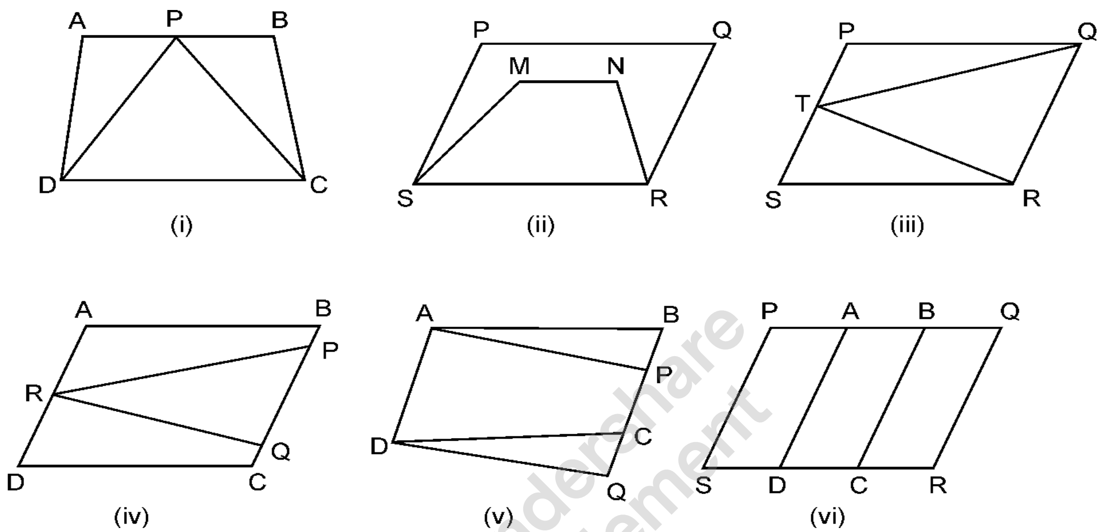
9 AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.1

Q.1. *Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.*



Sol. (i) Base DC, parallels DC and AB
(iii) Base QR, parallels QR and PS
(v) Base AD, parallels AD and BQ.

9 AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.2

Q.1. *In the figure, ABCD is a paralle-logram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*

Sol. Area of parallelogram ABCD

= AB × AE

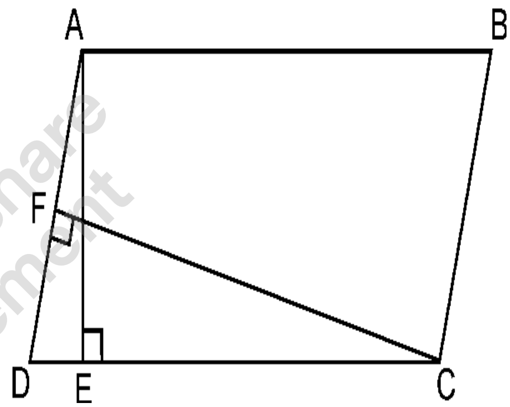
= 16 × 8 cm^2 = 128 cm^2

Also, area of parallelogram ABCD

= AD × FC = (AD × 10) cm^2

∴ AD × 10 = 128

⇒ AD = $\frac{128}{10}$ = **12.8 cm Ans.**

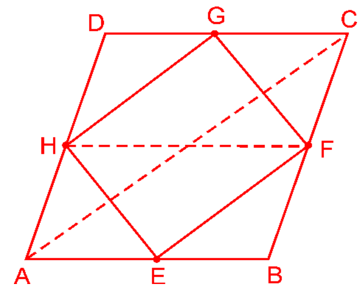


Q.2. *If E, F, G, and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).*

Sol. Given : A parallelogram ABCD · E, F, G, H are mid-points of sides AB, BC, CD, DA respectively

To Porve : ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Construction : Join AC and HF.



Proof : In ΔABC,
E is the mid-point of AB.
F is the mid-point of BC.

$\Rightarrow$ EF is parallel to AC and EF = $\frac{1}{2}$ AC ... (i)

Similarly, in ΔADC, we can show that

HG || AC and HG = $\frac{1}{2}$ AC ... (ii)

From (i) and (ii)
EF || HG and EF = HG
∴ EFGH is a parallelogram. [One pour of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

HA = FB and HA || FB [AD = BC $\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC $\Rightarrow$ HA = FB]

∴ ABFH is a parallelogram. [One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

∴ Area of ΔHEF = $\frac{1}{2}$ area of HABF ... (iii)

Similarly, area of ΔHGF = $\frac{1}{2}$ area of HFCD ... (iv)

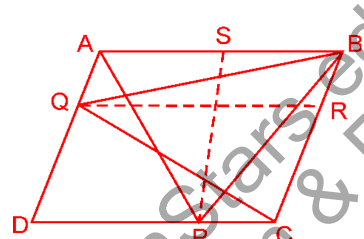
Adding (iii) and (iv),
Area of ΔHEF + area of ΔHGF

$$= \frac{1}{2} \text{(area of HABF + area of HFCD)}$$

$\Rightarrow$ ar (EFGH) = $\frac{1}{2}$ ar (ABCD) **Proved.**

Q.3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).*

Sol. **Given :** A parallelogram ABCD. P and Q are any points on DC and AD respectively.



To prove : ar (APB) = ar (BQC)

Construction : Draw PS || AD and QR || AB.

Proof : In parallelogram ABRQ, BQ is the diagonal.

