Ex 7.1

Q1

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{20+36}{65}$$

$$= \frac{56}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{15-48}{65}$$

$$= \frac{-33}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{4}{5}\times\frac{5}{13} - \frac{3}{5}\times\frac{12}{13}$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= \frac{20-36}{65}$$

$$= -\frac{16}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{15+48}{65}$$

$$= \frac{63}{65}$$

Q2

We have,

$$\sin A = \frac{12}{13} \text{ and } \sin B = \frac{4}{5}$$

$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \cos B = \sqrt{1-\sin^2 B}$

[$\because$ In the second quadrant $\cos\theta$ is negative]

$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{12}{13}\right)^2} \text{ and } \cos B = \sqrt{1-\left(\frac{4}{5}\right)^2}$

$\Rightarrow \quad \cos A = -\sqrt{1-\frac{144}{169}} \text{ and } \cos B = \sqrt{1-\frac{16}{25}}$

$\Rightarrow \quad \cos A = -\sqrt{\frac{25}{169}} \text{ and } \cos B = \sqrt{\frac{9}{25}}$

$\Rightarrow \quad \cos A = \frac{-5}{13} \text{ and } \cos B = \frac{3}{5}$

Now,

(i)

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$= \frac{12}{13}\times\frac{3}{5} - \frac{5}{13}\times\frac{4}{5}$$
$$= \frac{36}{65} - \frac{20}{65}$$
$$= \frac{16}{65}$$

(ii)

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$= \frac{-5}{13}\times\frac{3}{5} - \frac{12}{13}\times\frac{4}{5}$$
$$= \frac{-15}{65} - \frac{48}{65}$$
$$= \frac{-63}{65}$$

We have,

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{-12}{13}$$

$\therefore$ $$\cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

[$\because$ In the second quadrant $\cos\theta$ is negative]

$\Rightarrow$ $$\cos A = -\sqrt{1-\left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{-12}{13}\right)^2}$$

$\Rightarrow$ $$\cos A = -\sqrt{1-\frac{9}{25}} \text{ and } \sin B = \sqrt{1-\frac{144}{169}}$$

$\Rightarrow$ $$\cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$\Rightarrow$ $$\cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now,

$$\begin{aligned}\sin(A+B) &= \sin A\cos B + \cos A\sin B \\ &= \frac{3}{5}\times\left(\frac{-12}{13}\right) - \frac{4}{5}\times\frac{5}{13} \\ &= -\frac{36}{65} - \frac{20}{65} \\ &= -\frac{56}{65}\end{aligned}$$

$\therefore$ $$\sin(A+B) = -\frac{56}{65}$$

Q3

We have,

$$\cos A = -\frac{24}{25} \text{ and } \cos B = \frac{3}{5}$$

$$\therefore \quad \sin A = -\sqrt{1-\cos^2 A} \text{ and } \sin B = -\sqrt{1-\cos^2 B}$$

[$\because$ In the 3rd and 4th quadrant $\sin\theta$ is negative]

$$\Rightarrow \quad \sin A = -\sqrt{1-\left(-\frac{24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \quad \sin A = -\sqrt{1-\frac{576}{625}} \text{ and } \sin B = -\sqrt{1-\frac{9}{25}}$$

$$\Rightarrow \quad \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\Rightarrow \quad \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Now,

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$= -\frac{7}{25}\times\frac{3}{5} - \frac{24}{25}\times\left(-\frac{4}{5}\right)$$

$$= -\frac{21}{25} + \frac{96}{125}$$

$$= \frac{75}{125}$$

$$= \frac{3}{5}$$

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= -\frac{24}{25}\times\frac{3}{5} - \left(-\frac{7}{25}\right)\times\left(-\frac{4}{5}\right)$$

$$= -\frac{72}{125} - \frac{28}{125}$$

$$= \frac{-72-28}{125}$$

$$= \frac{-100}{125} = \frac{-4}{5}$$

Q4

We have,

$$\tan A = \frac{3}{4}, \text{ and } \cos B = \frac{9}{41}$$

$$\therefore \quad \sin B = \sqrt{1-\cos^2 B}$$

$$= \sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= \sqrt{1-\frac{81}{1681}}$$

$$= \sqrt{\frac{1600}{1681}}$$

$$= \frac{40}{41}$$

$$\therefore \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{40}{41}}{\frac{9}{41}} = \frac{40}{9}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{3}{4}+\frac{40}{9}}{1-\frac{3}{4}\times\frac{40}{9}}$$

$$= \frac{\frac{27+160}{36}}{\frac{36-120}{36}}$$

$$= \frac{\frac{187}{36}}{\frac{-84}{36}}$$

$$= -\frac{187}{84}$$

Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = -\sqrt{1-\cos^2 B}$$

$\left[\because \text{ cosine is negative in second quadrant and sine is negative in fourth quadrant}\right]$

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = -\sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$$

$$\text{Now,} \quad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{-5}{12}\right)}{1+\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{-5}{12}\right)}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{5}{12}}{1+\frac{5}{12\sqrt{3}}}$$

$$= \frac{\frac{-12+5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3}+5}{12\sqrt{3}}}$$

$$= \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

$$\Rightarrow \quad \tan(A-B) = \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

[∵ cosine is negative in second quadrant]

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = \sqrt{1-\frac{3}{4}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}}+\frac{1}{\sqrt{3}}}{1-\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{1}{\sqrt{3}}\right)}$$

$$= 0$$

$$\therefore \quad \tan(A+B) = 0$$

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$\therefore$ $\cos A = -\sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$ [$\because$ cosine is negative in second quadrant]

$\Rightarrow$ $\cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2}$ and $\sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow$ $\cos A = -\sqrt{1-\frac{1}{4}}$ and $\sin B = \sqrt{1-\frac{3}{4}}$

$\Rightarrow$ $\cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = \sqrt{\frac{1}{4}}$

$\Rightarrow$ $\cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = \frac{1}{2}$

$\therefore$ $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

and, $\tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Now, $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{-1}{\sqrt{3}}\right)\times\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-3}{\sqrt{3}}$$

$$= \frac{-\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$\therefore$ $\tan(A-B) = -\sqrt{3}$

Q7

(i)

$\sin 78° \cos 18° - \cos 78° \sin 18°$ $\quad [\sin(A-B) = \sin A \cos B - \cos A \sin B]$

