1$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -[1(0) - 1(-1)] = -(0+1) = -1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = [(-1)(-2) - 1(2)] = (2-2) = 0$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -[1(-2) - 3(2)] = -(-2-6) = 8$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = [1(1) - 3(-1)] = (1+3) = 4$$

$$\therefore \text{adj } A = \begin{bmatrix} 3 & -11 & -1 \\ 3 & 1 & -1 \\ 0 & 8 & 4 \end{bmatrix}' = \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$\Rightarrow |A| = 1 \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} + 1 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow |A| = 1(3-0) + 1(9+2) + 2(0-1)$$

$$\Rightarrow |A| = 3+11-2 \Rightarrow |A| = 12$$

$$\text{Now, L.H.S.} = A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+11-2 & 3-1-2 & 0-8+8 \\ 9-11-2 & 9+1+2 & 0+8-8 \\ 3-0-3 & 3+0-3 & 0+0+12 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\text{Now, M.H.S.} = (\text{adj } A) \cdot A = \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+9+0 & -3+3+0 & 6-6+0 \\ -11+3+8 & 11+1+0 & -22-2+24 \\ -1+(-3)+4 & 1-1+0 & -2+2+12 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\text{Now, R.H.S.} = |A| \cdot I.$$

$$12 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}. \text{ Hence, } A(\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$$

5. $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 5 \\ -2 & 2 \end{vmatrix} = (12-10) = 2; \quad C_{12} = (-1)^{1+2} \begin{vmatrix} -15 & -5 \\ 5 & 2 \end{vmatrix} = -(-30+25) = 5$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} -15 & 6 \\ 5 & -2 \end{vmatrix} = (30-30) = 0; \quad C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} = -(-2+2) = 0$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = (6-5) = 1; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -1 \\ 5 & -2 \end{vmatrix} = -1(-6+5) = 1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 6 & -5 \end{vmatrix} = (5-6) = -1; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ -15 & -5 \end{vmatrix} = -(-15+15) = 0$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -1 \\ -15 & 6 \end{vmatrix} = (18-15) = 3$$

$$\text{adj } A = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix} \Rightarrow |A| = 3 \begin{vmatrix} 6 & -5 \\ -2 & 2 \end{vmatrix} + 1 \begin{vmatrix} -15 & -5 \\ 5 & 2 \end{vmatrix} + 1 \begin{vmatrix} -15 & 6 \\ 5 & -2 \end{vmatrix}$$

$$\Rightarrow |A| = 3(12-10) + (-30+25) + (30-30) \Rightarrow |A| = 3(2) + (-5)$$

$$\Rightarrow |A| = 6-5 \Rightarrow |A| = 1$$

$$\begin{aligned} \text{Now, L.H.S.} &= A(\text{adj } A) = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6-5+0 & 0-1+1 & -3-0+3 \\ -30+30+0 & 0+6-5 & 15+0-15 \\ 10-10+0 & 0-2+2 & -5+0+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, M.H.S.} &= (\text{adj } A).A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 6-0-5 & -2+0+2 & 2-0-2 \\ 15-15+0 & -5+6-0 & 5-5+0 \\ 0-15+15 & 0+6-6 & 0-5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\text{Now, R.H.S.} = |A|.I = 1. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Hence, } A(\text{adj } A) = (\text{adj } A).A = |A|.I$$

6. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

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$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = (2-3) = -1; C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -(1-9) = 8$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (1-6) = -5; C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -(-1-2) = 1$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} = (0-6) = -6; C_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} = -(0-3) = 3$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3-4) = -1; C_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -(-0-2) = 2$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = (0-1) = -1$$

$$\text{adj } A = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} \Rightarrow |A| = 0 \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \Rightarrow |A| = 0 - 1(1-9) + 2(1-6)$$

$$\Rightarrow |A| = (8-10) \Rightarrow |A| = -2$$

$$\text{Now, L.H.S.} = A(\text{adj } A) = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+8-10 & 0-6+6 & 0+2-2 \\ -1+16-15 & 1-12+9 & -1+4-3 \\ -3+8-5 & 3-6+3 & -3+2-1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{Now, M.H.S.} = (\text{adj } A).A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1-3 & -1+2-1 & -2+3-1 \\ 0-6+6 & 8-12+2 & 16-18+2 \\ 0+3-3 & -5+6-1 & -10+9-1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{Now, R.H.S.} = |A|.I = (-2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{Hence, } A(\text{adj } A) = (\text{adj } A).A = |A|.I$$

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$$\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 4 \\ 8 & 2 \end{vmatrix} = (-2 - 32) = -34$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 4 \\ 6 & 2 \end{vmatrix} = -(10 - 24) = 14$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & -1 \\ 6 & 8 \end{vmatrix} = (40 + 6) = 46$; $C_{21} = (-1)^{2+1} \begin{vmatrix} 7 & 3 \\ 8 & 2 \end{vmatrix} = -(14 - 24) = 10$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 9 & 3 \\ 6 & 2 \end{vmatrix} = (18 - 18) = 0$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 9 & 7 \\ 6 & 8 \end{vmatrix} = -(72 - 42) = -30$

$C_{31} = (-1)^{3+1} \begin{vmatrix} 7 & 3 \\ -1 & 4 \end{vmatrix} = (28 + 3) = 31$; $C_{32} = \begin{vmatrix} 9 & 3 \\ 5 & 4 \end{vmatrix} = -(36 - 15) = -21$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 9 & 7 \\ 5 & -1 \end{vmatrix} = (-9 - 35) = -44$

$\text{adj } A = \begin{bmatrix} -34 & 14 & 46 \\ 10 & 0 & -30 \\ 31 & -21 & -44 \end{bmatrix}' = \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & 21 \\ 46 & -30 & -44 \end{bmatrix}$

Now, L.H.S. = $A \cdot (\text{adj } A) = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ 46 & -30 & -44 \end{bmatrix}$

