

Exercise – 8.1

1. Write the complement of each of the following angles:

(i) 20° (ii) 35° (iii) 90° (iv) 77° (v) 30°

Sol:

- (i) Given angle is 20°

Since, the sum of an angle and its complement is 90° .

\therefore its, complement will be $(90 - 20 = 70^\circ)$

- (ii) Given angle is 35°

Since, the sum of an angle and its complement is 90° .

\therefore its, complements will be $(90 - 35^\circ = 55^\circ)$

- (iii) The given angle is 90°

Since, the sum of an angle and its complement is 90° .

\therefore [its, complement will be $(90 - 90^\circ = 0^\circ)$]

- (iv) The given angle is 77°

Since, the sum of an angle and its complement is 90° .

\therefore its, complement will be $(90 - 77^\circ = 13^\circ)$

- (v) The given angle is 30° .

Since, the sum of an angle and its complement is 90° .

\therefore its, complement will be $(90 - 30^\circ = 60^\circ)$

2. Write the supplement of each of the following angles:

(i) 54° (ii) 132° (iii) 138°

Sol:

- (i) The given angle is 54°

Since, the sum of an angle and its supplement is 180° .

\therefore its, supplement will be $180^\circ - 54^\circ = 126^\circ$

- (ii) The given angle is 132°

Since, the sum of an angle and its supplement is 180° .

\therefore its, supplement will be $180^\circ - 132^\circ = 48^\circ$

- (iii) The given angle is 138°

Since, the sum of an angle and its supplement is 180° .

\therefore its, supplement will be $180^\circ - 138^\circ = 42^\circ$



3. If an angle is 28° less than its complement, find its measure.

Sol:

Angle measured will be 'x' say

\therefore its complement will be $(90 - x)^\circ$

It is given that

Angle = Complement $- 28^\circ$

$$\Rightarrow x = (90 - x)^\circ - 28^\circ$$

$$\Rightarrow x^\circ = 90^\circ - 28^\circ - x^\circ$$

$$\Rightarrow 2x^\circ = 62^\circ$$

$$\Rightarrow x = 31^\circ$$

\therefore Angle measured is 31°

4. If an angle is 30° more than one half of its complement, find the measure of the angle.

Sol:

Angle measured will be 'x' say.

\therefore its complement will be $(90 - x)^\circ$

It is given that

$$\text{Angle} = 30^\circ + \frac{1}{2} \text{Complement}$$

$$\Rightarrow x^\circ = 30^\circ + \frac{1}{2}(90 - x)$$

$$\Rightarrow 3\frac{x}{2} = 30^\circ + 45^\circ$$

$$\Rightarrow 3x = 150^\circ$$

$$\Rightarrow x = \frac{150}{3}$$

$$\Rightarrow x = 50^\circ$$

\therefore Angle is 50°

5. Two supplementary angles are in the ratio 4 : 5. Find the angles.

Sol:

Supplementary angles are in the ratio 4 : 5

Let the angles be $4x$ and $5x$

It is given that they are supplementary angles

$$\therefore 4x + 5x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = 20^\circ$$

$$\text{Hence, } 4x = 4(20) = 80^\circ$$

$$5(x) = 5(20) = 100^\circ$$

\therefore Angles are 80° and 100°



6. Two supplementary angles differ by 48° . Find the angles.

Sol:

Given that two supplementary angles are differ by 48°

Let the angle measured is x°

\therefore Its supplementary angle will be $(180 - x)^\circ$

It is given that

$$(180 - x) - x = 98^\circ$$

$$\Rightarrow 180 - 48^\circ = 2x$$

$$\Rightarrow 132 = 2x$$

$$\Rightarrow x = \frac{132}{2}$$

$$\Rightarrow x = 66^\circ$$

Hence, $180 - x = 114^\circ$

Therefore, angles are 66° and 114°

7. An angle is equal to 8 times its complement. Determine its measure.

Sol:

It is given that angle = 8 times its complement

Let ' x ' be measured angle

\Rightarrow angle = 8 complements

\Rightarrow angle = $8(90 - x)^\circ$ [\because complement = $(90 - x)^\circ$]

$$\Rightarrow x^\circ = 8(90) - 8x^\circ$$

$$\Rightarrow 9x^\circ = 720^\circ$$

$$\Rightarrow x = \frac{720}{9} = 80$$

\therefore The measured angle is 80°

8. If the angles $(2x - 10)^\circ$ and $(x - 5)^\circ$ are complementary angles, find x .

Sol:

Given that,

$(2x - 10)^\circ$ and $(x - 5)^\circ$ are complementary angles.

Let x be the measured angle.

Since the angles are complementary

\therefore Their sum will be 90°

$$\Rightarrow (2x - 10) + (x - 5) = 90^\circ$$

$$\Rightarrow 3x - 15 = 90$$

$$\Rightarrow 3x = 90^\circ + 15^\circ$$



$$\Rightarrow x = \frac{105^\circ}{3} = \frac{105^\circ}{3} = 35^\circ$$

$$\Rightarrow x = 35^\circ$$

9. If the complement of an angle is equal to the supplement of the thrice of it. Find the measure of the angle.

Sol:

The angle measured will be ' x ' say.

Its complementary angle is $(90^\circ - x^\circ)$ and

Its supplementary angle is $(180^\circ - 3x^\circ)$

Given that,

Supplementary of thrice of the angle $= (180^\circ - 3x^\circ)$

According to the given information

$$(90 - x)^\circ = (180 - 3x)^\circ$$

$$\Rightarrow 3x^\circ - x^\circ = 180^\circ - 90^\circ$$

$$\Rightarrow 2x^\circ = 90^\circ$$

$$\Rightarrow x = 45^\circ$$

The angle measured is 45°

10. If an angle differs from its complement by 10° , find the angle.

Sol:

The measured angle will be ' x ' say

Given that,

The angles measured will be differed by 10°

$$x^\circ - (90 - x)^\circ = 10^\circ$$

$$\Rightarrow x - 90 + x = 10$$

$$\Rightarrow 2x = 100$$

$$\Rightarrow x = 50^\circ$$

\therefore The measure of the angle will be $= 50^\circ$

11. If the supplement of an angle is three times its complement, find the angle.

Sol:

Given that,

Supplementary of an angle $= 3$ times its complementary angle.

The angles measured will be x°

Supplementary angle of x will be $180^\circ - x^\circ$ and

The complementary angle of x will be $(90^\circ - x^\circ)$.



It's given that

Supplementary of angle = 3 times its complementary angle

$$180^\circ - x^\circ = 3(90^\circ - x^\circ)$$

$$\Rightarrow 180^\circ - x^\circ = 270^\circ - 3x^\circ$$

$$\Rightarrow 3x^\circ - x^\circ = 270^\circ - 180^\circ$$

$$\Rightarrow 2x^\circ = 90^\circ$$

$$\Rightarrow x = 45^\circ$$

\therefore Angle measured is 45° .

12. If the supplement of an angle is two-third of itself. Determine the angle and its supplement.

Sol:

Given that

Supplementary of an angle = $\frac{2}{3}$ of angle itself.

The angle measured be ' x ' say.

Supplementary angle of x will be $(180 - x)^\circ$

It is given that

$$(180 - x)^\circ = \frac{2}{3}x^\circ$$

$$\Rightarrow 180^\circ - x^\circ = \frac{2}{3}x^\circ$$

$$\Rightarrow \frac{2}{3}x^\circ + x^\circ = 180^\circ$$

$$\Rightarrow 2x^\circ + 3x^\circ = 3 \times 180^\circ$$

$$\Rightarrow 5x^\circ = 540^\circ$$

$$\Rightarrow x = 108^\circ$$

Hence, supplement = $180 - 108 = 72^\circ$

\therefore Angle will be 108° and its supplement will be 72° .

13. An angle is 14° more than its complementary angle. What is its measure?

Sol:

Given that,

An angle is 14° more than its complementary angle

The angle measured is ' x ' say

The complementary angle of ' x ' is $(90 - x)$

It is given that

$$x - (90 - x) = 14$$



$$\Rightarrow x - 90 + x = 14$$

$$\Rightarrow 2x^\circ = 90^\circ + 14^\circ$$

$$\Rightarrow x^\circ = \frac{104^\circ}{2}$$

$$\Rightarrow x = 52^\circ.$$

\therefore The angle measured is 52°

14. The measure of an angle is twice the measure of its supplementary angle. Find its measure.

Sol:

Given that

The angle measure of an angle is twice of the measure of the supplementary angle.