∴ area of ΔBQR = $\frac{1}{2}$ area of ABRQ ... (i)

In parallelogram CDQR, CQ is a diagonal.

$\therefore$ area of ΔRQC = $\frac{1}{2}$ area of CDQR ... (ii)

Adding (i) and (ii), we have

area of ΔBQR + area of ΔRQC

= $\frac{1}{2}$ [area of ABRQ + area of CDQR]

$\Rightarrow$ area of ΔBQC = $\frac{1}{2}$ area of ABCD ... (iii)

Again, in parallelogram DPSA, AP is a diagonal.

$\therefore$ area of ΔASP = $\frac{1}{2}$ area of DPSA ... (iv)

In parallelogram BCPS, PB is a diagonal.

$\therefore$ area of ΔBPS = $\frac{1}{2}$ area of BCPS ... (v)

Adding (iv) and (v)

area of ΔASP + area of ΔBPS = $\frac{1}{2}$(area of DPSA + area of BCPS)

$\Rightarrow$ area of ΔAPB = $\frac{1}{2}$ (area of ABCD) ... (vi)

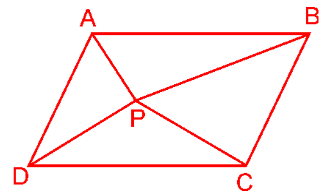
From (iii) and (vi), we have

area of ΔAPB = area of ΔBQC. **Proved.**

Q.4. *In the figure, P is a point in the interior of a parallelogram ABCD. Show that*

(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar(APB) + ar (PCD)



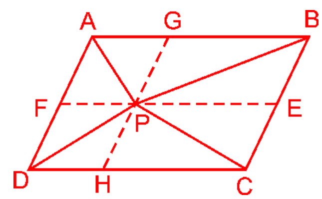
Sol. Given : A parallelogram ABCD. P is a point inside it.

To prove : (i) ar (APB) + ar(PCD)

= $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC)

= ar (APB) + ar (PCD)



Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$\therefore$ area of ΔAPG = area of ΔAPF ... (i)

In parallelogram BGPE, PB is a diagonal,

$\therefore$ area of ΔBPG = area of ΔEPB ... (ii)

In parallelogram DHPF, DP is a diagonal,

∴ area of ΔDPH = area of ΔDPF ... (iii)

In parallelogram HCEP, CP is a diagonal,

∴ area of ΔCPH = area of ΔCPE ... (iv)

Adding (i), (ii), (iii) and (iv)

area of ΔAPG + area of ΔBPG + area of ΔDPH + area of ΔCPH

= area of ΔAPF + area of ΔEPB + area of ΔDPF + area ΔCPE

⇒ [area of ΔAPG + area of ΔBPG] + [area of ΔDPH + area of ΔCPH]

= [area of ΔAPF + area of ΔDPF] + [area of ΔEPB + area of ΔCPE]

⇒ area of ΔAPB + area of ΔCPD = area of ΔAPD + area of ΔBPC ... (v)

But area of parallelogram ABCD

= area of ΔAPB + area of ΔCPD + area of ΔAPD + area of ΔBPC ... (vi)

From (v) and (vi)

area of ΔAPB + area of ΔPCD = $\frac{1}{2}$ area of ABCD

or, ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD) **Proved.**

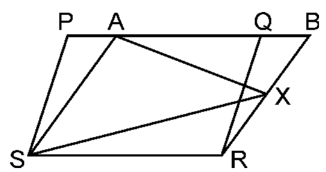
(ii) From (v),

⇒ ar (APD) + ar (PBC) = ar (APB) + ar (CPD) **Proved.**

Q.5. *In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that*

(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ *ar (PQRS)*



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.

To prove : (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof : (i) In ΔASP and BRQ, we have

∠SPA = ∠RQB [Corresponding angles] ...(1)

∠PAS = ∠QBR [Corresponding angles] ...(2)

∴ ∠PSA = ∠QRB [Angle sum property of a triangle] ...(3)

Also, PS = QR [Opposite sides of the parallelogram PQRS] ...(4)

So, ΔASP ≅ ΔBRQ [ASA axiom, using (1), (3) and (4)]

Therefore, area of ΔPSA = area of ΔQRB

[Congruent figures have equal areas] ...(5)

Now, ar (PQRS) = ar (PSA) + ar (ASRQ]

= ar (QRB) + ar (ASRQ]

= ar (ABRS)

So, ar (PQRS) = ar (ABRS) **Proved.**

(ii) Now, ΔAXS and ||gm ABRS are on the same base AS and between same parallels AS and BR

$\therefore$ area of $\Delta AXS = \frac{1}{2}$ area of ABRS

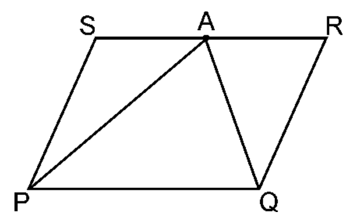
$\Rightarrow$ area of $\Delta AXS = \frac{1}{2}$ area of PQRS [$\because$ ar (PQRS) = ar (ABRS]

$\Rightarrow$ ar of (AXS) = $\frac{1}{2}$ ar of (PQRS) **Proved.**

Q.6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?*

Sol. The field is divided in three triangles.

Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.



$\therefore$ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

$\Rightarrow$ 2ar (APQ) = ar(PQRS)

But ar (PQRS) = ar(APQ) + ar (PSA) + ar (ARQ)

$\Rightarrow$ 2 ar (APQ) = ar(APQ) + ar(PSA) + ar (ARQ)

$\Rightarrow$ ar (APQ) = ar(PSA) + ar(ARQ)

Hence, area of ΔAPQ = area of ΔPSA + area of ΔARQ.

