Ratio and Proportion
Exercise 8A

Ratio and Proportion

- A **ratio** is a comparison of two values expressed as a quotient
  - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
  - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A **proportion** is an equation stating that two ratios are equal.
  - Example: $\frac{12}{18} = \frac{2}{3}$

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the first term or **antecedent** and b, the second term or **consequent**.

Eg. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

2. Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b.

Mean Proportional:

Mean proportional between a and b is $\sqrt{ab}$.

4. Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.   [componendo and dividendo]

6. Variations:

We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

Properties of proportions:

Convertendo: If $a : b :: c : d$,then $a : (a - b) :: c : (c - d)$.

Invertendo: If $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{b}{a} = \dfrac{d}{c}$.

Alternendo: If $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a}{c} = \dfrac{b}{d}$.

Componendo: If $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a+b}{b} = \dfrac{c+d}{d}$.

Dividendo: $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a-b}{b} = \dfrac{c-d}{d}$

Componendo and Dividendo: If $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$.

Q1

Answer:

(i) HCF of 24 and 40 is 8.

$\therefore 24 : 40 = \dfrac{24}{40} = \dfrac{24 \div 8}{40 \div 8} = \dfrac{3}{5} = 3 : 5$

Hence, 24 : 40 in its simplest form is 3 : 5.

(ii) HCF of 13.5 and 15 is 1.5.

$\dfrac{13.5}{15} = \dfrac{135}{150}$

The HCF of 135 and 150 is 15.

$= \dfrac{135 \div 15}{150 \div 15} = \dfrac{9}{10}$

Hence, 13.5 : 15 in its simplest form is 9 : 10.

(iii) $\dfrac{20}{3} : \dfrac{15}{2} = 40 : 45$

The HCF of 40 and 45 is 5.

$\therefore 40 : 45 = \dfrac{40}{45} = \dfrac{40 \div 5}{45 \div 5} = \dfrac{8}{9} = 8 : 9$

Hence, $6\dfrac{2}{3} : 7\dfrac{1}{2}$ in its simplest form is 8 : 9.

(iv) 9 : 6

The HCF of 9 and 6 is 3.

$\therefore 9 : 6 = \dfrac{9}{6} = \dfrac{9 \div 3}{6 \div 3} = 3 : 2$

Hence, $\dfrac{1}{6} : \dfrac{1}{9}$ in its simplest form is 3 : 2.

(v) LCM of the denominators is 2.

$\therefore 4 : 5 : \dfrac{9}{2} = 8 : 10 : 9$

The HCF of these 3 numbers is 1.

$\therefore 8 : 10 : 9$ is the simplest form.

(vi) 2.5 : 6.5 : 8 = 25 : 65 : 80

The HCF of 25, 65 and 80 is 5.

$\therefore 25 : 65 : 80 = \dfrac{25}{65}_{80} = \dfrac{25 \div 5}{65 \div 5}_{80 \div 5} = \dfrac{5}{13}_{16} = 5 : 13 : 16$

Q2

Answer :

(i) Converting both the quantities into the same unit, we have:

75 paise : (3×100) paise = 75 : 300

$= \frac{75}{300} = \frac{75 \div 75}{300 \div 75} = \frac{1}{4}$ ($\because$ HCF of 75 and 300 = 75)

= 1 paise : 4 paise

(ii) Converting both the quantities into the same unit, we have:

105 cm : 63 cm $= \frac{105}{63} = \frac{105 \div 21}{63 \div 21} = \frac{5}{3}$ ($\because$ HCF of 105 and 63 = 21)

= 5 cm : 3 cm

(iii) Converting both the quantities into the same unit

65 min : 45 min $= \frac{65}{45} = \frac{65 \div 5}{45 \div 5} = \frac{13}{9}$ ($\because$ HCF of 65 and 45 = 5)

= 13 min : 9 min

(iv) Converting both the quantities into the same unit, we get:

8 months : 12 months $= \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ ($\because$ HCF of 8 and 12 = 4)

= 2 months : 3 months

(v) Converting both the quantities into the same unit, we get:

2250g : 3000 g $= \frac{2250}{3000} = \frac{2250 \div 750}{3000 \div 750} = \frac{3}{4}$ ($\because$ HCF of 2250 and 3000 = 750)

= 3 g : 4 g

(vi) Converting both the quantities into the same unit, we get:

1000 m : 750 m $= \frac{1000}{750} = \frac{1000 \div 250}{750 \div 250} = \frac{4}{3}$ ($\because$ HCF of 1000 and 750 = 250)

= 4 m : 3 m

Q3

Answer :

$\frac{A}{B} = \frac{7}{5}$ and $\frac{B}{C} = \frac{9}{14}$

Therefore, we have:

$\frac{A}{B} \times \frac{B}{C} = \frac{7}{5} \times \frac{9}{14}$

$\frac{A}{C} = \frac{9}{10}$

$\therefore A : C = 9 : 10$

Q4

Answer :

$\frac{A}{B} = \frac{5}{8}$ and $\frac{B}{C} = \frac{16}{25}$

Now, we have : $\frac{A}{B} \times \frac{B}{C} = \frac{5}{8} \times \frac{16}{25} \Rightarrow \frac{A}{C} = \frac{2}{5}$

$\therefore A : C = 2 : 5$

Q5

Answer :

$A : B = 3 : 5$

$B : C = 10 : 13 = \frac{10 \div 2}{13 \div 2} = 5 : \frac{13}{2}$

Now, $A : B : C = 3 : 5 : \frac{13}{2}$

$\therefore A : B : C = 6 : 10 : 13$

Q6

Answer :

We have the following:

$A : B = 5 : 6$

$B : C = 4 : 7 = \frac{4}{7} = \frac{4 \times \frac{6}{4}}{7 \times \frac{6}{4}} = 6 : \frac{21}{2}$

$\therefore A : B : C = 5 : 6 : \frac{21}{2} = 10 : 12 : 21$

Q7

Answer :

Sum of the ratio terms $= 7 + 8 = 15$

Now, we have the following:

Kunal's share $= Rs \, 360 \times \frac{7}{15} = 24 \times 7 = Rs \, 168$

Mohit's share $= Rs \, 360 \times \frac{8}{15} = 24 \times 8 = Rs \, 192$

Q8

Answer :

Sum of the ratio terms $= \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

Now, we have the following:

Rajan's share $= Rs \, 880 \times \frac{\frac{1}{5}}{\frac{11}{30}} = Rs \, 880 \times \frac{6}{11} = Rs \, 80 \times 6 = Rs \, 480$

Kamal's share $= Rs \, 880 \times \frac{\frac{1}{6}}{\frac{11}{30}} = Rs \, 880 \times \frac{5}{11} = Rs \, 80 \times 5 = Rs \, 400$

Q9

Answer :

Sum of the ratio terms is (1 + 3 + 4) = 8

We have the following:

A's share = Rs $5600 \times \frac{1}{8}$ = Rs $\frac{5600}{8}$ = Rs 700

B's share = Rs $5600 \times \frac{3}{8}$ = Rs 700 × 3 = Rs 2100

C's share = Rs $5600 \times \frac{4}{8}$ = Rs 700 × 4 = Rs 2800

Q10

Answer :

Let x be the required number.
Then, (9 + x) : (16 + x) = 2 : 3

$\Rightarrow \frac{9+x}{16+x} = \frac{2}{3}$
$\Rightarrow 27 + 3x = 32 + 2x \Rightarrow x = 5$

Hence, 5 must be added to each term of the ratio 9 : 16 to make it 2 : 3.