$= \sin(78° - 18°)$

$= \sin 60°$

$= \frac{\sqrt{3}}{2}$

(ii)

$\cos 47° \cos 13° - \sin 47° \sin 13°$ $\quad [\cos(A+B) = \cos A \cos B - \sin A \sin B]$

$= \cos(47° + 13°)$

$= \cos 60°$

$= \frac{1}{2}$

(iii)

$\sin 36° \cos 9° + \cos 36° \sin 9°$ $\quad [\sin(A+B) = \sin A \cos B + \cos A \sin B]$

$= \sin(36° + 9°)$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$

(iv)

$\cos 80° \cos 20° + \sin 80° \sin 20°$ $\quad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$

$= \cos(80° - 20°)$

$= \cos 60°$

$= \frac{1}{2}$

Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\sin(A+B) = \sin A\cos B + \cos A \sin B$$

$$= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right)$$

$$= \frac{-120}{325} + \frac{84}{325}$$

$$= \frac{-120+84}{325}$$

$$= \frac{-36}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \left(\frac{-12}{13}\right)\times\left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right)\times\left(\frac{-7}{25}\right)$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \frac{323}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\therefore \quad \tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{-5}{12}+\frac{7}{24}}{1-\left(\frac{-5}{12}\right)\times\frac{7}{24}}$$

$$= \frac{\frac{-10+7}{24}}{1+\frac{35}{288}}$$

$$= \frac{\frac{-3}{24}}{\frac{288+35}{288}}$$

Q9

LHS: $\cos 105° + \cos 15°$

$= \cos(90° + 15°) + \cos(90° - 75°)$

$= -\sin 15° + \sin 75°$ $\quad\left[\because \cos(90+\theta) = -\sin\theta \text{ and } \cos(90-\theta) = \sin\theta\right]$

$= \sin 75° - \sin 15°$

$\therefore \cos 105° + \cos 15° = \sin 75° - \sin 15°$

Hence proved.

Q10

LHS: $\dfrac{\tan A + \tan B}{\tan A - \tan B}$

$= \dfrac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}$ $\quad\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}{\frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}}$

$= \dfrac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B}$

$= \dfrac{\sin(A+B)}{\sin(A-B)}$ $\quad\left[\because \sin(A+B) = \sin A\cos B + \cos A\sin B \text{ and, } \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$

$\therefore \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin(A+B)}{\sin(A-B)}$

Hence proved.

Q11

LHS: $\dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

Dividing numerator and denominator by $\cos 11°$, we get

$$\dfrac{\dfrac{\cos 11°}{\cos 11°} + \dfrac{\sin 11°}{\cos 11°}}{\dfrac{\cos 11°}{\cos 11°} - \dfrac{\sin 11°}{\cos 11°}}$$

$$= \dfrac{1 + \tan 11°}{1 - \tan 11°} \qquad \left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$$

$$= \dfrac{\tan 45° + \tan 11°}{1 - \tan 45° \times \tan 11°} \qquad [\tan 45° = 1]$$

$$= \tan(45° + 11°) \qquad \left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$= \tan 56°$$

$$\therefore \quad \dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$$

Hence proved.

LHS: $\dfrac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

$$\dfrac{\dfrac{\cos 9°}{\cos 9°} + \dfrac{\sin 9°}{\cos 9°}}{\dfrac{\cos 9°}{\cos 9°} - \dfrac{\sin 9°}{\cos 9°}}$$ [Dividing numerator and denominator by $\cos 9°$]

$$= \dfrac{1 + \tan 9°}{1 - \tan 9°}$$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$$= \dfrac{\tan 45° + \tan 9°}{1 - \tan 45° \times \tan 9°}$$ [$\tan 45° = 1$]

$$= \tan(45° + 9°)$$ $\left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$$= \tan 54°$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

LHS: $\dfrac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°}$

$$\dfrac{\dfrac{\cos 8°}{\cos 8°} - \dfrac{\sin 8°}{\cos 8°}}{\dfrac{\cos 8°}{\cos 8°} + \dfrac{\sin 8°}{\cos 8°}}$$ [Dividing numerator and denominator by $\cos 8°$]

$$= \dfrac{1 - \tan 8°}{1 + \tan 8°}$$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$$= \dfrac{\tan 45° - \tan 8°}{1 + \tan 45° \times \tan 8°}$$ [$\tan 45° = 1$]

$$= \tan(45° - 8°)$$ $\left[\because \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}\right]$

$$= \tan 37°$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

Q12

LHS: $\sin(60° - \theta)\cos(30° + \theta) + \cos(60° - \theta) \times \sin(30° + \theta)$

$= \sin[(60° - \theta) + (30° + \theta)]$ $[\sin(A+B) = \sin A \cos B + \cos A \sin B]$

$= \sin[60° - \theta + 30° + \theta]$

$= \sin(90°)$

$= 1$

$= \text{RHS}$

$\therefore$ LHS = RHS

Hence proved.

Q13

LHS: $\dfrac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°}$

$= \tan(69° + 66°)$ $\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$= \tan(135°)$

$= \tan(90° + 45°)$

$= -\cot 45°$ $[\because \tan\theta \text{ is negative in second quadrant}]$

$= -1$

$= \text{RHS}$

$\therefore$ LHS = RHS

Hence proved.

Q14

We have,

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{1 - \frac{5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{66-5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{61}{66}}$$

$$= \frac{61}{66} \times \frac{66}{61}$$

$$= 1$$

$$= \tan\frac{\pi}{4} \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad \tan(A+B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad A + B = \frac{\pi}{4}$$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \text{ and } \tan B = \frac{1}{2m-1}$$

Now, $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{1 + \frac{m}{(m-1)(2m-1)}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{\frac{(m-1)(2m-1)+(m)}{(m-1)(2m-1)}}$$

$$= \frac{m(2m-1)-(m-1)}{(m-1)(2m-1)+(m)}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - 2m + 1}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

$$= 1$$

$\therefore \quad \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$\Rightarrow \quad \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad A - B = \left(\frac{\pi}{4}\right)$

Q15

LHS: $\cos^2 45° - \sin^2 15°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15°$ $\quad \left[\because \cos 45 = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{2} - \left(\frac{1-\cos 2\times 15°}{2}\right)$ $\quad \left[\because \cos 2\theta = 1 - \sin^2\theta\right]$

$= \frac{1}{2} - \left(\frac{1-\cos 30°}{2}\right)$

$= \frac{1-1+\cos 30°}{2}$

$= \frac{\cos 30°}{2}$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4}$

= RHS

$\therefore$ LHS = RHS

Hence proved.