Let the angle measured will be ' x ' say

\therefore The supplementary angle of x is $180 - x$ as per question

$$x^\circ = 2(180 - x)^\circ$$

$$x^\circ = 2(180^\circ) - 2x^\circ$$

$$\Rightarrow 3x^\circ = 360^\circ$$

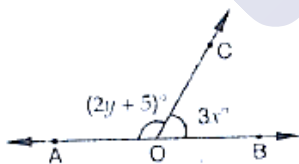
$$\Rightarrow x^\circ = 120^\circ$$

\therefore The angle measured is 120° .

Exercise – 8.2

1. In the below Fig, OA and OB are opposite rays:

- If $x = 25^\circ$, what is the value of y ?
- If $y = 35^\circ$, what is the value of x ?



Sol:

- Given that $x = 25^\circ$

Since $\angle AOC$ and $\angle BOC$ form a linear pair

$$\angle AOC + \angle BOC = 180^\circ$$

Given that

$$\angle AOC = 2y + 5 \text{ and } \angle BOC = 3x$$

$$\therefore \angle AOC + \angle BOC = 180^\circ$$

$$(2y + 5)^\circ + 3x = 180^\circ$$

$$(2y + 5)^\circ + 3(25^\circ) = 180^\circ$$



$$2y^\circ + 5^\circ + 75^\circ = 180^\circ$$

$$2y^\circ + 80^\circ = 180^\circ$$

$$2y^\circ = 180^\circ - 80^\circ = 100^\circ$$

$$y^\circ = \frac{100^\circ}{2} = 50^\circ$$

$$\Rightarrow \boxed{y = 50^\circ}$$

(ii) Given that if $y = 35^\circ$

$$\angle AOC + \angle BOC = 180^\circ$$

$$(2y + 5) + 3x = 180^\circ$$

$$(2(35) + 5) + 3x = 180^\circ$$

$$(70 + 5) + 3x = 180^\circ$$

$$3x = 180^\circ - 75^\circ$$

$$3x = 105^\circ$$

$$x = 35^\circ$$

$$\boxed{x = 35^\circ}$$

2. In the below fig, write all pairs of adjacent angles and all the linear pairs.



Sol:

Adjacent angles are

(i) $\angle AOC, \angle COB$

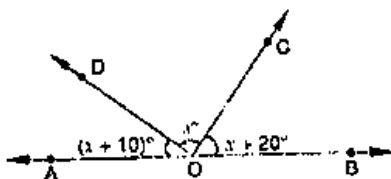
(ii) $\angle AOD, \angle BOD$

(iii) $\angle AOD, \angle COD$

(iv) $\angle BOC, \angle COD$

Linear pairs : $\angle AOD, \angle BOD; \angle AOC, \angle BOC$.

3. In the given below Fig, find x . Further find $\angle BOC$, $\angle COD$ and $\angle AOD$.



Sol:

Since $\angle AOD$ and $\angle BOD$ are form a line pair



$$\angle AOD + \angle BOD = 180^\circ$$

$$\angle AOD + \angle COD + \angle BOC = 180^\circ$$

Given that

$$\angle AOD = (x+10)^\circ, \angle COD = x^\circ, \angle BOC = (x+20)^\circ$$

$$\Rightarrow (x+10)^\circ + x^\circ + (x+20)^\circ = 180^\circ$$

$$\Rightarrow 3x + 30^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 30^\circ$$

$$\Rightarrow 3x = 150^\circ$$

$$\Rightarrow x = 50^\circ$$

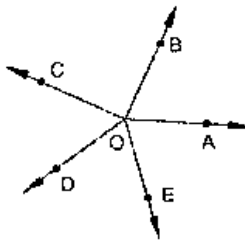
$$\therefore \angle AOD = x + 10^\circ$$

$$= 50^\circ + 10^\circ = 60^\circ$$

$$\angle COD = x^\circ = 50^\circ$$

$$\angle BOC = x + 20^\circ = 50 + 20 = 70^\circ$$

4. In the given below fig, rays OA, OB, OC, OP and OE have the common end point O. Show that $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360^\circ$.



Sol:

Given that

Rays OA, OB, OD and OE have the common end point O.

A ray of opposite to OA is drawn

Since $\angle AOB, \angle BOF$ are linear pairs

$$\angle AOB + \angle BOF = 180^\circ$$

$$\angle AOB + \angle BOC + \angle COF = 180^\circ \quad \dots\dots\dots(1)$$

Also

$\angle AOE, \angle EOF$ are linear pairs

$$\angle AOE + \angle EOF = 180^\circ$$

$$\angle AOE + \angle DOF + \angle DOE = 180^\circ \quad \dots\dots\dots(2)$$

By adding (1) and (2) equations we get

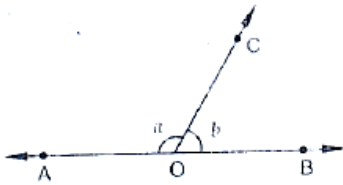
$$\angle AOB + \angle BOC + \angle COF + \angle AOE + \angle DOF + \angle DOE = 360^\circ$$

$$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360^\circ$$

Hence proved.



5. In the below Fig, $\angle AOC$ and $\angle BOC$ form a linear pair. if $a - 2b = 30^\circ$, find a and b .



Sol:

Given that,

$\angle AOC$ and $\angle BOC$ form a linear pair

If $a - 2b = 30^\circ$

$\angle AOC = a^\circ, \angle BOC = b^\circ$

$$\therefore a + b = 180^\circ \quad \dots\dots\dots(i)$$

$$\text{Given } a - 2b = 30^\circ \quad \dots\dots\dots(ii)$$

By subtracting (i) and (ii)

$$a + b - a + 2b = 180^\circ - 30^\circ$$

$$\Rightarrow 3b = 150^\circ$$

$$\Rightarrow b = \frac{150^\circ}{3}$$

$$\Rightarrow b = 50^\circ$$

Hence $a - 2b = 30^\circ$

$$a - 2(50)^\circ = 30^\circ \quad [\because b = 50^\circ]$$

$$a = 30^\circ + 100^\circ$$

$$a = 130^\circ$$

$$\therefore a = 130^\circ, b = 50^\circ.$$

6. How many pairs of adjacent angles are formed when two lines intersect in a point?

Sol:

Four pairs of adjacent angle formed when two lines intersect in a point they are

$\angle AOD, \angle DOB$

$\angle DOB, \angle BOC$

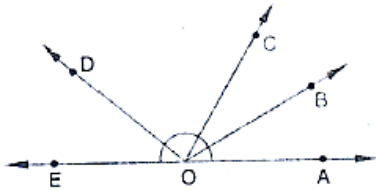
$\angle COA, \angle AOD$

$\angle BOC, \angle COA$

Hence 4 pairs



7. How many pairs of adjacent angles, in all, can you name in below fig.?



Sol:

Pairs of adjacent angles are

$\angle EOC, \angle DOC$

$\angle EOD, \angle DOB$

$\angle DOC, \angle COB$

$\angle EOD, \angle DOA$

$\angle DOC, \angle COA$

$\angle BOC, \angle BOA$

$\angle BOA, \angle BOD$

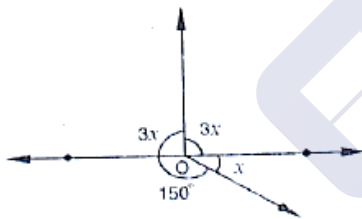
$\angle BOA, \angle BOE$

$\angle EOC, \angle COA$

$\angle EOC, \angle COB$

\therefore Hence 10 pairs of adjacent angles

8. In below fig, determine the value of x .



Sol:

Since sum of all the angles round a point is equal to 360° . Therefore

$$\Rightarrow 3x + 3x + 150 + x = 360^\circ$$

$$\Rightarrow 7x^\circ = 360^\circ - 150^\circ$$

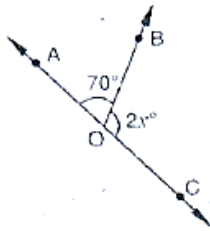
$$\Rightarrow 7x = 210^\circ$$

$$\Rightarrow x = \frac{210}{7}$$

$$\Rightarrow x = 30^\circ$$



9. In the below fig, AOC is a line, find x.



Sol:

Since $\angle AOB$ and $\angle BOC$ are linear pairs

$$\angle AOB + \angle BOC = 180^\circ$$

$$\Rightarrow 70^\circ + 2x^\circ = 180^\circ$$

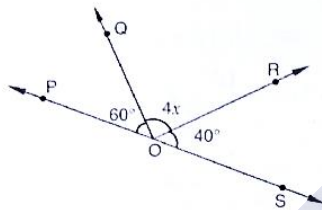
$$\Rightarrow 2x^\circ = 180^\circ - 70^\circ$$

$$\Rightarrow 2x = 110^\circ$$

$$\Rightarrow x = \frac{110}{2}$$

$$\Rightarrow x = 55^\circ$$

10. In the below fig, POS is a line, find x.



Sol:

Since $\angle POQ$ and $\angle QOS$ are linear pairs

$$\angle POQ + \angle QOS = 180^\circ$$

$$\Rightarrow \angle POQ + \angle QOR + \angle ROS = 180^\circ$$

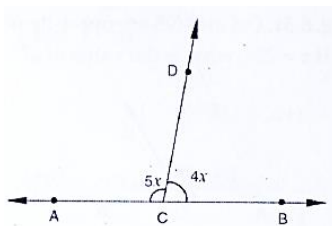
$$\Rightarrow 60^\circ + 4x^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow 4x^\circ = 180^\circ - 100^\circ$$

$$\Rightarrow 4x^\circ = 80^\circ$$

$$\Rightarrow \boxed{x = 20^\circ}$$

11. In the below fig, ACB is a line such that $\angle DCA = 5x$ and $\angle DCB = 4x$. Find the value of x.



**Sol:**Here, $\angle ACD + \angle BCD = 180^\circ$ [Since $\angle ACD, \angle BCD$ are linear pairs]

$$\angle ACD = 5x, \angle BCD = 4x$$

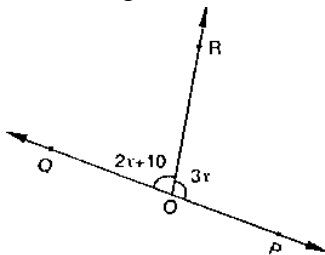
$$\Rightarrow 5x + 4x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = 20^\circ$$

$$\therefore x = 20^\circ$$

12. Given $\angle POR = 3x$ and $\angle QOR = 2x + 10$, find the value of x for which POQ will be a line. (Below fig).

**Sol:**Since $\angle QOR, \angle POR$ are linear pairs

$$\angle QOR + \angle POR = 180^\circ$$

$$\Rightarrow 2x + 10 + 3x = 180^\circ \quad [\because \angle QOR = 2x + 10, \angle POR = 3x]$$

$$\Rightarrow 5x + 10 = 180^\circ$$

$$\Rightarrow 5x = 180^\circ - 10$$

$$\Rightarrow 5x = 170^\circ$$

$$\Rightarrow x = 34^\circ$$

13. In Fig. 8.42, a is greater than b by one third of a right-angle. Find the values of a and b .

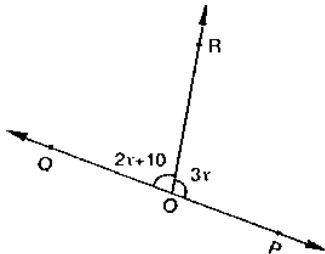


Fig. 8.41

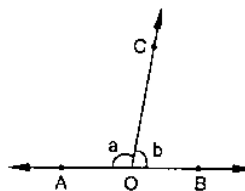


Fig. 8.42

Sol:Since a, b are linear pair

$$\Rightarrow a + b = 180^\circ$$

$$\Rightarrow a = 180 - b \quad \dots\dots(1)$$



Now,

$$\Rightarrow a = b + \frac{1}{3} \times 90^\circ \quad [\text{given}]$$

$$\Rightarrow a = b + 30^\circ \quad \dots\dots(2)$$

$$\Rightarrow a - b = 30^\circ$$

Equating (1) and (2) equations

$$180 - b = b + 30^\circ$$

$$\Rightarrow 180^\circ - 30^\circ = b + b$$

$$\Rightarrow 150^\circ = 2b$$

$$\Rightarrow b = \frac{150^\circ}{2}$$

$$\Rightarrow b = 75^\circ$$

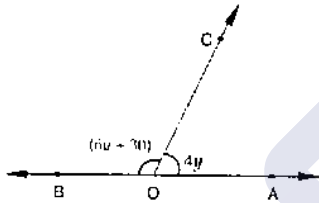
$$\text{Hence } a = 180 - b$$

$$= 180 - 75^\circ \quad [\because b = 75^\circ]$$

$$a = 105^\circ$$

$$\therefore a = 105^\circ, b = 75^\circ$$

14. What value of y would make AOB a line in below fig, if $\angle AOC = 4y$ and $\angle BOC = (6y + 30)$



Sol:

Since $\angle AOC, \angle BOC$ are linear pair

$$\Rightarrow \angle AOC + \angle BOC = 180^\circ$$

$$\Rightarrow 6y + 30 + 4y = 180^\circ$$

$$\Rightarrow 10y + 30 = 180^\circ$$

$$\Rightarrow 10y = 180^\circ - 30^\circ$$

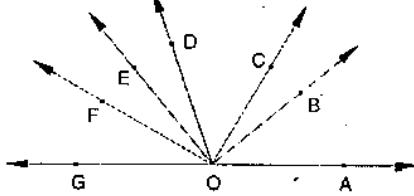
$$\Rightarrow 10y = 150^\circ$$

$$\Rightarrow y = \frac{150^\circ}{10}$$

$$\Rightarrow y = 15^\circ$$



15. If below fig, $\angle AOF$ and $\angle FOG$ form a linear pair.



$$\angle EOB = \angle FOC = 90^\circ \text{ and } \angle DOC = \angle FOG = \angle AOB = 30^\circ$$

- Find the measures of $\angle FOE$, $\angle COB$ and $\angle DOE$.
- Name all the right angles.
- Name three pairs of adjacent complementary angles.
- Name three pairs of adjacent supplementary angles.
- Name three pairs of adjacent angles.

Sol:

- $\angle FOE = x$, $\angle DOE = y$ and $\angle BOC = z$ sat

Since $\angle AOF$, $\angle FOG$ is Linear pair

$$\Rightarrow \angle AOF + 30^\circ = 180^\circ \quad [\angle AOF + \angle FOG = 180^\circ \text{ and } \angle FOG = 30^\circ]$$

$$\Rightarrow \angle AOF = 180^\circ - 30^\circ$$

$$\Rightarrow \angle AOF = 150^\circ$$

$$\Rightarrow \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOF = 150^\circ$$

$$\Rightarrow 30^\circ + z + 30^\circ + y + x = 150^\circ$$

$$\Rightarrow x + y + z = 150^\circ - 30^\circ - 30^\circ$$

$$\Rightarrow x + y + z = 90^\circ \quad \dots\dots(1)$$

Now $\angle FOC = 90^\circ$

$$\Rightarrow \angle FOE + \angle EOD + \angle DOC = 90^\circ$$

$$\Rightarrow x + y + 30^\circ = 90^\circ$$

$$\Rightarrow x + y = 90^\circ - 30^\circ$$

$$\Rightarrow x + y = 60^\circ \quad \dots\dots(2)$$

Substituting (2) in (1)

$$x + y + z = 90^\circ$$

$$\Rightarrow 60 + z = 90^\circ \Rightarrow z = 90^\circ - 60^\circ = 30^\circ$$

$$\text{i.e., } \angle BOC = 30^\circ$$

Given $\angle BOE = 90^\circ$

$$\Rightarrow \angle BOC + \angle COD + \angle DOE = 90^\circ$$

$$\Rightarrow 30^\circ + 30^\circ + \angle DOE = 90^\circ$$

$$\Rightarrow \angle DOE = 90^\circ - 60^\circ = 30^\circ$$

$$\therefore \angle DOE = x = 30^\circ$$

Now, also we have



$$x + y = 60^\circ$$

$$\Rightarrow y = 60^\circ - x = 60^\circ - 30^\circ = 30^\circ$$

$$\angle FOE = 30^\circ$$

(ii) Right angles are

$$\angle DOG, \angle COF, \angle BOF, \angle AOD$$

(iii) Three pairs of adjacent complementary angles are

$$\angle AOB, \angle BOD;$$

$$\angle AOC, \angle COD;$$

$$\angle BOC, \angle COE.$$

(iv) Three pairs of adjacent supplementary angles are

$$\angle AOB, \angle BOG;$$

$$\angle AOC, \angle COG;$$

$$\angle AOD, \angle DOG.$$

(v) Three pairs of adjacent angles

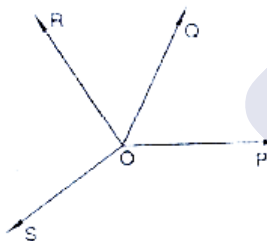
$$\angle BOC, \angle COD;$$

$$\angle COD, \angle DOE;$$

$$\angle DOE, \angle EOF,$$

16. In below fig, OP, OQ, OR and OS are four rays. Prove that:

$$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$$



Sol:

Given that

OP, OQ, OR and OS are four rays

You need to produce any of the ray OP, OQ, OR and OS backwards to a point in the figure.