Q11

Answer :

Suppose that x is the number that must be subtracted.
Then, (17 − x) : (33 − x) = 7 : 15

$\Rightarrow \frac{17-x}{33-x} = \frac{7}{15}$
$\Rightarrow 255 - 15x = 231 - 7x \Rightarrow 8x = 255 - 231 = 24 \Rightarrow x = 3$

Hence, 3 must be subtracted from each term of ratio 17 : 33 so that it becomes 7 : 15.

Q12

Answer :

Suppose that the numbers are 7x and 11x.

Then, (7x + 7) : (11x + 7) = 2 : 3
$\Rightarrow \frac{7x+7}{11x+7} = \frac{2}{3}$

⇒ 21x + 21 = 22x + 14

⇒ x = 7

Hence, the numbers are (7 × 7 =) 49 and (11 × 7 =) 77.

Q13

Answer :

Suppose that the numbers are 5x and 9x.
Then, (5x − 3) : (9x − 3) = 1 : 2

$\Rightarrow \frac{5x-3}{9x-3} = \frac{1}{2}$

⇒ 10x − 6 = 9x − 3
⇒ x = 3

Hence, the numbers are (5 × 3 =) 15 and (9 × 3 =) 27.

Q14

Answer :

Let the numbers be $3x$ and $4x$.
Their LCM is $12x$.
Then, $12x = 180$
$\Rightarrow x = 15$

∴ The numbers are $(3 \times 15 =)$ 45 and $(4 \times 15 =)$ 60.

Q15

Answer :

Suppose that the present ages of A and B are $8x$ yrs and $3x$ yrs.
Then, $(8x + 6) : (3x + 6) = 9 : 4$
$\Rightarrow \frac{8x+6}{3x+6} = \frac{9}{4}$
$\Rightarrow 32x + 24 = 27x + 54$
$\Rightarrow 5x = 30$
$\Rightarrow x = 6$

Now, present age of A = 8×6 yrs = 48 yrs
Present age of B = 3×6 yrs = 18 yrs

Q16

Answer :

Suppose that the weight of zinc is x g.

Then, $48.6 : x = 9 : 5$

$\Rightarrow x = \frac{48.6 \times 5}{9} = \frac{243}{9} = 27$

Hence, the weight of zinc in the alloy is 27 g.

Q17

Answer :

Suppose that the number of boys is x.
Then, $x : 375 = 8 : 3$

$\Rightarrow x = \frac{8 \times 375}{3} = 8 \times 125 = 1000$

Hence, the number of girls in the school is 1000.

Q18

Answer :

Suppose that the monthly income of the family is Rs x.

Then, $x : 2500 = 11 : 2$

$\Rightarrow x = \frac{11 \times 2500}{2} = 11 \times 1250$
$\Rightarrow x = $ Rs 13750

Hence, the income is Rs 13,750.
∴ Expenditure = (monthly income − savings)
$\qquad\qquad$ =Rs (13750 − 2500)
$\qquad\qquad\quad$ = Rs 11250

Q19

Answer :

Let the numbers one rupee, fifty paise and twenty-five paise coins be 5*x*, 8*x* and 4*x*, respectively.

Total value of these coins = $\left(5x \times \frac{100}{100} + 8x \times \frac{50}{100} + 4x \times \frac{25}{100}\right)$

$\Rightarrow 5x + \frac{8x}{2} + \frac{4x}{4}$

$= \frac{20x + 16x + 4x}{4} = \frac{40x}{4} = 10x$

However, the total value is Rs 750.