To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in ΔAPQ and pulses in other two triangles or pulses in ΔAPQ and wheat in other two triangles. **Ans.**

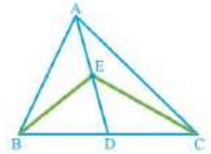
Mathematics

(Chapter – 9)(Areas of Parallelograms and Triangles)
(Class – 9)

Exercise 9.3

Question 1:

In Figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).



Answer 1:

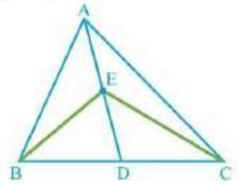
In ΔABC, AD is median. [∵ Given]
Hence, $ar(\text{ABD}) = ar(\text{ACD})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔEBC, ED is median. [∵ Given]
Hence, $ar(\text{EBD}) = ar(\text{ECD})$ ... (2)

Subtracting equation (2) from (1), we get
$ar(\text{ABD}) - ar(\text{EBD}) = ar(\text{ACD}) - ar(\text{ECD})$
$\Rightarrow ar(\text{ABE}) = ar(\text{ACE})$

Question 2:

In a triangle ABC, E is the mid-point of median AD. Show that $ar(\text{BED}) = \frac{1}{4}ar(\text{ABC})$.



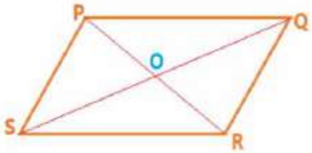
Answer 2:

In ΔABC, AD is median. [∵ Given]
Hence, $ar(\text{ABD}) = ar(\text{ACD})$
$\Rightarrow ar(\text{ABD}) = \frac{1}{2}ar(\text{ABC})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔABD, BE is median. [∵ E is the mid-point of AD]
Hence, $ar(\text{BED}) = ar(\text{ABE})$
$\Rightarrow ar(\text{BED}) = \frac{1}{2}ar(\text{ABD})$
$\Rightarrow ar(\text{BED}) = \frac{1}{2}\left[\frac{1}{2}ar(\text{ABC})\right]$ [$\because ar(\text{ABD}) = \frac{1}{2}ar(\text{ABC})$]
$\Rightarrow ar(\text{BED}) = \frac{1}{4}ar(\text{ABC})$

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.



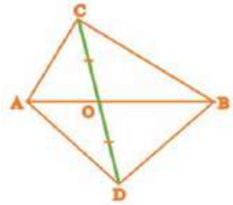
Answer 3:

Diagonals of parallelogram bisect each other.
Therefore, PO = OR and SO = OQ
In ΔPQS, PO is median. [∵ SO = OQ]
Hence, $ar(\text{PSO}) = ar(\text{PQO})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]
Similarly, in ΔPQR, QO is median. [∵ PO = OR]
Hence, $ar(\text{PQO}) = ar(\text{QRO})$ ... (2)
And in ΔQRS, RO is median. [∵ SO = OQ]
Hence, $ar(\text{QRO}) = ar(\text{RSO})$ ... (3)
From the equations (1), (2) and (3), we get
$ar(\text{PSO}) = ar(\text{PQO}) = ar(\text{QRO}) = ar(\text{RSO})$

Hence, in parallelogram PQRS, diagonals PR and QS divide it into four triangles in equal area.

Question 4:

In Figure, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).



Answer 4:

In ΔADC, AO is median. [$\because$ CO = OD]

Hence, $ar(\text{ACO}) = ar(\text{ADO})$ ... (1)

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔBDC, BO is median. [$\because$ CO = OD]

Hence, $ar(\text{BCO}) = ar(\text{BDO})$ ... (2)

Adding equation (1) and (2), we get

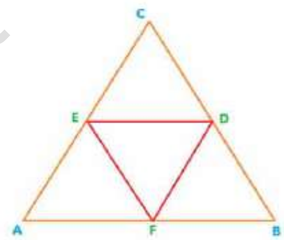
$ar(\text{ACO}) + ar(\text{BCO}) = ar(\text{ADO}) + ar(\text{BDO})$

$\Rightarrow ar(\text{ABC}) = ar(\text{ABD})$

Question 5:

D, E and F are respectively the mid-points of the sides BC, CA and AB of a Δ ABC. Show that

(i) BDEF is a parallelogram. **(ii)** $ar(\text{DEF}) = \frac{1}{4}ar(\text{ABC})$ **(iii)** $ar(\text{BDEF}) = \frac{1}{2}ar(\text{ABC})$



Answer 5:

(i) In ΔABC, E and D are mid-points of CA and BC respectively।

Hence, ED || AB and ED = $\frac{1}{2}$AB [$\because$ Mid-point theorem]

$\Rightarrow$ ED || AB and ED = FB [$\because$ F is mid-point of AB]

$\Rightarrow$ BDEF is a parallelogram.

(ii) BDEF is a parallelogram. [$\because$ Proved above]

$ar(\text{DEF}) = ar(\text{BDF})$ ... (1)

[$\because$ Diagonal of a parallelogram divide it into two triangles, equal in area]

Similarly,

AEDF is a parallelogram.

$ar(\text{DEF}) = ar(\text{AEF})$ ... (2)

तथा AEDF is a parallelogram.