$= \begin{bmatrix} -306 + 98 + 138 & 90 + 0 - 90 & 279 - 147 - 132 \\ -70 - 14 + 184 & 50 - 0 - 120 & 155 + 21 - 176 \\ -204 + 112 + 92 & 60 + 0 - 60 & 186 - 168 - 88 \end{bmatrix} = \begin{bmatrix} -70 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -70 \end{bmatrix}$

Now, M.H.S. = $(\text{adj } A) \cdot A = \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & 21 \\ 46 & -30 & -44 \end{bmatrix} \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$

$= \begin{bmatrix} -306 + 50 + 186 & 238 - 10 + 248 & -132 - 40 + 62 \\ 126 + 0 - 126 & 98 - 0 - 168 & 42 + 0 - 142 \\ 414 - 150 - 264 & 322 + 30 - 352 & 138 - 120 - 88 \end{bmatrix} = \begin{bmatrix} -70 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -70 \end{bmatrix}$

$A = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix} \Rightarrow |A| = 9 \begin{vmatrix} -1 & 4 \\ 8 & 2 \end{vmatrix} - 7 \begin{vmatrix} 5 & 4 \\ 6 & 2 \end{vmatrix} + 3 \begin{vmatrix} 5 & -1 \\ 6 & 8 \end{vmatrix}$

$\Rightarrow |A| = 9(-2 - 32) - 7(10 - 24) + 3(40 + 6) \Rightarrow |A| = -306 + 98 + 138 \Rightarrow |A| = -70$

Now, R.H.S. = $|A| \cdot I = (-70) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -70 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -70 \end{bmatrix}$

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Hence, $A(\text{adj } A) = (\text{adj } A)A = |A|I$ <https://millionstar.godaddysites.com/>

8.

$$\begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix}$$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 6 \\ 7 & 9 \end{vmatrix} = (0-42) = -42$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 6 \\ 2 & 9 \end{vmatrix} = -(9-12) = 3$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 2 & 7 \end{vmatrix} = (7-0) = 7$; $C_{21} = (-1)^{2+1} \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = -(45-21) = -24$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 4 & 3 \\ 2 & 9 \end{vmatrix} = (36-6) = 30$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix} = -(28-10) = -18$

$C_{31} = (-1)^{3+1} \begin{vmatrix} 5 & 3 \\ 0 & 6 \end{vmatrix} = (30-0) = 30$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 4 & 3 \\ 1 & 6 \end{vmatrix} = -(24-3) = -21$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 4 & 5 \\ 1 & 0 \end{vmatrix} = (0-5) = -5$

$\text{adj } A = \begin{bmatrix} -42 & 3 & 7 \\ -24 & 30 & -18 \\ 30 & -21 & -5 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -42 & -24 & 30 \\ 3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{vmatrix} \Rightarrow |A| = 4 \begin{vmatrix} 0 & 6 \\ 7 & 9 \end{vmatrix} - 5 \begin{vmatrix} 1 & 6 \\ 2 & 9 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 \\ 2 & 7 \end{vmatrix} \Rightarrow |A| = 4(0-42) - 5(9-12) + 3(7+0)$

$\Rightarrow |A| = 4(-42) - 5(-3) + 3(7) \Rightarrow |A| = -168 + 15 + 21 \Rightarrow |A| = -132$

Now, L.H.S. $= A \cdot (\text{adj } A) = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix} \begin{bmatrix} -42 & -24 & 30 \\ 3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix}$

$= \begin{bmatrix} -168+15+21 & -96+150-54 & 120-105-15 \\ -42+0+42 & -24+0-108 & 30-0-30 \\ -84+21+63 & -48+210-162 & 60-147-45 \end{bmatrix} = \begin{bmatrix} -132 & 0 & 0 \\ 0 & -132 & 0 \\ 0 & 0 & -132 \end{bmatrix}$

Now, M.H.S. $= (\text{adj } A) \cdot A = \begin{bmatrix} -42 & -24 & 30 \\ 3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix} \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix}$

$= \begin{bmatrix} -168-24+60 & -210-0+21 & -126-144+270 \\ 12+30-42 & 15+0-147 & 9+180-189 \\ 28-18-10 & 35-0-35 & 21-108-45 \end{bmatrix} = \begin{bmatrix} -132 & 0 & 0 \\ 0 & -132 & 0 \\ 0 & 0 & -132 \end{bmatrix}$

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$$\text{Now, R.H.S.} = |A| \cdot I = (-132) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -132 & 0 & 0 \\ 0 & -132 & 0 \\ 0 & 0 & -132 \end{bmatrix}$$

$$\text{Hence, } A(\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$$

9. $\begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} \cos \alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos \alpha; C_{12} = (-1)^{1+2} \begin{vmatrix} \sin \alpha & 0 \\ 0 & 1 \end{vmatrix} = -\sin \alpha$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} \sin \alpha & \cos \alpha \\ 0 & 0 \end{vmatrix} = -(0-0) = 0; C_{21} = (-1)^{2+1} \begin{vmatrix} -\sin \alpha & 0 \\ 0 & 1 \end{vmatrix} = -(-\sin \alpha) = \sin \alpha$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} \cos \alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos \alpha; C_{23} = (-1)^{2+3} \begin{vmatrix} \cos \alpha & -\sin \alpha \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -\sin \alpha & 0 \\ \cos \alpha & 0 \end{vmatrix} = (0-0) = 0; C_{32} = (-1)^{3+2} \begin{vmatrix} \cos \alpha & 0 \\ \sin \alpha & 0 \end{vmatrix} = (0-0) = 0$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} = (\cos^2 \alpha + \sin^2 \alpha) = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow |A| = \cos \alpha \begin{vmatrix} \cos \alpha & 0 \\ 0 & 1 \end{vmatrix} + \sin \alpha \begin{vmatrix} \sin \alpha & 0 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} \sin \alpha & \cos \alpha \\ 0 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = \cos^2 \alpha + \sin^2 \alpha + 0 \Rightarrow |A| = 1$$

$$\text{Now, L.H.S.} = A(\text{adj } A) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & 0 \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, M.H.S.} = (\text{adj } A) \cdot A = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha & 0 \\ -\sin \alpha \cos \alpha + \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, R.H.S. = $|A| \cdot I = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Hence, $A(\text{adj } A) = (\text{adj } A) \cdot A = |A| \cdot I$