We have,

LHS $\sin^2(n+1)A - \sin^2 nA$

$= \sin[(n+1)A + nA]\sin[(n+1)A - nA]$

$\left[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)\right]$

$= \sin[nA + A + nA]\sin[nA + A - nA]$

$= \sin(2nA + A)\sin(A)$

$= \sin(2n+1)A \sin A$

= RHS

$\because$ LHS = RHS

Hence proved.

Q16

We have,

$$\text{LHS} = \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B} \qquad \left[\begin{array}{l}\because \quad 2\sin A\cos B = \sin(A+B)+\sin(A-B) \\ \text{and, } 2\cos A\cos B = \cos(A+B)+\cos(A-B)\end{array}\right]$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

$$\text{LHS} = \frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}$$

$$= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C-\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A-\cos C\sin A}{\cos C\cos A}$$

$$= \frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C}{\cos B\cos C}-\frac{\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A}{\cos C\cos A}-\frac{\cos C\sin A}{\cos C\cos A}$$

$$= \tan A-\tan B+\tan B-\tan C+\tan C-\tan A$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A}$$

$$= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A}$$

$$= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} - \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A} - \frac{\cos C}{\sin C}$$

$$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{RHS} = \sin^2 A + \sin^2(A-B) - 2\sin A \cos B \sin(A-B)$$

$$= \sin^2 A + \sin(A-B)\left[\sin(A-B) - 2\sin A \cos B\right]$$

$$= \sin^2 A + \sin(A-B)\left[\sin A \cos B - \cos A \sin B - 2\sin A \cos B\right]$$

$$= \sin^2 A + \sin(A-B)\left[-\sin A \cos B - \cos A \sin B\right]$$

$$= \sin^2 A - \sin(A-B)(\sin A \cos B + \cos A \sin B)$$

$$= \sin^2 A - \sin(A-B)(\sin(A+B))$$

$$= \sin^2 A - \sin(A-B)\sin(A+B)$$

$$= \sin^2 A - (\sin^2 A - \sin^2 B) \qquad \left[\because \sin(A-B)\sin(A+B) = \sin^2 A - \sin^2 B\right]$$

$$= \sin^2 A - \sin^2 A + \sin^2 B$$

$$= \sin^2 B$$

$$= \text{LHS}$$

$\therefore$ LHS = RHS

Hence proved.

$$\begin{aligned}
\text{RHS} &= \cos^2 A + \cos^2 B - 2\cos A \cos B \cos(A+B) \\
&= \cos^2 A + \left(1 - \sin^2 B\right) - 2\cos A \cos B \cos(A+B) \\
&= \left[\cos^2 A - \sin^2 B\right] - 2\cos A \cos B \cos(A+B) + 1 \\
&= \left[\cos(A+B)\cos(A-B)\right] - 2\cos A \cos B \cos(A+B) + 1 \\
&= \cos(A+B)\left[\cos(A-B) - 2\cos A \cos B\right] + 1 \\
&= \cos(A+B)\left[\cos A \cos B + \sin A \sin B - 2\cos A \cos B\right] + 1 \\
&= \cos(A+B)\left[-\cos A \cos B + \sin A \sin B\right] + 1 \\
&= -\cos(A+B)\left[\cos A \cos B - \sin A \sin B\right] + 1 \\
&= -\cos(A+B)\left[\cos(A+B)\right] + 1 \\
&= -\cos^2(A+B) + 1 \\
&= 1 - \cos^2(A+B) \\
&= \sin^2(A+B) \qquad \left[\sin^2\theta = 1 - \cos^2\theta\right] \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\tan(A+B)}{\cot(A-B)}$$

$$= \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$= \tan(A+B)\tan(A-B)$$

$$= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B}\right]\left[\frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)}$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$= \text{RHS}$

$\therefore$ LHS = RHS

Hence proved.

Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$$\Rightarrow \tan 8\theta = \tan(6\theta + 2\theta)$$

$$\Rightarrow \tan 8\theta = \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \tan 8\theta(1 - \tan 6\theta \tan 2\theta) = \tan 6\theta + \tan 2\theta$$

$$\Rightarrow \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta = \tan 6\theta + \tan 2\theta$$

$$\Rightarrow \tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \tan 6\theta \tan 2\theta$$

Hence proved.

We have,

$$45° = 30° + 15°$$

$$\Rightarrow \quad \tan 45° = \tan(30° + 15°)$$

$$\Rightarrow \quad 1 = \frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad 1 - \tan 30° \tan 15° = \tan 15° + \tan 30°$$

$$\Rightarrow \quad 1 = \tan 15° + \tan 30° + \tan 30° \tan 15°$$

$$\Rightarrow \quad \tan 15° + \tan 30° + \tan 15° \tan 30° = 1$$

Hence proved.

We have,

$$45° = 9° + 36°$$

$$\Rightarrow \quad \tan 45° = \tan(9° + 36°)$$

$$\Rightarrow \quad 1 = \frac{\tan 9° + \tan 36°}{1 - \tan 9° \tan 36°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad 1 - \tan 9° \tan 36° = \tan 9° + \tan 36°$$

$$\Rightarrow \quad 1 = \tan 9° + \tan 36° + \tan 9° \tan 36°$$

$$\Rightarrow \quad \tan 9° + \tan 36° + \tan 9° \tan 36° = 1$$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \quad \tan 13\theta = \tan(9\theta + 4\theta)$$

$$\Rightarrow \quad \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad \tan 13\theta(1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$$

Hence proved.