$ar(\text{DEF}) = ar(\text{CDE})$ ... (3)

From the equation (1), (2) and (3), we get

$ar(\text{DEF}) = ar(\text{BDF}) = ar(\text{AEF}) = ar(\text{CDF})$

Let $ar(\text{DEF}) = ar(\text{BDF}) = ar(\text{AEF}) = ar(\text{CDF}) = x$

Therefore, $ar(\text{ABC}) = ar(\text{DEF}) + ar(\text{BDF}) + ar(\text{AEF}) + ar(\text{CDF})$

$\Rightarrow ar(\text{ABC}) = x + x + x + x = 4x = 4ar(\text{DEF})$

$\Rightarrow ar(\text{DEF}) = \frac{1}{4}ar(\text{ABC})$

(iii) $ar(\text{BDEF}) = ar(\text{DEF}) + ar(\text{BDF}) = x + x = 2x$

$\Rightarrow ar(\text{BDEF}) = \frac{1}{2} \times 4x$

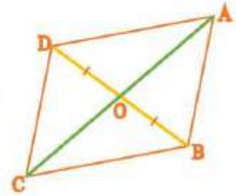
$\Rightarrow ar(\text{BDEF}) = \frac{1}{2} \times ar(\text{ABC})$ [$\because ar(\text{ABC}) = 4x$]

Question 6:

In Figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB) **(ii)** ar (DCB) = ar (ACB) **(iii)** DA || CB or ABCD is a parallelogram.

[*Hint*: From D and B, draw perpendiculars to AC.]



Answer 6:

(i) Construction: Draw perpendiculars DM and BN form D and B respectively to AC.

In ΔDMO and ΔBNO,

∠DMO = ∠BNO [∵ Each 90°]

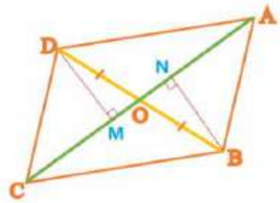
∠DOM = ∠BON [∵ Vertically opposite angles]

DO = BO [∵ Given]

Hence, ΔDMO ≅ΔBNO [∵ AAS Congruency rule]

DM = BN ... (1) [∵ CPCT]

And $ar(\text{DMO}) = ar(\text{BNO})$ ... (2) [∵ CPCT]



In ΔDMC and ΔBNA,

∠DMC = ∠BNA [∵ Each 90°]

DM = BN [∵ From the equation (1)]

CD = AB [∵ Given]

Hence, ΔDMC ≅ΔBNA [∵ RHS Congruency rule]

And $ar(\text{DMC}) = ar(\text{BNA})$ ... (3) [∵ Congruent triangles area equal in area]

Adding the equation (2) and (3), we get

$ar(\text{DMO}) + ar(\text{DMC}) = ar(\text{BNO}) + ar(\text{BNA})$

$\Rightarrow ar(\text{DOC}) = ar(\text{AOB})$

(ii) $ar(\text{DOC}) = ar(\text{AOB})$ [∵ Proved above]

Adding $ar(\text{BOC})$ both sides

$ar(\text{DOC}) + ar(\text{BOC}) = ar(\text{AOB}) + ar(\text{BOC})$

$\Rightarrow ar(\text{DCB}) = ar(\text{ACB})$

(iii) ΔDMC ≅ΔBNA [∵ Proved above]

∠DCM = ∠BAN [∵ CPCT]

Here, the alternate angles (∠DCM = ∠BAN) are equal, Hence,

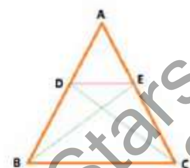
CD || AB

And CD = AB [∵ Given]

Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE||BC.



Answer 7:

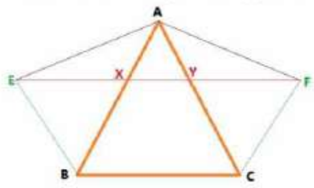
ΔDBC and ΔEBC are on the same base BC and $ar(\text{DBC}) = ar(\text{EBC})$.

Therefore, DE || BC

[∵Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that: $ar(ABE) = ar(ACF)$



Answer 8:

In quadrilateral BCYE, BE || CY [∵ BE || AC]

BC || EY [∵ BC || XY]

Therefore, BCYE is a parallelogram.

Triangle ABE and parallelogram BCYE are on the same base BE and between same parallels, BE || AC.

Hence, $ar(ABE) = \frac{1}{2} ar(BCYE)$ ... (1)

[∵If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Similarly, triangle ACF and parallelogram BCFX are on the same base CF and between same parallels CF || AB.

Hence, $ar(ACF) = \frac{1}{2} ar(BCFX)$ ... (2)

And, $ar(BCYE) = ar(BCFX)$ ... (3)

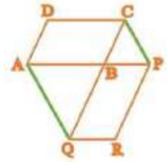
[∵ On the same base (BC) and between same parallels (BC || EF), area of parallelograms are equal]

From the equation (1), (2) and (3), $ar(ABE) = ar(ACF)$

Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Figure). Show that ar (ABCD) = ar (PBQR).

[*Hint*: Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]



Answer 9:

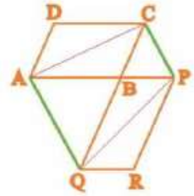
Construction: Join AC and PQ.

Triangles ACQ and APQ lie on the same base AQ and between same parallels, AQ || CP.

Hence, $ar(ACQ) = ar(APQ)$

[∵ Triangles on the same base (or equal) and between the same parallels are equal in area.]

