Division Of Algebraic Expres

Ex-8.1

8. DIVISION OF ALGEBRAIC EXPRESSIONS.

1. Degree of a polynomial in one variable:-

In a polynomial in one variable, the highest power of the variable is called its degree.

(i) $2x^3+5x^2-7$.

highest power of a polynomial $=3$.

degree $=3$.

(ii) $5x^2+3x+2$.

highest power of a polynomial $=2$.

degree $=2$.

(iii) $2x+x^2-8$.

highest power of a polynomial $=2$.

degree $=2$.

(iv) $\frac{1}{2}y^7-12y^6+48y^5-10$.

highest power of a polynomial $=7$.

degree $=7$.

(v) $3x^3+1$

highest power of a polynomial $=3$.

degree $=3$.

(vi) 5

highest power of a polynomial $=0$, degree $=0$.

(vii) $20x^3+12x^2y^2-10y^2+20$.

highest power of a polynomial $=4$, degree $=4$.

2. Polynomials :-

An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial.

In the given expressions (i), (iv), (v) are not polynomials.

because. The expressions in which the variables involved have only non-negative integral powers.

3. (i) $3+6x+x^2+5x^4$ (or) $5x^4+x^2+6x+3$.

degree = 4.

(ii) $4+a^2+5a^6$ (or) $5a^6+a^2+4$.

degree = 6.

(iii) x^6-5x^3+4 (or) $4-5x^3+x^6$

degree = 6.

(iv) y^6+9y^3-22 (or) $-22+9y^3+y^6$

degree = 6.

(v) $a^6+\frac{27}{136}a^3-\frac{48}{136}$ (or), $-\frac{48}{136}+\frac{27}{136}a^3+a^6$.

degree = 6.

(vi) $a^2+\frac{25}{12}a+1$ (or), $1+\frac{25}{12}a+a^2$

degree = 2.

Division Of Algebraic Express

Ex 8.2

Exercise - 8.2.

Divide:

1. $6x^3y^2z^2$ by $3x^2yz$.

we have, $\Rightarrow \dfrac{6x^3y^2z^2}{3x^2yz} = 2xyz$.

$$\frac{6x^3y^2z^2}{3x^2yz} = 2xyz$$

2. $15m^2n^3$ by $5m^2n^2$

$$\frac{15m^2n^3}{5m^2n^2} = \frac{15 \cdot m \cdot m \cdot n \cdot n \cdot n}{5 \cdot m \cdot m \cdot n \cdot n}$$

$$= 3n.$$

3. $24a^3b^3$ by $-8ab$.

$$\frac{24a^3b^3}{-8ab} = \frac{24 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b}{-8 \cdot a \cdot b}$$

$$= -3a^2b^2.$$

4. $-21abc^2$ by $7abc$.

$$\frac{-21abc^2}{7abc} = \frac{-21 a \cdot b \cdot c \cdot c}{7 \cdot a \cdot b \cdot c}$$

$$= -3c.$$

5. $72xyz^2$ by $-9xz$

$$\frac{72xyz^2}{-9xz} = \frac{8 \times 9 \cdot x \cdot y \cdot z \cdot z}{-9 \cdot x \cdot z} = -8yz.$$

Solution-06:-

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3} = \frac{+72 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c}{+9 \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c}$$

$$= 8a^2b^3c^5$$

Solution-07:-

$$\frac{16m^3y^2}{4m^2y} = \frac{16 \cdot m \cdot m \cdot m \cdot y \cdot y}{4 \cdot m \cdot m \cdot y}$$

$$= 4m \cdot y.$$

Solution-08:-

$$\frac{32m^2n^3p^2}{4m \cdot n \cdot p} = \frac{32\, m \cdot m \cdot n \cdot n \cdot n \cdot p \cdot p}{4 \cdot m \cdot n \cdot p}$$

$$= 8mn^2p.$$

Division Of Algebraic Expre

Ex 8.3

Exercise-8.3

Divide.

