Exercise – 9.1

1. Write the first terms of each of the following sequences whose n[th] term are

(i) $a_n = 3n + 2$

(ii) $a_n = \frac{n-2}{3}$

(iii) $a_n = 3^n$

(iv) $a_n = \frac{3n-2}{5}$

(v) $a_n = (-1)^n 2^n$

(vi) $a_n = \frac{n(n-2)}{2}$

(vii) $a_n = n^2 - n + 1$

(viii) $a_n = n^2 - n + 1$

(ix) $a_n = \frac{2n-3}{6}$

Sol:

We have to write first five terms of given sequences

(i) $a_n = 3n + 2$

Given sequence $a_n = 3n + 2$

To write first five terms of given sequence put n = 1, 2, 3, 4, 5, we get

$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$

$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$

$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$

$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$

$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$

∴ The required first five terms of given sequence $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = \frac{n-2}{3}$

Given sequence $a_n = \frac{n-2}{3}$

To write first five terms of given sequence $a_n = \frac{n-2}{3}$

put n = 1, 2, 3, 4, 5 then we get

$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$

$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$

$a_5 = \frac{5-2}{3} = 1$

∴ The required first five terms of given sequence $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

(iii) $a_n = 3^n$

Given sequence $a_n = 3^n$

To write first five terms of given sequence, put n = 1, 2, 3, 4, 5 in given sequence.

Then,

$a_1 = 3^1 = 3; a_2 = 3^2 = 9; a_3 = 27; a_4 = 3^4 = 81; a_5 = 3^5 = 243.$

(iv) $a_n = \frac{3n-2}{5}$

Given sequence, $a_n = \frac{3n-2}{5}$

To write first five terms, put n = 1, 2, 3, 4, 5 in given sequence $a_n = \frac{3n-2}{5}$

Then, we ger

$a_1 = \frac{3\times1-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

$a_2 = \frac{3\times2-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

$a_3 = \frac{3\times3-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

$a_4 = \frac{3\times4-2}{5} = \frac{12-2}{5} = \frac{10}{5}$

$a_5 = \frac{3\times5-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

∴ The required first five terms are $\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \frac{13}{5}$

(v) $a_n = (-1)^n 2^n$

Given sequence is $\mathrm{a_n} = (-1)^{\mathrm{n}} 2^{\mathrm{n}}$

To get first five terms of given sequence an, put n = 1, 2, 3, 4, 5.

$a_1 = (-1)^1 . 2^1 = (-1). 2 = -2$

$a_2 = (-1)^2 . 2^2 = (-1). 4 = 4$

$a_3 = (-1)^3 . 2^3 = (-1). 8 = -8$

$a_4 = (-1)^4 . 2^4 = (-1). 16 = 16$

$a_5 = (-1)^5 . 2^5 = (-1). 32 = -32$

∴ The first five terms are -2, 4, -8, 16, -32.

(vi) $a_n = \frac{n(n-2)}{2}$

The given sequence is, $a_n = \frac{n(n-2)}{2}$

To write first five terms of given sequence $a_n = \frac{n(n-2)}{2}$

Put n = 1, 2, 3, 4, 5. Then, we get

$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$

$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$

$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$

$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$

$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$

∴ The required first five terms are $\frac{-1}{2}, 0, \frac{3}{2}, 4, \frac{15}{2}$.

(vii) $a_n = n^2 - n + 1$

The given sequence is, $a_n = n^2 - n + 1$

To write first five terms of given sequence a_{n1} we get put n = 1, 2, 3, 4, 5. Then we get $a_1 = 1^2 - 1 + 1 = 1$

$a_2 = 2^2 - 2 + 1 = 3$

$a_3 = 3^2 - 3 + 1 = 7$

$a_4 = 4^2 - 4 + 1 = 13$

$a_5 = 5^2 - 5 + 1 = 21$

∴The required first five terms of given sequence $a_n = n^2 - n + 1$ are 1, 3, 7, 13, 21

(viii) $a_n = 2n^2 - 3n + 1$

The given sequence is $a_n = 2n^2 - 3n + 1$

To write first five terms of given sequence a_n, we put n = 1, 2, 3, 4, 5. Then we get

$a_1 = 2.1^2 - 3.1 + 1 = 2 - 3 + 1 = 0$

$a_2 = 2.2^2 - 3.2 + 1 = 8 - 6 + 1 = 3$

$a_3 = 2.3^2 - 3.3 + 1 = 18 - 9 + 1 = 10$

$a_4 = 2.4^2 - 3.4 + 1 = 32 - 12 + 1 = 21$

$a_5 = 2.5^2 - 3.5 + 1 = 50 - 15 + 1 = 36$

∴ The required first five terms of given sequence $a_n - 2n^2 - 3n + 1$ are 0, 3, 10, 21, 36

(ix) $a_n = \frac{2n-3}{6}$

Given sequence is, $a_n = \frac{2n-3}{6}$

To write first five terms of given sequence we put n = 1, 2, 3, 4, 5. Then, we get,

$a_1 = \frac{2.1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$

$a_2 = \frac{2.2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$

$a_3 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$

$a_4 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$

$a_5 = \frac{2.5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$

∴ The required first five terms of given sequence $a_n = \frac{2n-3}{6}$ are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

2. Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

(iii) $a_n = n(n-1)(n-2); a_5\ and\ a_8$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

Sol:

We have to find the required term of a sequence when nth term of that sequence is given.

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

Given nth term of a sequence $a_n = 5n - 4$

To find 12th term, 15th terms of that sequence, we put n = 12, 15 in its nth term.

Then, we get

$a_{12} = 5.12 - 4 = 60 - 4 = 56$

$a_{15} = 5.15 - 4 = 15 - 4 = 71$

∴ The required terms $a_{12} = 56, a_{15} = 71$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

Given nth term is (a_n) = $\frac{3n-2}{4n+5}$

To find 7th, 8th terms of given sequence, we put n = 7, 8.

$a_7 = \frac{(3.7)-2}{(4.7)+5} = \frac{19}{33}$

$a_8 = \frac{(3.8)-2}{(4.8)+5} = \frac{22}{37}$

∴ The required terms $a_7 = \frac{19}{33}$ and $a_8 = \frac{22}{37}$.

(iii) $a_n = n(n-1)(n-2); a_5\ and\ a_8$

Given nth term is $a_n = n(n-1)(n-2)$

To find 5th, 8th terms of given sequence, put n= 5, 8 in an then, we get

$a_5 = 5(5-1).(5-2) = 5.4.3 = 60$

$a_8 = 8.(8-1).(8-2) = 8.7.6 = 336$

∴ The required terms are $a_5 = 60\ and\ a_8 = 336$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

The given nth term is $a_n = (n+1)(2-n)(3+n)$

To find a_1, a_2, a_3 of given sequence put n = 1, 2, 3 is an

$a_1 = (1-1)(2-1)(3+1) = 0.1.4 = 0$

$a_2 = (2-1)(2-2)(3+2) = 1.0.5 = 0$

$a_3 = (3-1)(2-3)(3+3) = 2.-1.6 = -12$

∴ The required terms a_1 = 0, $a_2 = 0$, $a_3 = -12$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

The given nth term is, $a_n = (-1)^n.n$

To find $a_3.a_5, a_8$ of given sequence put n = 3, 5, 8, in a_n.

$a_3 = (-1)^3.3 = -1.3 = -3$
$a_5 = (-1)^5.5 = -1.5 = -5$
$a_8 = (-1)^8 = 1.8 = 8$
∴ The required terms $a_3 = -3, a_5 = -5, a_8 = 8$

3. Find the next five terms of each of the following sequences given by:
(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$
(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$
(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$
(iv) $a_1 = 4, a_n = 4\,a_{n-1} + 3, n > 1$
Sol:
We have to find next five terms of following sequences.
(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$
Given, first term (a_1) = 1,
n^th term $a_n = a_{n-1} + 2, n \geq 2$
To find 2^nd, 3^rd, 4^th, 5^th, 6^th terms, we use given condition n ≥ 2 for n^th term $a_n = a_{n-1} + 2$
$a_2 = a_{2-1} + 2 = a_1 + 2 = 1 + 2 = 3\ (\because a_1 = 1)$
$a_3 = a_{3-1} + 2 = a_2 + 2 = 3 + 2 = 5$
$a_4 = a_{4-1} + 2 = a_3 + 2 = 5 + 2 = 7$
$a_5 = a_{5-1} + 2 = a_4 + 2 = 7 + 2 = 9$
$a_6 = a_{6-1} + 2 = a_5 + 2 = a + 2 = 11$
∴ The next five terms are,
$a_2 = 3, a_3 = 5, a_4 = 7, a_5 = a, a_6 = 11$
(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$
Given,
First term $(a_1) = 2$
Second term $(a_2) = 2$
n^th term $(a_n) = a_{n-1} - 3$
To find next five terms i.e., a_3, a_4, a_5, a_6, a_7 we put n = 3, 4, 5, 6, 7 is a_n
$a_3 = a_{3-1} - 3 = 2 - 3 = -1$
$a_4 = a_{4-1} - 3 = a_3 - 3 = -1 - 3 = -4$
$a_5 = a_{5-1} - 3 = a_4 - 3 = -4 - 3 = -7$
$a_6 = a_{6-1} - 3 = a_5 - 3 = -7 - 3 = -10$
$a_7 = a_{7-1} - 3 = a_6 - 3 = -10 - 3 = -13$
∴ The next five terms are, $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$
Given, first term $(a_1) = -1$

n^{th} term $(a_n) = \frac{a_n - 1}{n}, n \geq 2$

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n = 2, 3, 4, 5, 6 is an

$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$

$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$

∴ The next five terms are,

$a_2 = \frac{-1}{2}, a_3 = \frac{-1}{6}, a_4 = \frac{-1}{24}, a_5 = \frac{-1}{120}, a_6 = \frac{-1}{720}$

(iv) $a_1 = 4, a_n = 4\, a_{n-1} + 3, n > 1$

Given,

First term $(a_1) = 4$

n^{th} term $(a_n) = 4\, a_{n-1} + 3$, n > 1

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n= 2, 3, 4, 5, 6 is a_n

Then, we get

$a_2 = 4a_{2-1} + 3 = 4.a_1 + 3 = 4.4 + 3 = 19$ ($\therefore a_1 = 4$)

$a_3 = 4a_{3-1} + 3 = 4.a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4\,a_{4-1} + 3 = 4.a_3 + 3 = 4(79) + 3 = 319$

$a_5 = 4\,a_{5-1} + 3 = 4.a_4 + 3 = 4(319) + 3 = 1279$

$a_6 = 4.a_{6-1} + 3 = 4.a_5 + 3 = 4(1279) + 3 = 5119$

∴ The required next five terms are,

$a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$

Exercise – 9.2

1. For the following arithmetic progressions write the first term a and the common difference d:

(i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

(iii) 0.3, 0.55, 0.80, 1.05,.........