Subtracting $ar(ABQ)$ from both the sides



$ar(ACQ) - ar(ABQ) = ar(APQ) - ar(ABQ)$

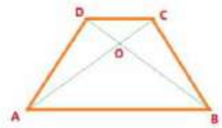
$\Rightarrow ar(ABC) = ar(PBQ) \Rightarrow \frac{1}{2} ar(ABCD) = \frac{1}{2} ar(PBQR)$

[∵ Diagonal divides the parallelogram in two triangles equal in area]

$\Rightarrow ar(ABCD) = ar(PBQR)$

Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).



Answer 10:

Triangles ABD and ABC are on the same base AB and between same parallels, AB || CD.

Hence, $ar(ABD) = ar(ABC)$

[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

Subtracting $ar(ABO)$ form both the sides

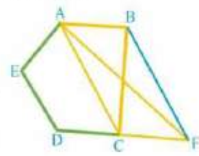
$ar(ABD) - ar(ABO) = ar(ABC) - ar(ABO)$

$\Rightarrow ar(AOD) = ar(BOC)$

Question 11:

In Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) $ar(ACB) = ar(ACF)$ **(ii)** $ar(AEDF) = ar(ABCDE)$



Answer 11:

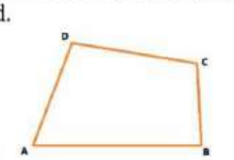
(i) Triangles ACB and ACF are on the same base AC and between same parallels AC || FB.
Hence, $ar(ACB) = ar(ACF)$
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

(ii) $ar(ACB) = ar(ACF)$ [∵ Proved above]
Adding $ar(AEDC)$ both the sides
$ar(ACB) + ar(AEDC) = ar(ACF) + ar(AEDC)$
$\Rightarrow ar(ABCDE) = ar(AEDF)$

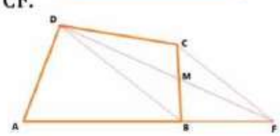
Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

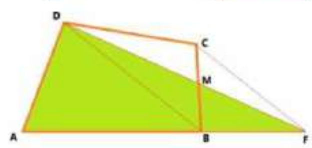
Answer 12:



Let ABCD be the Itwaari's plot.
Join BD and through C draw a line CF parallel to BD which meet AB produced at F.
Now join D and F.
Triangles CBD and FBD are on the same base BD and between same parallels BD || CF.
Hence, $ar(CBD) = ar(FBD)$



[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]
Subtracting $ar(BDM)$ from both the sides
$ar(CBD) - ar(BDM) = ar(FBD) - ar(BDM)$
$\Rightarrow ar(CMD) = ar(BFM)$

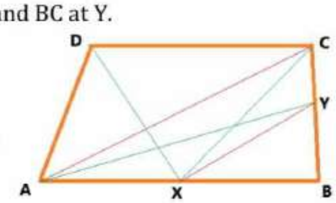


Hence, in place of ΔCMD, if ΔBFM be given to Itwaari, his plot become triangular (ΔADF).

Question 13:

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY). [*Hint*: Join CX.]

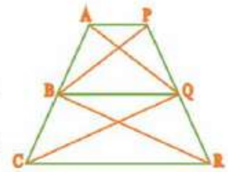
Answer 13:



Construction: Join CX.
Triangles ADX and ACX are on the same base AX and between same parallels AB || DC.
Hence, $ar(ADX) = ar(ACX)$ ... (1)
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]
Similarly, triangles ACY and ACX are on the same base AC and between same parallels AC || XY.
Hence, $ar(ACY) = ar(ACX)$ ... (2)
From the equation (1) and (2), $ar(ADX) = ar(ACY)$

Question 14:

In Figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

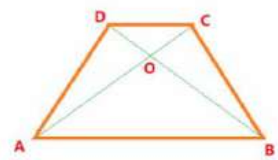


Answer 14:

Triangles ABQ and PBQ are on the same base BQ and between same parallels BQ || AP.
Hence, $ar(ABQ) = ar(PBQ)$ ... (1)
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]
Similarly,
Triangles BQC and BQR are on the same base BQ and between same parallels BQ || CR.
Hence, $ar(BQC) = ar(BQR)$ ... (2)
Adding equation (1) and (2), we get
$ar(ABQ) + ar(BQC) = ar(PBQ) + ar(BQR)$
$\Rightarrow ar(AQC) = ar(PBR)$

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

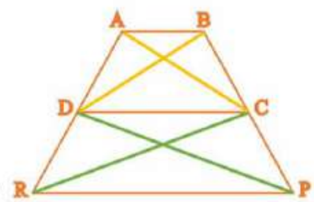


Answer 15:

$ar(AOD) = ar(BOC)$ [∵ Given]
Adding $ar(AOB)$ both the sides
$ar(AOD) + ar(AOB) = ar(BOC) + ar(AOB)$
$\Rightarrow ar(ABD) = ar(ABC)$
ΔABD and ΔABC are on the same base AB and $ar(ABD) = ar(ABC)$.
Therefore, AB || DC
[∵Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, ABCD is a trapezium.

Question 16:

In Figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Answer 16:

$ar(DRC) = ar(DPC)$ ... (1) [∵ Given]
ΔDRC and ΔDPC are on the same base DC and $ar(DRC) = ar(DPC)$.
Therefore, DC || RP
[∵ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, DCPR is a trapezium.

And $ar(ARC) = ar(BDP)$ ... (2) [∵ Given]

Subtracting equation (1) form equation (2), we get
$ar(ARC) - ar(DRC) = ar(BDP) - ar(DPC)$

$\Rightarrow ar(ADC) = ar(BDC)$

ΔADC and ΔBDC are on the same base DC and $ar(ADC) = ar(BDC)$.