1. $x+2x^2+3x^4-x^5$ by $2x$

$$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x(1+2x+3x^3-x^4)}{2x}$$

$$= \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^3.x}{2.x} - \frac{x^4.x}{2x}$$

$$= \frac{1}{2} + x + \frac{3}{2}x^3 - \frac{1}{2}x^4.$$

2. $y^4-3y^3+\frac{1}{2}y^2$ by $3y$.

$$\frac{y^4-3y^3+\frac{1}{2}y^2}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{\frac{1}{2}.y^2}{3.y}$$

$$= \frac{1}{3}.y^3 - y^2 + \frac{1}{6}.y.$$

$\boxed{3.\ -2a^2+2a+\frac{1}{2}}$

3. $-4a^3+4a^2+a$ by $2a$.

$$\frac{-4a^3+4a^2+a}{2a} = \frac{-4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a}$$

$$= -2a^2+2a+\frac{1}{2}.$$

4. $-x^6+2x^4+4x^3+2x^2$ by $\sqrt{2}\,x^2$.

$$\frac{-x^6+2x^4+4x^3+2x^2}{\sqrt{2}\,x^2} = \frac{-x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} + \frac{2x^2}{\sqrt{2}x^2}$$

$$= \frac{-1}{\sqrt{2}}x^4 + \sqrt{2}\,x^2 + 2\sqrt{2}\,x + \sqrt{2}.$$

5. $5z^3-6z^2+7z$ by $2z$.

$$\frac{5z^3-6z^2+7z}{2z} = \frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z}$$

$$= \frac{5}{2}z^2 - 3z + \frac{7}{2}.$$

6. $\sqrt{3}a^4+2\sqrt{3}a^3+3a^2-6a$ by $3a$.

$$\frac{\sqrt{3}a^4+2\sqrt{3}a^3+3a^2-6a}{3a} = \frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{3a^2}{3a} - \frac{6a}{3a}$$

$$= \frac{1}{\sqrt{3}}a^2 + \frac{2}{\sqrt{3}}a^2 + a - 2.$$

Division Of Algebraic Expressions

Ex 8.4

Exercise-8.4.

Divide.

1. $5x^3 - 15x^2 + 25x$ by $5x$.

$$\frac{5x^3 - 15x^2 + 25x}{5x} = \frac{5x^3}{5x} + \left(\frac{-15}{5}\right) \cdot \frac{x^2}{x} + \frac{25}{5} \cdot \frac{x}{x}$$

$$= x^2 - 3x + 5.$$

2. $4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

$$\frac{4z^3 + 6z^2 - z}{-\frac{1}{2}z} = \frac{4z^3 \cdot (2)}{-z} - \frac{6z^2 \cdot 2}{z} + \frac{z \cdot 2}{z}$$

$$= -8z^2 - 12z + 2.$$

3. $9x^2y - 6xy + 12xy^2$ by $-\frac{3}{2}xy$.

$$\frac{9x^2y - 6xy + 12xy^2}{-\frac{3}{2}xy} = \frac{9x^2y}{-3x \cdot y} \cdot 2 + \frac{6xy \cdot 2}{3 \cdot x \cdot y} + \frac{12x \cdot y^2 \cdot 2}{-3xy}$$

$$= -6x + 4 - 8y.$$

4. $3x^3 \cdot y^2 + 2x^2 \cdot y + 15xy$ by $3xy$.

$$\frac{3x^3 \cdot y^2 + 2x^2 \cdot y + 15xy}{3xy} = \frac{3 \cdot x^3 \cdot y^2}{3xy} + \frac{2x^2 \cdot y}{3xy} + \frac{15xy}{3xy}$$

$$= x^2y + \frac{2}{3}x + 5.$$

5. $x^2+7x+12$ by $x+4$.

step 1 :-

we devide the first term x^2 of the dividend by the first term x of the divisor and obtain $\frac{x^2}{x} = x$ as the first term of the quotient.

$$\begin{array}{r|l} & \quad x+3 \\ \hline x+4 & x^2+7x+12 \\ & \underline{x^2+4x} \\ & \quad 3x+12 \\ & \quad \underline{3x+12} \\ & \quad\quad (0) \end{array}$$

Step -2 :-

we multiply the divisor $x+4$ by the first term x of the quotient and subtract the result from the dividend $x^2+7x+12$. we obtain $3x+12$ as the remainder

step-3 :-

Now we treat $3x+12$ as the new dividend and divide the first term $3x$ by the first term x of the divisor to obtain $\frac{3x}{x} = 3$ as the third term of the quotient.

step- IV :-

we multiply the divisor $x+4$ and the Second ~~third~~ term 3 of the quotient and subtract the resur $3x+12$ from the new dividena. we obtain 0 as the remainder.