10. If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, show that $\text{adj } A = A$.

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = (0-4) = -4$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = -(3-4) = 1$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = (4-0) = 4$; $C_{21} = (-1)^{2+1} \begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = -(-9+12) = -3$

$C_{22} = (-1)^{2+2} \begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix} = (-12+12) = 0$; $C_{23} = (-1)^{2+3} \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} = -(-16+12) = 4$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} = (-3-0) = -3$; $C_{32} = (-1)^{3+2} \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} = -(-4+3) = 1$

$C_{33} = (-1)^{3+3} \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = (0+3) = 3$;

$\text{adj } A = \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A$. Hence, $\text{adj } A = A$.

11. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, show that $\text{adj } A = 3A'$.

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1-4) = -3$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = (-4-2) = -6$; $C_{21} = (-1)^{2+1} \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2-4) = 6$

$C_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1+4) = 3$; $C_{23} = (-1)^{2+3} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = (4+2) = 6; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1+4) = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$3A' = 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \quad \text{Hence, } \text{adj } A = 3A'$$

Find the inverse of each of the matrices given below :

12. $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} \Rightarrow |A| = (6-5) \Rightarrow |A| = 1, |A| \neq 0$

$\therefore A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(2) = 2$; $C_{12} = (-1)^{1+2}(-1) = 1$; $C_{21} = (-1)^{2+1}(-5) = 5$; $C_{22} = (-1)^{2+2}(3) = 3$

$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{1} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} \Rightarrow |A| = (12-2) = 10$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(3) = 3$; $C_{12} = (-1)^{1+2}(2) = -2$; $C_{21} = (-1)^{2+1}(1) = -1$; $C_{22} = (-1)^{2+2}(4) = 4$

$\therefore \text{adj } A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10} \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 3/10 & -1/10 \\ -1/5 & 2/5 \end{bmatrix}$

$$\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$$

Sol. $|A| = \begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix} \Rightarrow |A| = (12 + 12) \Rightarrow |A| = 24$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(6) = 6$; $C_{12} = (-1)^{1+2}(4) = -4$; $C_{21} = (-1)^{2+1}(-3) = 3$; $C_{22} = (-1)^{2+2}(2) = 2$

$$\text{adj}(A) = \begin{bmatrix} 6 & -4 \\ 3 & 2 \end{bmatrix}' \Rightarrow \text{adj}(A) = \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix};$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{24} \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/8 \\ -1/6 & 1/12 \end{bmatrix}$$

15. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $(ad - bc) \neq 0$

Sol. $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \Rightarrow |A| = (ad - bc)$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(d) = d$; $C_{12} = (-1)^{1+2}(c) = -c$; $C_{21} = (-1)^{2+1}(b) = -b$; $C_{22} = (-1)^{2+2}(a) = a$

$$\text{adj}(A) = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}' \Rightarrow \text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

16. $\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{vmatrix} \Rightarrow |A| = 1 \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + 5 \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$

$$\Rightarrow |A| = 1(1+3) - 2(-1+2) + 5(3+2) \Rightarrow |A| = 1(4) - 2(1) + 5(5) = 4 - 2 + 25 \Rightarrow |A| = 27$$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} = (1+3) = 4$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1+2) = -1$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = (3+2) = 5; C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} = -(-2-15) = 17$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 5 \\ 2 & -1 \end{vmatrix} = (-1-10) = -11; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -(3-4) = 1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} = (-2+5) = 3; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 5 \\ 1 & -1 \end{vmatrix} = -(-1-5) = 6$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = (-1-2) = -3$$

$$\text{adj } A = \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

17. $\begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{vmatrix} \Rightarrow |A| = 2 \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} + 1 \begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix}$

$$\Rightarrow |A| = 2(0+6) + (0+2) + (18-0) \Rightarrow |A| = 2(0+6) + (0+2) + (18-0)$$

$$\Rightarrow |A| = 12 + 2 + 18 \Rightarrow |A| = 32$$

$$\therefore |A| \neq 0, A^{-1} \text{ exist.} \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} = (0+6) = 6; \quad C_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -(0+2) = -2$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = (18-0) = 18; \quad C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 6 & 0 \end{vmatrix} = -(0-6) = 6$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = (0-2) = -2; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 2 & 6 \end{vmatrix} = -(12+2) = -14$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = (1-0) = 1; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -(-2-3) = 5$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = (0+3) = 3$$

$$\text{adj } A = \begin{bmatrix} 6 & -2 & 18 \\ 6 & -2 & -14 \\ 1 & 5 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{32} \begin{bmatrix} 6 & 6 & 6 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

18. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix} \Rightarrow |A| = 2 \begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix}$
 $\Rightarrow |A| = 2(4+6) + 3(4-9) + 3(-4-6) \Rightarrow |A| = 20 - 15 - 30 \Rightarrow |A| = -25$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} = (4+6) = 10$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -(4-9) = 5$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix} = (-4-6) = -10$; $C_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 3 \\ -2 & 2 \end{vmatrix} = -(-6+6) = 0$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = (4-9) = -5$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = -(-4+9) = -5$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 3 \\ 2 & 2 \end{vmatrix} = (-9-6) = -15$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = -(6-6) = 0$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -3 \\ 2 & 2 \end{vmatrix} = (4+6) = 10$

$\text{adj } A = \begin{bmatrix} 10 & 5 & -10 \\ 0 & -5 & -5 \\ -15 & 0 & 10 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-25} \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} -10 & 0 & 15 \\ -5 & 5 & 0 \\ 10 & 5 & -10 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$