Q18

We have,

RHS $= \tan 3\theta \tan\theta$

$= \tan(2\theta+\theta)\times\tan(2\theta-\theta)$

$= \left[\dfrac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}\right]\times\left[\dfrac{\tan 2\theta-\tan\theta}{1+\tan 2\theta\tan\theta}\right]$

$= \dfrac{(\tan 2\theta+\tan\theta)(\tan 2\theta-\tan\theta)}{(1-\tan 2\theta\tan\theta)(1+\tan 2\theta\tan\theta)}$

$= \dfrac{\tan^2 2\theta-\tan^2\theta}{1-\tan^2 2\theta\tan^2\theta}$ $\left[\because (a-b)(a+b)=a^2-b^2\right]$

$=$ LHS

$\therefore$ LHS = RHS

Hence proved

Q19

$\dfrac{\sin x.\cos y+\sin y.\cos x}{\sin x.\cos y-\sin y.\cos x}=\dfrac{a+b}{a-b}$

$\Rightarrow \dfrac{\sin x.\cos y+\sin y.\cos x+\sin x.\cos y-\sin y.\cos x}{\sin x.\cos y+\sin y.\cos x-\sin x.\cos y+\sin y.\cos x}=\dfrac{a+b+a-b}{a+b-a+b}$ [Using Componendo and Dividendo]

$\Rightarrow \dfrac{2\sin x.\cos y}{2\sin y.\cos x}=\dfrac{2a}{2b}$

$\Rightarrow \dfrac{\tan x}{\tan y}=\dfrac{a}{b}$

Hence Proved

Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x\frac{\sin B}{\cos B} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\Rightarrow \quad \sin A\cos B = x\cos A\sin B \qquad ---(i)$$

Now,

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A\cos B - \sin B\cos A}{\sin A\cos B + \cos A\sin B}$$

$$= \frac{x\cos A\sin B - \cos A\sin B}{x\cos A\sin B + \cos A\sin B} \qquad [\text{Using equation (i)}]$$

$$= \frac{\cos A\sin B(x-1)}{\cos A\sin B(x+1)}$$

$$= \frac{x-1}{x+1}$$

$$\therefore \quad \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$$

Hence proved.

Q21

We have,

$\tan(A+B) = x$ and $\tan(A-B) = y$

Now, $\tan 2A = \tan[(A+B)+(A-B)]$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

$\therefore \quad \tan 2A = \dfrac{x+y}{1-xy}$

Now, $\tan 2B = \tan[(A+B)-(A-B)]$

$$= \frac{\tan(A+B)-\tan(A-B)}{1+\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x-y}{1+xy}$$

$\therefore \quad \tan 2B = \dfrac{x-y}{1+xy}$

Q22

We have,

$\cos A + \sin B = m$ and $\sin A + \cos B = n$

Now, $m^2 + n^2 - 2$

$= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$

$= \cos^2 A + \sin^2 B + 2\cos A \sin B + \sin^2 A + \cos^2 B + 2\sin A \cos B - 2$

$= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 1 + 1 + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 2 + 2(\sin A \cos B + \cos A \sin B) - 2$

$= 2(\sin A \cos B + \cos A \sin B)$

$= 2\sin(A + B)$ $\quad [\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$

$\therefore \quad 2\sin(A+B) = m^2 + n^2 - 2$

Hence proved

Q23

We have,

$$\tan A + \tan B = a \text{ and } \cot A + \cot B = b$$

Now, $\cot A + \cot B = b$

$$\Rightarrow \frac{1}{\tan A} + \frac{1}{\tan B} = b \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$\Rightarrow \frac{\tan B + \tan A}{\tan A \tan B} = b$$

$$\Rightarrow \frac{a}{\tan A \tan B} = b \qquad [\because \tan A + \tan B = a]$$

$$\Rightarrow \frac{a}{b} = \tan A \tan B$$

$$\because \cot(A+B) = \frac{1}{\tan(A+B)}$$

$$= \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \frac{a}{b}}{a} \qquad \left[\because \tan A \tan B = \frac{a}{b}\right]$$

$$= \frac{b-a}{ab}$$

$$= \frac{b}{ab} - \frac{a}{ab}$$

$$= \frac{1}{a} - \frac{1}{b}$$

$$\therefore \cot(A+B) = \frac{1}{a} - \frac{1}{b}$$

Hence proved.

Q24

We have,

$$\cos\theta = \frac{8}{17}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$= \frac{15}{17}$$

Now, $\cos\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{4}-\theta\right)+\cos\left(\frac{2\pi}{3}-\theta\right)$

$$=\left[\cos\frac{\pi}{6}\cos\theta-\sin\frac{\pi}{6}\sin\theta\right]+\left[\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta\right]+\left[\cos\frac{2\pi}{3}\cos\theta+\sin\frac{2\pi}{3}\sin\theta\right]$$

$$=\left[\cos\frac{\pi}{6}+\cos\frac{\pi}{4}+\cos\frac{2\pi}{3}\right]\cos\theta+\sin\theta\left[-\sin\frac{\pi}{6}+\sin\frac{\pi}{4}+\sin\frac{2\pi}{3}\right]$$

$$=\left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}+\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right]$$

$$=\left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\sin\frac{\pi}{6}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\cos\frac{\pi}{6}\right]$$

$$=\left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\right]$$

[$\because$ $\cos A$ is negative in second quadrant]

$$=\left[\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right]$$

$$=\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\left(\frac{8}{17}+\frac{15}{17}\right)$$

$$=\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\left(\frac{8+15}{17}\right)$$

$$=\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\times\frac{23}{17}$$

$\therefore$ $\cos\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{4}-\theta\right)+\cos\left(\frac{2\pi}{3}-\theta\right)=\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\times\frac{23}{17}$

Hence proved.

Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \tan\frac{\pi}{3}}{1 - \tan x \tan\frac{\pi}{3}}\right] + \left[\frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan x \tan\frac{2\pi}{3}}\right] = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}\right] + \left[\frac{\tan x + \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}{1 - \tan x \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}\right] = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \cot\frac{\pi}{3}}{1 + \tan x \cot\frac{\pi}{3}} = 3 \qquad \left[\because \tan\theta \text{ is negative in second quadrant}\right]$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = 3$$

$$\Rightarrow \quad \tan x + \frac{\left(\tan x + \sqrt{3}\right)\left(1 + \sqrt{3}\tan x\right) + \left(\tan x - \sqrt{3}\right)\left(1 - \sqrt{3}\tan x\right)}{\left(1 - \sqrt{3}\tan x\right)\left(1 + \sqrt{3}\tan x\right)} = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}\tan^2 x + \sqrt{3} + 3\tan x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1 - \left(\sqrt{3}\tan x\right)^2} = 3$$

$$\Rightarrow \quad \tan x + \frac{8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x\left(1 - 3\tan^2 x\right) + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x - 3\tan^3 x + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{9\tan x - 3\tan^3 x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\left(3\tan x - \tan^3 x\right)}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 1$$

Hence proved.