$\therefore 750 = 10x$

$\Rightarrow x = 75$

Hence, number of one rupee coins = 5 × 75 = 375
Number of fifty paise coins = 8 × 75 = 600
Number of twenty-five paise coins = 4 × 75 = 300

Q20
Answer :

$(4x + 5) : (3x + 11) = 13 : 17$

$\Rightarrow \frac{4x+5}{3x+11} = \frac{13}{17}$
$\Rightarrow 68x + 85 = 39x + 143 \Rightarrow 29x = 58 \Rightarrow x = 2$

Q21
Answer :

$\frac{x}{y} = \frac{3}{4}$

$\Rightarrow x = \frac{3y}{4}$

Now, we have $(3x + 4y) : (5x + 6y)$

$= \frac{3x+4y}{5x+6y} = \frac{3 \times \frac{3y}{4} + 4y}{5 \times \frac{3y}{4} + 6y}$

$= \frac{9y+16y}{15y+24y} = \frac{25y}{39y} = \frac{25}{39}$

$= 25 : 39$

Q22
Answer :

$\frac{x}{y} = \frac{6}{11}$

$\Rightarrow x = \frac{6y}{11}$

Now, we have:

$\frac{8x-3y}{3x+2y}$

$= \frac{8 \times \frac{6y}{11} - 3y}{3 \times \frac{6y}{11} + 2y}$

$= \frac{48y-33y}{18y+22y}$

$= \frac{15y}{40y} = \frac{3}{8}$

$\therefore (8x - 3y) : (3x + 2y) = 3 : 8$

Q23

Answer :

Suppose that the numbers are $5x$ and $7x$.
The sum of the numbers is 720.
i.e., $5x + 7x = 720$
$\Rightarrow 12x = 720$
$\Rightarrow x = 60$

Hence, the numbers are $(5 \times 60 =) 300$ and $(7 \times 60 =) 420$.

Q24
Answer :

(i) The LCM of 6 and 9 is 18.

$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$
$\frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$ **Clearly,** $\frac{14}{18} < \frac{15}{18}$

$\therefore (7 : 9) < (5 : 6)$

(ii) The LCM of 3 and 7 is 21.

$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$
$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$
Clearly, $\frac{12}{21} < \frac{14}{21}$

$\therefore (4 : 7) < (2 : 3)$

(iii) The LCM of 2 and 7 is 14.

$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$
$\frac{4 \times 2}{7 \times 2} = \frac{8}{14}$

Clearly, $\frac{7}{14} < \frac{8}{14}$

$\therefore (1 : 2) < (4 : 7)$

(iv) The LCM of 5 and 13 is 65.

$\frac{3}{5} = \frac{3 \times 13}{5 \times 13} = \frac{39}{65}$
$\frac{8}{13} = \frac{8 \times 5}{13 \times 5} = \frac{40}{65}$
Clearly, $\frac{39}{65} < \frac{40}{65}$

$\therefore (3 : 5) < (8 : 13)$

Q25
Answer :

(i) We have $\frac{5}{6}$, $\frac{8}{9}$ and $\frac{11}{18}$.

$$\begin{array}{c|l} 2 & 6, 9, 18 \\ \hline 3 & 3, 9, 9 \\ \hline 3 & 1, 3, 3 \\ \hline & 1, 1, 1 \end{array}$$

The LCM of 6, 9 and 18 is 18. Therefore, we have:

$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$
$\frac{8}{9} = \frac{8 \times 2}{9 \times 2} = \frac{16}{18}$ $\frac{11}{18} = \frac{11}{18}$ **Clearly,** $\frac{11}{18} < \frac{15}{18} < \frac{16}{18}$

Hence, $(11 : 18) < (5 : 6) < (8 : 9)$

(ii) We have $\frac{11}{14}$, $\frac{17}{21}$, $\frac{5}{7}$ and $\frac{2}{3}$.

$$\begin{array}{c|l} 2 & 14, 21, 7, 3 \\ \hline 7 & 7, 21, 7, 3 \\ \hline 3 & 1, 3, 1, 3 \\ \hline & 1, 1, 1, 1 \end{array}$$

The LCM of 14, 21, 7 and 3 is 42.