Therefore, AB || DC

[∵ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, ABCD is a trapezium.

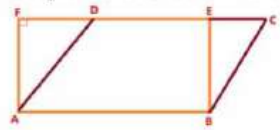
Mathematics

(Chapter – 9)(Areas of Parallelograms and Triangles)
(Class – 9)

Exercise 9.4 (Optional)

Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.



Answer 1:

In ΔAFD,

∠F = 90° [∵ Angle of a rectangle]

AD > AF [∵ In a right triangle, hypotenuse is the longest side]

Adding AB on both the sides, AD + AB > AF + AB

Multiplying both sides by 2,

2[AD + AB] > 2[AF + AB]

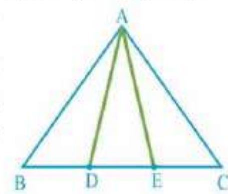
⇒ Perimeter of parallelogram > Perimeter of rectangle

Question 2:

In Figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?

[**Remark:** Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into *n* equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]



Answer 2:

In ΔABE, AD is median. [∵ BD = DE]

Hence, $ar(ABD) = ar(AED)$ ... (1)

[∵ A median of a triangle divides it into two triangles of equal areas.]

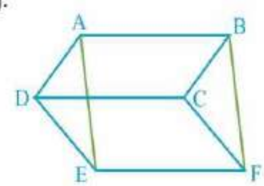
Similarly, in ΔADC, AE is median. [∵ DE = EC]

Hence, $ar(ADE) = ar(AEC)$ ... (2)

From the equation (1) and (2), $ar(ABD) = ar(ADE) = ar(AEC)$

Question 3:

In Figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Answer 3:

In ΔADE and ΔBCF,

AD = BC [∵ Opposite sides of parallelogram ABCD]

DE = CF [∵ Opposite sides of parallelogram DCFE]

AE = BF [∵ Opposite sides of parallelogram ABFE]

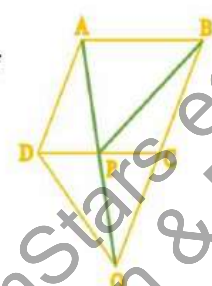
Hence, ΔADE ≅ ΔBCF [∵ SSS Congruency rule]

Hence, $ar(ADE) = ar(BCF)$ [∵ Congruent triangles are equal in area also]

Question 4:

In Figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).

[*Hint* : Join AC.]



Answer 4:

In ΔADP and ΔQCP,

∠APD = ∠QPC [∵ Vertically Opposite Angles]

∠ADP = ∠QCP [∵ Alternate angles]

AD = CQ [∵ Given]

Hence, $\Delta ABD \cong \Delta ACD$ [∵ AAS Congruency rule]
Therefore, DP = CP [∵ CPCT]
In ΔCDQ, QP is median. [∵ DP = CP]
Hence, $ar(DPQ) = ar(QPC)$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

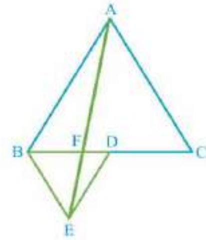
Similarly,
In ΔPBQ, PC is median. [∵ AD = CQ and AD = BC ⇒ BC = QC]
Hence, $ar(QPC) = ar(BPC)$ ... (2)
From the equation (1) and (2),
$ar(BPC) = ar(DPQ)$

Question 5:

In Figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

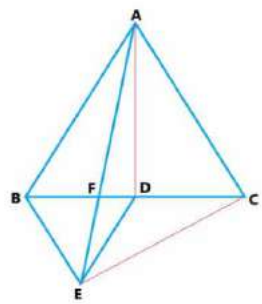
(i) $ar(BDE) = \frac{1}{4} ar(ABC)$ **(ii)** $ar(BDE) = \frac{1}{2} ar(BAE)$
(iii) $ar(ABC) = 2\, ar(BEC)$ **(iv)** $ar(BFE) = ar(AFD)$
(v) $ar(BFE) = 2\, ar(FED)$ **(vi)** $ar(FED) = \frac{1}{8} ar(AFC)$

[*Hint*: Join EC and AD. Show that BE || AC and DE || AB, etc.]



Answer 5:

(i) Construction: Join EC and AD.
Let, $BC = x$
Therefore, $ar(ABC) = \frac{\sqrt{3}}{4}x^2$ [∵ Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$]
And $ar(BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2$ [∵ D is mid-point of BC]
$= \frac{1}{4}\left[\frac{\sqrt{3}}{4}x^2\right] = \frac{1}{4}[ar(ABC)]$



(ii) In ΔBEC, ED is median. [∵ D is mid-point of BC]
Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]
$\angle EBC = 60°$ and $\angle BCA = 60°$ [∵ Angles of equilateral triangles]
Therefore, $\angle EBC = \angle BCA$
Here, Alternate angles ($\angle EBC = \angle BCA$) are equal, Hence, BE || AC
Triangles BEC and BAE are on the same base BE and between same parallels, BE || AC.
Hence, $ar(BEC) = ar(BAE)$ ... (2)
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in]
From the equation (1) and (2),
$ar(BDE) = \frac{1}{2} ar(BAE)$