(iv) $-1.1, -3.1, -5.1, -7.1, \ldots..$

Sol:

We know that if a is the first term and d is the common difference, the arithmetic progression is $a, a+d, a+2d+a+3d, \ldots\ldots\ldots\ldots$

(i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

Given arithmetic series is

$-5, -1, 3, 7 \ldots\ldots\ldots\ldots$

This is in the form of $a, a+d, a+2d+a+3d, \ldots\ldots\ldots\ldots$ by comparing these two

$a=-5, a+d=1, a+2d=3, a+3d=7, \ldots\ldots$

First term $(a)=-5$

By subtracting second and first term, we get

$(a+d)-(a)=d$

$-1-(-5)=d$

$4=d$

Common difference $(d)=4.$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

Given arithmetic series is,

$\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

This is in the form of $\frac{1}{5}, \frac{2}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

$a, a+d, a+2d, a+3d, \ldots\ldots$

By comparing this two, we get

$a=\frac{1}{5}, a+d=\frac{3}{5}, a+2d=\frac{5}{5}, a+3d=\frac{7}{5}$

$\boxed{First\ term\ \cos=\frac{1}{5}}$

By subtracting first term from second term, we get

$d=(a+d)-(a)$

$d=\frac{3}{5}-\frac{1}{5}$

$d=\frac{2}{5}$

$\boxed{common\ difference(d)=\frac{2}{5}}$

(iii) 0.3, 0.55, 0.80, 1.05,.........

Given arithmetic series,

0.3, 0.55, 0.80, 1.05,.........

General arithmetic series

$a, a+d, a+2d, a+3d, \ldots\ldots$

By comparing,

$a=0.3, a+d=0.55, a+2d=0.80, a+3d=1.05$

First term $(a) = 0.3.$

By subtracting first term from second term. We get

$d = (a+d) - (a)$

$d = 0.55 - 0.3$

$d = 0.25$

Common difference (d) $= 0.25$

(iv) $-1.1, -3.1, -5.1, -7.1, ...$

General series is

$a, a+d, a+2d, a+3d, ...$

By comparing this two, we get

$a = -1.1, a+d = -3.1, a+2d = -5.1, a+3d = -71$

First term $(a) = -1.1$

Common difference $(d) = (a+d) - (a)$

$= -3.1 - (-1.1)$

Common difference (d) $= -2$

2. Write the arithmetic progressions write first term a and common difference d are as follows:

(i) $a = 4, d = -3$

(ii) $a = -1, d = \frac{1}{2}$

(iii) $a = -1.5, d = -0.5$

Sol:

We know that, if first term $(a) = a$ and common difference $= d$, then the arithmetic series is, $a, a+d, a+2d, a+3d, ...$

(i) $a = 4, d = -3$

Given first term $(a) = 4$

Common difference $(d) = -3$

Then arithmetic progression is,

$a, a+d, a+2d, a+3d, ...$

$\Rightarrow 4, 4-3, a+2(-3), 4+3(-3), ...$

$\Rightarrow 4, 1, -2, -5, -8, ...$

(ii) $a = -1, d = \frac{1}{2}$

Given,

First term $(a) = -1$

Common difference $(d) = \frac{1}{2}$

Then arithmetic progression is,

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1, -1+\frac{1}{2}, -1+2\frac{1}{2}, -1+3\frac{1}{2}, ...$

$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}, ...$

(iii) $a = -1.5, d = -0.5$

Given

First term $(a) = -1.5$

Common difference $(d) = -0.5$

Then arithmetic progression is

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1.5, -1.5-0.5, -1.5+2(-0.5), -1.5+3(-0.5)$

$\Rightarrow -1.5, -2, -2.5, -3, ...$

Then required progression is

$-1.5, -2, -2.5, -3, ...$

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.

(ii) The amount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Sol:

(i) Given,

Cost of digging a well for the first meter $(c_1) = Rs.150.$

Cost rises by $Rs.20$ for each succeeding meter

Then,

Cost of digging for the second meter $(c_2) = Rs.150 + Rs\ 20$

$= Rs\ 170$

Cost of digging for the third meter $(c_3) = Rs.170 + Rs\ 20$

$= Rs\ 210$

Thus, costs of digging a well for different lengths are

$150, 170, 190, 210, ...$

Clearly, this series is in $A\cdot p.$

With first term $(a)=150,$ common difference $(d)=20$

(ii) Given

Let the initial volume of air in a cylinder be V liters each time $\frac{3}{4}^{th}$ of air in a remaining i.e.,

$$1-\frac{1}{4}$$

First time, the air in cylinder is $\frac{3}{4}V.$

Second time, the air in cylinder is $\frac{3}{4}V.$

Third time, the air in cylinder is $\left(\frac{3}{4}\right)^2 V.$

Therefore, series is $V,\ \frac{3}{4}V,\left(\frac{3}{4}\right)^2 V,\left(\frac{3}{4}\right)^3 V,.........$

4. Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Sol:

Given sequence is

$a_n = 5n-7$

n^{th} term of given sequence $(a_n)=5n-7$

$(n+1)^{th}$ term of given sequence $(a_n+1)-a_n$

$=(5n-2)-(5n-7)$

$=5$

$\therefore d=5$

5. Show that the sequence defined by $a_n = 3n^2 - 5$ is not an A.P.

Sol:

Given sequence is,

$a_n = 3n^2-5.$

n^{th} term of given sequence $(a_n)=3n^2-5.$

$(n+1)^{th}$ term of given sequence $(a_n+1)=3(n+1)^2-5$

$=3(n^2+1^2+2n.1)-5$

$=3n^2+6n-2$

$\therefore$ The common difference $(d) = a_n + 1 - an$

$d = (3n^2 + 6n - 2) - (3n^2 - 5)$

$= 3a^2 + 6n - 2 - 3n^2 + 5$

$= 6n + 3$

Common difference (d) depends on $'n'$ value

$\therefore$ given sequence is not in $A.p$

6. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15th term and the common difference.

Sol:

Given sequence is,

$a_n = -4n + 15.$

n^{th} term is $(a_n) = -4n + 15$

$(n+1)^{th}$ term is $(a_{n+1}) = -4(n+1) + 15$

$= -4n - 4 + 15$

$= -4n + 11$

Common difference $(d) = a_{n+1} - an$

$= (-4n + 11) - (-4n + 15)$

$= -4n + 11 + 4n - 15$

$d = -4$

Common difference $(d) = a_{n+1} - an$

$= (-4n + 11) + (-4n + 15)$

$= -4n + 11 + 4n - 15$

$d = -4.$

Common difference (d) does not depend on $'n'$ value

$\therefore$ given sequence is in $A.P$

$\Rightarrow 15^{th}$ term $a_{15} = -4(15) + 15$

$= -60 + 15$

$= -45$

$a_{15} = -45$

7. Find the common difference and write the next four terms of each of the following arithmetic progressions:

(i) $1, -2, -5, -8, \ldots\ldots.$

(ii) $0, -3, -6, -9, \ldots\ldots\ldots..$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots\ldots\ldots$

(iv) $-1, \frac{-5}{6}, \frac{-2}{3}, \ldots\ldots\ldots$

Sol:

(i) $1, -2, -5, -8, \ldots\ldots.$

Given arithmetic progression is,

$a_1 = 1, a_2 = -2, a_3 = -5, a_4 = -8 \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= -2 - 1$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -8 - 3 = -11$

$a_6 = a_5 + d = -11 - 3 = -14$

$a_7 = a_6 + d = -14 - 3 = -17$

$a_8 = a_7 + d = -17 - 3 = -20$

$\therefore d = -3, a_5 = -11, a_6 = -16, a_7 = -17, a_8 = -20$

(ii) $0, -3, -6, -9, \ldots\ldots\ldots\ldots$

Given arithmetic progression is.

$0, -3, -6, a_4 = -9 \ldots\ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= -3 - 0$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -9 - 3 = -12$

$a_6 = a_5 + d = -12 - 3 = -15$

$a_7 = a_6 + d = -15 - 3 = -18$

$a_8 = a_7 + d = -18 - 3 = -21$

$\therefore d = -3, a_5 = -12, a_6 = -15, a_7 = -17, a_8 = -21$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots\ldots\ldots$

Given arithmetic progression is,

$-1, \frac{1}{4}, \frac{3}{2}, \ldots\ldots\ldots$

$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= \frac{1}{4} - (-1)$

$= \frac{1+4}{4}$

$d = \frac{5}{4}$

To find next four terms,

$a_4 = a_3 + d = \frac{3}{2} + \frac{5}{4} = \frac{6+4}{4} = \frac{11}{4}$

$a_5 = a_4 + d = \frac{11}{4} + \frac{5}{4} = \frac{16}{4}$

$a_6 = a_5 + d = \frac{16}{4} + \frac{5}{4} = \frac{21}{4}$

$a_7 = a_6 + d = \frac{21}{4} + \frac{5}{4} = \frac{26}{4}$

$\therefore d = \frac{5}{4}, a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}.$

(iv) Given arithmetic progression is,

$-1, \frac{-5}{6}, \frac{-2}{3}, \ldots\ldots\ldots$

$a_1 = -1, a_2 = \frac{-5}{6}, a_3 = \frac{-2}{3}, \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= \frac{-5}{6} - (-1)$

$= \frac{-5+6}{6}$

$= \frac{1}{6}$

To find next four terms,

$a_4 = a_3 + d = \frac{-2}{3} + \frac{1}{6} = \frac{-4+1}{6} = \frac{-\cancel{3}^{1}}{\cancel{6}_{3}} = -\frac{1}{2}.$

$$a_5 = a_4 + d = \frac{-1}{2} + \frac{1}{6} = \frac{-3+1}{6} = \frac{-\cancel{2}^1}{\cancel{6}_3} = -\frac{1}{2}.$$

$$a_6 = a_5 + d = \frac{-1}{3} + \frac{1}{6} = \frac{-2+1}{6} = -\frac{1}{16}.$$

$$a_7 = a_6 + d = \frac{-1}{6} + \frac{1}{6} = 0.$$

$$\therefore d = \frac{1}{6}, a_4 = -\frac{1}{2}, a_5 = -\frac{1}{3}, a_6 = -\frac{1}{6}, a_7 = 0$$

8. Prove that no matter what the real numbers a and b are, the sequence with nth term a + nb is always an A.P. What is the common difference?