$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{28}$

$\frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$

$\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$

$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42}$

Clearly, $\frac{28}{42} < \frac{30}{42} < \frac{33}{28} < \frac{34}{42}$

Hence, $\left(2 : 3\right) < \left(5 : 7\right) < \left(11 : 14\right) < \left(17 : 21\right)$

Ratio and Proportion
Exercise 8B

Q1

Answer :

We have:

Product of the extremes = 30 × 60 = 1800
Product of the means = 40 × 45 = 1800
Product of extremes = Product of means

Hence, 30 : 40 :: 45 : 60

Q2

Answer :

We have:
Product of the extremes = 36 × 7 = 252
Product of the means = 49 × 6 = 294
Product of the extremes ≠ Product of the means

Hence, 36, 49, 6 and 7 are not in proportion.

Q3

Answer :

Product of the extremes = 2 × 27 = 54
Product of the means = 9 × x = 9x

Since 2 : 9 :: x : 27, we have:
Product of the extremes = Product of the means
⇒ 54 = 9x
⇒ x = 6

Q4

Answer :

Product of the extremes = 8 × 35 = 280
Product of the means = 16 × x = 16x

Since 8 : x :: 16 : 35, we have:
Product of the extremes = Product of the means
⇒ 280 = 16x
⇒ x = 17.5

Q5
Answer :

Product of the extremes = x × 60 = 60x
Product of the means = 35 × 48 = 1680

Since x : 35 :: 48 : 60, we have:
Product of the extremes = Product of the means
⇒ 60x= 1680
⇒ x = 28

Q6
Answer :

(i) Let the fourth proportional be x.
Then, 8 : 36 :: 6 : x

$$8 \times x = 36 \times 6 \qquad \text{[Product of extremes = Product of means]}$$
⇒ 8x = 216
⇒ x = 27
Hence, the fourth proportional is 27.

(ii) Let the fourth proportional be x.
Then, 5 : 7 :: 30 : x
$$\Rightarrow 5 \times x = 7 \times 30 \qquad \text{[Product of extremes = Product of means]}$$
⇒ 8x = 216
⇒ 5x = 210
⇒ x = 42

Hence, the fourth proportional is 42.

(iii) Let the fourth proportional be x.
Then, $2.8 \times x = 14 \times 3.5 \qquad$ [Product of extremes = Product of means]
⇒ 8x = 216
⇒ 2.8x = 49
⇒ x = 17.5
Hence, the fourth proportional is 17.5.

Q7
Answer :

36, 54 and x are in continued proportion.
Then, 36 : 54 :: 54 : x
$$\Rightarrow 36 \times x = 54 \times 54 \qquad \text{[Product of extremes = Product of means]}$$

⇒ 36x = 2916
⇒ x = 81

Q8

Answer :

27, 36 and x are in continued proportion.

Then, 27 : 36 :: 36 : x

$\Rightarrow$ 27×x = 36 ×36   [Product of extremes = Product of means]

$\Rightarrow$ 27x = 1296

$\Rightarrow$ x = 48

Hence, the value of x is 48.

Q9

Answer :

(i) Suppose that x is the third proportional to 8 and 12.

Then, 8 :12 :: 12 : x

$\Rightarrow 8 \times x = 12 \times 12$ (Product of extremes = Product of means)

$\Rightarrow$ 8x = 144

$\Rightarrow$ x = 18

Hence, the required third proportional is 18.

(ii) Suppose that x is the third proportional to 12 and 18.

Then, 12 : 18 :: 18 : x

$\Rightarrow 12 \times x = 18 \times 18$ (Product of extremes = Product of means)

$\Rightarrow$ 12x = 324

$\Rightarrow$ x = 27

Hence, the third proportional is 27.

(iii) Suppose that x is the third proportional to 4.5 and 6.

Then, 4.5 : 6:: 6 : x

$\Rightarrow 4.5 \times x = 6 \times 6$ (Product of extremes = Product of means)

$\Rightarrow$ 4.5x = 36

$\Rightarrow$ x = 8

Hence, the third proportional is 8.