Q26

We have,

$$\sin(\alpha+\beta)=1$$

$$\Rightarrow \quad \sin(\alpha+\beta)=\sin\frac{\pi}{2}$$

$$\Rightarrow \quad \alpha+\beta=\frac{\pi}{2} \qquad ---(i)$$

and, $\sin(\alpha-\beta)=\frac{1}{2}$

$$\Rightarrow \quad \sin(\alpha-\beta)=\sin\frac{\pi}{6}$$

$$\Rightarrow \quad \alpha-\beta=\frac{\pi}{6} \qquad ---(ii)$$

Adding equations (i) and (ii), we get

$$2\alpha=\frac{\pi}{2}+\frac{\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$$

$$\Rightarrow \quad \alpha=\frac{\pi}{3}$$

Putting $\alpha=\frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3}+\beta=\frac{\pi}{2}$$

$$\Rightarrow \quad \beta=\frac{\pi}{2}-\frac{\pi}{3}$$

$$\Rightarrow \quad \beta=\frac{3\pi-2\pi}{6}$$

$$=\frac{\pi}{6}$$

$$\Rightarrow \quad \beta=\frac{\pi}{6}$$

Now, $\tan(\alpha + 2\beta) = \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$

$= \tan\frac{2\pi}{3}$

$= \tan\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$= -\cot\frac{\pi}{6}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\sqrt{3}$

$\therefore$ $t(\alpha + 2\beta) = -\sqrt{3}$

and, $\tan(2\alpha + \beta) = \tan\left(2 \times \frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \tan\left(\frac{2\pi}{3} + \frac{\pi}{6}\right)$

$= \tan\left(\frac{4\pi + \pi}{6}\right)$

$= \tan\left(\frac{5\pi}{6}\right)$

$= \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= -\cot\frac{\pi}{3}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\frac{1}{\sqrt{3}}$

$\therefore$ $\tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$

Q27

We have,

$$6\cos\theta + 8\sin\theta = 9 \quad ---(i)$$

$\Rightarrow \quad 8\sin\theta = 9 - 6\cos\theta$

$\Rightarrow \quad (8\sin\theta)^2 = (9 - 6\cos\theta)^2 \quad [\because \text{Squaring both sides}]$

$\Rightarrow \quad 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow \quad 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow \quad 64(1 - \cos^2\theta) = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow \quad 64 - 64\cos^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow \quad 36\cos^2\theta + 64\cos^2\theta - 108\cos\theta + 81 - 64 = 0$

$\Rightarrow \quad 100\cos^2\theta - 108\cos\theta + 17 = 0 \quad ---(ii)$

Since α, β are roots of equation (ii).

Therefore, $\cos\alpha$ and $\cos\beta$ are roots of equation (ii)

$\therefore \quad \cos\alpha + \cos\beta = \dfrac{17}{100} \quad ---(iii)$

Again, $6\cos\theta + 8\sin\theta = 9$

$\Rightarrow \quad 6\cos\theta = 9 - 8\sin\theta$

$\Rightarrow \quad (6\cos\theta)^2 = (9 - 8\sin\theta)^2 \quad [\because \text{Squaring both sides}]$

$\Rightarrow \quad 36\cos^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow \quad 36(1 - \sin^2\theta) = 81 + 64\sin^2\theta - 144\cos\theta$

$\Rightarrow \quad 36 - 36\sin^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow \quad 64\sin^2\theta + 36\sin^2\theta - 144\sin\theta + 81 - 36 = 0$

$\Rightarrow \quad 100\sin^2\theta - 144\sin\theta + 45 = 0 \quad ---(iv)$

$\therefore \quad \sin\alpha \times \sin\beta = \frac{45}{100} \qquad ---(v)$

Now, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{17}{100} - \frac{45}{100}$ [Using equation (iii) and (v)]

$= -\frac{28}{100}$

$= -\frac{7}{25}$

Now, $\sin(\alpha+\beta) = \sqrt{1-(\cos\theta)^2}$

$= \sqrt{1-\left(-\frac{7}{25}\right)^2}$

$= \sqrt{1-\frac{49}{625}}$

$= \sqrt{\frac{625-49}{625}}$

$= \sqrt{\frac{576}{625}}$

$= \frac{24}{25}$

$\therefore \quad \sin(\alpha+\beta) = \frac{24}{25}$

Q28

$$\sin(\alpha+\beta)=\sqrt{1-\cos^2(\alpha+\beta)}$$

$$b^2+a^2=(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2$$

$$\Rightarrow\quad b^2+a^2=\left(\cos^2\alpha+\sin^2\alpha\right)+\left(\cos^2\beta+\sin^2\beta\right)+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$$

$$\Rightarrow\quad b^2+a^2=1+1+2\cos(\alpha-\beta)=2+2\cos(\alpha-\beta)$$

and, $b^2-a^2=(\cos\alpha+\cos\beta)^2-(\sin\alpha+\sin\beta)^2$

$$b^2-a^2=\cos^2\alpha+\cos^2\beta-\sin^2\alpha-\sin^2\beta+2(\cos\alpha\cos\beta-\sin\alpha\sin\beta)$$

$$\Rightarrow\quad b^2-a^2=\left(\cos^2\alpha-\sin^2\beta\right)+\left(\cos^2\beta-\sin^2\alpha\right)+2\cos(\alpha+\beta)$$

$$\Rightarrow\quad b^2-a^2=\cos(\alpha+\beta)\cos(\alpha-\beta)+\cos(\beta+\alpha)\cos(\beta-\alpha)+2\cos(\alpha+\beta)$$

$$\Rightarrow\quad b^2-a^2=2\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos(\alpha+\beta)$$

$$\left[\because\cos(\beta-\alpha)=\cos\{-(\alpha-\beta)\}=\cos(\alpha-\beta)\right]$$

$$\Rightarrow\quad b^2-a^2=\cos(\alpha+\beta)\{2\cos(\alpha-\beta)+2\}$$

$$\Rightarrow\quad b^2-a^2=\cos(\alpha+\beta)\left(b^2+a^2\right)\qquad[\text{Using (i)}]$$