19. $\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{vmatrix} \Rightarrow |A| = 0 \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1 \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$
 $\Rightarrow |A| = 0 - 1(-12+8) = 4$

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$$\therefore |A| \neq 0, A^{-1} \text{ exist.} \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} = (-28 + 20) = -8; \quad C_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} = -(-21 + 10) = 11$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix} = (-12 + 8) = -4; \quad C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ -4 & -7 \end{vmatrix} = -(0 - 4) = 4$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 0 & -1 \\ -2 & -7 \end{vmatrix} = (0 - 2) = -2; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 0 \\ -2 & -2 \end{vmatrix} = -(0 + 0) = 0$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = (0 + 4) = 4; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 0 & -1 \\ 3 & 5 \end{vmatrix} = -(0 + 3) = -3$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix} = (0 - 0) = 0$$

$$\therefore \text{adj } A = \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

20. $\begin{bmatrix} 2 & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 2 & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \Rightarrow |A| = 2 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -3 & 1 \\ -1 & 2 \end{vmatrix} + 4 \begin{vmatrix} -3 & 0 \\ -1 & 1 \end{vmatrix}$

$$\Rightarrow |A| = 2(0 - 1) + 1(-6 + 1) + 4(-3 + 0) \Rightarrow |A| = -2 - 5 - 12 \Rightarrow |A| = -19$$

$$\therefore |A| \neq 0, A^{-1} \text{ exist.} \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = (0 - 1) = -1; \quad C_{12} = (-1)^{1+2} \begin{vmatrix} -3 & 1 \\ -1 & 2 \end{vmatrix} = -(-6 + 1) = 5$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} -3 & 0 \\ -1 & 1 \end{vmatrix} = (-3 + 0) = -3; \quad C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 4 \\ 1 & 2 \end{vmatrix} = -(-2 - 4) = 6$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = (4 + 4) = 8; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = -(2 - 1) = -1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix} = (-1 - 0) = -1; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = -(2 + 12) = -14$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ -3 & 0 \end{vmatrix} = (0-3) = -3$$

$$\text{adj } A = \begin{bmatrix} -1 & 5 & -3 \\ 6 & 8 & -1 \\ -1 & -14 & -3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & -1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-19} \begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & -1 & -3 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 1 & -6 & 1 \\ -5 & -8 & 14 \\ 3 & 1 & 3 \end{bmatrix}$$

21. $\begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$

Sol. $|A| = \begin{vmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{vmatrix} \Rightarrow |A| = 8 \begin{vmatrix} 0 & 6 \\ 1 & 6 \end{vmatrix} + 4 \begin{vmatrix} 10 & 6 \\ 8 & 6 \end{vmatrix} + 1 \begin{vmatrix} 10 & 0 \\ 8 & 1 \end{vmatrix}$

$$\Rightarrow |A| = 8(0-6) + 4(60-48) + 1(10-0) \Rightarrow |A| = -48 + 48 + 10 \Rightarrow |A| = 10$$

$\therefore |A| \neq 0, A^{-1}$ exist. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 6 \\ 1 & 6 \end{vmatrix} = (0-6) = -6$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 10 & 6 \\ 8 & 6 \end{vmatrix} = -(60-48) = -12$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 10 & 0 \\ 8 & 1 \end{vmatrix} = (10-0) = 10$; $C_{21} = (-1)^{2+1} \begin{vmatrix} -4 & 1 \\ 1 & 6 \end{vmatrix} = -(-24-1) = 25$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 8 & 1 \\ 8 & 6 \end{vmatrix} = (48-8) = 40$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 8 & -4 \\ 8 & 1 \end{vmatrix} = -(8+32) = -40$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix} = (-24+0) = -24$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 8 & 1 \\ 10 & 6 \end{vmatrix} = -(48-10) = -38$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 8 & -4 \\ 10 & 0 \end{vmatrix} = (0+40) = 40$

$$\text{adj } A = \begin{bmatrix} -6 & -12 & 10 \\ 25 & 40 & -40 \\ -24 & -38 & 40 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{10} \begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$$

22. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19} A$.

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 5 \end{vmatrix} = -2$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 2 \end{vmatrix} = -3$;

$$C_{21} = (-1)^{2+1} (3) = -3; C_{22} = (-1)^{2+2} (2) = 2$$

$$\therefore \text{adj } A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}. \text{ Hence, } A^{-1} = \frac{1}{19} A$$

23. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, show that $A^{-1} = A^2$.

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = (0-0) = 0$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = -(0-0) = 0$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = (0+1) = 1; C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = (0-1) = -1; C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(1-0) = -1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = (0+1) = 1; C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -(0-2) = 2$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = (-1+2) = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}. \text{ Also, } A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \Rightarrow |A| = 1 \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = 1(0-0) + 1(-0-0) + 1(0+1) \Rightarrow |A| = 0+0+1 \Rightarrow |A| = 1$$

$$A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1-2+1 & -1+1+0 & 1-0+0 \\ 2-2+0 & -2+1+0 & 2-0+0 \\ 1+0+0 & -1-0+0 & 1+0+0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = A^{-1}$$

Hence, $A^{-1} = A^2$.

If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, prove that $A^{-1} = A^3$. <https://millionstar.godaddysites.com/>

Sol. $A^3 = A^2 \cdot A \Rightarrow A^3 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \cdot A$

$$\Rightarrow A^3 = \begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 0+3-1 & 0-4+1 \end{bmatrix} \cdot A \Rightarrow A^3 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 9-8+0 & -9+12-4 & 12-16+4 \\ 0-2+0 & 0+3-0 & 0-4+0 \\ -6+4-0 & 6-6+3 & -8+8-3 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = (-3+4) = 1$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = -(2-0) = -2$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = (-2+0) = -2$; $C_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -(-3+4) = -1$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = (3-0) = 3$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = -(-3+0) = 3$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = (-12+12) = 0$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -(12-8) = -4$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = (-9+6) = -3$

$\therefore \text{adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} \Rightarrow |A| = 3 \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} + 4 \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$

$\Rightarrow |A| = 3(-3+4) + 3(2-0) + 4(-2+0) \Rightarrow |A| = 3+6-8 \Rightarrow |A| = 1$

$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) \Rightarrow A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = A^3$. Hence, $A^{-1} = A^3$.