Thus, we can say that

$$\frac{x^2+7x+12}{x+4} = x+3.$$

Solution-06:-

$4y^2+3y+\frac{1}{2}$ by $2y+1$

$$
\begin{array}{r|l}
 & 2y+\frac{1}{2} \\
\hline
2y+1 & 4y^2+3y+\frac{1}{2} \\
 & 4y^2+2y \\
\hline
 & y+\frac{1}{2} \\
 & y+\frac{1}{2} \\
\hline
 & (0)
\end{array}
$$

Solution-07.

$3x^3+4x^2+5x+18$ by $x+2$.

$$
\begin{array}{r|l}
 & 3x^2-2x+9 \\
\hline
x+2 & 3x^3+4x^2+5x+18 \\
 & 3x^3+6x^2 \\
\hline
 & -2x^2+5x \\
 & -2x^2-4x \\
\hline
 & 9x+18 \\
 & 9x+18 \\
\hline
 & (0)
\end{array}
$$

Solution-08:-

$$
\begin{array}{r|l}
 & 2x-5 \\
\hline
7x-9 & 14x^2-53x+45 \\
 & 14x^2-18x \\
\hline
 & -35x+45 \\
 & -35x+45 \\
\hline
 & (0)
\end{array}
$$

Solution - 09.

$$\frac{-(-21+71x-31x^2-24x^3)}{-(3-8x)} = \frac{21-71x+31x^2+24x^3}{8x-3}$$

$$\begin{array}{r|l} & 3x^2+5x-7 \\ \hline 8x-3 & 24x^3+31x^2-71x+21 \\ & 24x^3-9x^2 \\ & \quad - \quad + \\ \hline & 40x^2-71x \\ & 40x^2-15x \\ & \quad - \quad + \\ \hline & -56x+21 \\ & -56x+21 \\ & \quad + \quad - \\ \hline & (0) \end{array}$$

Solution-10:-

$3y^4-3y^3-4y^2-4y$ by y^2-2y.

$$\begin{array}{r|l} & 3y^2+3y+2 \\ \hline y^2-2y & 3y^4-3y^3-4y^2-4y \\ & 3y^4-6y^3 \\ & - \quad + \\ \hline & 3y^3-4y^2 \\ & 3y^3-6y^2 \\ & - \quad + \\ \hline & 2y^2-4y \\ & 2y^2-4y \\ & - \quad + \\ \hline & (0) \end{array}$$

$(y^2-2y)(3y^2+3y+2) = 3y^4-3y^3-4y^2-4y$.

Solution-11

$2y^5+10y^4+6y^3+y^2+5y+3$ by $2y^3+1$

$$
\begin{array}{r|l}
 & y^2+5y+3 \\
\hline
2y^3+1 & 2y^5+10y^4+6y^3+y^2+5y+3 \\
 & \underline{2y^5+0+0+y^2} \\
 & 10y^4+6y^3+0+5y \\
 & \underline{10y^4+0+0+5y} \\
 & 6y^3+0+0+3 \\
 & \underline{6y^3+0+0+3} \\
 & (0)
\end{array}
$$

Solution-12:-

$x^4-2x^3+2x^2+x+4$ by x^2+x+1

$$
\begin{array}{r|l}
 & x^2-3x+4 \\
\hline
x^2+x+1 & x^4-2x^3+2x^2+x+4 \\
 & \underline{x^4+x^3+x^2} \\
 & -3x^3+x^2+x \\
 & \underline{-3x^3-3x^2-3x} \\
 & 4x^2+4x+4 \\
 & \underline{4x^2+4x+4} \\
 & (0)
\end{array}
$$

Solution-13:-

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$.

$$\begin{array}{r|l} & m-2 \\ \hline m^2-12m+13 & m^3-14m^2+37m-26 \\ & m^3-12m^2+13m \\ & - \quad + \quad - \\ \hline & -2m^2+24m-26 \\ & -2m^2+24m-26 \\ & + \quad - \quad + \\ \hline & (0) \end{array}$$

Solution-014 :-

$$\begin{array}{r|l} & x^2-x+1 \\ \hline x^2+x+1 & x^4+x^2+1+0 \\ & x^4+x^2+0+x^3 \\ & - \quad - \quad - \quad - \\ \hline \end{array}$$