Thus, $b^2-a^2=\left(b^2+a^2\right)\cos(\alpha+\beta)$

$$\Rightarrow\quad \cos(\alpha+\beta)=\frac{b^2-a^2}{b^2+a^2}$$

$$\Rightarrow\quad \sin(\alpha+\beta)=\sqrt{1-\left(\frac{b^2-a^2}{b^2-a^2}\right)^2}=\sqrt{\frac{4a^2b^2}{\left(a^2+b^2\right)^2}}=\frac{2ab}{a^2+b^2}$$

$b^2 + a^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$\Rightarrow \quad b^2 + a^2 = (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\Rightarrow \quad b^2 + a^2 = 1 + 1 + 2\cos(\alpha - \beta) = 2 + 2\cos(\alpha - \beta)$

and, $\quad b^2 - a^2 = (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2$

$b^2 - a^2 = \cos^2\alpha + \cos^2\beta - \sin^2\alpha - \sin^2\beta + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$\Rightarrow \quad b^2 - a^2 = (\cos^2\alpha - \sin^2\beta) + (\cos^2\beta - \sin^2\alpha) + 2\cos(\alpha + \beta)$

$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\cos(\alpha - \beta) + \cos(\beta + \alpha)\cos(\beta - \alpha) + 2\cos(\alpha + \beta)$

$\Rightarrow \quad b^2 - a^2 = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta)$

$[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)]$

$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\{2\cos(\alpha - \beta) + 2\}$

$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)(b^2 + a^2)$ $\quad$ [Using (i)]

Thus, $\quad b^2 - a^2 = (b^2 + a^2)\cos(\alpha + \beta)$

$\Rightarrow \quad \cos(\alpha + \beta) = \dfrac{b^2 - a^2}{b^2 + a^2}$

Q29

LHS $\frac{1}{\sin(x-a)\sin(x-b)}$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)}-\frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\cot(x-a)-\cot(x-b)\right]$$

$$= \frac{\cot(x-a)-\cot(x-b)}{\sin(a-b)}$$

= RHS

$\therefore$ LHS=RHS

Hence proved

LHS $\dfrac{1}{\sin(x-a)\cos(x-b)}$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)}+\frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-a)}{\sin(x-a)}+\frac{\sin(x-b)}{\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\cot(x-a)+\tan(x-b)\right]$$

$$= \frac{\cot(x-a)+\tan(x-b)}{\cos(a-b)}$$

= RHS

$\therefore$ LHS=RHS

Hence proved

LHS $\frac{1}{\cos(x-a)\cos(x-b)}$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)}-\frac{\cos(x-b)\ \sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)}{\cos(x-b)}-\frac{\sin(x-a)}{\cos(x-a)}\right]$

$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-s)\right]$

$= \frac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$

= RHS

$\therefore$ LHS=RHS

Hence proved

Q30

We have,

$$\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$$

$$\Rightarrow \quad -\left(\cos\alpha\cos\beta - \sin\alpha\sin\beta\right) = -1$$

$$\Rightarrow \quad \cos\left(\alpha+\beta\right) = 1 \qquad ---(i)$$

$$\therefore \quad \sin\left(\alpha+\beta\right) = \sqrt{1-\cos^2\left(\alpha+\beta\right)}$$

$$= \sqrt{1-1^2}$$

$$= 0$$

$$\Rightarrow \quad \sin\left(\alpha+\beta\right) = 0 \qquad ---(ii)$$

Now,

$$1+\cot\alpha\tan\beta = 1+\frac{\cos\alpha}{\sin\alpha}\times\frac{\sin\beta}{\cos\beta}$$

$$= \frac{\sin\alpha\times\cos\beta+\cos\alpha\times\sin\beta}{\sin\alpha\times\cos\beta}$$

$$= \frac{\sin\left(\alpha+\beta\right)}{\sin\alpha\times\cos\beta}$$

$$= \frac{0}{\sin\alpha\times\cos\beta} \qquad \left[\text{Using equation (ii)}\right]$$

$$= 0$$

$$\therefore \quad 1+\cot\alpha\tan\beta = 0$$

Hence proved

Q31

We have,

$\tan\alpha = x + 1$ and $\tan\beta = x - 1$

Now, $2\cot(\alpha - \beta)$

$= \dfrac{2}{\tan(\alpha - \beta)}$

$= \dfrac{2}{\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$

$= \dfrac{2(1 + \tan\alpha\tan\beta)}{\tan\alpha - \tan\beta}$

$= \dfrac{2\left[1 + (x+1)(x-1)\right]}{x + 1 - (x - 1)}$

$= \dfrac{2\left[1 + x^2 - 1\right]}{x + 1 - x + 1}$

$= \dfrac{2 \times x^2}{2} = x^2$

$\therefore$ $2\cot(\alpha - \beta) = x^2$

Hence proved

Q32

Let the two parts of the angle be θ and θ − Ø.

$\tan(\theta - \varnothing) = \lambda \tan \varnothing$ [*According to question*]

$\Rightarrow \dfrac{\tan(\theta - \varnothing)}{\tan \varnothing} = \dfrac{\lambda}{1}$

$\Rightarrow \dfrac{\tan(\theta - \varnothing)}{\tan \varnothing} = \dfrac{\lambda}{1}$

$\Rightarrow \dfrac{\tan(\theta - \varnothing) + \tan \varnothing}{\tan(\theta - \varnothing) - \tan \varnothing} = \dfrac{\lambda + 1}{\lambda - 1}$ [*Using Componendo and Dividendo*]

$\Rightarrow \dfrac{\dfrac{\tan\theta - \tan\varnothing}{1 + \tan\theta.\tan\varnothing} + \tan\varnothing}{\dfrac{\tan\theta - \tan\varnothing}{1 + \tan\theta.\tan\varnothing} - \tan\varnothing} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\dfrac{\tan\theta - \tan\varnothing + \tan\varnothing(1 + \tan\theta.\tan\varnothing)}{1 + \tan\theta.\tan\varnothing}}{\dfrac{\tan\theta - \tan\varnothing - \tan\varnothing(1 + \tan\theta.\tan\varnothing)}{1 + \tan\theta.\tan\varnothing}} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan\theta - \tan\varnothing + \tan\varnothing + \tan\theta.\tan^2\varnothing}{\tan\theta - \tan\varnothing - \tan\varnothing - \tan\theta.\tan^2\varnothing} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan\theta + \tan\theta.\tan^2\varnothing}{\tan\theta - 2\tan\varnothing - \tan\theta.\tan^2\varnothing} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan}{}$