Let us produce ray OQ backwards to a point

T so that TOQ is a line

Ray OP stands on the TOQ

Since $\angle TOP, \angle POQ$ is linear pair

$$\angle TOP + \angle POQ = 180^\circ \quad \dots\dots(1)$$

Similarly, ray OS stands on the line TOQ

$$\therefore \angle TOS + \angle SOQ = 180^\circ \quad \dots\dots(2)$$



$$\text{But } \angle SOQ = \angle SOR + \angle QOR$$

So, (2), becomes

$$\angle TOS + \angle SOR + \angle QOR = 180^\circ$$

Now, adding (1) and (3) you get

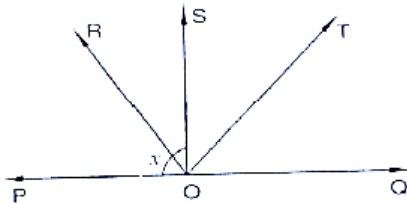
$$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360^\circ$$

$$\Rightarrow \angle TOP + \angle TOS = \angle POS$$

\therefore (4) becomes

$$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$$

17. In below fig, ray OS stand on a line POQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$ respectively. If $\angle POS = x$, find $\angle ROT$.



Sol:

Given,

Ray OS stand on a line POQ

Ray OR and Ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$ respectively

$$\angle POS = x$$

$\angle POS$ and $\angle QOS$ is linear pair

$$\angle POS + \angle QOS = 180^\circ$$

$$x + \angle QOS = 180^\circ$$

$$\angle QOS = 180 - x$$

Now, ray or bisector $\angle POS$

$$\therefore \angle ROS = \frac{1}{2} \angle POS$$

$$= \frac{1}{2} \times x \quad [\because \angle POS = x]$$

$$\angle ROS = \frac{x}{2}$$

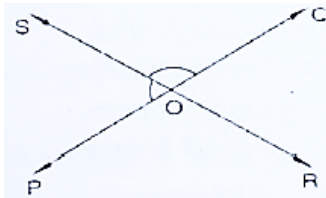
Similarly ray OT bisector $\angle QOS$

$$\therefore \angle TOS = \frac{1}{2} \angle QOS$$



$$\begin{aligned}
 &= \frac{180-x}{2} && [\because \angle QOS = 180-x] \\
 &= 90 - \frac{x}{2} \\
 \therefore \angle ROT &= \angle ROS + \angle ROT \\
 &= \frac{x}{2} + 90 - \frac{x}{2} \\
 &= 90^\circ \\
 \therefore \angle ROT &= 90^\circ
 \end{aligned}$$

18. In the below fig, lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5 : 7$, find all the angles.



Sol:

Given $\angle POR$ and $\angle ROP$ is linear pair

$$\angle POR + \angle ROP = 180^\circ$$

Given that

$$\angle POR : \angle ROP = 5 : 7$$

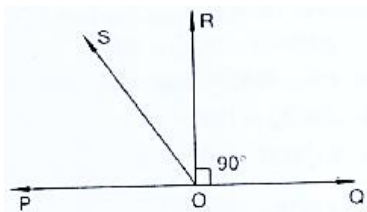
$$\therefore \angle POR = \frac{5}{12} \times 180 = 75^\circ$$

$$\text{Similarly } \angle ROQ = \frac{7}{5+7} \times 180^\circ = 105^\circ$$

$$\text{Now, } \angle POS = \angle ROQ = 105^\circ \quad [\because \text{Vertically opposite angles}]$$

$$\therefore \angle SOQ = \angle POR = 75^\circ \quad [\because \text{Vertically opposite angles}]$$

19. In the below fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Sol:

Given that, OR perpendicular

$$\therefore \angle POR = 90^\circ$$



$$\angle POS + \angle SOR = 90^\circ \quad [\because \angle POR = \angle POS + \angle SOR]$$

$$\angle ROS = 90^\circ - \angle POS \quad \dots\dots\dots(1)$$

$$\angle QOR = 90^\circ \quad (\because OR \perp PQ)$$

$$\angle QOS - \angle ROS = 90^\circ$$

$$\angle ROS = \angle QOS - 90^\circ \quad \dots\dots\dots(2)$$

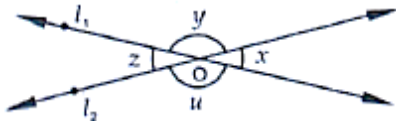
By adding (1) and (2) equations, we get

$$2\angle ROS = \angle QOS - \angle POS$$

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Exercise – 8.3

1. In the below fig, lines l_1 and l_2 intersect at O, forming angles as shown in the figure. If $x = 45$, Find the values of x , y , z and u .



Sol:

Given that

$$x = 45^\circ, y = ?, z = ?, u = ?$$

Vertically opposite sides are equal

$$\therefore z = x = 45^\circ$$

z and u angles are linear pair of angles

$$\therefore z + u = 180^\circ$$

$$z = 180^\circ - 4$$

$$\Rightarrow u = 180^\circ - x$$

$$\Rightarrow u = 180^\circ - 45^\circ \quad [\because x = 45^\circ]$$

$$\Rightarrow u = 135^\circ$$

x and y angles are linear pair of angles

$$\therefore x + y = 180^\circ$$

$$y = 180^\circ - x$$

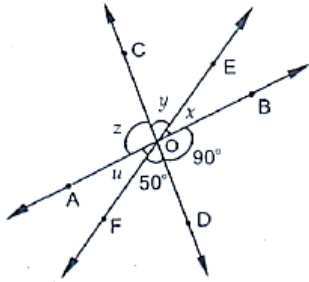
$$y = 180^\circ - 45^\circ$$

$$y = 135^\circ$$

$$\therefore x = 45^\circ, y = 135^\circ, z = 45^\circ \text{ and } u = 135^\circ$$



2. In the below fig, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x , y , z and u .



Sol:

Vertically opposite angles are equal

$$\text{So } \angle BOD = z = 90^\circ$$

$$\angle DOF = y = 50^\circ$$

$$\text{Now, } x + y + z = 180^\circ \quad [\text{Linear pair}]$$

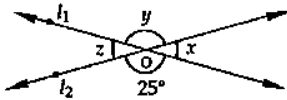
$$\Rightarrow x + y + z = 180^\circ$$

$$\Rightarrow 90^\circ + 50^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 140^\circ$$

$$\Rightarrow x = 40^\circ$$

3. In the given fig, find the values of x , y and z .



Sol:

From the given figure

$$\angle y = 25^\circ \quad [\because \text{Vertically opposite angles are equal}]$$

Now

$$\angle x + \angle y = 180^\circ \quad [\text{Linear pair of angles are } x \text{ and } y]$$

$$\Rightarrow \angle x = 180^\circ - 25^\circ$$

$$\Rightarrow \angle x = 155^\circ$$

Also

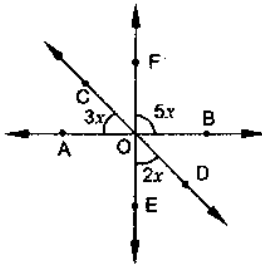
$$\angle z = \angle x = 155^\circ \quad [\text{Vertically opposite angle}]$$

$$\angle y = 25^\circ$$

$$\angle z = \angle x = 155^\circ$$



4. In the below fig, find the value of x .



Sol:

Vertically opposite angles are equal

$$\angle AOE = \angle BOF = 5x$$

Linear pair

$$\angle COA + \angle AOE + \angle EOD = 180^\circ$$

$$\Rightarrow 3x + 5x + 2x = 180^\circ$$

$$\Rightarrow 10x = 180^\circ$$

$$\Rightarrow x = 18^\circ$$

5. Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Sol:

Given,

Lines AOB and COD intersect at point O such that

$$\angle AOC = \angle BOD$$

Also OE is the bisector $\angle ADC$ and OF is the bisector $\angle BOD$

To prove: EOF is a straight line vertically opposite angles is equal

$$\angle AOD = \angle BOC = 5x \quad \dots\dots\dots(1)$$

Also $\angle AOC + \angle BOD$

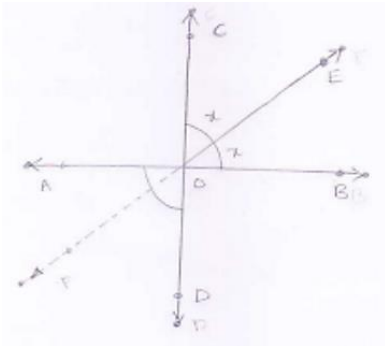
$$\Rightarrow 2\angle AOE = 2\angle DOF \quad \dots\dots\dots(2)$$

Sum of the angles around a point is 360°

$$\Rightarrow 2\angle AOD + 2\angle AOE + 2\angle DOF = 360^\circ$$

$$\Rightarrow \angle AOD + \angle AOF + \angle DOF = 180^\circ$$

From this we conclude that EOF is a straight line.