Q10

Answer :

The third proportional to 7 and x is 28.

Then, 7 : x :: x : 28

$\Rightarrow 7 \times 28 = x^2$ (Product of extremes = Product of means)

$\Rightarrow$ x = 14

Q11

Answer :

(i) Suppose that x is the mean proportional.

Then, 6 : x :: x : 24

$\Rightarrow 6 \times 24 = x \times x$ (Product of extremes = Product of means)

$\Rightarrow x^2 = 144$

$\Rightarrow$ x = 12

Hence, the mean proportional to 6 and 24 is 12.

(ii) Suppose that x is the mean proportional.

Then, 3 : x :: x : 27

$\Rightarrow 3 \times 27 = x \times x$ (Product of extremes =Product of means)

$\Rightarrow x^2 = 81$

$\Rightarrow$ x = 9

Hence, the mean proportional to 3 and 27 is 9.

(iii) Suppose that *x* is the mean proportional.

Then, 0.4 : *x* :: *x* : 0.9

$$\Rightarrow 0.4 \times 0.9 = x \times x \qquad \text{(Product of extremes = Product of means)}$$
$$\Rightarrow x^2 = 0.36$$
$$\Rightarrow x = 0.6$$

Hence, the mean proportional to 0.4 and 0.9 is 0.6.

Q12
Answer :

Suppose that the number is *x*.

Then, (5 + *x*) : (9 + *x*) :: (7 + *x*) : (12 + *x*)

$$\Rightarrow (5 + x) \times (12 + x) = (9 + x) \times (7 + x)$$
$$\text{(Product of extremes = Product of means)}$$
$$\Rightarrow 60 + 5x + 12x + x^2 = 63 + 9x + 7x + x^2$$
$$\Rightarrow 60 + 17x = 63 + 16x$$
$$\Rightarrow x = 3$$

Hence, 3 must be added to each of the numbers: 5, 9, 7 and 12, to get the numbers which are in proportion.

Q13
Answer :

Suppose that x is the number that is to be subtracted.

Then, (10 − *x*) : (12 − *x*) :: (19 − *x*) : (24 − *x*)

$$\Rightarrow (10 - x) \times (24 - x) = (12 - x) \times (19 - x)$$
$$\text{(Product of extremes = Product of means)}$$
$$\Rightarrow 240 - 10x - 24x + x^2 = 228 - 12x - 19x + x^2$$
$$\Rightarrow 240 - 34x = 228 - 31x$$
$$\Rightarrow 3x = 12$$
$$\Rightarrow x = 4$$

Hence, 4 must be subtracted from each of the numbers: 10, 12, 19 and 24, to get the numbers which are in proportion.

Q14
Answer :

Distance represented by 1 cm on the map = 5000000 cm = 50 km

Distance represented by 3 cm on the map = 50 × 4 km = 200 km

∴ The actual distance is 200 km.

Q15
Answer :

(Height of tree) : (height of its shadow) = (height of the pole) : (height of its shadow)

Suppose that the height of pole is *x* cm.

Then, 6 : 8 = *x* : 20

$$\Rightarrow x = \frac{6 \times 20}{8} = 15$$
∴ Height of the pole = 15 cm

Ratio and Proportion
Exercise 8C

Q1

Answer:

The correct option is (d).

$$\frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{3}{4} \times \frac{8}{9}$$
$$= \frac{2}{3}$$

Hence, $a : c = 2 : 3$

Q2

Answer:

(a) 15 : 8

$$\frac{A}{B} = \frac{2}{3}$$
$$\frac{B}{C} = \frac{4}{5}$$

Then, $\frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Hence, $C : A = 15 : 8$

Q3

Answer:

The correct option is (d).

$$A = \frac{3B}{2}$$
$$C = \frac{4B}{5}$$

$$\therefore A : C = \frac{A}{C} = \frac{\frac{3B}{2}}{\frac{4B}{5}} = \frac{15}{8}$$