Q33

$$\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - 1}{\tan\alpha + 1} \text{[Dividing both Numerator and Denominator by } \cos\alpha]$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4}.\tan\alpha}$$

$$\Rightarrow \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \theta = \alpha - \frac{\pi}{4} \quad \text{[Removing tan from both sides]}$$

$$\Rightarrow \cos\theta = \cos\left(\alpha - \frac{\pi}{4}\right) \text{[Taking cos on both sides]}$$

$$\Rightarrow \cos\theta = \cos\alpha.\cos\frac{\pi}{4} + \sin\alpha.\sin\frac{\pi}{4}$$

$$\Rightarrow \cos\theta = \cos\alpha.\frac{1}{\sqrt{2}} + \sin\alpha.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\theta = \frac{\cos\alpha + \sin\alpha}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}\cos\theta = \sin\alpha + \cos\alpha$$

Hence Proved

Q34

RHS,

$$\frac{p+q}{1-pq}$$

$$=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B).\tan(A-B)}$$

$$=\frac{\frac{\tan A+\tan B}{1-\tan A.\tan B}+\frac{\tan A-\tan B}{1+\tan A.\tan B}}{1-\frac{\tan A+\tan B}{1-\tan A.\tan B}.\frac{\tan A-\tan B}{1+\tan A.\tan B}}$$

$$=\frac{\frac{(\tan A+\tan B)(1-\tan A.\tan B)+(\tan A-\tan B)(1-\tan A.\tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}{\frac{(1-\tan A.\tan B)(1+\tan A.\tan B)-(\tan A+\tan B).(\tan A-\tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}$$

$$=\frac{\tan A+\tan B+\tan^2 A.\tan B-\tan A.\tan^2 B+\tan A-\tan B-\tan^2 A.\tan B+\tan A.\tan^2 B}{1-\tan^2 A.\tan^2 B-\tan^2 A+\tan^2 B}$$

$$=\frac{2\tan A+2\tan A.\tan^2 B}{(1-\tan^2 A)(1+\tan^2 B)}=\frac{2\tan A(1+\tan^2 B)}{(1-\tan^2 A)(1-\tan^2 B)}=\frac{2\tan A}{1-\tan^2 A}=\tan 2A=LHS$$

Hence Proved

Ex 7.2

Q1

Let $f(\theta) = 12\sin\theta - 5\cos\theta$

We know that

$$-\sqrt{(12)^2 + (-5)^2} \le f(\theta) \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144+25} \le f(\theta) \le \sqrt{144+25}$$

$$\Rightarrow \quad -\sqrt{169} \le f(\theta) \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le f(\theta) \le 13$$

Hence, minimum and maximum values of $12\sin\theta - 5\cos\theta$ are -13 and 13 respectively.

Let $f(\theta) = 12\cos\theta + 5\sin\theta + 4$

We know that

$$-\sqrt{(12)^2 + (5)^2} \le 12\cos\theta + 5\sin\theta \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144+25} \le 12\cos\theta + 5\sin\theta \le \sqrt{144+25}$$

$$\Rightarrow \quad -\sqrt{169} \le 12\cos\theta + 5\sin\theta \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le 12\cos\theta + 5\sin\theta \le 13$$

$$\Rightarrow \quad -13+4 \le 12\cos\theta + 5\sin\theta + 4 \le 13+4$$

$$\Rightarrow \quad -9 \le 12\cos\theta + 5\sin\theta + 4 \le 17$$

$$\Rightarrow \quad -9 \le f(\theta) \le 17$$

Hnece, minimum and maximum values of $12\cos\theta + 5\sin\theta + 4$ are -9 and 17 respectively.

Let $f(\theta) = 5\cos\theta + 3\sin\left(\frac{\pi}{6} - \theta\right) + 4$

Then, $f(\theta) = 5\cos\theta + 3\left[\sin\frac{\pi}{6}\cos\theta - \cos\frac{\pi}{6}\sin\theta\right] + 4$

$$= 5\cos\theta + 3\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] + 4$$

$$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \left(5 + \frac{3}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4$$

We know that

$$-\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad -\sqrt{\frac{169}{4} + \frac{27}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{169}{4} + \frac{27}{4}}$$

$$\Rightarrow \quad -\sqrt{\frac{196}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{196}{4}}$$

$$\Rightarrow \quad -\frac{14}{2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \frac{14}{2}$$

$$\Rightarrow \quad -7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$$

$$\Rightarrow \quad -7 + 4 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4 \le 7 + 4$$

$$\Rightarrow \quad -3 \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4 \le 11$$

$$\Rightarrow \quad -3 \le f(\theta) \le 11$$

Let $f(\theta) = \sin\theta - \cos\theta + 1$. Then,

$$f(\theta) = \sin\theta + (-1)\cos\theta + 1$$
$$= (-1)\cos\theta + \sin\theta + 1$$

We know that

$$-\sqrt{(-1)^2 + (1)^2} \le -\cos\theta + \sin\theta \le \sqrt{(-1)^2 + (1)^2}$$
$$\Rightarrow \quad -\sqrt{1+1} \le -\cos\theta + \sin\theta \le \sqrt{1+1}$$
$$\Rightarrow \quad -\sqrt{2} \le -\cos\theta + \sin\theta \le \sqrt{2}$$
$$\Rightarrow \quad -\sqrt{2} + 1 \le -\cos\theta + \sin\theta + 1 \le \sqrt{2} + 1$$
$$\Rightarrow \quad 1 - \sqrt{2} \le f(\theta) \le 1 + \sqrt{2}$$

Hence, minimum and maximum values of $\sin\theta - \cos\theta + 1$ are $1 - \sqrt{2}$ and $1 + \sqrt{2}$ respectively.