(iii) In ΔBEC, ED is median. [∵ D is mid-point of BC]
Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$ ... (3)
[∵ A median of a triangle divides it into two triangles of equal areas.]
$ar(BDE) = \frac{1}{4} ar(ABC)$ ... (4) [∵ Proved above in **(i)**]
From the equation (3) and (4),
$ar(ABC) = 2\, ar(BEC)$

(iv) ∠ABD = 60° and ∠BDE = 60° [∵ Angles of equilateral triangle]
Therefore, ∠ABD =∠BDE
Here, Alternate angles (∠ABD =∠BDE) are equal,
Hence, BA || ED
Triangles BDE and AED are on the same base ED and between same parallels BA || ED.
Hence, $ar(\text{BDE}) = ar(\text{AED})$
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in]
Subtracting $ar(\text{FED})$ form both the sides
$ar(\text{BDE}) - ar(\text{FED}) = ar(\text{AED}) - ar(\text{FED})$
$\Rightarrow ar(\text{BEF}) = ar(\text{AFD})$

(v) In ΔBEC, $AD^2 = AB^2 - BD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Rightarrow AD = \frac{\sqrt{3}a}{2}$

In ΔLED, $EL^2 = DE^2 - DL^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16} \Rightarrow EL = \frac{\sqrt{3}a}{4}$

Therefore, $ar(\text{AFD}) = \frac{1}{2} \times \text{FD} \times \text{AD} = \frac{1}{2} \times \text{FD} \times \frac{\sqrt{3}a}{2}$ ... (5)

And $ar(\text{EFD}) = \frac{1}{2} \times \text{FD} \times \text{EL} = \frac{1}{2} \times \text{FD} \times \frac{\sqrt{3}a}{4}$ ... (6)

From the equation (5) and (6),
$ar(\text{AFD}) = 2\, ar(\text{FED})$ ... (7)
$\Rightarrow ar(\text{BFE}) = 2\, ar(\text{FED})$ [∵ Comparing with **(iv)**]

(vi) $ar(\text{BDE}) = \frac{1}{4}\, ar(\text{ABC})$ [∵ From the equation **(i)**]

$\Rightarrow ar(\text{BEF}) + ar(\text{FED}) = \frac{1}{4}\, ar(\text{ABC})$

$\Rightarrow ar(\text{BEF}) + ar(\text{FED}) = \frac{1}{4}[2\, ar(\text{ADC})]$ [∵ $ar(\text{ABC}) = 2\, ar(\text{ABC})$]

$\Rightarrow 2\, ar(\text{FED}) + ar(\text{FED}) = \frac{1}{2}[ar(\text{ADC})]$ [∵ From the equation **(v)**]

$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2}[ar(\text{AFC}) - ar(\text{AFD})]$

$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2}[ar(\text{AFC}) - 2ar(\text{FED})]$ [∵ From the equation (7)]

$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC}) - \frac{1}{2} \times 2ar(\text{FED})$

$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC}) - ar(\text{FED})$

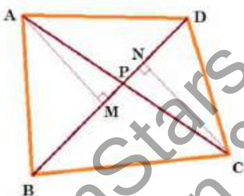
$\Rightarrow 4\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC})$

$\Rightarrow ar(\text{FED}) = \frac{1}{8} ar(\text{AFC})$

Question 6:

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar (APB) × ar (CPD) = ar (APD) × ar (BPC). [*Hint*: From A and C, draw perpendiculars to BD.]

Answer 6:



Construction: From the points A and C, draw perpendiculars AM and CN on BD.

$ar(\text{APB}) \times ar(\text{CPD}) = \frac{1}{2} \times \text{BP} \times \text{AM} \times \frac{1}{2} \times \text{PD} \times \text{CN}$ ... (1)

$ar(\text{APD}) \times ar(\text{BPC}) = \frac{1}{2} \times \text{PD} \times \text{AM} \times \frac{1}{2} \times \text{BP} \times \text{CN}$ ... (2)

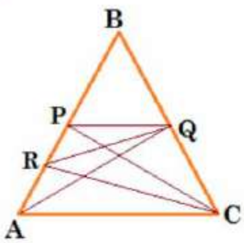
From the equation (1) and (2), $ar(\text{APB}) \times ar(\text{CPD}) = ar(\text{APD}) \times ar(\text{BPC})$

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) $ar(PRQ) = \frac{1}{2}ar(ARC)$ **(ii)** $ar(RQC) = \frac{3}{8}ar(ABC)$ **(iii)** $ar(PBQ) = ar(ARC)$

Answer 7:



Construction: Join AQ, PC, RC and RQ.

(i) In ΔAPQ, QR is median. [∵ Given]

Hence, $ar(PQR) = \frac{1}{2}ar(APQ)$ ... (1)

[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In ΔAQB, QP is median. [∵ Given]

Hence, $ar(APQ) = \frac{1}{2}ar(ABQ)$ ... (2)

And, in ΔABC, AQ is median. [∵ Given]

Hence, $ar(ABQ) = \frac{1}{2}ar(ABC)$ ... (3)

From the equation (1), (2) and (3),

$ar(PQR) = \frac{1}{8}ar(ABC)$ ... (4)

In ΔARC, CR is median. [∵ Given]

Hence, $ar(ARC) = \frac{1}{2}ar(APC)$ ... (5)

[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In ΔABC, CP is median. [∵ Given]

Hence, $ar(APC) = \frac{1}{2}ar(ABC)$ ... (6)

From the equation (5) and (6),

$ar(ARC) = \frac{1}{4}ar(ABC)$ ... (7)

From the equation (4) and (7),

$$ar(PQR) = \frac{1}{8}ar(ABC) = \frac{1}{2}\left[\frac{1}{4}ar(ABC)\right] = \frac{1}{2}ar(ARC)$$

(ii) $ar(RQC) = ar(RQA) + ar(AQC) - ar(ARC)$ ... (8)

In ΔPQA, QR is median. [∵ Given]

Hence, $ar(RQA) = \frac{1}{2}ar(PQA)$ ... (9)

In ΔAQB, PQ is median.