25. If $A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, show that $A^{-1} = A'$.

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Sol. $A = \begin{bmatrix} -\frac{8}{9} & \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & -\frac{8}{9} & \frac{4}{9} \end{bmatrix}$. Now, $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} \frac{4}{9} & \frac{7}{9} \\ -\frac{8}{9} & \frac{4}{9} \end{vmatrix} = \left(\frac{16}{81} + \frac{56}{81} \right) = \frac{8}{9}; C_{12} = (-1)^{1+2} \begin{vmatrix} \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & \frac{4}{9} \end{vmatrix} = -\left(\frac{16}{81} - \frac{7}{81} \right) = \frac{-1}{9}$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} \frac{4}{9} & \frac{4}{9} \\ \frac{1}{9} & -\frac{8}{9} \end{vmatrix} = \left(\frac{-32}{81} - \frac{4}{81} \right) = \frac{-4}{9}; C_{21} = (-1)^{2+1} \begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ -\frac{8}{9} & \frac{4}{9} \end{vmatrix} = -\left(\frac{4}{81} + \frac{32}{81} \right) = \frac{-4}{9}$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} -\frac{8}{9} & \frac{4}{9} \\ \frac{1}{9} & \frac{4}{9} \end{vmatrix} = \left(\frac{-32}{81} - \frac{4}{81} \right) = \frac{-4}{9}; C_{23} = (-1)^{2+3} \begin{vmatrix} -\frac{8}{9} & \frac{1}{9} \\ \frac{1}{9} & -\frac{8}{9} \end{vmatrix} = -\left(\frac{64}{81} - \frac{1}{81} \right) = \frac{-7}{9}$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{7}{9} \end{vmatrix} = \left(\frac{7}{81} - \frac{16}{81} \right) = \frac{-1}{9}; C_{32} = (-1)^{3+2} \begin{vmatrix} -\frac{8}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{7}{9} \end{vmatrix} = -\left(\frac{-56}{81} - \frac{16}{81} \right) = \frac{8}{9}$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} -\frac{8}{9} & \frac{1}{9} \\ \frac{4}{9} & \frac{4}{9} \end{vmatrix} = \left(\frac{-32}{81} - \frac{4}{81} \right) = \frac{-4}{9}$$

$$\text{adj } A = \begin{bmatrix} \frac{8}{9} & -\frac{1}{9} & -\frac{4}{9} \\ -\frac{4}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{1}{9} & \frac{8}{9} & -\frac{4}{9} \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} \frac{8}{9} & -\frac{4}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{-1}{81} \begin{bmatrix} \frac{8}{9} & -\frac{4}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \end{bmatrix} = \frac{1}{-9} \begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = A' \text{ proved}$$

26. Let $D = \text{diag}[d_1, d_2, d_3]$, where none of d_1, d_2, d_3 is 0; prove that $D^{-1} = \text{diag}[d_1^{-1}, d_2^{-1}, d_3^{-1}]$.


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$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} d_2 & 0 \\ 0 & d_3 \end{vmatrix} = d_2 d_3; \quad C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 0 & d_3 \end{vmatrix} = 0; \quad C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & d_2 \\ 0 & 0 \end{vmatrix} = 0;$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0 & d_3 \end{vmatrix} = 0; \quad C_{22} = (-1)^{2+2} \begin{vmatrix} d_1 & 0 \\ 0 & d_3 \end{vmatrix} = d_1 d_3; \quad C_{23} = (-1)^{2+3} \begin{vmatrix} d_1 & 0 \\ 0 & 0 \end{vmatrix} = 0;$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ d_2 & 0 \end{vmatrix} = 0; \quad C_{32} = (-1)^{3+2} \begin{vmatrix} d_1 & 0 \\ 0 & 0 \end{vmatrix} = 0; \quad C_{33} = (-1)^{3+3} \begin{vmatrix} d_1 & 0 \\ 0 & d_2 \end{vmatrix} = d_1 d_2$$

$$\therefore \text{adj } D = \begin{bmatrix} d_2 d_3 & 0 & 0 \\ 0 & d_1 d_3 & 0 \\ 0 & 0 & d_1 d_2 \end{bmatrix} \Rightarrow \text{adj } D = \begin{bmatrix} d_2 d_3 & 0 & 0 \\ 0 & d_1 d_3 & 0 \\ 0 & 0 & d_1 d_2 \end{bmatrix}$$

$$|D| = \begin{vmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_1 d_2 \end{vmatrix} \Rightarrow |D| = d_1 \begin{vmatrix} d_2 & 0 \\ 0 & d_3 \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ 0 & d_3 \end{vmatrix} + 0 \begin{vmatrix} 0 & d_2 \\ 0 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = d_1 (d_2 d_3) - 0 + 0 \Rightarrow |A| = (d_1 d_2 d_3)$$

$$\text{Now, } D^{-1} = \frac{1}{|D|} (\text{adj } D) = \frac{1}{(d_1 d_2 d_3)} \begin{bmatrix} d_2 d_3 & 0 & 0 \\ 0 & d_1 d_3 & 0 \\ 0 & 0 & d_1 d_2 \end{bmatrix} = \begin{bmatrix} d_1^{-1} & 0 & 0 \\ 0 & d_2^{-1} & 0 \\ 0 & 0 & d_3^{-1} \end{bmatrix}$$

$$\therefore D^{-1} = \text{diag} [d_1^{-1} \quad d_2^{-1} \quad d_3^{-1}]$$

27. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

Sol. $AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 18+16 & 21+18 \\ 42+40 & 49+45 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 34 & 39 \\ 82 & 94 \end{bmatrix}$