Hence, $A : C = 15 : 8$

Q4

Answer :

The correct option is (b).

$$\frac{15}{100}A = \frac{20}{100}B$$
$$\Rightarrow \frac{A}{B} = \frac{4}{3}$$

Hence, $A : B = 4 : 3$

Q5

Answer :

(a) $1 : 3 : 6$

$$A = \frac{1}{3}B$$
$$C = 2B$$
$$\therefore A : B : C = \frac{1}{3}B : B : 2B = 1 : 3 : 6$$

Q6

Answer :

(b) $30 : 42 : 77$

$$\frac{A}{B} = \frac{5}{7}$$
$$\Rightarrow A = \frac{5B}{7} \quad \frac{B}{C} = \frac{6}{11} \Rightarrow C = \frac{11B}{6}$$
$$\therefore A : B : C = \frac{5B}{7} : B : \frac{11B}{6} = 30 : 42 : 77$$

Q7

Answer :

(c) $6 : 4 : 3$

$$2A = 3B = 4C$$
Then, $A = \frac{3B}{2}$ and $C = \frac{3B}{4}$
$$\therefore A : B : C = \frac{3B}{2} : B : \frac{3B}{4} = 6 : 4 : 3$$

Q8

Answer :

(a) $3 : 4 : 5$

$$A = \frac{3B}{4}$$
$$C = \frac{5B}{4}$$
$$\therefore A : B : C = \frac{3B}{4} : B : \frac{5B}{4}$$

$$= 3 : 4 : 5$$

Q9

Answer :

(b) $15 : 10 : 6$

$$\frac{1}{x} : \frac{1}{y} = 2 : 3$$
Then, $y : x = 2 : 3$ and $y = \frac{2}{3}x$
$$\frac{1}{y} : \frac{1}{z} = 3 : 5$$
Then, $z : y = 3 : 5$ and $z = \frac{3}{5}y$
$$\therefore x : y : z = x : \frac{2}{3}x : \frac{3}{5}y = x : \frac{2}{3}x : \frac{3}{5} \times \frac{2}{3}x$$
$$= x : \frac{2}{3}x : \frac{2}{5}x = 15 : 10 : 6$$

Q10

Answer :

$$\frac{x}{y} = \frac{3}{4}$$

$$x = \frac{3y}{4}$$

$$\therefore \frac{7x + 3y}{7x - 3y} = \frac{7\frac{3y}{4} + 3y}{7\frac{3y}{4} - 3y}$$

$$= \frac{21y + 12y}{21y - 12y} = \frac{33y}{9y} = \frac{11}{3}$$

Hence, $(7x + 3y) : (7x - 3y) = 11 : 3$

The correct option is (c).

Q11

Answer :

(c) 5 : 2

$$\frac{3a + 5b}{3a - 5b} = \frac{5}{1}$$
$$3a + 5b = 15a - 25b$$
$$12a = 30b$$
$$\frac{a}{b} = \frac{30}{12} = \frac{5}{2}$$

$\therefore a : b = 5 : 2$

Q12

Answer :

(c) 9

$$7 \times 45 = x \times 35 \quad \text{(Product of extremes} = \text{Product of means)}$$
$$\Rightarrow 35x = 315$$
$$\Rightarrow x = 9$$

Q13

Answer :

(b) 7

Suppose that *x* is the number that is to be added.

Then, $(3 + x) : (5 + x) = 5 : 6$

$$\Rightarrow \frac{3 + x}{5 + x} = \frac{5}{6}$$
$$\Rightarrow 18 + 6x = 25 + 5x$$
$$\Rightarrow x = 7$$

Q14

Answer :

(d) 40

Suppose that the numbers are *x* and *y*.