Hence, $ar(PQA) = \frac{1}{2}ar(AQB)$ ... (10)

In ΔABC, AQ is median. [∵ Given]

Hence, $ar(AQB) = \frac{1}{2}ar(ABC)$ ... (11)

From the equation (9), (10) and (11),

$ar(RQA) = \frac{1}{8}ar(ABC)$ ... (12)

In ΔABC, AQ is median. [∵ Given]

Hence, $ar(AQC) = \frac{1}{2}ar(ABC)$ ... (13)

In ΔAPC, CR is median.

Hence, $ar(ARC) = \frac{1}{2}ar(APC)$ ... (14)

In ΔABC, CP is median. [∵ Given]

Hence, $ar(APC) = \frac{1}{2} ar(ABC)$ ... (15)

From the equation (14) and (15),

$ar(ARC) = \frac{1}{4} ar(ABC)$ ... (16)

From the equation (8), (12), (13) and (16),

$$ar(RQC) = \frac{1}{8} ar(ABC) + \frac{1}{2} ar(ABC) - \frac{1}{4} ar(ABC) = \frac{3}{8} ar(ABC)$$

(iii) In ΔABQ, PQ is median. [∵ Given]

Hence, $ar(PBQ) = \frac{1}{2} ar(ABQ)$ ... (17)

In ΔABC, AQ is median.

Hence, $ar(ABQ) = \frac{1}{2} ar(ABC)$ ... (18)

From the equation (16), (17) and (18),

$ar(PQB) = ar(ARC)$

Question 8:

In Figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX⊥ DE meets BC at Y. Show that:

(i) ΔMBC ≅ ΔABD

(ii) $ar(BYXD) = 2\ ar(MBC)$

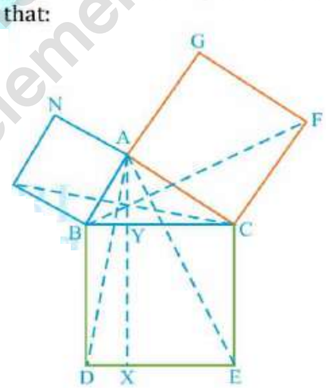
(iii) $ar(BYXD) = ar(ABMN)$

(iv) ΔFCB ≅ ΔACE

(v) $ar(CYXE) = 2\ ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$



Answer 8:

(i) In ΔMBC and ΔABD,

AB = AC [∵ Sides of square]

∠MBC = ∠ABD [∵ Each 90° + ∠ABC]

MB = AB [∵ Sides of square]

Hence, ΔMBC ≅ ΔABD [∵ SAS Congruency rule]

(ii) Triangle ABD and parallelogram BYXD are on the same base BD and between same parallels AX || BD.

Hence, $ar(ABD) = \frac{1}{2} ar(BYXD)$ ... (1)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

But, ΔMBC ≅ ΔABD [∵ Proved above]

Therefore, $ar(MBC) = ar(ABD)$ ... (2)

From the equation (1) and (2),

$ar(MBC) = \frac{1}{2} ar(BYXD)$ ... (3)

$\Rightarrow 2\ ar(MBC) = ar(BYXD)$

(iii) Triangle MBC and square ABMN are on the same base MB and between same parallels MB || NC.

Hence, $ar(\text{MBC}) = \frac{1}{2} ar(\text{ABMN})$ ... (4)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (3) and (4),

$ar(\text{BYXD}) = ar(\text{ABMN})$

(iv) In ΔACE and ΔBCF,

CE = BC [∵ Sides of square]

∠ACE = ∠BCF [∵ Each 90° + ∠BCA]

AC = CF [∵ Sides of square]

Hence, ΔACE ≅ ΔBCF [∵ SAS Congruency rule]

(v) Triangle ACE and square CYXE are on the same base CE and between same parallels CE || AX.

Hence, $ar(\text{ACE}) = \frac{1}{2} ar(\text{CYXE})$

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

$\Rightarrow ar(\text{FCB}) = \frac{1}{2} ar(\text{CYXE})$ ... (5) [∵ $ar(\text{FCB}) = ar(\text{ACE})$]

$\Rightarrow 2\, ar(\text{FCB}) = ar(\text{CYXE})$

(vi) Triangle BCF and square ACFG are on the same base CF and between same parallels CF || FG.

Hence, $ar(\text{BCF}) = \frac{1}{2} ar(\text{ACFG})$ ... (6)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (5) and (6),

$\Rightarrow ar(\text{CYXE}) = ar(\text{ACFG})$

(vii) From the result of **(iii)**, we have

$ar(\text{BYXD}) = ar(\text{ABMN})$ ... (7)

From the result of **(vi)**, we have

$ar(\text{CYXE}) = ar(\text{ACFG})$ ... (8)

Adding (7) and (8), we get

$ar(\text{BYXD}) + ar(\text{CYXE}) = ar(\text{ABMN}) + ar(\text{ACFG})$

$\Rightarrow ar(\text{BCED}) = ar(\text{ABMN}) + ar(\text{ACFG})$