Solution-14 :-

$$\begin{array}{r|l} & x^2-x+1 \\ \hline x^2+x+1 & x^4+0+x^2+0+1 \\ & x^4+x^3+x^2 \\ & - \quad - \quad - \\ \hline & -x^3+0+0+1 \\ & -x^3-x^2-x+0 \\ & + \quad + \quad + \\ \hline & x^2+x+1 \\ & x^2+x+1 \\ & - \quad - \quad - \\ \hline & (0) \end{array}$$

Solution-15:-

$$
\begin{array}{r|l}
 & x^2+x+1 \\
\hline
x^3+1 & x^5+x^4+x^3+x^2+x+1 \\
 & x^5+0+0+x^2 \\
 & \underline{-\quad\;\; -} \\
 & x^4+x^3+0+x \\
 & x^4+0+0+x \\
 & \underline{-\quad -\quad -\quad -} \\
 & x^3+0+0+1 \\
 & x^3+0+0+1 \\
 & \underline{-\quad -\quad -\quad -} \\
 & (0)
\end{array}
$$

16:- Divide the following and find the quotient and remainder.

Solution-16:-

$14x^3-5x^2+9x-1$ by $2x-1$.

$$
\begin{array}{r|l}
 & 7x^2+x+5 \\
\hline
2x-1 & 14x^3-5x^2+9x-1. \\
 & 14x^3-7x^2 \\
 & \underline{-\qquad +} \\
 & 2x^2+9x \\
 & 2x^2-x \\
 & \underline{-\qquad +} \\
 & 10x-1 \\
 & 10x-5 \\
 & \underline{-\qquad +} \\
 & (4).
\end{array}
$$

$\therefore$ $Q = 7x^2+x+5$; $R = 4$.

Solution-17:-

$$
\begin{array}{r|l}
 & 3x^2+4x+1 \\
\hline
2x-3 & 6x^3-x^2-10x-3 \\
 & \underline{6x^3-9x^2} \\
 & \quad 8x^2-10x \\
 & \quad \underline{8x^2-8x} \\
 & \qquad 2x-3 \\
 & \qquad \underline{2x-3} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore$ Quotient $= 3x^2+4x+1$; Remainder $= 0$.

Solution-18:-

$6x^3+11x^2-39x-65$ by $3x^2+13x+13$

$$
\begin{array}{r|l}
 & 2x-5 \\
\hline
3x^2+13x+13 & 6x^3+11x^2-39x-65. \\
 & \underline{6x^3+26x^2+26x} \\
 & \quad -15x^2-65x-65 \\
 & \quad \underline{-15x^2-65x-65} \\
 & \qquad 0
\end{array}
$$

$\therefore$ Quotient $= 2x-5$; Remainder $= 0$.

Solution-19:-

$30x^4+11x^3-82x^2-12x+48$ by $3x^2+2x-4$.

$$
\begin{array}{r|l}
 & 10x^2-3x-12 \\
\hline
3x^2+2x-4 & 30x^4+11x^3-82x^2-12x+48 \\
 & \underline{\overset{-}{30x^4}\overset{-}{+20x^3}\overset{+}{-40x^2}} \\
 & -9x^3-42x^2-12x \\
 & \underline{\overset{+}{-9x^3}\overset{+}{-6x^2}\overset{-}{+12x}} \\
 & -36x^2-24x+48 \\
 & \underline{\overset{+}{-36x^2}\overset{+}{-24x}\overset{-}{+48}} \\
 & 0
\end{array}
$$

$\therefore$ Quotient $= 10x^2-3x-12$; Remainder $=0$.

Solution-20:-

$9x^4-4x^2+4$ by $3x^2-4x+2$.

$$
\begin{array}{r|l}
 & 3x^2+4x+2 \\
\hline
3x^2-4x+2 & 9x^4-0x^3+(-4x^2)+0x+4 \\
 & \underline{\overset{-}{9x^4}\overset{+}{-12x^3}\overset{-}{+6x^2}} \\
 & +12x^3-10x^2+0x+4 \\
 & \underline{\overset{-}{12x^3}\overset{+}{-16x^2}\overset{-}{+8x}\overset{-}{+0}} \\
 & 6x^2-8x+4 \\
 & \underline{\overset{-}{6x^2}\overset{+}{-8x}\overset{-}{+4}} \\
 & 0
\end{array}
$$

Quotient $= 3x^2-4x+2$; Remainder $=0$.