Given that :- AB and CD intersect each other at O

OE bisects $\angle COB$

To prove: $\angle AOF = \angle DOF$

Proof: OE bisects $\angle COB$

$$\angle COE = \angle EOB = x$$

Vertically opposite angles are equal

$$\angle BOE = \angle AOF = x \quad \dots\dots(1)$$

$$\angle COE = \angle DOF = x \quad \dots\dots(2)$$

From (1) and (2)

$$\angle AOF = \angle DOF = x$$

6. If one of the four angles formed by two intersecting lines is a right angle, then show that each of the four angles is a right angle.

Sol:

Given,

AB and CD are two lines intersecting at O such that

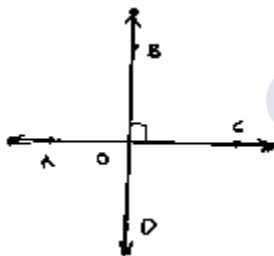
$$\angle BOC = 90^\circ$$

$$\angle AOC = 90^\circ, \angle AOD = 90^\circ \text{ and } \angle BOD = 90^\circ$$

Proof:

Given that $\angle BOC = 90^\circ$

Vertically opposite angles are equal



$$\angle BOC = \angle AOD = 90^\circ$$

$\angle AOC, \angle BOC$ are Linear pair of angles

$$\angle AOC + \angle BOC = 180^\circ \quad [\text{LinearPair}]$$

$$\Rightarrow \angle AOC + 90^\circ = 180^\circ$$

$$\Rightarrow \angle AOC = 90^\circ$$

Vertically opposite angles

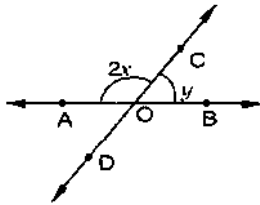
$$\therefore \angle AOC = \angle BOD = 90^\circ$$

Hence, $\angle AOC = \angle BOC = \angle BOD = \angle AOD = 90^\circ$



7. In the below fig, rays AB and CD intersect at O.

- (i) Determine y when $x = 60^\circ$
 (ii) Determine x when $y = 40^\circ$



Sol:

- (i) Given $x = 60^\circ$

$$y = ?$$

$\angle AOC, \angle BOC$ are linear pair of angles

$$\angle AOC + \angle BOC = 180^\circ$$

$$\Rightarrow 2x + y = 180^\circ$$

$$\Rightarrow 2 \times 60 + y = 180^\circ \quad [\because x = 60^\circ]$$

$$\Rightarrow y = 180^\circ - 120^\circ$$

$$\Rightarrow \boxed{y = 60^\circ}$$

- (ii) Given $y = 40^\circ, x = ?$

$\angle AOC$ and $\angle BOC$ are linear pair of angles

$$\angle AOC + \angle BOC = 180^\circ$$

$$\Rightarrow 2x + y = 180^\circ$$

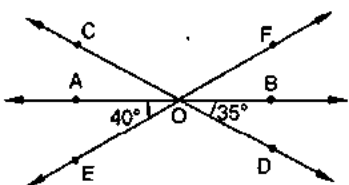
$$\Rightarrow 2x + 40 = 180^\circ$$

$$\Rightarrow 2x = 140^\circ$$

$$\Rightarrow x = \frac{140^\circ}{2}$$

$$\Rightarrow \boxed{y = 70^\circ}$$

8. In the below fig, lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle COF$, $\angle DOE$ and $\angle BOF$.



Sol:

$\angle AOE$ and $\angle EOB$ are linear pair of angles



$$\angle AOE + \angle EOB = 180^\circ$$

$$\angle AOE + \angle DOE + \angle BOD = 180^\circ$$

$$\Rightarrow \angle DOE = 180^\circ - 40^\circ - 35^\circ = 105^\circ$$

Vertically opposite side angles are equal

$$\angle DOE = \angle COF = 105^\circ$$

$$\text{Now, } \angle AOE + \angle AOF = 180^\circ \quad [\because \text{Linear pair}]$$

$$\Rightarrow \angle AOE + \angle AOC + \angle COF = 180^\circ$$

$$\Rightarrow 40^\circ + \angle AOC + 105^\circ = 180^\circ$$

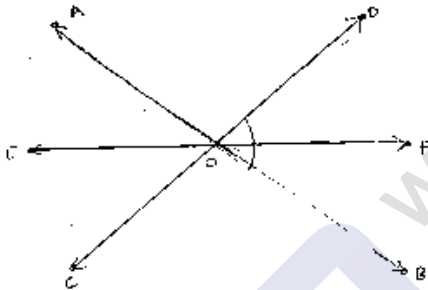
$$\Rightarrow \angle AOC = 180^\circ - 145^\circ$$

$$\Rightarrow \angle AOC = 35^\circ$$

$$\text{Also, } \angle BOF = \angle AOE = 40^\circ \quad [\because \text{Vertically opposite angle are equal}]$$

9. AB, CD and EF are three concurrent lines passing through the point O such that OF bisects $\angle BOD$. If $\angle BOF = 35^\circ$, find $\angle BOC$ and $\angle AOD$.

Sol:



Given

OF bisects $\angle BOD$

OF bisects $\angle BOD$

$$\angle BOF = 35^\circ$$

$$\angle BOC = ?$$

$$\angle AOD = ?$$

$$\therefore \angle BOD = 2\angle BOF = 70^\circ$$

$[\because \text{OF bisects } \angle BOD]$

$$\angle BOD = \angle AOC = 70^\circ$$

$[\angle BOD \text{ and } \angle AOC \text{ are vertically opposite angles}]$

Now,

$$\angle BOC + \angle AOC = 180^\circ$$

$$\Rightarrow \angle BOC + 70^\circ = 180^\circ$$

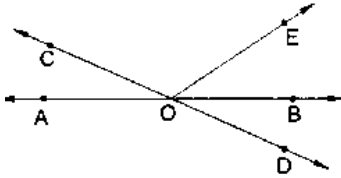
$$\Rightarrow \angle BOC = 110^\circ$$

$$\therefore \angle AOD = \angle BOC = 110^\circ$$

$[\text{Vertically opposite angles}]$



10. In below figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.



Sol:

Given that

$$\angle AOC + \angle BOE = 70^\circ \text{ and } \angle BOD = 40^\circ$$

$$\angle BOE = ?$$

Here, $\angle BOD$ and $\angle AOC$ are vertically opposite angles

$$\angle BOD = \angle AOC = 40^\circ$$

$$\text{Given } \angle AOC + \angle BOE = 70^\circ$$

$$40^\circ + \angle BOE = 70^\circ$$

$$\angle BOE = 70^\circ - 40^\circ$$

$$\angle BOE = 30^\circ$$

$\angle AOC$ and $\angle BOC$ are linear pair of angles

$$\Rightarrow \angle AOC + \angle COE + \angle BOE = 180^\circ$$

$$\Rightarrow \angle COE = 180^\circ - 30^\circ - 40^\circ$$

$$\Rightarrow \angle COE = 110^\circ$$

$$\therefore \text{Reflex } \angle COE = 360^\circ - 110^\circ = 250^\circ.$$

11. Which of the following statements are true (T) and which are false (F)?
- Angles forming a linear pair are supplementary.
 - If two adjacent angles are equal, and then each angle measures 90° .
 - Angles forming a linear pair can both be acute angles.
 - If angles forming a linear pair are equal, then each of these angles is of measure 90° .

Sol:

- True
- False
- False
- true

12. Fill in the blanks so as to make the following statements true:

- If one angle of a linear pair is acute, then its other angle will be _____
- A ray stands on a line, then the sum of the two adjacent angles so formed is _____
- If the sum of two adjacent angles is 180° , then the _____ arms of the two angles are opposite rays.