Sol:

Given sequence $(a_n) = a_n + 6n$

n^{th} term $(a_n) = a + nb$

$(n+1)^{th}$ term $(a_{n+1}) = a + (n+1)b.$

Common difference (d) $= a_{n+1} - an$

$d = (a + (n+1)b) - (a + nb)$

$= \cancel{a} + \cancel{n}b + b - \cancel{a} - \cancel{nb}$

$= b$

$\therefore$ common difference (d) does not depend on n^{th} value so, given sequence I sin Ap with $(d) = b$

9. Write the sequence with nth term :

(i) $a_n = 3 + 4n$

Sol:

(i) $a_n = 3 + 4n$

Given, n^{th} term $a_n = 3 + 4n$

$(n+1)^{th}$ term $a_{n+1} = 3 + 4(n+1)$

Common difference $(d) = a_{n+1} - a_n$

$= (3 + 4(n+1)) - 3 + 4n$

$= 4.$

$d = 4$ does not depend on n value so, the given series is in $A.p$ and the sequence is

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = a_1 + d = 7 + 4 = 11; a_3 = a_2 + d = 11 + 4 = 15$

$\Rightarrow 7, 11, 15, 19, ...$

(ii) $a_n = 5 + 2n$

Given, n^{th} term $(a_n) = 5 + 2n$

$(n+1)^{th}$ term $(a_{n+1}) = 5 + 2(n+1)$

$= 7 + 2n$

Common difference $(d) = 7 + 2n - 5 - 20$

$= 2.$

$\because d = 2$ does not depend on n value given sequence is in $A.p$ and the sequence is B_1

$a_1 = 5 + 2.1 = 7$

$a_2 = 7 + 2 = 9, a_3 = 9 + 2 = 11, a_4 = 11 + 2 = 13$

$\Rightarrow 7, 9, 11, 13, ...$

(iii) $a_n = 6 - n$

Given, n^{th} term $a_n = 6 - n$

$(n+1)^{th}$ term $a_{n+1} = 6 - (n+1)$

$= 5 - n$

Common difference $(d) = a_{n+1} - a_n$

$= (5 - n) - (6 - n)$

$= -1$

$\therefore d = -1$ does not depend on n value given sequence is in $A.p$ the sequence is

$a_1 = 6 - 1 = 5, a_2 = 5 - 1 = 4, a_3 = 4 - 1 = 3, a_4 = 3 - 1 = 2$

$\Rightarrow 5, 4, 3, 2, 1, ...$

(iv) $a_n = 9 - 5n$

Given, n^{th} term $a_n = 9 - 5n$

$(n+1)^{th}$ term $a_{n+1} = 9 - 5(n+1)$

$= 4 - 5n$

Common difference $(d) = a_{n+1} - a_n$

$= (4 - 5n) - (4 - 5n)$

$= -5$

$\because d = -1$ does not depend on n value given sequence is in $A.p$

The sequence is,

$a_1 = 9 - 1.1 = 4$
$a_2 = 9 - 5.2 = -1$
$a_3 = 9 - 5.3 = -6$
$\Rightarrow 4, -1, -6, -11, ...$

10. Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common differences.

Sol:

(i) $3, 6, 12, 24, ...$

General arithmetic progression is $a, a+d, a+2d, a+3d, ...$

Common difference $(d) =$ Second term – first term

$= (d+d) - a = d$ (or)

$=$ Third term – second term

$= (a+2d) - (a+d) = d$

To check given sequence is in $A.p$ or not we use this condition.

Second term – First term = Third term – Second term

$a_1 = 3, a_2 = 6, a_3 = 12, a_4 = 24$

Second term – First term = $6 - 3 = 3$

Third term – Second term $= 12 - 6 = 6$

This two are not equal so given sequence is not in $A.p$

(ii) $0, -4, -8, -12, ...$

In the given sequence

$a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

Check the condition

Second term – first term = third term – second term

$a_2 - a_1 = a_8 - a_2$

$-4 - 0 = -8 - (-4)$

$-4 = +8 + 4$

$-4 = -4$

Condition is satisfied $\therefore$ given sequence is in $A.p$ with common difference

$(d) = a_2 - a_1 = -4$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, ...$

In the given sequence

$a_1 = \frac{1}{2}, a_2 = \frac{1}{4}, a_3 = \frac{1}{6}, a_4 = \frac{1}{8}.$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\frac{1}{4} - \frac{1}{2} = \frac{1}{6} - \frac{1}{4}$

$\frac{1-2}{4} = \frac{4-6}{24}$

$\frac{-1}{4} = -\frac{2}{24}$

$\frac{-1}{4} \neq \frac{-1}{12}$

Condition is not satisfied

$\therefore$ given sequence not in $A.p$

(iv) $12, 2, -8, -18, ...$

In the given sequence

$a_1 = 12, a_2 = 2, a_3 = -8, a_4 = -18$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$2 - 12 = -8 - 2$

$-10 = -10$

$\therefore$ given sequence is in $A.p$ with common difference $d = -10$

(v) $3, 3, 3, 3, ...$

In the given sequence

$a_1 = 3, a_2 = 3, a_3 = 3, a_4 = 3$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$3 - 3 = 3 - 3$

$0 = 0$

$\therefore$ given sequence is in $A.p$ with common difference $d = 0$

(vi) $p, p+90, p+80, p+270, ...$ where $p = (999)$

In the given sequence

$a_1 = p, a_2 = p+90, a_3 = p+180, a_4 = p+270$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\cancel{p} + 90 \cancel{+p} = \cancel{R} + 180 - \cancel{p} - 90$

$90 = 180 - 90$

$90 = 90$

(vii) 1.0, 1.7, 2.4, 3.1….,
In the given sequence
$a_1 = 1.0, a_2 = 1.7, a_3 = 2.4, a_4 = 31$
Check the condition
$a_2 - a_1 = a_3 - a_2$
$1.7 - 1.0 = 2.4 - 1.7$
$0.7 = 0.7$
$\therefore$ The given sequence is in $A.p$ with $d = 0.7$

(viii) $-225, -425, -625, -825, ...$
In the given sequence
$a_1 = 225, a_2 = -425, a_3 = -625, a_4 = -825$
Check the condition
$a_2 - a_1 = a_3 - a_2$
$-425 + 225 = -625 + 425$
$-200 = -200$
$\therefore$ The given sequence is in $A.p$ with $d = -200$

(ix) $10, 10 + 2^5, 10 + 2^6, 10 + 2^7, ...$
In the given sequence
$a_1 = 10, a_2 = 10 + 2^5, a_3 = 10 + 2^6, a_4 = 10 + 2^7$
Check the condition
$a_2 - a_1 = a_3 - a_2$
$10 + 2^5 - 10 = 10 + 2^6 - 10 - 2^5$
$2^5 \neq 2^6 - 2^5.$
$\therefore$ The given sequence is not in $A.p$

Exercise – 9.3

1. Find:
   (i) 10th term of the AP 1,4 , 7, 10….
   (ii) 18th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$
   (iii) nth term of the AP $13, 8, 3, -2, ...$
   (iv) 10th term of the AP $-40, -15, 10, 35, ...$
   (v) 8th term of the AP 11, 104, 91, 78………
   (vi) 11th term of the AP 10.0, 10.5, 11.0, 11.2…….
   (vii) 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, ...$

Sol:

(i) Given $A.p$ is

$1, 4, 7, 10, ...$

First term $(a) = 1$

Common difference (d) = second term first term

$= 4 - 1$

$= 3.$

n^{th} term in an $A.p = a + (n-1)d$

10^{th} term in an $1 + (10-1)3$

$= 1 + 9.3$

$= 1 + 27$

$= 28$

(ii) Given $A.p$ is

$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

First term $(a) = \sqrt{2}$

Common difference $=$ Second term – First term

$= 3\sqrt{2} - \sqrt{2}$

$d = 2\sqrt{2}$

n^{th} term in an $A.p = a + (n-1)d$

18^{th} term of $A.p = \sqrt{2} + (18-1)2\sqrt{2}$

$= \sqrt{2} + 17.2\sqrt{2}$

$= \sqrt{2}(1 + 34)$

$= 35\sqrt{2}$

$\therefore 18^{th}$ term of $A.p$ is $35\sqrt{2}$

(iii) Given $A.p$ is

$13, 8, 3, -2, ...$

First term $(a) = 13$

Common difference $(d) =$ Second term first term

$= 8 - 13$

$= -5$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

$= 13 + (n-1) - 5$

$= 13 - 5n + 5$

$a_n = 18 - 5n$

(iv) Given $A.p$ is

$-40, -15, 10, 35, ...$

First term $(a) = -40$

Common difference $(d) =$ Second term – first term

$= -15 - (-40)$

$= 40 - 15$

$= 25$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

10^{th} term of $A.p$ $a_{10} = -40 + (10-1)25$

$= -40 + 9.25$

$= -40 + 225$

$= 185$

(v) Given sequence is

$117, 104, 91, 78, ...$

First learn can $= 117$

Common difference $(d) =$ Second term – first term

$= 104 - 117$

$= -13$

n^{th} term $a_n = a + (n-1)d$

8^{th} term $a_8 = a + (8-1)d$

$= 117 + 7(-13)$

$= 117 - 91$

$= 26$

(vi) Given $A.p$ is

$10.0, 10.5, 11.0, 11.5, ...$

First term $(a) = 10.0$

Common difference (d) $=$ Second term – first term

$= 10.5 - 10.0$

$= 0.5$

n^{th} term $a_n = a + (n-1)d$

11^{th} term $a_{11} = 10.0 + (11-1)0.5$

$= 10.0 + 10 \times 0.5$

$= 10.0 + 5$

$=15.0$

(vii) Given $A.p$ is

$\frac{3}{4},\frac{5}{4},\frac{7}{4}+\frac{9}{4},$..............

First term $(a)=\frac{3}{4}$

Common difference $(d)=$ Second term – first term

$=\frac{5}{4}-\frac{3}{4}$

$=\frac{2}{4}$

n^{th} term $a_n=a+(n-1)d$

9^{th} term $a_9=a+(9-1)d$

$=\frac{3}{4}+8.\frac{2}{4}$

$=\frac{3}{4}+\frac{16}{4}$

$=\frac{19}{4}$

2. (i) Which term of the AP 3, 8, 13, …. is 248?
(ii) Which term of the AP 84, 80, 76, … is 0?
(iii) Which term of the AP 4, 9, 14, …. is 254?
(iv) Which term of the AP 21, 42, 63, 84, … is 420?
(v) Which term of the AP 121, 117, 113, … is its first negative term?