$$\text{adj } AB = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} (94) = 94; \quad C_{12} = (-1)^{1+2} (82) = -82;$$

$$C_{21} = (-1)^{2+1} (39) = -39, \quad C_{22} = (-1)^{2+2} (34) = 34$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = (15-14) = 1; \quad |B| = \begin{vmatrix} 6 & 7 \\ 8 & 9 \end{vmatrix} = (54-56) = -2$$

$$\therefore |AB| = \begin{vmatrix} 34 & 39 \\ 82 & 94 \end{vmatrix} = (3196 - 3198) = -2$$

$$\therefore (AB)^{-1} = \frac{1}{|AB|} (\text{adj } AB) = \frac{1}{-2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -94 & 39 \\ 82 & -34 \end{bmatrix}$$

$$\text{R.H.S.} = B^{-1} A^{-1} \Rightarrow B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Cofactor of $C_{11} = (-1)^{1+1} (9) = 9$; $C_{12} = (-1)^{1+2} (8) = -8$; $C_{21} = (-1)^{2+1} (7) = -7$; $C_{22} = (-1)^{2+2} (6) = 6$

$$\therefore \text{adj } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}' \Rightarrow \text{adj } B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}. \quad \text{Also, } A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$$

Cofactor of $C_{11} = (-1)^{1+1} (5) = 5$; $C_{12} = (-1)^{1+2} (7) = -7$; $C_{21} = (-1)^{2+1} (2) = -2$; $C_{22} = (-1)^{2+2} (3) = 3$

$$\therefore \text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = (15 - 14) = 1; \quad |B| = \begin{vmatrix} 6 & 7 \\ 8 & 9 \end{vmatrix} = (54 - 56) = -2$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \text{ and } B^{-1} = \frac{1}{|B|} (\text{adj } B) = \frac{1}{(-2)} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -9 & 7 \\ 8 & -6 \end{bmatrix}$$

$$\text{Now, } B^{-1}A^{-1} = \frac{1}{2} \begin{bmatrix} -9 & 7 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -45 - 49 & 18 + 21 \\ 40 + 42 & -16 - 18 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -94 & 39 \\ 82 & -34 \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$.

28. If $A = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

$$\text{Sol. L.H.S.} = (AB)^{-1} \Rightarrow AB = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} -36 - 5 & 27 + 4 \\ -24 - 10 & 18 + 8 \end{bmatrix} = \begin{bmatrix} -41 & 31 \\ -34 & 26 \end{bmatrix}$$

$$\text{adj } AB = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$$

Cofactor of $C_{11} = (-1)^{1+1} (26) = 26$; $C_{12} = (-1)^{1+2} (-34) = 34$

$C_{21} = (-1)^{2+1} (31) = -31$; $C_{22} = (-1)^{2+2} (-41) = -41$

$$\therefore \text{adj } AB = \begin{bmatrix} 26 & 34 \\ -31 & -41 \end{bmatrix}' \Rightarrow \text{adj } AB = \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -41 & 31 \\ -34 & 26 \end{vmatrix} = (1066 - 1054) = 12. \quad \therefore (AB)^{-1} = \frac{1}{|AB|} (\text{adj } AB) = \frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

$$\text{R.H.S.} = B^{-1}A^{-1} \Rightarrow B = \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix} \Rightarrow \text{adj } B = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$$

Cofactor of $C_{11} = (-1)^{1+1} (-4) = -4$; $C_{12} = (-1)^{1+2} (5) = -5$

$C_{21} = (-1)^{2+1} (3) = -3$; $C_{22} = (-1)^{2+2} (-4) = -4$

$$\text{adj } B = \begin{bmatrix} -4 & -5 \\ -3 & -4 \end{bmatrix}' \Rightarrow \text{adj } B = \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} -4 & 3 \\ 5 & -4 \end{vmatrix} = (16 - 15) = 1$$

$$\therefore B^{-1} = \frac{1}{|B|} (\text{adj } B) = \frac{1}{1} \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}$$

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$$A = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1}(-1) = -2; C_{12} = (-1)^{1+2}(6) = -6; C_{21} = (-1)^{2+1}(-1) = 1, C_{22} = (-1)^{2+2}(9) = 9$$

$$|A| = \begin{vmatrix} 9 & -1 \\ 6 & -2 \end{vmatrix} = (-18 + 6) = -12 \quad \therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-12} \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}$$

$$\text{Now, } B^{-1}A^{-1} = \frac{1}{12} \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 8+18 & -4-27 \\ 10+24 & -5-36 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

$$\text{Hence, } (AB)^{-1} = B^{-1} \cdot A^{-1}$$

29. Compute $(AB)^{-1}$ when $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$

Sol. $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = (8-6) = 2; C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0+9) = -9$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = (0-6) = -6; C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = -(4+4) = -8$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (4-6) = -2; C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2-3) = 5$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = (-3-4) = -7; C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3-0) = 3$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = (2-0) = 2$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & -9 & -6 \\ -8 & -2 & 5 \\ -7 & 3 & 2 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix} \Rightarrow |A| = 1 \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} - 1 \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} + 2 \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= 1(8-6) - 1(0+9) + 2(0-6) = 2-9-12 = (-19)$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-19} \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} -2 & 8 & 7 \\ 9 & 2 & -3 \\ 6 & -5 & -2 \end{bmatrix}$$

$$\Rightarrow (AB)^{-1} = B^{-1} \cdot A^{-1} = \frac{1}{19} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} -2 & 0 & 7 \\ 9 & 2 & -3 \\ 6 & -5 & -2 \end{bmatrix}$$

$$= \frac{1}{19} \begin{bmatrix} -2+18+0 & 0+4+0 & 7-8+0 \\ 0+27-6 & 0+6+5 & 0-9+2 \\ -2+0+12 & 0+0-10 & 7-0-4 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 16 & 4 & -1 \\ 21 & 11 & -7 \\ 10 & -10 & 3 \end{bmatrix}$$