Solution - 21.

(i) Dividend = $14x^2+13x-15$.

Divisor = $7x-4$

$$
\begin{array}{r|l}
 & 2x+3 \\
\hline
7x-4 & 14x^2+13x-15 \\
 & 14x^2-8x \\
 & \quad - \quad + \\
\hline
 & \quad 21x-15 \\
 & \quad 21x-12 \\
 & \quad - \quad + \\
\hline
 & \qquad (-3)
\end{array}
$$

$\therefore$ Quotient $= 2x+3$; Remainder $= -3$.

$$14x^2+13x-15 = (7x-4)(2x+3)-3$$

$$= 14x^2+21x-8x-12-3$$

$$= 14x^2+13x-15.$$

$\therefore$ LHS $=$ RHS.

(ii)

$$
\begin{array}{r|l}
 & 5z^2+\frac{10}{3}z+11 \\
\hline
3z-6 & 15z^3-20z^2+13z-12 \\
 & 15z^3-30z^2 \\
 & - \quad + \\
\hline
 & \quad 10z^2+13z \\
 & \quad 10z^2-20z \\
 & \quad - \quad + \\
\hline
 & \qquad 33z-12 \\
 & \qquad 33z-66 \\
 & \qquad - \quad + \\
\hline
 & \qquad (54)
\end{array}
$$

$$\therefore \quad 15z^3-20z^2+13z-12 = (3z-6)\left(5z^2+\frac{10z}{3}+11\right)+54$$

$$= 15z^3-20z^2+13z-66+54$$

$$= 15z^3-20z^2+13z-12.$$

Solution - 21 :-

(iii)

$2y^2 - 6$ $\downarrow$ divisor

$6y^5 - 28y^3 + 3y^2 + 30y - 9$ $\downarrow$ dividend

Quotient: $3y^3 - 5y + \frac{3}{2}$

Divisor: $2y^2 - 6$

$$
\begin{array}{l}
6y^5 + 0 - 28y^3 + 3y^2 + 30y - 9 \\
6y^5 + 0 - 18y^3 \\
\quad - \;\; + \\
\hline
-10y^3 + 3y^2 + 30y \\
-10y^3 + 0 + 30y \\
\; + \quad\;\; - \\
\hline
3y^2 - 9 \\
3y^2 - 9 \\
- \quad + \\
\hline
(0)
\end{array}
$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = (2y^2 - 6)(3y^3 - 5y + \tfrac{3}{2}) + 0$$

$$= 6y^5 - 10y^3 + 3y^2 - 18y^3 + 30y - 9$$

$$= 6y^5 - 28y^3 + 3y^2 + 30y - 9$$

(iv)

Quotient: $-4x^3 + 2x^2 - 8x + 30$

Divisor: $3x + 7$

$$
\begin{array}{l}
-12x^4 - 22x^3 - 10x^2 + 34x - 75 \\
-12x^4 - 28x^3 \\
\; + \quad\;\; + \\
\hline
6x^3 - 10x^2 + 34x \\
6x^3 + 14x^2 + 0 \\
\; - \quad\;\; - \\
\hline
-24x^2 + 34x - 75 \\
-24x^2 - 56x \\
\; + \quad\;\; + \\
\hline
90x - 75 \\
90x + 210 \\
\hline
(-285)
\end{array}
$$

Quotient = $-4x^3+2x^2-8x+30$

$R = -285$

$\therefore\ 34x-22x^3-12x^4-10x^2-75 = (3x+7)(-4x^3+2x^2-8x+30) - 285$

$= -12x^4-22x^3-10x^2+34x-75.$

(v)

$$
\begin{array}{r|l}
3y-2 & \overline{5y^3-2y^2+\frac{5}{3}y} \\
 & 15y^4-16y^3+9y^2-\frac{10}{3}y+6 \\
 & \underline{15y^4-10y^3} \\
 & -6y^3+9y^2 \\
 & \underline{-6y^3+4y^2} \\
 & 5y^2-\frac{10}{3}y+6 \\
 & \underline{5y^2-\frac{10}{3}y+0} \\
 & (6)
\end{array}
$$

$\therefore\ 15y^4-16y^3+9y^2-\frac{10}{3}y+6 = (3y-2)(5y^3-2y^2+\frac{5}{3}y)+6$

$= 15y^4-6y^3+5y^2-10y^3+4y^2-\frac{10y}{3}+6$

$= 15y^4-16y^3+9y^2-\frac{10}{3}y+6.$

$\therefore$ LHS = RHS.