Sol:

(i) Given $A.p$ is $3,8,13,$.........

First term $(a)=3$

Common difference $(d)=$ Second term – first term

$=8-3$

$=5$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term $a_n=248$

$248=3+(n-1).5$

$248=-2+5n$

$5n=250$

$n=\frac{250}{5}=50$

50^{th} term is 248.

(ii) Given $A.p$ is 84, 80, 76, ...

First term $(a)=84$

Common difference $(d)=a_2-a$

$=80-84$

$=-4$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 0

$0=84+(n-1)-4$

$+84=+4(n-1)$

$n-1=\frac{84}{4}=21$

$n=21+1=22$

22^{nd} term is 0.

(iii) Given $A.p$ 4, 9, 14, ...

First term $(a)=4$

Common difference $(d)=a^2-a$

$=9-4$

$=5$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 254

$4+(n-1)5=254$

$(n-1)\cdot 5=250$

$n-1=\frac{250}{5}=50$

$n=51$

$\therefore 51^{st}$ term is 254.

(iv) Given $A.p$

21, 42, 63, 84, ...

$a=21, d=a_2-a$

$=42-21$

$=21$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $= 420$

$21 + (n-1)21 = 420$

$(n-1)21 = 399$

$n - 1 = \frac{399}{21} = 19$

$n = 20$

$\therefore 20^{th}$ term is 420.

(v) Given $A.p$ is $121, 117, 113, \ldots\ldots\ldots\ldots$

First term $(a) = 121$

Common difference $(d) = 117 - 121$

$= -4$

n^{th} term $(a) = a + (n-1)d$

Given n^{th} term is negative i.e., $a_n < 0$

$121 + (n-1) - 4 < 0$

$121 + 4 - 4n < 0$

$125 - 4n < 0$

$4n > 125$

$n > \frac{125}{4}$

$n > 31.25$

The integer which comes after 31.25 is 32.

$\therefore 32^{nd}$ term is first negative term

3. (i) Is 68 a term of the AP 7, 10, 13,….?
(ii) Is 302 a term of the AP 3, 8, 13, …. ?
(iii) Is −150 a term of the AP 11, 8, 5, 2, …?

Sol:

In the given problem, we are given an $A.p$ and the Value of one of its term

We need to find whether it is a term of the $A.p$ or not so here we will use the formula

$a_n = a + (n-1)d$

(i) Here, $A.p$ is $7, 10, 13, \ldots\ldots\ldots\ldots$

$a_n = 68, a = 7$ and $d = 10 - 7 = 3$

Using the above mentioned formula, we get

$68 = 7 + (n-1)3$

$\Rightarrow 68-7=3n-3$

$\Rightarrow 31+3=3n$

$\Rightarrow 64=3n$

$\Rightarrow n=\dfrac{64}{3}$

Since, the value of n is a fraction.

Thus, 68 is not the team of the given $A.p$

(ii) Here, $A.p$ is $3,8,13,..........$

$a_n=302, a=3$

Common difference $(d)=8-3=5$ using the above mentioned formula, we get

$302=3+(n-1)5$

$\Rightarrow 302-3=5n-5$

$\Rightarrow 299=5n-5$

$\Rightarrow 5n=304$

$\Rightarrow n=\dfrac{305}{5}$

Since, the value of $'n'$ is a fraction. Thus, 302 is not the term of the given $A.p$

(iii) Here, $A.p$ is $11,8,5,2,...........$

$a_n=-150, a=1$ and $d=8-11=-3$

Thus, using the above mentioned formula, we get

$-150=11+(x-1)(-3)$

$\Rightarrow -150-11=-34+3$

$\Rightarrow -161-3=-34$

$\Rightarrow -34=-164$

$\Rightarrow n=\dfrac{164}{3}$

Since, the value of n is a fraction. Thus, -150 is not the term of the given $A.p$

4. How many terms are there in the AP?

(i) $7,10,13,..............43$

(ii) $-1,\dfrac{-5}{6},\dfrac{-2}{3},\dfrac{-1}{2},.........,\dfrac{10}{3}.$

(iii) $7,13,19,...........05$

(iv) $18,15\dfrac{1}{2},13,..........-47$

Sol:

(i) $7,10,13,..............43$

From given $A.p$

$a=7, d=10-7=3, a_n=a+(n-1)d.$

Let, $a_n=43$ (last term)

$7+(n-1)3=43$

$(n-1)=\frac{\cancel{26}^{12}}{\cancel{3}}=12$

$n=13$

$\therefore 13$ terms are there in given $A.p$

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ..., \frac{10}{3}.$

From given $A.p$

$a=-1, d=-\frac{5}{6}+1, \quad a_n=a+(n-1)d$

$=\frac{1}{6}$

Let, $a_n=\frac{10}{3}$ (last term)

$-1+(n-1)\frac{1}{6}=\frac{10}{3}$

$(n-1)\times\frac{1}{6}=\frac{10}{3}$

$(n+1)=\frac{13\times\cancel{6}^{2}}{\cancel{3}}=26$

$n=27$

$\therefore 27$ terms are there in given $A.p$

(iii) $7, 13, 19, ...05$

From the given $A.p$

$a=7, d=13-7=6, a_n=a+(n-1)d$

Let, $a_n=205$ (last term)

$7+(n-1)6=205$

$(n-1).6=198$

$n-1=33$

$n=34$

$\therefore 34$ terms are there in given $A.p$

(iv) $18, 15\frac{1}{2}, 13, ...-47$

From the given $A.p.$,

$a = 18, d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = 15\cdot5 - 18 = -2\cdot5$

$a_n = a + (n-1).d$

Let $a_n = -47$ (last term)

$18 + (n-1), 2.5 = -47$

$12.5(n-1) = +65$

$n - 1 = \frac{65}{2\times5} = \frac{65\times10}{25} = 26$

$n = 27$

$\therefore 27$ terms are there in given $A.p$

5. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.

Sol:

Given

First term $(a) = 5$

Common difference $(d) = 3$

Last term $(1) = 80$

To calculate no of terms in given $A.p$

$a_n = a + (n-1)d$

Let $a_n = 80$,

$80 = 5 + (n-1)\cdot3$

$75 = (n-1)\cdot3$

$n - 1 = \frac{75}{3} = 25$

$n = 26$

$\therefore$ There are 26 terms.

6. The 6th and 17 terms of an A.P. are 19 and 41 respectively, find the 40th term.

Sol:

Given, $a_6 = 19, a_{17} = 41$

$\Rightarrow a_6 = a + (6-1)d$

$19 = a + 5d$ ……….(1)

$\Rightarrow a_{17} = a + (17-1)\cdot d$

$41 = a + 16d$ ………..(2)

Subtract (1) from (2)

$a + 16d = 41$

$a + 5d = 19$

$0 + 11d = 22$

$d = \frac{22}{11} = 2$

Substitute $d = 2$ in (1)

$19 = a + 5(2)$

$9 = a$

$\therefore 40^{th}$ term $a_{40} = a + (40-1)\cdot d$

$= 9 + 39\cdot 2$

$= 9 + 78$

$= 87$

$\therefore a_{40} = 87$

7. If 9th term of an A.P. is zero, prove that its 29th term is double the 19th term.

Sol:

Given

9^{th} term of an $A.p$ $a_9 = 0, a_n = a + (n-1)d$

$a + (a-1)\cdot d = 0$

$a + 8d = 0$

$a = -8d$

We have to prove

24^{th} term is double the 19^{th} term $a_{29} = 2\cdot a_{19}$

$a + (29-1)d = 2[a + (1a-1).d]$

$a + 28d = 2[a + 18a]$

Put $a = -8d$

$-8d + 28d = 2[-8d + 18d]$

$20d = 2\times 10d$

$20d = 20d$

Hence proved

8. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that 25th term of the A.P. is zero.

Sol:

Given,

10 times of 10^{th} term is equal to 15 times of 15^{th} term.

$10a_{10} = 15.a_{15}$

$10\left[a+(10-1)d\right] = 15\left[a+(15-1).d\right]\left(\because a_n = a+(n-1)d\right)$

$10(a+9d) = 15(a+14\cdot d)$

$a+9d = \frac{15}{10}(a+14d)$

$a-\frac{3}{2}a = \frac{42d}{2}-9d$

$-\frac{1}{2}a = \frac{24^{12}}{2}\cdot d$

$-a = +24\cdot d$

$a = -24\cdot d$

We have to prove 25^{th} term of $A.p$ is 0

$a_{25} = 0$

$a+(25-1)d = 0$

$a+24d = 0$

Put $a = -24d$

$-24\times d + 24d = 0$

$0 = 0$

Hence proved.

9. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Sol:

Given,

$a_{10} = 41, a_{18} = 73, a_n = a+(n-1)\cdot d$

$\Rightarrow a_{10} = a+(10-1)\cdot d$

$41 = a+9d$ ....(1)

$\Rightarrow a_{18} = a+(18-1)d$

$73 = a+17d$ ....(2)

Subtract (1) from (2)

(2) (1)

$a+17d=73$
$a+9d=41$
$0+8d=32$
$d=\frac{32}{8}=4$
Substitute $d=4$ in (1)
$a+9\cdot4=41$
$a=41-36$
$a=5$
26^{th} term $a_{26}=a+(26-1)d$
$=5+25.4$
$=5+100$
$=105$
$\therefore 26^{th}$ term $a_{26}-105.$

10. In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Sol:
Given
24^{th} term is twice the 10^{th} term
$a_{24}=2\ a_{10}$
Let, first term of a square $=a$
Common difference $=d$
n^{th} term $a_n=a+(n-1)d$
$a+(24-1)d=(a+(10-1).d)2$
$a+23d=2(a+9d)$
$(23-18)d=a$
$a=5d$
We have to prove
72^{nd} term is twice the 34^{th} term
$a_{12}=2a_{34}$
$a+(12-1)d=2[a+(34-1)d]$
$a+71d=2a+66d$
Substitute $a=5d$
$5d+71d=2(5d)+66d$
$76d=10d+66d$

$76d = 76d$

Hence proved.

11. If (m + 1)th term of an A.P. is twice the (n + 1)th term, prove that (3m + 1)th term is twice the (m+n+l)th term.

Sol:

Given

$(m+1)^{th}$ term is twice the $(m+1)^{th}$ term.