30. Obtain the inverses of the matrices $\begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$ And, hence find the inverse of the

matrix $\begin{bmatrix} 1+pq & p & 0 \\ q & 1+pq & p \\ 0 & q & 1 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$ $R = \begin{bmatrix} 1+pq & p & 0 \\ q & 1+pq & p \\ 0 & q & 1 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} = (1-0) = 1$; $C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & p \\ 0 & 1 \end{vmatrix} = 0$; $C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$;

$C_{21} = (-1)^{2+1} \begin{vmatrix} p & 0 \\ 0 & 1 \end{vmatrix} = -p$; $C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & p \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = (-1)^{3+1} \begin{vmatrix} p & 0 \\ 1 & p \end{vmatrix} = p^2$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 0 & p \end{vmatrix} = -p$; $C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} = 1$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ -p & 1 & 0 \\ p^2 & -p & 1 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix} \Rightarrow \text{adj } B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

Cofactor of $C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} = (1-0) = 1$; $C_{12} = (-1)^{1+2} \begin{vmatrix} q & 0 \\ 0 & 1 \end{vmatrix} = -q$; $C_{13} = (-1)^{1+3} \begin{vmatrix} q & 1 \\ 0 & q \end{vmatrix} = q^2$;

$C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ q & 1 \end{vmatrix} = 0$; $C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$; $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & q \end{vmatrix} = -q$

$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$; $C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ q & 0 \end{vmatrix} = 0$; $C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} = 1$

$$\therefore \text{adj } B = \begin{bmatrix} 1 & -q & q^2 \\ 0 & 1 & -q \\ 0 & 0 & 1 \end{bmatrix}' \Rightarrow \text{adj } B = \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow |A| = 1 \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} - p \begin{vmatrix} 0 & p \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 1(1-0) - p(0-0) + 0(0-0) = 1$$

$$|B| = \begin{vmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{vmatrix} \Rightarrow |B| = 1 \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} - 0 \begin{vmatrix} q & 0 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} q & 1 \\ 0 & q \end{vmatrix} = 1(1-0) - 0(q-0) - 0(q-0) = 1$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{1} \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B^{-1} = \frac{1}{|B|}(\text{adj } B) = \frac{1}{1} \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1+pq & p & 0 \\ q & 1+pq & p \\ 0 & q & 1 \end{bmatrix} \Rightarrow |R| = (1+pq) \begin{vmatrix} 1+pq & p \\ q & 1 \end{vmatrix} - p \begin{vmatrix} q & p \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} q & 1+pq \\ 0 & q \end{vmatrix}$$

$$\Rightarrow |R| = (1+pq)(1+pq-pq) - p(q-0) + 0 \Rightarrow |R| = (1+pq)(1) - pq$$

$$\Rightarrow |R| = 1 + pq - pq \Rightarrow |R| = 1$$

$$\text{adj } R = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{Cofactor of } C_{11} = (-1)^{1+1} \begin{vmatrix} 1+pq & p \\ q & 1 \end{vmatrix} = (1+pq-pq) = 1; C_{12} = (-1)^{1+2} \begin{vmatrix} q & 0 \\ 0 & 1 \end{vmatrix} = -(q-0) = -q$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} q & 1+pq \\ 0 & q \end{vmatrix} = (q^2-0) = q^2; C_{21} = (-1)^{2+1} \begin{vmatrix} p & 0 \\ q & 1 \end{vmatrix} = -(p-0) = -p$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1+pq & 0 \\ 0 & 1 \end{vmatrix} = (1+pq-0) = 1+pq; C_{23} = (-1)^{2+3} \begin{vmatrix} 1+pq & p \\ 0 & q \end{vmatrix} = -(q+p^2q)$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} p & 0 \\ 1+pq & p \end{vmatrix} = (p^2-0) = p^2;$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1+pq & 0 \\ q & p \end{vmatrix} = -(p+p^2q-0) = -(p+p^2q)$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1+pq & p \\ q & 1+pq \end{vmatrix} = (1+2pq+p^2q^2-pq) = (1+pq+p^2q^2)$$

$$\text{adj } R = \begin{bmatrix} 1 & -q & q^2 \\ -p & 1+pq & -(q+p^2q) \\ p^2 & -(p+p^2q) & (1+pq+p^2q^2) \end{bmatrix} \Rightarrow \text{adj } R = \begin{bmatrix} 1 & -p & p^2 \\ -q & 1+pq & -(p+q^2p) \\ q^2 & -(q+p^2q) & (1+pq+p^2q^2) \end{bmatrix}$$

$$\therefore R^{-1} = \begin{bmatrix} 1 & -p & p^2 \\ -q & (1+pq) & -(p+q^2p) \\ q^2 & -(q+p^2q) & (1+pq+p^2q^2) \end{bmatrix}$$

If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ verify that $A^2 - 4A - I = 0$ and hence find A^{-1} <https://millionstar.godaddysites.com/>

Sol. $A^2 - 4A - I = 0$, multiplying both side A^{-1}

$$\Rightarrow A^{-1}(A^2 - 4A - I) = 0.A^{-1} \Rightarrow A^{-1}A^2 - 4A.A^{-1} - I.A^{-1} = 0$$

$$\Rightarrow A^{-1}.A.A - 4A.A^{-1} - I.A^{-1} = 0 \Rightarrow AI - 4I - IA^{-1} = 0$$

$$\Rightarrow A - 4I - IA^{-1} = 0 \Rightarrow IA^{-1} = A - 4I \Rightarrow IA^{-1} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 3-4 & 2-0 \\ 2-0 & 1-4 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}.$$

32. Show that the matrix $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$ and hence find A^{-1}

Sol. $A^2 + 4A - 42 = 0$, multiplying both side by A^{-1} .