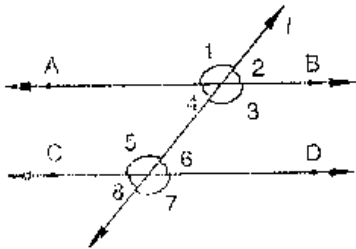
Sol:

- Obtuse angle
- 180°
- uncommon



Exercise – 8.4

1. In below fig, AB CD and $\angle 1$ and $\angle 2$ are in the ratio 3 : 2. Determine all angles from 1 to 8.

**Sol:**Let $\angle 1 = 3x$ and $\angle 2 = 2x$ $\angle 1$ and $\angle 2$ are linear pair of angleNow, $\angle 1 + \angle 2 = 180^\circ$

$$\Rightarrow 3x + 2x = 180^\circ$$

$$\Rightarrow 5x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{5}$$

$$\Rightarrow x = 36^\circ$$

$$\therefore \angle 1 = 3x = 108^\circ, \angle 2 = 2x = 72^\circ$$

Vertically opposite angles are equal

$$\angle 1 = \angle 3 = 108^\circ$$

$$\angle 2 = \angle 4 = 72^\circ$$

$$\angle 6 = \angle 7 = 108^\circ$$

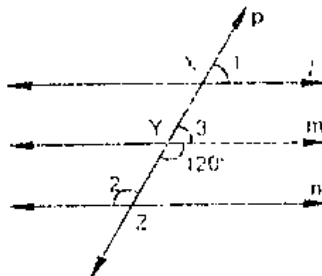
$$\angle 5 = \angle 8 = 72^\circ$$

Corresponding angles

$$\angle 1 = \angle 5 = 108^\circ$$

$$\angle 2 = \angle 6 = 72^\circ$$

2. In the below fig, l , m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$.

**Sol:**

From the given figure:



$$\angle 3 + \angle m YZ = 180^\circ \quad [\text{Linear pair}]$$

$$\Rightarrow \angle 3 = 180^\circ - 120^\circ$$

$$\Rightarrow \angle 3 = 60^\circ$$

Now line l parallel to m

$$\angle 1 = \angle 3 \quad [\text{Corresponding angles}]$$

$$\Rightarrow \angle 1 = 60^\circ$$

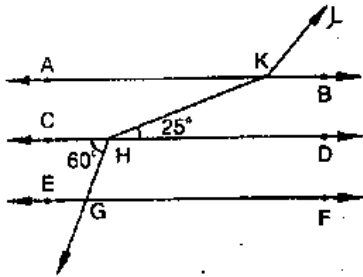
Also $m \parallel n$

$$\Rightarrow \angle 2 = 120^\circ \quad [\text{Alternative interior angle}]$$

$$\therefore \angle 1 = \angle 3 = 60^\circ$$

$$\angle 2 = 120^\circ$$

3. In the below fig, $AB \parallel CD \parallel EF$ and $GH \parallel KL$. Find $\angle HKL$



Sol:

Produce LK to meet GF at N .

Now, alternative angles are equal

$$\angle CHG = \angle HGN = 60^\circ$$

$$\angle HGN = \angle KNF = 60^\circ \quad [\text{Corresponding angles}]$$

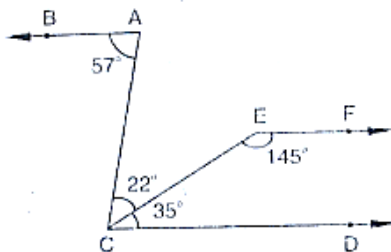
$$\therefore \angle KNG = 180^\circ - 60^\circ = 120^\circ$$

$$\angle GKN = \angle AKL = 120^\circ \quad [\text{Corresponding angles}]$$

$$\angle AKH = \angle KHD = 25^\circ \quad [\text{Alternative angles}]$$

$$\therefore \angle HKL = \angle AKH + \angle AKL = 25^\circ + 120^\circ = 145^\circ.$$

4. In the below fig, show that $AB \parallel EF$.



Sol:

Produce EF to intersect AC at K .

$$\text{Now, } \angle DCE + \angle CEF = 35^\circ + 145^\circ = 180^\circ$$

$$\therefore EF \parallel CD \quad [\because \text{Sum of Co-interior angles is } 180^\circ] \quad \dots\dots(1)$$



Now, $\angle BAC = \angle ACD = 57^\circ$

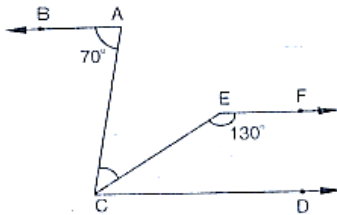
$\Rightarrow BA \parallel CD$ [\because Alternative angles are equal](2)

From (1) and (2)

$AB \parallel EF$ [Lines parallel to the same line are parallel to each other]

Hence proved.

5. If below fig, if $AB \parallel CD$ and $CD \parallel EF$, find $\angle ACE$.



Sol:

Since $EF \parallel CD$

$\therefore \angle EFC + \angle ECD = 180^\circ$ [co-interior angles are supplementary]

$\Rightarrow \angle ECD = 180^\circ - 130^\circ = 50^\circ$

Also $BA \parallel CD$

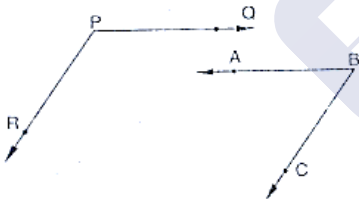
$\Rightarrow \angle BAC = \angle ACD = 70^\circ$ [alternative angles]

But

$\angle ACE + \angle ECD = 70^\circ$

$\Rightarrow \angle ACE = 70^\circ - 50^\circ = 20^\circ$

6. In the below fig, $PQ \parallel AB$ and $PR \parallel BC$. If $\angle QPR = 102^\circ$, determine $\angle ABC$. Give reasons.



Sol:

AB is produce to meet PR at K

Since $PQ \parallel AB$

$\angle QPR = \angle BKR = 102^\circ$ [corresponding angles]

Since $PR \parallel BC$

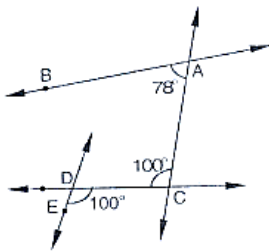
$\therefore \angle RKB + \angle CBK = 180^\circ$ [\because Corresponding angles are supplementary]

$\Rightarrow \angle CKB = 180 - 102 = 78^\circ$

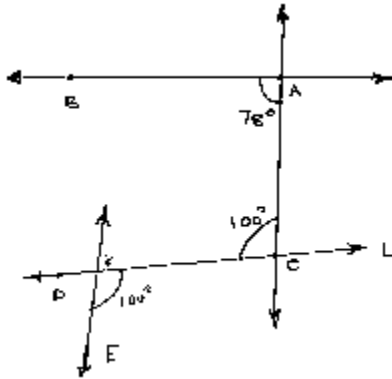
$\therefore \angle CKB = 78^\circ$.



7. In the below fig, state which lines are parallel and why?



Sol:



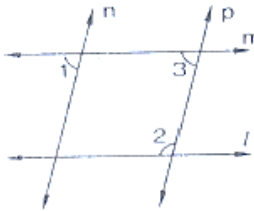
Vertically opposite angles are equal

$$\angle EOC = \angle DOK = 100^\circ$$

$$\text{Angle } \angle DOK = \angle ACO = 100^\circ$$

Here two lines EK and CA cut by a third line 'l' and the corresponding angles to it are equal $\therefore EK \parallel AC$.

8. In the below fig, if $l \parallel m$, $n \parallel p$ and $\angle 1 = 85^\circ$, find $\angle 2$.



Sol:

Corresponding angles are equal

$$\angle 1 = \angle 3 = 85^\circ$$

By using the property of co-interior angles are supplementary

$$\angle 2 + \angle 3 = 180^\circ$$

$$\angle 2 + 85^\circ = 180^\circ$$

$$\angle 2 = 180^\circ - 85^\circ$$

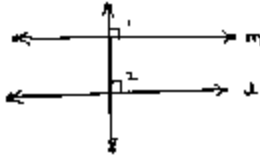
$$\angle 2 = 95^\circ$$

$$\therefore \angle 2 = 95^\circ$$



9. If two straight lines are perpendicular to the same line, prove that they are parallel to each other.

Sol:



Given m perpendicular t and $l \perp t$

$$\angle 1 = \angle 2 = 90^\circ$$

$\therefore l$ and m are two lines and t is transversal and the corresponding angles are equal

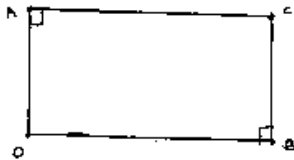
$$\therefore l \parallel m$$

Hence proved

10. Prove that if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

Sol:

Consider be angles AOB and ACB



Given $OA \perp AC, OB \perp BC$

To prove: $\angle AOB = \angle ACB$ (or)

$$\angle AOB + \angle ACB = 180^\circ$$

Proof:- In a quadrilateral

[Sum of angles of quadrilateral]

$$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360^\circ$$

$$\Rightarrow 180 + \angle O + \angle C = 360^\circ$$

$$\Rightarrow \angle O + \angle C = 360 - 180 = 180^\circ$$

$$\text{Hence, } \angle AOB + \angle ACB = 180^\circ \quad \dots (i)$$

Also,

$$\angle B + \angle ACB = 180^\circ \quad \dots (i)$$

Also,

$$\angle B + \angle ACB = 180^\circ \quad \dots (i)$$

Also,

$$\angle B + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - 90^\circ$$

$$\Rightarrow \angle ACB = 90^\circ \quad \dots (ii)$$

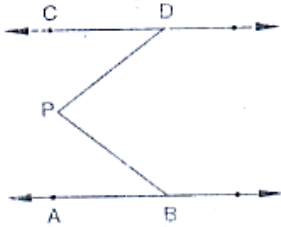


From (i) and (ii)

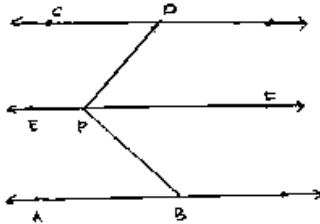
$$\therefore \angle ACB = \angle AOB = 90^\circ$$

Hence, the angles are equal as well as supplementary.