First term $= a$

Common difference $= d$

n^{th} term $a_n = a + (n-1).d$

$a_{m+1} = 2\ a_n + 1$

$a + (m+1-1)\cdot d = 2(a + (n+1-1)\cdot d)$

$a + md = 2(a + nd)$

$a = (m-2n)d$

We have to prove

$(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term

$a_{3m+1} = 2\cdot a_{m+n+1}$

$a + (3m+1-1)\cdot d = (a + (m+n+1-1)\cdot d)$

$a + 3m\cdot d = 2a + 2(m+n)d$

Substitute $a = (m-2n)\cdot d$

$(m-2n)\not{d} + 3m\not{d} = 2(m-2n)\not{d} + 2(m+n)\cdot\not{d}$

$4m - 2n = 4m - 4n + 2n$

$4m - 2n = 4m - 2n$

Hence proved.

12. If the n term of the A.P. 9, 7, 5, ... is same as the th term of the A.P. 15, 12, 9, ... find n.

Sol:

Given,

First sequence is $9, 7, 5, ...$

$a = 9, d = \neq -9 = -2, a_n = a + (n+1)d$

$a_n = 9 + (n-1). - 2$

Second sequence is $15, 12, 9, ...$

$a = 15, d = 12 - 15 = -3, a_n = a + (n-1)d$

$a_n = 15 + (n-1) - 3$

Given an. a_n are equal

$9 - 2(n-1) = 15 - 3(n-1)$

$3(n-1) - 2(n-1) - 15 - 9$

$n - 1 = 6$

$n = 7$

$\therefore 7^{th}$ term of two sequence are equal

13. Find the 12th term from the end of the following arithmetic progressions:
(i) 3 51 7, 9, ... 201 (ii) 3, 8, 13, ... , 253 (iii) 1, 4, 7, 10, ..., 88

Sol:

(i) $3, 5, 7, 9, \ldots\ldots\ldots\ldots 2d$

First term $(a) = 3$

Common difference $(d) = 5 - 3 = 2$

12^{th} term from the end is can be considered as (1) last term = first term and common differnce $= d^1 = -d$ n^{th} term from the end = last term $+(n-1) - d$

12^{th} term from end $= 201 + (12-1)(-2)$

$= 201 - 22$

$= 179$

(ii) $3, 8, 13, \ldots\ldots\ldots\ldots 253$

First term $= a = 3$

Common difference $d = 8 - 3 = 5$

Last term $(1) = 253$

n^{th} term of a sequence on $= a + (n-1) \cdot d$

To find n^{th} term from the end, we put last term (1) as 'a'and common difference as $-d$

n^{th} term from the end = last term $+(n-1) \cdot -d$

12^{th} term from the end $= 253 + (12-1) - 5$

$= 253 - 55$

$= 198$

$\therefore 12^{th}$ term from the end $= 198$

(iii) $1, 4, 7, 10, \ldots\ldots\ldots\ldots\ldots 88$

First term $a = 1$

Common differnce $d = 4 - 1 = 3$

Last term $(1) = 88$

n^{th} term $a_n = a + (n+1)\cdot d$

n^{th} term from the end = last term $+(n-1)\cdot -d$

12^{th} term from the end $= 88 + (12-1)\cdot -3$

$= 88 - 33$

$= 55$

$\therefore 12^{th}$ term from the end $= 55$.

14. The 4th term of an A.P. is three times the first and the 7 term exceeds twice the third term by 1. Find the first term and the common difference.

Sol:

Given,

4^{th} term of an $A.p$ is three times the times the first term

$a_4 = 3.\ a$

n^{th} term of a sequence $a_n = a + (n-1)\cdot d$

$a + (4-1)\cdot d = 3a$

$a + 3d = 3a$

$3d = 2a$

$a = \frac{3}{2}d.$ ………….(1)

Seventh term exceeds twice the third term by 1.

$a_7 + 1 = 2.a_3$

$a + (7-1)\cdot d + 1 = 2(\alpha + \beta - 1\cdot d)$

$a + 6d + 1 = 2a + 4d$

$a = 2d + 1$ ………….(2)

By equating (1), (2)

$\frac{3}{2}d = 2d + 1$

$\frac{3}{2}d - 2d = 1$

$\frac{3d - 4d}{2} = 1$

$-d = 2$

$d = -2$

Put $d = -2$ in $a = \frac{3}{2}d$

$=\frac{3}{2}.\,x$

$=-3$

$\therefore$ First term $a=-3,$ common difference $d=-2.$

15. Find the second term and nth term of an A.P. whose 6th term is 12 and the 8th term is 22.

Sol:

Given

$a_6=12, a_8=22$

n^{th} term of an $A.p$ $a_n=a+(n-1)d$

$a_6=a+(n-1)\cdot d=a+(6-1)d=a+5d=12$ ....(1)

Subtracting (1) from (2)

$$a+7d=22$$

$$(2)\ (1)\Rightarrow \frac{a+5d=12}{0+2d=10}$$

$$d=5$$

Put $d=5$in

$$a+5d=12$$
$$a+55=12$$
$$a=12-25$$
$$a=-13$$

Second term $a_2=a+(2-1)\cdot d$

$=a+d$

$=-13+5$

$a_1=-8$

n^{th} term $a_n=a+(n-1)d$

$=-13+(n-1)-5$

$a_n=-18+5n$

n^{th} term $a_n=a+(n-1)d$

$=-13+(n-1)-5$

$a_n=-18+5n$

$\therefore a_2=-8, a_n=-18+5n$

16. How many numbers of two digits are divisible by 3?

Sol:

We observe that 12 is the first two-digit number divisible by 3 and 99 is the last two digit number divisible by 3. Thus, the sequence is

$12, 15, 18, \ldots\ldots\ldots\ldots\ldots 99$

This sequence is in $A.p$ with

First term $(9) = 12$

Common difference $(d) = 15 - 12 = 3$

n^{th} term $a_n = 99$

n^{th} term of an $A.p(a_n) = a + (n-1) \cdot d$

$99 = 12 + (n-1) \cdot 3$

$99 - 12 = n - 1 \cdot 3$

$\frac{87}{3} = n = 1$

$n = 30$

$\therefore 30$ term are there in the sequence

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Sol:

Given

No. of terms $= n = 60$

First term $(a) = 7$

Last term $a_{10} = 125$

$a_{60} = a + (60-1) \cdot d$ $\quad (\therefore a_n = a + (n-1)d)$

$125 = 7 + 59 \cdot d$

$118 = 59d$

$d = \frac{118}{59} = 2$

52^{nd} term $a_{32} = a + (32-1)d$

$= 7 + 31.2$

$= 7 + 62$

$= 69$

18. The sum of 4 and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Sol:
Given

$a_4 + a_8 = 24$

$a_6 + a_{10} = 34$

$\Rightarrow a + (4-1)d + a + (18-1)d = 24$

$2a + 10d = 24$

$a + 5d = 12$ ....(1)

$\Rightarrow a_6 + a_{10} = 34$

$a + (6-1)d + a + (10-1)d = 34$

$2a + 14d = 34$

$a + 7d = 17$ ....(2)

Subtract (1) from (2)

$a + 7d = 17$

$a + 5d = 12$

$2d = 5$

$d = \frac{5}{2}$

Put $d = \frac{5}{2}$ in $a + 5d = 12$

$a = 12 - 5 \cdot \frac{5}{2}$

$a = \frac{24-25}{2} = \frac{-1}{2}$

$\therefore a = -\frac{1}{2}, d = \frac{b}{2}$

19. The first term of an A.P. is 5 and its 100th term is — 292. Find the 50th term of this A.P.

Sol:
Given,

$a_{30} - a_{20} = a + (30-1)d - (a + (20-1)d) (\because a_n = a + (n-1)d)$

$= a + 29d - a - 19d$

$= 10d$

(i) $-9, -14, -19, -24, ...$

Common difference (d) = second tern – first term

$= -14 - (-9)$

$= -14 + 9$

$d = 5$

Then $a_{30}-a_{20}=10d$

$=10.5$

$a_{30}-a_{20}=50$

(ii) $a, a+d, a+2d, a+3d$

First term $(a)=a$

Common difference $(d)=d$

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)$

$=a+29d-0-19d$

$a_{30}-a_{20}=10d$

20. Find a_{30} — a_{20} for the A.P.

(i) $-9,-14,-19,-24,........$

(ii) a, a + d, a + 2d, a + 3d,...

Sol:

Given,

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)(\therefore a_n=a+(n-1)d)$

$=a+29d-a-19d$

$=10d$

(i) $-9,-14,-19,-24,........$

Common difference (d) = second term – first term

$=-14-(-9)$

$=-14+9$

$d=5$

Then $a_{30}-a_{20}=10d$

$=10.5$

$a_{30}-a_{20}=50$

(ii) $a, a+d, a+2d, a+3d,............$

First term $(a)=a$

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)$

$=a+29d-a-19d$

$a_{30}-a_{20}=10d$

21. Write the expression $a_n - a_k$ for the A.P. a, a + d, a + 2d, ...
Hence, find the common difference of the AP for which

(i) 11^{th} term $a_n=5$ and 13^{th} term $a_{13}=79$

(ii) $a_{10}-a_5=200$

(iii) 20^{th} term is 10 more than the 18^{th} term.