$$\Rightarrow A^{-1}(A^2 + 4A - 42) = 0.A^{-1} \Rightarrow A^{-1}A^2 + 4AA^{-1} - 42A^{-1} = 0$$

$$\Rightarrow IA + 4I - 42A^{-1} = 0 \Rightarrow A + 4I - 42A^{-1} = 0 \Rightarrow 42A^{-1} = A + 4I$$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 42A^{-1} = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{42} \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}. \text{ Hence, } A^{-1} = \frac{1}{42} \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}.$$

33. If $A = \begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$ show that $A^2 + 3A + 4I_2 = 0$ and hence find A^{-1}

Sol. $A^2 + 3A + 4I_2 = 0$, Multiplying both side by A^{-1} .

$$\Rightarrow A^{-1}(A^2 + 3A + 4I_2) = 0.A^{-1} \Rightarrow A^{-1}A^2 + 3AA^{-1} + 4I_2A^{-1} = 0$$

$$\Rightarrow A^{-1}A.A + 3A.A^{-1} + 4I_2A^{-1} = 0 \Rightarrow IA + 3I + 4I_2A^{-1} = 0 \Rightarrow 4A^{-1} = -A - 3I$$

$$\Rightarrow 4A^{-1} = -\begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 4A^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} -2 & 1 \\ -2 & -1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} -2 & 1 \\ -2 & -1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -1/2 & 1/4 \\ -1/2 & -1/4 \end{bmatrix}$$

34. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$ hence find A^{-1}

Sol. $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

$$\therefore A^2 + xI = yA \Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow 16 + x = 3y \Rightarrow x - 3y = -16 \dots (1)$$

$$\Rightarrow y = 8 \dots (2)$$

Putting the value of y in equation (1), then, $x - 3(8) = -16$

$\Rightarrow x = -16 + 24 \Rightarrow x = 8$. Hence, $x = 8$ and $y = 8$.

Now, $A^2 + 8I = 8A \Rightarrow A^2 + 8I - 8A = 0$

Multiplying both side by A^{-1} , we get, $A^{-1}(A^2 + 8I - 8A) = 0.A^{-1} \Rightarrow A^{-1}A^2 + 8IA^{-1} - 8AA^{-1} = 0$

$\Rightarrow A^{-1}A.A + 8IA^{-1} - 8I = 0 \Rightarrow 1A + 8IA^{-1} - 8I = 0 \Rightarrow A + 8A^{-1} - 8I = 0$

$\Rightarrow 8A^{-1} = 8I - A \Rightarrow 8A^{-1} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \Rightarrow 8A^{-1} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$

$\Rightarrow 8A^{-1} = \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{8} \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

35. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. Hence, find A^{-1}

Sol. $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & -2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$\therefore A^2 = \lambda A - 2I \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow 3\lambda - 2 = 1 \Rightarrow 3\lambda = 1 + 2 \Rightarrow 3\lambda = 3 \therefore \lambda = 1$

$\Rightarrow A^2 = A - 2I \Rightarrow A^2 - A + 2I = 0$

Multiplying both side by A^{-1} , we get, $A^{-1}(A^2 - A + 2I) = 0.A^{-1} \Rightarrow A^{-1}A^2 - A.A^{-1} + 2IA^{-1} = 0$

$\Rightarrow A^{-1}.A.A - A.A^{-1} + 2A^{-1} = 0 \Rightarrow 1A - I + 2A^{-1} = 0 \Rightarrow 2A^{-1} = I - A$

$\Rightarrow 2A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow 2A^{-1} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$

36. Show that the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation $A^3 - A^2 - 3A - I = 0$, and hence find

A^{-1} .

Sol. $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1-0-6 & 0-0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} -5+16-12 & 0+8-16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2-0+9 & 0-0+12 & 4+0+3 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} -1 & -8 & -20 \\ 0 & 7 & 2 \\ 7 & 12 & 7 \end{bmatrix}$. $\therefore A^3 - A^2 - 3A - I = 0$, multiplying both side by A^{-1}

We get, $A^{-1}(A^3 - A^2 - 3A - I) = 0.A^{-1} \Rightarrow A^{-1}.A.A.A - A^{-1}A.A - 3A.A^{-1} - IA^{-1} = 0$

$$\Rightarrow IAA - IA - 3I - A^{-1} = 0 \Rightarrow A^2 - A - 3I - A^{-1} = 0$$

$$\Rightarrow A^{-1} = A^2 - A - 3I \Rightarrow A^{-1} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -6 & -8 & -2 \\ 8 & 10 & 2 \\ -5 & -4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

37. Prove that : (i) $\text{adj } I = I$ (ii) $\text{adj } O = O$ (iii) $I^{-1} = I$.

Sol. (i) R.H.S. Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\text{adj } I = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(1) = 1$; $C_{12} = (-1)^{1+2}(0) = 0$; $C_{21} = (-1)^{2+1}(0) = 0$; $C_{22} = (-1)^{2+2}(1) = 1$

$$\text{adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}' \Rightarrow \text{adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore \text{adj } I = I$$

(ii) Let the zero matrix $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\text{adj } O = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(0) = 0$; $C_{12} = (-1)^{1+2}(0) = 0$; $C_{21} = (-1)^{2+1}(0) = 0$; $C_{22} = (-1)^{2+2}(0) = 0$

$$\text{adj } O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}' \Rightarrow \text{adj } O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \therefore \text{adj } O = O$$

(iii) Let I be the identify matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow |I| = (1-0) = 1$ $\text{adj } I = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

Cofactor of $C_{11} = (-1)^{1+1}(1) = 1$; $C_{12} = (-1)^{1+2}(0) = 0$; $C_{21} = (-1)^{2+1}(0) = 0$; $C_{22} = (-1)^{2+2}(1) = 1$

$$\text{adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}' \Rightarrow \text{adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$I^{-1} = \frac{1}{|I|} \text{adj } I \Rightarrow I^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore I^{-1} = I$$