Then, $x : y = 3 : 5$ and $(x + 10) : (y + 10) = 5 : 7$

$$\frac{x}{y} = \frac{3}{5}$$
$$x = \frac{3y}{5}$$
$$\Rightarrow \frac{x + 10}{y + 10} = \frac{5}{7} \Rightarrow 7x + 70 = 5y + 50 \Rightarrow 7 \times \frac{3y}{5} + 70 = 5y + 50 \Rightarrow 5y \frac{21y}{5} =$$
$$20 \Rightarrow \frac{4y}{5} = 20 \Rightarrow y = 25 \text{ Therefore, } x = \frac{3 \times 25}{5} = 15$$

Hence, sum of numbers = 15 + 25 = 40

Q15

Answer:

(a) 3

Suppose that x is the number that is to be subtracted.

Then, $(15 - x) : (19 - x) = 3 : 4$

$$\Rightarrow \frac{15 - x}{19 - x} = \frac{3}{4}$$

Cross multiplying, we get :

$$60 - 4x = 57 - 3x$$
$$\Rightarrow x = 3$$

Q16

Answer:

(a) Rs 180

A's share = $\frac{3}{7} \times 420 = 180$

Q17

Answer:

(d) 416

Let x be the number of boys.

Then, $8 : 5 = x : 160$

$$\Rightarrow \frac{8}{5} = \frac{x}{160}$$
$$\Rightarrow x = \frac{8 \times 160}{5} = 256$$

$\therefore$ Total strength of the school $= 256 + 160 = 416$

Q18

Answer:

(a) (2 :3)

LCM of 3 and 7 = $7 \times 3 = 21$

$$\frac{2 \times 7}{3 \times 7} = \frac{14}{21} \text{ and } \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

Clearly, $\frac{12}{21} < \frac{14}{21}$

Hence, $\left(4 : 7\right) < \left(2 : 3\right)$

Q19

Answer:

(c) 16

Suppose that the third proportional is x.

Then, $9 : 12 :: 12 : x$

$$\Rightarrow 9 \times x = 12 \times 12 \qquad \text{(Product of extremes = Product of means)}$$
$$\Rightarrow 9x = 144$$
$$\Rightarrow x = 16$$

Q20

Answer :

(b) 12

Suppose that the mean proportional is x.

Then, $9 : x :: x : 16$

$9 \times 16 = x \times x$ (Product of extremes = Product of means)

$\Rightarrow x^2 = 144$

$\Rightarrow x = 12$

Q21

Answer :

(a) 18 years

Suppose that the present ages of A and B are $3x$ yrs and $8x$ yrs, respectively.
After six years, the age of A will be $(3x+6)$ yrs and that of B will be $(8x+6)$ yrs.
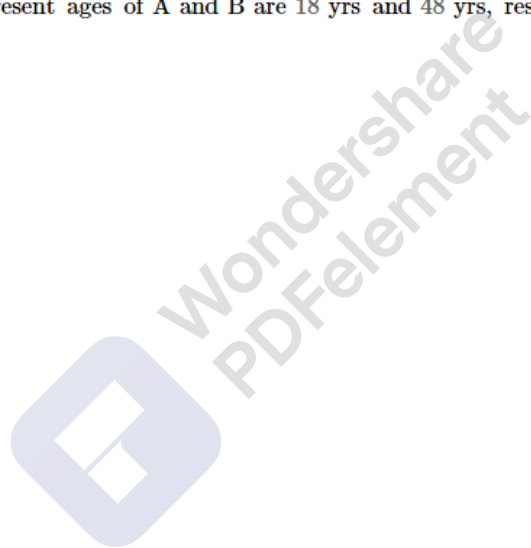Then, $(3x +6) : (8x + 6) = 4 : 9$

$\Rightarrow \frac{3x + 6}{8x+6} = \frac{4}{9}$

$\Rightarrow 27x + 54 = 32x + 24$

$\Rightarrow 5x = 30$

$\Rightarrow x = 6$

Hence, the present ages of A and B are 18 yrs and 48 yrs, respectively.