Sol:

General arithmetic progression

$a, a+d, a+2d, \ldots\ldots\ldots$

$a_b-a_k=a+(n-1)d-(a+(k-1)d)(\therefore a_n=a+(n-1)d)$

$=a+(n-1)d-2d+(k-1)d$

$a_n-a_k=(n-k)d$ ....(1)

(i) Given

11^{th} term $a_n=5$

13^{th} term $a_{13}=79$

By using (1) put $n=13, k=11$

$a_n-a_k=(n-k)\cdot d$

$79-5=(13-11)\cdot d$

$74=2\times d$

$d=\frac{74}{2}=37$

(ii) Given

$a_{10}-a_5=200$

From (1) $a_{10}-a_5=(10-5)d$

$200=5\cdot d$

$d=\frac{200}{5}=40 \Rightarrow d=40$

(iii) Given

$a_{20}\neq 10=a_{18}$

$a_{20}-a_{18}=10$

By (1) $a_n-a_k=(n-k)\cdot d$

$a_{20}-a_{18}=(20-18)\cdot d$

$10=2\cdot d$

$d=\frac{10}{2}=5$

$\therefore d=5$

22. Find n if the given value of x is the n term of the given A.P.

(i) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, ..., x = \frac{141}{11}$

(ii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, ...: x = 550$

(iii) $-1, -3, -5, -7, ...: x = -151$

(iv) $25, 50, 75, 100.........: c = 1000$

Sol:

(i) $25, 50, 75, 100.........: c = 1000$

First term $(a) = 25$

Common difference $(d) = 50 - 25 = 25$

n^{th} term $a_n = a + (n-1) \times d$

Given, $a_n = 1000$

$1000 = 25 + (n-1) \cdot 25$

$975 = (n-1) \times 25$

$n - 1 = \frac{975}{25} = 39$

$n = 40$

(ii) Given sequence $-1, -3, -5, -7, ...: x = -151$

First term $(a) = -1$

Common difference $(d) = -3 - (-1) = -3 + 1 = -2$

n^{th} term $a_n = a + (n-1)d$

Given $a_n = -151$,

$-151 = -1 + (n-1) - 2$

$-150 = 1(n+1)2$

$n + 1 = \frac{150}{2} = 75$

$n = 74$

(iii) Given sequence is

$5\frac{1}{2}, 11, 16\frac{1}{2}, 22, ...: x = 550$

First term (a) $= 5\frac{1}{2} = \frac{11}{2}$

$= \frac{22-11}{2}$

$=\frac{11}{2}$

n^{th} term $a_n=a+(n-1)d$

$550=\frac{11}{2}+(n-1)\cdot\frac{11}{2}$

$550=\frac{11}{2}[1+n-1]$

$n=550\times\frac{2}{11}$

$n=100$

(iv) Given sequence is

$1,\frac{21}{11},\frac{31}{11},\frac{41}{11},........,x=\frac{141}{11}$

First term $(a)=1$

Common difference $(d)=\frac{21}{11}-1$

$=\frac{21-11}{11}$

$=\frac{10}{11}$

n^{th} term $a_n=a+(n-1)\times d$

$\frac{171}{11}=1+(n-1).\frac{10}{11}$

$\frac{171}{11}-1=(n-1)\frac{10}{11}$

$\frac{171-11}{11}=(n-1)\frac{10}{11}$

$\frac{160}{11}=(n-1).\frac{10}{11}$

$n-1=\frac{160}{11}\times\frac{11}{10}$

$n=17$

23. If an A.P. consists of n terms with first term a and nth term 1 show that the sum of the mth term from the beginning and the mth term from the end is (a + l).

Sol:

First term of a sequence is a

Last term $=1$

Total no. of terms $= n$

Common difference $= d$

m^{th} term from the beginning $a_m = a + (n-1) \cdot d$

m^{th} term from the end = last term $+(n-1) - d$

$a_n - m + 1 = 1 - (n-1) \times d$

$\Rightarrow a_m + a_n - m + 1 = a + (n-1)d + (l - (n-1)d)$

$= a + (n-1)d + l - (n-1)d$

$a_m + a_n - m + 1 = a + l$

Hence proved

24. Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Sol:

Given, $a_3 = 16$

$a + (3-1)d = 16$

$a + 2d = 16.$ ....(1)

And $a_7 - 12 = a_5$

$a + (7-1)d - 12 = a + (5-1)d$ $(\because a_n = a + (n-1)d)$

$\not{a} + 6d - 12 = \not{a} + 4d$

$2d = +12$

$d = +\frac{12}{2} = +6$

Put $d = -6$ in (1)

$a + 2(+6) = 16$

$a + 12 = 6$

$a = 284$

Then the sequence is $a, a+d, a+2d, a+3d, ...$

$\Rightarrow 28, 4, 10, 16, 22, ...$

25. The 7th term of an A.P. is 32 and its 13th term is 62. Find the A.P.

Sol:

Given,

$a + = 32$

$a + (7-1)d = 32$

$a + 6d = 32$ ....(1)

And $a_{13} = 62$

$a + (13-1)d = 62$

$a + 12d = 62$ ....(2)

Subtract (1) from (2)

$$\begin{array}{rl} & a+12d = 62 \\ (2)-(1) \Rightarrow & a+6d = 32 \\ \hline & 0+6d = 32 \end{array}$$

$d = \frac{30}{6} = 5$

Put $d = 5$ in $a + 6d = 32$

$a + 6 \cdot 5 = 32$

$a = 2$

Then the sequence is $a, a+d, a+2d, a+3d, ...$

$\Rightarrow 2, 7, 12, 17, ...$

26. Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13th term?

Sol:

Given $A.p$ is $3, 10, 17, ...$

First term $(a) = 3$, Common difference $(d) = 10 - 3$

$= 7$

Let, n^{th} term of $A.p$ will be 84 more than 13^{th} term

$a_n = 84 + a_{13}$

$\cancel{a} + (n-1)d = \cancel{a} + (13-1)d + 84$

$(n-1)7 = 12.7 + 84$

$(n-1) \cdot 7 = 168$

$n - 1 = \frac{168}{7} = 24$

$n = 25$

Hence 25^{th} term of given $A.p$ is 84 more than 13^{th} term

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their l000th terms?

Sol:

Let the two $A.p$ is be $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$

$a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1) \cdot d$

Since common difference of two equations is same given difference between 100^{th} terms is 100

$a_{100} - b_{100} = 100$

$a_1 + (99)d - b_1 - 99d = 100$

$a_1 - b_1 = 100$ ....(1)

Difference between. 1000^{th} terms is

$a_{1000} - b_{1000} = a_1 + (1000-1)d - (b_1 + (1000-1)d)$

$= a_1 + 999d - b_1 - 999d$

$= a_1 - b_1$

$= 100$ (from (1))

$\therefore$ Hence difference between 1000^{th} terms of two $A.p$ is 100.

28. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, . . . and 3, 10, 17, . . . are equal?

Sol:

Given two $A.p$ is are

$63, 65, 67..........$ and $3, 10, ...$

First term of sequence 1 is $a_1 = 63$

Common difference $d_1 = 65 - 63$

$= 2.$

n^{th} term $(a_n) = a_1 + (n-1)d$

$= 63 + (n-1)d$

First term of sequence 2 is $b_1 = 3.$

Common difference $d_2 = 10 - 3$

$= 7$

n^{th} term $(b_n) = b_1 + (n-1)d_2$

$= 3 + (n-1) \cdot 7$

Let n^{th} terms of two sequence is equal

$63 + (n-1)2 = 3 + (n-1) \times 7$

$60 = 5(n-1)$

$n - 1 = \frac{60}{5} = 12$

$n = 13$

$\therefore$ 13^{th} term of both the sequence are equal.

29. How many multiples of 4 lie between 10 and 250?

Sol:

Multiple of 4 after 10 is 12 and multiple of 4 before 250 is $\frac{250}{4}$ remainder is 2, so,

$250-2=248$

248 is the last multiple of 4 before 250.

The sequence is

$12,........,248$

With first term $(a)=12$

Last term $(1)=248$

Common difference $(d)=4$

n^{th} term $a_n=a+(n-1)\cdot d$

Here, n^{th} term $a_n=248$

$248=12+(n-1)\times 4$

$236=(n-1)\times 4$

$n-1=\frac{236}{4}=59$

$n=60$

$\therefore$ There are 60 terms between 10 and 250 which are multiples of 4

30. How many three digit numbers are divisible by 7?

Sol:

The three digit numbers are $100,......999$ 105 is the first 3 digit number which is divisible by 7 when we divide 999 with 7 remainder is 5. So, 999 – 5 = 994 is the last three digits divisible by 7 so, the sequence is

$105,..........,994$

First term $(a)=105$

Last term $(1)=994$

Common difference $(d)=-7$

Let there are n numbers in the sequence

$a_n=994$

$a+(n-1)d=994$

$a+(n-1)d=994$

$105+(n-1)7=994$

$(n-1)\cdot 7=889$

$n-1=\frac{889}{7}=127$

$n=128$

$\therefore$ there are 128 numbers between 105, 994 which are divisible by 7

31. Which term of the arithmetic progression 8, 14, 20, 26, . . . will be 72 more than its 41st term?

Sol:

Given sequence

$8, 14, 20, 26, ...$

Let n^{th} term is 72 more than its 41^{st} term

$a_n = a_{41}+72$

For the given sequence

$a=8, d=14-8=6$

$a+(n-1)d=8+(a+1)6+72$

$8+(n-1)6=8+(90)\cdot 6+72$

$(n-1)6=312$

$n-1=\frac{312}{6}=52$

$n=53$

$\therefore 53^{rd}$ term is 72 more than 41^{st} term

32. Find the term of the arithmetic progression 9, 12, 15, 18,. . . which is 39 more than its 36th term.

Sol:

Given $A.p$ is $9, 12, 15, ...$

For this $a=9, d=12-9=3$

Let n^{th} term is 39 more than its 36^{th} term

$a_n = 39+a_{36}$

$\not{a}+(n-1)3=39+\not{9}+(36-1)\cdot 3 \qquad (\therefore a_n = a+(n-1)d)$

$(n-1)3=39+35\cdot 3$

$(n-1)\times 3=144$

$n-1=\frac{144}{3}=48$

$n=49$

$\therefore 49^{th}$ term is 39 more than its 36^{th} term

33. Find the 8th term from the end of the A.P. 7,1õ13, . . ., 184

Sol:

Given $A.p$ is $7,10,13,.........184$

$a=7, d=10-7=3, l=184$

n^{th} term from the end $=l+(n-1)-d$

8^{th} term from the end $=184+(8-1)-3$

$=184-21$

$=163$

$\therefore 8^{th}$ term from the end -163

34. Find the 10th term from the end of the A.P. 8, 10, 12,. . ., 126.

Sol:

Given $A.p$ is $8,10,12,........126$

$a=8, d=10-8=2, l=126$

n^{th} term from the end $=l+(n-1)-d$

10^{th} term from the end $=126+(10-1)-2$

$=126-18$

$=108$

$\therefore 10^{th}$ term from the end $=108$

35. The sum of 4th and 8th terms of an A.P. is 24 and the sum of 6th and 10th terms is 44. Find the A.P.

Sol:

Given, $a_4+a_8=24$

$(a+(4-1)d)+(a+(8-1)d)=24 (\because a_n=a+(n-1)d)$

$2a+10d=24$

$a+5d=12$ ....(1)

And $a_6+a_{10}=44$

$a+(6-1)+a+(10-1)d=44 \quad (\because a_n=a+(n-1)d)$

$2a+14d=44$

$a+7d=22$ ....(2)

Subtract (1) from (2)

$$\begin{aligned} a+7d &= 22 \\ (2)-(1) \Rightarrow \quad a+5d &= 12 \\ \hline 0+2d &= 10 \end{aligned}$$

$d=5$

Put $d=5$ in (1) $a+5\cdot 5=12$

$a=-13$

36. Which term of the A.P. 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Sol:

Given $A.p$ is

$3, 15, 27, 39, \ldots\ldots..$

Let n^{th} term is 120 more than 21^{st} term

Then $a_n = 120 + a_{21}$

For the given sequence

$a=3, d=15-3=12$

$\cancel{a}+(n-1)d = 120+\cancel{a}+(21-1)d$

$(n-1)12 = 120+20(2)$

$(n-1)12 = 360$

$(n-1) = \frac{360}{12} = 30$

$n=31$

$\therefore$ 31^{st} term is 120 more than 21^{st} term

37. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, find the nth term.