11. In the below fig, lines AB and CD are parallel and P is any point as shown in the figure. Show that $\angle ABP + \angle CDP = \angle DPB$.



Sol:



Given that $AB \parallel CD$

Let EF be the parallel line to AB and CD which passes through P .

It can be seen from the figure

Alternative angles are equal

$$\angle ABP = \angle BPF$$

Alternative angles are equal

$$\angle CDP = \angle DPF$$

$$\Rightarrow \angle ABP + \angle CDP = \angle BPF + \angle DPF$$

$$\Rightarrow \angle ABP + \angle CDP = \angle DPB$$

Hence proved

AB parallel to CD , P is any point

To prove: $\angle ABP + \angle BPD + \angle CDP = 360^\circ$

Construction: Draw $EF \parallel AB$ passing through P

Proof:

Since $AB \parallel EF$ and $AB \parallel CD$

$$\therefore EF \parallel CD \quad [\text{Lines parallel to the same line are parallel to each other}]$$

$$\angle ABP + \angle EPB = 180^\circ \quad [\text{Sum of co-interior angles is } 180^\circ \text{ } AB \parallel EF \text{ and } BP \text{ is the transversal}]$$

$$\angle EPD + \angle COP = 180^\circ$$

$$[\text{Sum of co-interior angles is } 180^\circ \text{ } EF \parallel CD \text{ and } DP \text{ is transversal}] \quad \dots\dots(1)$$

$$\angle EPD + \angle CDP = 180^\circ$$



[Sum of Co-interior angles is 180° $EF \parallel CD$ and DP is the transversal]

...(2)

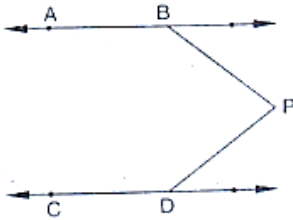
By adding (1) and (2)

$$\angle ABP + \angle EPB + \angle EPD + \angle CDP = 180^\circ + 180^\circ$$

$$\angle ABP + \angle EPB + \angle COP = 360^\circ$$

12. In the below fig, $AB \parallel CD$ and P is any point shown in the figure. Prove that:

$$\angle ABP + \angle BPD + \angle CDP = 360^\circ$$



Sol:

Through P , draw a line PM parallel to AB or CD .

Now,

$$AB \parallel PM \Rightarrow \angle ABP + \angle BPM = 180^\circ$$

And

$$CD \parallel PM \Rightarrow \angle MPD + \angle CDP = 180^\circ$$

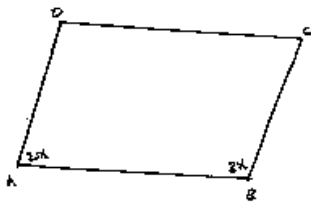
Adding (i) and (ii), we get

$$\angle ABP + (\angle BPM + \angle MPD) + \angle CDP = 360^\circ$$

$$\Rightarrow \angle ABP + \angle BPD + \angle CDP = 360^\circ$$

13. Two unequal angles of a parallelogram are in the ratio 2 : 3. Find all its angles in degrees.

Sol:



Let $\angle A = 2x$ and $\angle B = 3x$

Now,

$$\angle A + \angle B = 180^\circ$$

$$2x + 3x = 180^\circ$$

$$\Rightarrow 5x = 180^\circ$$

[Co-interior angles are supplementary]

[$AD \parallel BC$ and AB is the transversal]



$$\Rightarrow x = \frac{180^\circ}{5} = 36^\circ$$

$$\therefore \angle A = 2 \times 36^\circ = 72^\circ$$

$$\angle B = 3 \times 36^\circ = 108^\circ$$

Now,

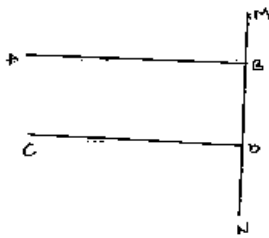
$$\angle A = \angle C = 72^\circ$$

[Opposite side angles of a parallelogram are equal]

$$\angle B = \angle D = 108^\circ$$

14. If each of the two lines is perpendicular to the same line, what kind of lines are they to each other?

Sol:



Let AB and CD be perpendicular to MN

$$\angle ABD = 90^\circ \quad [AB \perp MN] \quad \dots (i)$$

$$\angle CDN = 90^\circ \quad [CD \perp MN] \quad \dots (ii)$$

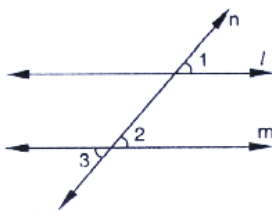
Now,

$$\angle ABD = \angle CDN = 90^\circ \quad [\text{From (i) and (ii)}]$$

$\therefore AB$ parallel to CD ,

Since corresponding angle are equal

15. In the below fig, $\angle 1 = 60^\circ$ and $\angle 2 = \left(\frac{2}{3}\right)^{\text{rd}}$ of a right angle. Prove that $l \parallel m$.



Sol:

Given:

$$\angle 1 = 60^\circ, \angle 2 = \left(\frac{2}{3}\right)^{\text{rd}} \text{ to right angle}$$

To prove: parallel to m

Proof $\angle 1 = 60^\circ$

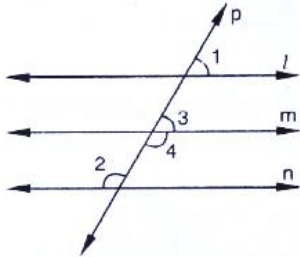


$$\angle 2 = \frac{2}{3} \times 90^\circ = 60^\circ$$

Since, $\angle 1 = \angle 2 = 60^\circ$

\therefore Parallel to m as pair of corresponding angles are equal

16. In the below fig, if $l \parallel m \parallel n$ and $\angle 1 = 60^\circ$, find $\angle 2$.



Sol:

Since l parallel to m and p is the transversal

\therefore Given: $l \parallel m \parallel n$, $\angle 1 = 60^\circ$

To find $\angle 2$

$\angle 1 = \angle 3 = 60^\circ$ [Corresponding angles]

Now,

$\angle 3$ and $\angle 4$ are linear pair of angles

$$\angle 3 + \angle 4 = 180^\circ$$

$$60^\circ + \angle 4 = 180^\circ$$

$$\angle 4 = 180^\circ - 60^\circ$$

$$\angle 4 = 120^\circ$$

Also, $m \parallel n$ and P is the transversal

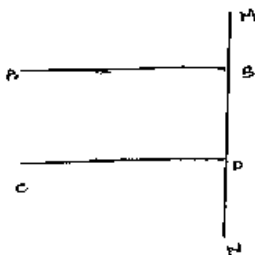
$\therefore \angle 4 = \angle 2 = 120^\circ$ [Alternative interior angle]

Hence $\angle 2 = 120^\circ$

17. Prove that the straight lines perpendicular to the same straight line are parallel to one another.

Sol:

Let AB and CD perpendicular to the Line MN



$$\angle ABD = 90^\circ \quad [\because AB \perp MN] \quad \dots\dots(i)$$



$$\angle CON = 90^\circ \quad [\because CD \perp MN] \quad \dots\dots(ii)$$

Now,

$$\angle ABD = \angle CDN = 90^\circ \quad [\text{From (i) and (ii)}]$$

$\therefore AB \parallel CD$, Since corresponding angles are equal.

18. The opposite sides of a quadrilateral are parallel. If one angle of the quadrilateral is 60° , find the other angles.

Sol:



Given $AB \parallel CD$

$AD \parallel BC$

Since $AB \parallel CD$ and AD is the transversal

$$\therefore \angle A + \angle D = 180^\circ \quad [\text{Co-interior angles are supplementary}]$$

$$60^\circ + \angle D = 180^\circ$$

$$\angle D = 180^\circ - 60^\circ$$

$$\angle D = 120^\circ$$

Now, $AD \parallel BC$ and AB is the transversal

$$\angle A + \angle B = 180^\circ \quad [\text{Co-interior angles are supplementary}]$$

$$60^\circ + \angle B = 180^\circ$$

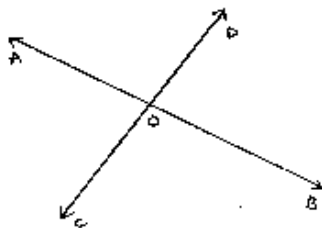
$$\angle B = 180^\circ - 60^\circ = 120^\circ$$

Hence $\angle A = \angle C = 60^\circ$

$$\angle B = \angle D = 120^\circ$$

19. Two lines AB and CD intersect at O . If $\angle AOC + \angle COB + \angle BOD = 270^\circ$, find the measures of $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle DOA$.