Q2

Let $f(\theta) = \sqrt{3}\sin\theta - \cos\theta$

Multiplying and dividing by $\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}$, we get

$$f(\theta) = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}} - \frac{\cos\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}}\right]$$

$$= \sqrt{3+1}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{3+1}} - \frac{\cos\theta}{\sqrt{3+1}}\right]$$

$$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}\sin\theta}{2} - \frac{\cos\theta}{2}\right] \qquad ---(i)$$

$$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}}{2}\times\sin\theta - \frac{1}{2}\times\cos\theta\right]$$

$$= 2\left[\cos\frac{\pi}{6}\times\sin\theta - \sin\frac{\pi}{6}\times\cos\theta\right]$$

$$= 2\left[\sin\theta\times\cos\frac{\pi}{6} - \cos\theta\times\sin\frac{\pi}{6}\right]$$

$$= 2\sin\left(\theta - \frac{\pi}{6}\right) \qquad \left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$$

$$\Rightarrow \quad f(\theta) = 2\sin\left(\theta - \frac{\pi}{6}\right)$$

Again,

$$f(\theta) = 2\left[\frac{\sqrt{3}}{2}\sin\theta - \frac{\cos\theta}{2}\right]$$

$$= -2\left[\frac{1}{2}\times\cos\theta - \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= -2\left[\cos\frac{\pi}{3}\times\cos\theta - \sin\frac{\pi}{3}\times\sin\theta\right]$$

$$= -2\cos\left(\frac{\pi}{3} + \theta\right)$$

Let $f(\theta) = \cos\theta - \sin\theta$

Multiplying and dividing by $\sqrt{1^2+1^2}$, we get

$$f(\theta) = \sqrt{1^2+1^2}\left[\frac{\cos\theta}{\sqrt{1^2+1^2}} - \frac{\sin\theta}{\sqrt{1^2+1^2}}\right]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\left[\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right] \qquad ---(i)$$

Now, $f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$

$$= \sqrt{2}\left[\sin\frac{\pi}{4}\times\cos\theta - \cos\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right) \qquad [\because \sin(A-B) = \sin A\cos B - \cos A\sin B]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)$$

Again,

$$f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4}\times\cos\theta - \sin\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right) \qquad [\because \cos(A+B) = \cos A\cos B - \sin A\sin B]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)$$

Let $f(\theta) = 24\cos\theta + 7\sin\theta$

Multiplying and dividing by $\sqrt{(24)^2 + (7)^2}$, we get

$$f(\theta) = \sqrt{(24)^2 + 7^2}\left[\frac{24\cos\theta}{\sqrt{24^2+7^2}} + \frac{7\sin\theta}{\sqrt{24^2+7^2}}\right]$$

$$= \sqrt{576+49}\left[\frac{24\cos\theta}{\sqrt{576+49}} + \frac{7\sin\theta}{\sqrt{576+49}}\right]$$

$$= \sqrt{625}\left[\frac{24\cos\theta}{\sqrt{625}} + \frac{7\sin\theta}{\sqrt{625}}\right]$$

$$= 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$\Rightarrow \quad f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right] \qquad ---(i)$$

Now, $f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$

$$= 25\left[\sin\alpha\times\cos\theta + \cos\alpha\times\sin\theta\right]$$

where $\sin\alpha = \frac{24}{25}$ and $\cos\alpha = \frac{7}{25}$

$$\Rightarrow \quad f(\theta) = 25\sin(\alpha+\theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{24}{7}$$

Again,

$$f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$= 25\left[\cos\alpha\times\cos\theta + \sin\alpha\times\sin\theta\right], \text{ where } \cos\alpha = \frac{24}{25} \text{ and } \sin\alpha = \frac{7}{25}$$

$$\Rightarrow \quad f(\theta) = 25\cos(\alpha-\theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{7}{24}$$

Q3

We have,

$\sin 100° - \sin 10°$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \times \sin 100° - \frac{1}{\sqrt{2}} \times \cos 100°\right)$ $\left[\begin{array}{l}\text{Multiplying and dividing} \\ \text{by } \sqrt{1^2+1^2} \text{ ie., by } \sqrt{2}\end{array}\right]$

$= \sqrt{2}\left(\cos 45° \times \sin 100° - \sin 45° \times \cos 100°\right)$

$= \sqrt{2}\left(\sin 100° \times \cos 45° - \cos 100° \times \sin 45°\right)$

$= \sqrt{2}\left(\sin\left(100° - 45°\right)\right)$

$= \sqrt{2}\sin 55°$, which is positive real number.

$[\because \sin\theta$ is positive in first quadrant$]$

Q4

$(2\sqrt{3}+3)\sin\theta + 2\sqrt{3}\cos\theta$

assume $a = 2\sqrt{3}+3$, $b = 2\sqrt{3}$

$\sqrt{a^2+b^2} = \sqrt{12+9+12\sqrt{3}+12} = \sqrt{33+12\sqrt{3}}$

Dividing and multiplying the above equation with above value

we get, $\sqrt{33+12\sqrt{3}}\left(\frac{2\sqrt{3}+3}{\sqrt{33+12\sqrt{3}}}\sin\theta + \frac{2\sqrt{3}}{\sqrt{33+12\sqrt{3}}}\cos\theta\right)$

Assume $\tan\phi = \frac{a}{b}$, we have $\sin\phi = \frac{a}{\sqrt{a^2+b^2}}$, $\cos\phi = \frac{b}{\sqrt{a^2+b^2}}$

so above expressions changes to $\sqrt{33+12\sqrt{3}}(\sin\phi\sin\theta + \cos\phi\cos\theta)$

which is equal to $\sqrt{33+12\sqrt{3}}\cos(\theta-\phi)$

We know that maximum and minimum value of any cosine term is +1 and -1

$\sqrt{33+12\sqrt{3}} = \sqrt{15+12+6+12\sqrt{3}}$

we know that $12\sqrt{3}+6 < 12\sqrt{5}$ becasue value of $\sqrt{5}-\sqrt{3}$ is more than 0.5

so if we replace $12\sqrt{3}+6$ with $12\sqrt{5}$ the above inequality still holds

So range of above expression can be $\sqrt{15+12+12\sqrt{5}} = 2\sqrt{3}+\sqrt{15}$

$-(2\sqrt{3}+\sqrt{15}) < \sqrt{33+12\sqrt{3}}\cos(\theta-\phi) < 2\sqrt{3}+\sqrt{15}$