Sol:

Given

17^{th} term of an $A.p$ is 5 more than twice its 8^{th} term

$a_{17} = 5+2a_8$

$a+(17-1)d = 5+2(a+(8-1)\cdot d)$

$a+16d = 5+2a+14d$

$a+5=2d$ …………(1)

And 11^{th} term of the $A.p$ is 43

$a_{11}=43$

$a+(11-1)d=43$

$a_{11} = 43$

$a + (11-1)d = 43$

$a + 10d = 43$ ....(2)

$$(2)-(1) \Rightarrow \begin{array}{r} a+10d=43 \\ a-2d=+5 \\ \hline a+12d=48 \end{array}$$

$d = \frac{48}{12} = 4$

Put $d = 4$ in (1)

$a + 5 = 2(4)$

$a = 3$

$\therefore n^{th}$ term of given sequence is an $= a = 1 + (n-1)d$

$= 3 + (n-1)4$

$= 3 + 4n - 4$

$= 4n - 1$

$\therefore n^{th}$ term of given sequence $a_n = 4n - 1$

Exercise – 9.4

1. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

Sol:

Given,

Sum of three terms of on A.P is 21.

Product of first and the third term exceeds the second term by 6.

Let, the three numbers be a-d, a, a+d, with common difference d: then,

(a – d) + a + (a + d) = 21

3a = 21

a = $\frac{21}{3}$ = 7

and (a – d) (a + d) = a + 6

$a^2 - d^2 = a + 6$

Put a = 7 $\Longrightarrow 7^2 - d^2 = 7 + 6$

49 – 13 = d^2

d = $\pm$6

∴ The three terms are a – d, a, a + d, i.e., 1, 7, 13.

2. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Sol:

Let, the three numbers are a – d, a, a + d.

Given,

(a – d) + a + (a + d) = 27

3a = 27

a = $\frac{27}{3} = 9$

and, $(a-d)(a)(a+d) = 648$

$(a^2 - d^2)(a) = 648$

Put a = 9, then

$(9^2 - d^2)\ 9 = 648$

81 – d^2 = $\frac{648}{9} = 72$

$d^2 = 81 - 72$

$d^2 = 9$

d = 3

∴ The three terms are a – d, a, a + d i.e. 6, 9, 12.

3. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Sol:

Let, the four numbers be a – 3d, a – d, a + d, a + 3d, with common difference 2d.

Given, sum is 50.

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 50

4a = 50

a = 12.5

greater number is 4 time the least

(a + 3d) = 4(a – 3d)

a + 3d = 4a – 12d

15d = 3a

Put a = 12.5

d = $\frac{3}{15} \times 12.5$

d = 2.5

∴ The four numbers are a – 3d, a – d, a + 3d i.e., 12.5 – 3(2.5), 12.5 – 2.5, 12.5 + 2.5, 12.5 + 3(2.5)

⟹ 5, 10, 15, 20

4. The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

Sol:

A quadrilateral has four angles. Given, four angles are in A.P with common difference 10.
Let, the four angles be, a – 3d, a – d, a + d, a + 3d with common difference = 2d.
2d = 10
d = $\frac{10}{2}$ = 5
In a quadrilateral, sum of all angles = 360°
(a – 3d) + (a – d) + (a + d) + (a + 3d) = 360
4a = 360
a = 360/4 = 90°
∴ The angles are a – 3d, a – d, a + d, a + 3d with a = 90, d = 5
i.e. 90 – 3(5), 90 – 5, 90 + 3(5)
⟹ 75°, 85°, 95°, 105°.

5. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.

Sol:

2, 4, 6, or 6, 4, 2.

6. Find the value of x for which (8x + 4), (6x — 2) and (2x + 7) are in A.P.

Sol:

Given,
8x + 4, 6x – 2, 2x + 7 are are A.P.
If the numbers a, b, c are in A.P. then condition is 2b = a + c.
Then, 2(6x – 2) = 8x + 4 + 2x + 7
12x – 4 = 10 + 11
2x = 15
x = $\frac{15}{2}$

7. If x + 1 , 3x and 4x + 2 are in A.P., find the value of x.

Sol:

Given numbers
x + 1, 3x, 4x + 2 are in AP
If a, b, c are in AP then 2b = a + c
Then 2(3x) = x + 1 + 4x + 2
6x = 5x + 3
x = 3

8. Show that $(a - b)^2, (a^2 + b^2)$ and $(a + b)^2$ are in A.P.

Sol:

We have to show, (a – b)2, (a2 + b2) and (a + b)2 are in AP.

If they are in AP. Then they have to satisfy the condition

2b = a + c

$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$

$2a^2 + 2b^2 = a^2 + 2ab + b^2 + a^2 + 2ab + b^2$

$2a^2 + 2b^2 = 2a^2 + 2b^2$.

They satisfy the condition means they are in AP.

Exercise – 9.5

1. Find the sum of the following arithmetic progressions:
   (i) 50, 46, 42, ... to 10 terms
   (ii) 1, 3, 5, 7, ... to 12 terms
   (iii) 3, 9/2, 6, 15/2, ... to 25 terms
   (iv) 41, 36, 31, ... to 12 terms
   (v) a + b, a – b, a − 3b, ... to 22 terms
   (vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots,$ to n terms
   (vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots . to\ n\ terms$
   (viii) $-26, -24, -22, \ldots . to\ 36\ terms$

Sol:

In an A.P let first term = a, common difference = d, and there are n terms. Then, sum of n terms is,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

(i) Given progression is,

50, 46, 42, ……to 10 term.

First term (a) = 50

Common difference (d) = 46 – 50 = −4

n^{th} term = 10

Then $S_{10} = \frac{10}{2}\{2.50 + (10-1) - 4\}$

$= 5\{100 - 9.4\}$

$= 5\{100 - 36\}$

$= 5 \times 64$

$\therefore S_{10} = 320$

(ii) Given progression is,

1, 3, 5, 7, …..to 12 terms

First term difference (d) = 3 – 1 = 2

n^{th} term = 12

Sum of n[th] terms $S_{12} = \frac{12}{2} \times \{2.1 + (12 - 1).2\}$

$= 6 \times \{2 + 22\} = 6.24$

$\therefore S_{12} = 144.$

(iii) Given expression is

$3, \frac{9}{2}, 6, \frac{15}{2}, \dots \dots .\, to\ 25\ terms$

First term (a) = 3

Common difference (d) $= \frac{9}{2} - 3 = \frac{3}{2}$

Sum of n[th] terms $S_n, given\ n = 25$

$S_{25} = \frac{n}{2}(2a + (n - 1).d)$

$S_{25} = \frac{25}{2}\left(2.3 + (25 - 1).\frac{3}{2}\right)$

$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$

$= \frac{25}{2}(6 + 36)$

$= \frac{25}{2}(42)$

$\therefore S_{25} = 525$

(iv) Given expression is,

41, 36, 31, …… to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = – 5

Sum of n[th] terms $S_n, given\ n = 12$

$S_{12} = \frac{n}{2}(2a(n - 1).d)$

$= \frac{12}{6}(2.41 + (12 - 1).-5)$

$= 6(82 + 11.(-5))$

$= 6(27)$

= 162

$\therefore S_{12} = 162.$

(v) a + b, a – b, a – 3b, …… to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = –2b

Sum of n[th] terms $S_n = \frac{n}{2}\{2a(n - 1).d\}$

Here n = 22

$S_{22} = \frac{22}{2}\{2.(a + b) + (22 - 1).-2b\}$

$= 11\{2(a + b) - 22b\}$

= 11 {2a – 20b}

= 22a – 440b

$\therefore S_{22} = 22a - 440b$

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots\ldots to\ n\ terms$

First term (a) = $(x-y)^2$

Common difference (d) = $x^2+y^2-(x-y)^2$

= $x^2+y^2-(x^2+y^2-2xy)$

= $x^2+y^2-x^2+y^2+2xy$

= 2xy

Sum of n[th] terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

= $\frac{n}{2}\{2(x-y)^2+(n-1).2xy\}$

= $n\{(x-y)^2+(n-1)xy\}$

$\therefore S_n = n\{(x-y)^2+(n-1).xy\}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots.. to\ n\ terms$

$First\ term\ (a) = \frac{x-y}{x+y}$

Common difference (d) = $\frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x+y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1).d\}$

$= \frac{n}{2}\left\{2.\frac{x-y}{x+y}+(n-1).\frac{2x-y}{x+y}\right\}$

$= \frac{n}{2(x+y)}\{2(x-y)+(n-1)(2x-y)\}$

$= \frac{n}{2(x+y)}\{2x-2y+2nx-ny-2x+y\}$

$= \frac{n}{2(x+y)}\{n(2x-y)-y\}$

$\therefore S_n = \frac{n}{2(x+y)}\{n(2x-y)-y\}$

(viii) Given expression −26, −24, −22, ……. To 36 terms

First term (a) = −26

Common difference (d) = −24 – (−26) = −24 + 26 = 2

Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1)d\}$

Sum of n terms $S_n = \frac{36}{2}\{2.-26+(36-1)2\}$

= $18[-52+70]$

= 18.18

= 324

∴ S_n = 324

2. Find the sum to n term of the A.P. 5, 2, — 1, —4, —7, ...

Sol:

Given AP is 5, 2, -1, -4, -7, …..

a = 5, d = 2 – 5 = −3

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{2.5 + (n-1) - 3\}$

$= \frac{n}{2}\{10 - 3(n-1)\}$

$= \frac{n}{2}\{13 - 3n\}$

$\therefore S_n = \frac{n}{2}(13 - 3n)$

3. Find the sum of n terms of an A.P. whose th terms is given by $a_n = 5 - 6n$.

Sol:

Given nth term $a_n = 5 - 6n$

Put n = 1, $a_1 = 5 - 6.1 = -1$

We know, first term $(a_1) = -1$

Last term $(a_n) = 5 - 6n = 1$

Then $S_n = \frac{n}{2}(-1 + 5 - 6n)$

$= \frac{n}{2}(4 - 6n) = \frac{n}{2}(2 - 3n)$

4. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Sol:

Given AP is 25, 22, 19, ……

First term (a) = 25, d = 22 – 25 = −3.