(vi)

$$
\begin{array}{r|l}
 & 2y+5 \\
2y^2-y+1 & 4y^3+8y^2+8y+7 \\
 & \underline{4y^3-2y^2+2y} \\
 & \quad 10y^2+6y+7 \\
 & \quad \underline{10y^2-5y+5} \\
 & \qquad 11y+2
\end{array}
$$

$(2y^2-y+1)(2y+5)+11y+2 = 4y^3+10y^2-2y^2-5y$
$+2y+5+11y+2$
$= 4y^3+8y^2+8y+7.$

(vii)

$$
\begin{array}{r|l}
 & 3y^2+2y+2 \\
2y^3+1 & 6y^5+4y^4+4y^3+7y^2+27y+6 \\
 & \underline{6y^5+0+0+3y^2} \\
 & \quad 4y^4+4y^3+4y^2+27y+6 \\
 & \quad \underline{4y^4+0+0+2y} \\
 & \qquad 4y^3+4y^2+25y+6 \\
 & \qquad \underline{4y^3+0+0+2} \\
 & \qquad\quad 4y^2+25y+4.
\end{array}
$$

$\therefore$ Quotient $= 4y^2+25y+4$; Divisor $= 2y^3+1$.

Remainder $= 4y^2+25y+4$.

$\Rightarrow 6y^5+4y^4+4y^3+7y^2+27y+6 = (2y^3+1)(4y^2+25y+4)+$
$4y^2+25y+4$
$= 6y^5+4y^4+4y^3+7y^2+27y+6.$

Solution-22 :-

$$Q = 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$$

$$3y-2 \;\big)\; 15y^4 + 16y^3 + \frac{10}{3}y - 9y^2 - 6$$

can be written as

$$15y^4 + 16y^3 - 9y^2 + \frac{10}{3}y - 6$$

$$15y^4 \mp 10y^3$$

$$26y^3 - 9y^2$$

$$26y^3 - \frac{52}{3}y^2$$

$$- \quad +$$

$$\frac{25}{3}y^2 + \frac{10}{3}y$$

$$\frac{25y^2}{3} - \frac{50}{9}y$$

$$- \quad +$$

$$\frac{80}{9}y - 6.$$

$$\frac{80}{9}y - 6$$

$$- \quad +$$

$$(0).$$

$\therefore$ Quotient $= 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$.

coefficient of $y^3 = 5$

" $y^2 = \frac{26}{3}$

" $y = \frac{25}{9}$.

constant term $= \frac{80}{27}$.

23. (i)

$$
\begin{array}{r|l}
 & x-7 \\
\hline
x+6 & x^2-x-42 \\
 & \underline{x^2+6x} \\
 & \quad -7x-42 \\
 & \quad \underline{-7x-42} \\
 & \qquad (0)
\end{array}
$$

Yes, $x+6$ is a factor of x^2-x-42.

(ii)

$$
\begin{array}{r|l}
 & x-3 \\
\hline
4x-1 & 4x^2-13x-12 \\
 & \underline{4x^2-x} \\
 & \quad -12x-12 \\
 & \quad \underline{-12x+3} \\
 & \qquad (-15)
\end{array}
$$

$4x-1$ is not a factor of $4x^2-13x-12$.

23. (iii)

$$
\begin{array}{r|l}
 & 2y^3+5y \\
\hline
2y-5 & 4y^4-10y^3-10y^2+30y-15 \\
 & \underline{4y^4-10y^3} \\
 & \quad -10y^2+30y \\
 & \quad \underline{10y^2-25y} \\
 & \qquad 5y-15
\end{array}
$$

$2y-5$ is not a factor of $4y^4-10y^3-10y^2+30y-15$

(iv)

$$
\begin{array}{r|l}
 & 2y^3+5y^2+2y-7 \\
\hline
3y^2+5 & 6y^5+15y^4+16y^3+4y^2+10y-35 \\
 & \underline{6y^5+10y^3} \\
 & \quad 15y^4+6y^3+4y^2 \\
 & \quad \underline{15y^4+0+25y^2} \\
 & \qquad 6y^3-21y^2+10y \\
 & \qquad \underline{6y^3+0+10y} \\
 & \qquad\quad -21y^2-35 \\
 & \qquad\quad \underline{-21y^2-35} \\
 & \qquad\qquad (0)
\end{array}
$$

$\therefore$ $3y^2+5$ is a factor of given polynomial.