Sol:



$$\text{Given: } \angle AOC + \angle COB + \angle BOD = 270^\circ$$

To find: $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle DOA$



Here, $\angle AOC + \angle COB + \angle BOD + \angle AOD = 360^\circ$ [Complete angle]

$$\Rightarrow 270 + \angle AOD = 360^\circ$$

$$\Rightarrow \angle AOD = 360^\circ - 270^\circ$$

$$\Rightarrow \angle AOD = 90^\circ$$

Now,

$$\angle AOD + \angle BOD = 180^\circ \quad [\text{Linear pair}]$$

$$90 + \angle BOD = 180^\circ$$

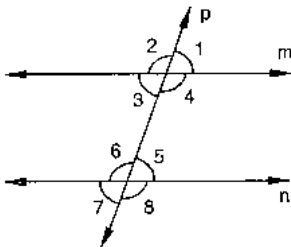
$$\Rightarrow \angle BOD = 180^\circ - 90^\circ$$

$$\therefore \angle BOD = 90^\circ$$

$$\angle AOD = \angle BOC = 90^\circ \quad [\text{Vertically opposite angles}]$$

$$\angle BOD = \angle AOC = 90^\circ \quad [\text{Vertically opposite angles}]$$

20. In the below fig, p is a transversal to lines m and n , $\angle 2 = 120^\circ$ and $\angle 5 = 60^\circ$. Prove that $m \parallel n$.



Sol:

Given that

$$\angle 2 = 120^\circ, \angle 5 = 60^\circ$$

To prove

$$\angle 2 + \angle 1 = 180^\circ$$

[\because Linear pair]

$$120^\circ + \angle 1 = 180^\circ$$

$$\angle 1 = 180^\circ - 120^\circ$$

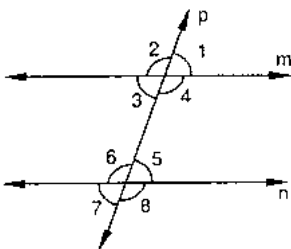
$$\angle 1 = 60^\circ$$

Since $\angle 1 = \angle 5 = 60^\circ$

$$\therefore m \parallel n$$

[As pair of corresponding angles are equal]

21. In the below fig, transversal l intersects two lines m and n , $\angle 4 = 110^\circ$ and $\angle 7 = 65^\circ$. Is $m \parallel n$?



Sol:

Given:



$$\angle 4 = 110^\circ, \angle 7 = 65^\circ$$

To find: Is $m \parallel n$

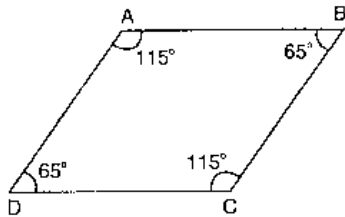
Here, $\angle 7 = \angle 5 = 65^\circ$ [Vertically opposite angle]

Now,

$$\angle 4 + \angle 5 = 110 + 65^\circ = 175^\circ$$

$\therefore m$ is not parallel to n as the pair of co-interior angles is not supplementary.

22. Which pair of lines in the below fig, is parallel? Given reasons.



Sol:

$$\angle A + \angle B = 115 + 65 = 180^\circ$$

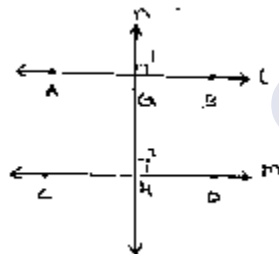
$\therefore AB \parallel BC$ [As sum of co-interior angles we supplementary]

$$\angle B + \angle C = 65 + 115 = 180^\circ$$

$\therefore AB \parallel CD$ [As sum of interior angles are supplementary]

23. If l, m, n are three lines such that $l \parallel m$ and $n \perp l$, prove that $n \perp m$.

Sol:



Given $l \parallel m, n$ perpendicular l

To prove: $n \perp m$

Since $l \parallel m$ and n intersects them at G and H respectively

$\therefore \angle 1 = \angle 2$ [Corresponding angles]

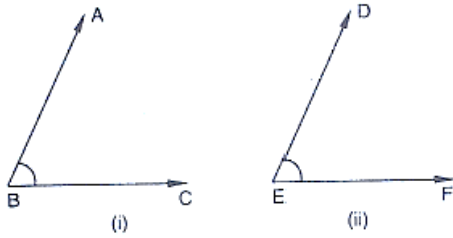
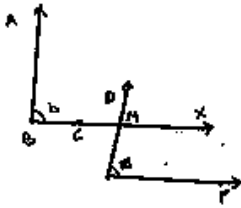
But, $\angle 1 = 90^\circ$ [$n \perp l$]

$$\Rightarrow \angle 2 = 90^\circ$$

Hence n perpendicular m



24. In the below fig, arms BA and BC of $\angle ABC$ are respectively parallel to arms ED and EF of $\angle DEF$. Prove that $\angle ABC = \angle DEF$.

**Sol:**

Given $AB \parallel DE$ and $BC \parallel EF$

To prove: $\angle ABC = \angle DEF$

Construction: Produce BC to x such that it intersects DE at M.

Proof: Since $AB \parallel DE$ and BX is the transversal

$$\therefore \angle ABC = \angle DMX \quad [\text{Corresponding angle}] \quad \dots\dots(i)$$

Also,

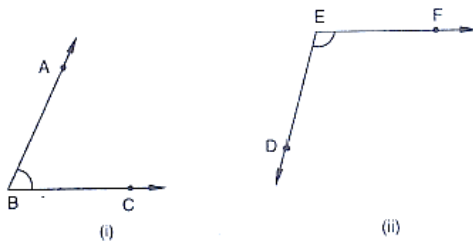
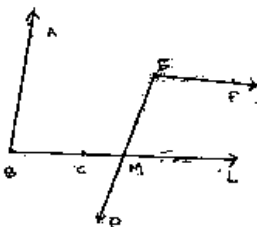
$BX \parallel EF$ and DE is the transversal

$$\therefore \angle DMX = \angle DEF \quad [\text{Corresponding angles}] \quad \dots\dots(ii)$$

From (i) and (ii)

$$\therefore \angle ABC = \angle DEF$$

25. In the below fig, arms BA and BC of $\angle ABC$ are respectively parallel to arms ED and EF of $\angle DEF$. Prove that $\angle ABC + \angle DEF = 180^\circ$.

**Sol:**



Given $AB \parallel DE, BC \parallel EF$

To prove: $\angle ABC + \angle DEF = 180^\circ$

Construction: produce BC to intersect DE at M

Proof: Since $AB \parallel EM$ and BL is the transversal

$$\angle ABC = \angle EML \quad [\text{Corresponding angle}] \quad \dots\dots(1)$$

Also,

$EF \parallel ML$ and EM is the transversal

By the property of co-interior angles are supplementary

$$\angle DEF + \angle EML = 180^\circ$$

From (i) and (ii) we have

$$\therefore \angle DEF + \angle ABC = 180^\circ$$

26. Which of the following statements are true (T) and which are false (F)? Give reasons.

- (i) If two lines are intersected by a transversal, then corresponding angles are equal.
- (ii) If two parallel lines are intersected by a transversal, then alternate interior angles are equal.
- (iii) Two lines perpendicular to the same line are perpendicular to each other.
- (iv) Two lines parallel to the same line are parallel to each other.
- (v) If two parallel lines are intersected by a transversal, then the interior angles on the same side of the transversal are equal.

Sol:

- | | | |
|-----------|-------------|-----------|
| (i) False | (iii) False | (v) False |
| (ii) True | (iv) True | |

27. Fill in the blanks in each of the following to make the statement true:

- (i) If two parallel lines are intersected by a transversal, then each pair of corresponding angles are _____
- (ii) If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are _____
- (iii) Two lines perpendicular to the same line are _____ to each other.
- (iv) Two lines parallel to the same line are _____ to each other.
- (v) If a transversal intersects a pair of lines in such a way that a pair of alternate angles are equal, then the lines are _____
- (vi) If a transversal intersects a pair of lines in such a way that the sum of interior angles on the same side of transversal is 180° , then the lines are _____.

Sol:

- | | |
|--------------------|---------------|
| (i) Equal | (iv) Parallel |
| (ii) Supplementary | (v) Parallel |
| (iii) Parallel | (vi) Parallel |