(v)

$$\begin{array}{r|l} & z^3+3z \\ \hline z^2+3 & z^5-9z = z^5+0+0+0-9z+0 \\ & -z^5+0\pm 3z^3 \\ \hline & -3z^3-9z \\ & 3z^3+9z \\ \hline & (0) \end{array}$$

z^2+3 is a factor of polynomial z^5-9z.

(vi)

$$\begin{array}{r|l} & 3x^3+x^2-2x+5 \\ \hline 2x^2+x+3 & 6x^5-x^4+4x^3-5x^2-x-15 \\ & 6x^5-3x^4+9x^3 \\ & -\quad +\quad - \\ \hline & 2x^4-5x^3-5x^2 \\ & 2x^4-x^3+3x^2 \\ & -\quad +\quad - \\ \hline & -4x^3-8x^2-x \\ & -4x^3+2x^2-6x \\ & +\quad -\quad + \\ \hline & -10x^2+5x-15 \\ & 10x^2-5x+15 \\ \hline & (0) \end{array}$$

$2x^2-x+3$ is factor of given polynomial.

26.

$$\begin{array}{r|l} & x^2+1 \\ \hline x^2+2x-3 & x^4+2x^3-2x^2+x+1 \\ & -x^4+2x^3\mp 3x^2 \\ \hline & x^2+x-1 \\ & x^2+2x-3 \\ & -\quad -\quad + \\ \hline & -x+2 \end{array}$$

$x-2$ added to $x^4+2x^3-2x^2+x-1$ so that the resulting polynomial exactly divisible by x^2+2x-3.

24.

$$\begin{array}{r|l} & 4x^3-6x^2+9x+10 \\ \hline x+2 & 4x^4+2x^3-3x^2+8x+5a \\ & 4x^3+8x^3 \\ \hline & -6x^3-3x^2 \\ & \mp 6x^3 \mp 12x^2 \\ \hline & 9x^2+8x \\ & 9x^2+18x \\ \hline & -10x+5a \\ & -10x+20 \\ \hline & 5a+20 \end{array}$$

$5a+20=0$

$a = -\frac{20}{5}$

$\boxed{a=-4}$

Division Of Algebraic Expr

Ex 8.5

Exercise - 8.5.

1. Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and the remainder:

(i) $3x^2+4x+5$, $x-2$.

we have,

$$3x^2+4x+5 = x(3x+10)-2(3x+10)+5+20$$
$$= (x-2)(3x+10)+20+5$$

Quotient $= 3x+10$, Remainder $= 25$.

(ii) $10x^2-7x+8$, $5x-3$

we have

$$10x^2-7x+8 = 5x\left(2x-\frac{1}{5}\right)-3\left(2x-\frac{1}{5}\right)+8-\frac{3}{5}$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{40-3}{5}$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{37}{5}$$

Quotient $= \left(2x-\frac{1}{5}\right)$, Remainder $= \frac{37}{5}$.

(iii) $5y^3-6y^2+6y-1$, $5y-1$.

$$5y^3-6y^2+6y-1 = 5y(y^2-y+1)-1(y^2-y+1)-1+1$$
$$= (5y-1)(y^2-y+1)+0$$

$\therefore$ Quotient $= y^2-y+1$, Remainder $= 0$.

② (iii)

$$\begin{array}{r|l} 4x^2-5 & 4x^4+0+7x^2+15 \quad (2x^2+3) \\ & 4x^4+0-5x^2 \\ \hline & 12x^2+15 \\ & 12x^2+15 \\ \hline & 30 \end{array}$$

No, $4x^2-5$ is not a factor of $4x^4+7x^2+15$.

② ⓥ

$$\begin{array}{r|l} 2a-3 & 10a^2-9a-5 \quad (5a+3) \\ & 10a^2-15a \\ \hline & 6a-5 \\ & 6a-9 \\ \hline & 4 \end{array}$$

$10a^2-9a-5 / 2a-5 = 5a+3$

No, $(2a-3)$ is not a factor of $10a^2-9a-5$

1>

(iv) x^4-x^3+5x, $x-1$.

We have,

$$x^4-x^3+5x = x(x^3+5)-1(x^3+5)+5$$

$$= (x-1)(x^3+5)+5.$$

$\therefore$ Quotient $= (x^3+5)$, Remainder $= 5$

ⓥ y^4+y^2, y^2-2.

$\boxed{y^2(y^2-2)-2(y^2-2)+4.}$

We have,

$$y^4+y^2 = y^2(y^2+3)-2(y^2+3)+6$$

$$= (y^2-2)(y^2+3)+6.$$

$\therefore$ Quotient $= y^2+3$, Remainder $= 6$.

②
ⓘ

$$\begin{array}{r|l} x+1 & 2x^2+5x+4 \quad (2x+3) \\ & 2x^2+2x \\ \hline & 3x+4 \\ & 3x+3 \\ \hline & 1. \end{array}$$

No, $(x+1)$ is not a factor of the second.

(ii)

$$\begin{array}{r|l} y-2 & 3y^3+5y^2+5y+2 \quad (3y^2+11y+27) \\ & 3y^3-6y^2 \\ \hline & 11y^2+5y \\ & 11y^2-22y \\ \hline & 27y+2 \\ & 27y-54 \\ \hline & 56 \end{array}$$

No, $(y-2)$ is not a factor of the second.

3 (vi)

$$
\begin{array}{r|l}
 & 2y - 1 \\
\hline
4y+1 & 8y^2 - 2y + 1 \\
 & \underline{-\,8y^2 \mp 2y} \\
 & \quad -4y + 1 \\
 & \quad \underline{\cancel{4}y + 1} \\
 & \qquad (2)
\end{array}
$$

No, 4y+1 is not a factor of given Polynomial

Division Of Algebraic Expres

Ex 8.6

Exercise - 8.6.

1. x^2-5x+6 by $x-3$.

$$x^2-5x+6 = x^2-2x-3x+6$$
$$= x(x-2)-3(x-2)$$
$$= (x-3)(x-2)$$

$$\therefore \frac{x^2-5x+6}{(x-3)} = \frac{(x-3)(x-2)}{(x-3)} = x-2.$$

2. ax^2-ay^2 by $ax+ay$.

$$ax^2-ay^2 = a(x^2-y^2)$$
$$= a(x+y)(x-y)$$

$$\therefore \frac{ax^2-ay^2}{ax+ay} = \frac{a(x+y)(x-y)}{a(x+y)} = (x-y)$$

3. x^4-y^4 by x^2-y^2

$$x^4-y^4 = (x^2)^2-(y^2)^2 = (x^2+y^2)(x^2-y^2)$$

$$\therefore \frac{x^4-y^4}{x^2-y^2} = \frac{(x^2+y^2)(x^2-y^2)}{(x^2-y^2)} = (x^2+y^2).$$

4. $[acx^2+(bc+ad)x+bd]$ by $[ax+b]$

$$acx^2+(bc+ad)x+bd = \underline{(ax+b)(cx+d)}$$

$$\therefore \frac{acx^2+(bc+ad)x+bd}{(ax+b)} = \frac{(ax+b)(cx+d)}{(ax+b)}$$

$$= cx+d.$$

5. $(a^2+2ab+b^2)-(a^2+2ac+c^2)$ by $2a+b+c$.

$$\cancel{a^2}+2ab+b^2-\cancel{a^2}-2ac-c^2 = (b^2-c^2)+2ab(1)-2ac$$

$$= (b-c)(b+c)+2a(b-c)$$

$$= (b-c)(2a+b+c)$$

$$\therefore \frac{(a^2+2ab+b^2)-(a^2+2ac+c^2)}{(2a+b+c)} = \frac{(2a+b+c)(b-c)}{(2a+b+c)}$$

$$= b-c.$$

6. $\frac{1}{4}x^2-\frac{1}{2}x-12$ by $\frac{1}{2}x-4$.

$$\frac{1}{4}x^2-\frac{1}{2}x-12 = \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{1}{2}x+3\right)$$

$$= \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{x}{2}+3\right)$$

$$= \left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)$$

$$\therefore \frac{\frac{1}{4}x^2-\frac{1}{2}x-12}{\frac{x}{2}-4} = \frac{\left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)}{\frac{x}{2}-4} = \frac{x}{2}+3.$$