Given, $S_n = \frac{n}{2}(2a + (n-1)d)$

$116 = \frac{n}{2}(2 \times 25 + (n-1) - 3)$

$232 = n(50 - 3(n-1))$

232 = n(53 – 3n)

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

(3n – 29) (n – 8) = 0

∴ n = 8

$\Longrightarrow a_8 = 25 + (8 - 1) - 3$

∴ n = 8, $a_8 = 4$

= 25 – 21 = 4

5. (i) How many terms of the sequence 18, 16, 14, ... should be taken so that their

(ii) How many terms are there in the A.P. whose first and fifth terms are −14 and 2 respectively and the sum of the terms is 40?

(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

Sol:

(i) Given sequence, 18, 16, 14, …

$a = 18, d = 16 – 18 = -2.$

Let, sum of n terms in the sequence is zero

$S_n = 0$

$\frac{n}{2}(2a + (n-1)d) = 0$

$\frac{n}{2}(2.18 + (n-1) - 2) = 0$

$n(18 - (n-1)) = 0$

$n(19 - n) = 0$

n = 0 or n = 19

$\because n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

(ii) Given, a = -14, a 5 = 2

a + (5 – 1)d = 2

−14 + 4d = 2

$4d = 16 \implies d = 4$

Sequence is −14, −10, -6, −2, 2, ……

Given $S_n = 40$

$40 = \frac{n}{2}\{2(-14) + (n-1)4\}$

$80 = n(-28 + 4n - 4)$

$80 = n(-32 + 4n)$

4(20) = 4n(−8 + n)

$n^2 - 8n - 20 = 0$

$(n-10)(n+2) = 0$

$n = 10 \; or \; n = -2$

∴ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25, …….

a = 9, d = 17 – 9 = 8, and $S_n = 636$

$636 = \frac{n}{2}(2.9 + (n-1)8)$

$1272 = n(18 - 8 + 8n)$

1272 = n(10 + 8n)

2 × 636 = 2n(5 + 4n)

$636 = 5n + 4n^2$

$4n^2 + 5n – 636 = 0$

(4n + 53) (n – 12) = 0

∴ n = 12 (Since n $\frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57, ……

a =63, d = 60 – 63 = -3 $S_n = 693$

$S_n = \frac{n}{2}(2a + (n-1)d)$

693 = $\frac{n}{2}(2.63 + (n-1) - 3)$

1386 = n(126 – 3n + 3)

1386 = (129 – 3n)n

$3n^2 - 129n + 1386 = 0$

$n^2 - 43n + 462 = 0$

n = 21, 22

∴ Sum of 21 or 22 term is 693

6. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol:

Given, a = 17, l = 350, d = 9

l = $a_n = a + (n-1)d$

350 = 17 + (n – 1)9

333 = (n – 1)9

n – 1 = $\frac{333}{9} = 37$

n = 38

∴ 38 terms are there

$S_n = \frac{n}{2}\{a + l\}$

= $\frac{38}{2}\{17 + 350\}$

= 19.367

∴ $S_n = 6973$

7. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Sol:

Given, $a_3 = 7$ and $3a_3 + 2 = a_7$

$a_7 = 3.7 + 2$

$a_7 = 21 + 2 = 23$

∴ $a_n = a + (n-1)d$

$a_3 = a(3-1)d \ and \ a_7 = a + (7-1)d$

$7 = a + 2d \ldots..(i) \qquad 23 = a + 6d \quad \ldots.(ii)$

$Subtract\ (i) from\ (ii)$

$(ii) - (i) \Longrightarrow$ a + 6d = 23

a + 2d = 7 _

4d = 16

d = 4

Put d = 4 in (i) ⟹ 7 = a + 2.4

a = 7 – 8 = −1

Given to find sum of first 20 terms.

$S_{20} = \frac{20}{2}\{-2 + (10 - 1)4\}$

$= 10(-2 + 76)$

$\therefore S_{20} = 740$

8. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Sol:

Given a = 2, l = 50, $S_n = 442$

$S_n = \frac{n}{2}(a + 1)$

$442 = \frac{n}{2}(2 + 50)$

$442 = \frac{n}{2}.52$

$\therefore$ n = $\frac{442}{26} = 17$

Given, $a_n = 1 = 50$

50 = 2 + (17 – 1) d

48 = 16 × d

d = $\frac{48}{16} = 3$

$\therefore$ d = 3

9. If 12[th] term of an A.P. is—13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Sol:

Given, $a_{12} = -13, a + a_2 + a_3 + a_4 = 24$

$S_4 = \frac{4}{2}(2a + 3d) = 24$

2a + 3d = $\frac{24}{2} = 12 \quad ....(i)$

⟹ a + $(12 - 1)d = -13$

a + 11d = -13 ... ... (ii)

Subtract (i) from (ii) × 2

2 × (ii) – (i) ⟹ 2a + 22d = −28

2a + 3d = 12

198d = −38

d = $\frac{-38}{19} = -2$

put d = -2 in (ii)

a + 11 (−2) = −13
a = −13 + 22
a = 9
Given to find sum of first 10 terms.
$S_{10} = \frac{10}{2}\{2(a) + (10 - 1) - 2\}$
= 5(18 – 18)
= 0
$\therefore S_{10} = 0$

10. Find the sum of first 22 terms of an A.P. in which d = 22 and a = 149.
Sol:
Given, d = 22, $a_{22} = 149$
$a + (22 - 1)\ d$ = 149
$a = -313$
Given, to find $S_{22} = \frac{22}{2}[2a + (22 - 1)d]$
= $11[2(-313) + 21.22]$
= $11[-626 + 462]$
= 11 – 164
= -1804
$\therefore S_{22} = -1804$

11. Find the sum of all natural numbers between 1 and 100, which are divisible by 3.
Sol:
The numbers between 1 and 100 which are divisible by 3 are 3, 6, 9, ….99.
In this sequence, a = 3, d = 3, $a_n = 99$
99 = $a + (n - 1)d$
99 = 3 + (n – 1)3
99 = 3[1 + n – 1]
n = $\frac{99}{3} = 33$
∴ There are 33 numbers in the given sequence
$S_{33} = \frac{33}{2}(2.3 + (33 - 1)3)\ \left(\therefore S_n = \frac{n}{2}(2a + (n - 1)d)\right)$
= $\frac{33}{2}(6 + 96)$
= $\frac{33}{2} \times 102$
= 1683
∴ Sum of all natural numbers between 1 and 100, which are divisible by 3 is 1683.

12. Find the sum of first n odd natural numbers.

Sol:

The sequence is, 1, 3, 5, …..n.

In this first term (a) = 1, common difference (d) = 2

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2.1 + (n-1)2)$

$= \frac{n}{2} \times 2(1 + n - 1)$

$= n^2.$

∴ Sum of first n odd natural numbers is n^2.

13. Find the sum of all odd numbers between (i) 0 and 50 (ii) 100 and 200.

Sol:

(i) Odd numbers between 0 and 50 are 1, 3, 5, ….., 49

In this a = 1, d = 2, l = 49 = a_n

49 = 1 + (n – 1)2 (∴ a_n = a + (n – 1)d)

48 = (n – 1)2

n – 1 = $\frac{48}{2} = 24$

n = 25.

∴ There are 25 terms

$S_{25} = \frac{25}{2}(1 + 49)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

$= \frac{25}{2} \times 50 = 625$

∴ Sum of all odd numbers between 0 and 50 is 625.

(ii) Odd numbers between 100 and 200 are 101, 103, …. 199

In this a = 101, d = 2, l = a_n = 199

199 = 101 + (n – 1)2

n – 1 = $\frac{98}{2} = 49$

n = 50

∴ There are 50 terms.

$S_{50} = \frac{50}{2}(101 + 199)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

$= \frac{50}{2} \times 300$

$= 7500$

∴ Sum of all odd numbers between 100 and 200 is 7500.

14. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Sol:

Odd integers between 1 and 1000 which are divisible by 3 are 3, 6, 9, 15 …… 999.

In this a = 3, d = 5, l = $a_n = 999$

999 = 3 + (n – 1)5 $(\therefore a_n = a + (n-1)d)$

999 = 3[1 + (n – 1)2]

∴ 2n – 1 = $\frac{999}{3} = 333 \Rightarrow n = \frac{334}{2} = 167$

∴ There are 333 numbers 167.

$S_{167} = \frac{333}{2}[3 + 999]$

= $\frac{333}{2} \times 100$ = 83667

∴ $S_{167} = 83667$

∴ Sum of all odd integers between 1 and 4000 which are divisible by 3 is 83667.

15. Find the sum of all integers between 84 and 719, which are multiples of 5.

Sol:

The numbers between 84 and 719, which are multiples of 5 are 85, 90, 95,…..715.

In this, a = 85, d = 5, $a_n = l = 715$

715 = 85 + (n – 1)5 $(\therefore a_n = a + (n-1)d)$

630 = (n – 1)5

$n - 1 = 126$

n = 127

∴ $S_n = \frac{127}{2}(85 + 115)$ $\left(\therefore S_n = \frac{n}{2}(a+1)\right)$

= $\frac{127}{2} \times 800 = 50800$

∴ Sum of all integers between 84 and 719, which are multiples of 5 is 50800.

16. Find the sum of all integers between 50 and 500, which are divisible by 7.

Sol:

Numbers between 50 and 500, which are divisible by 7 are 56, 63, ….., 497.

In this a = 56, d = 7, l = $a_n = 497$

497 = 56 + (n – 1)7

441 = (n – 1)7

n – 1 = $\frac{441}{7}$ = 63

n = 64

∴ There are 64 terms.

$S_{64} = \frac{64}{2}(56 + 497)$

$= 32 \times 553 = 17696$

∴ Sum of all integers between 50 and 500, which are divisible by 7 is 17696.

17. Find the sum of all even integers between 101 and 999.

Sol:

Even integers between 101 and 999 are 102, 104, …..998

a = 102, d = 2, $a_n = l = 998$

998 = 102 + (n – 1)× 2 ($\therefore a_n = a + (n-1)d$)

896 = (n – 1)(2)

n – 1 = 448

n = 449.

∴ 449 terms are there

$S_{449} = \frac{449}{2}[102 + 998]$

$= \frac{449}{2} \times 1100 = 246950$

∴ Sum of all even integers between 101 and 999 is 24690

18. Find the sum of all integers between 100 and 550, which are divisible by 9.

Sol:

Integers between 100 and 550 which are divisible by 9 are 108, 117, …., 549.

In this a = 108, d = 9, $a_n = l = 549$

549 = 108 + (n – 1)× 9 $(\therefore a_n = a + (n+1)d)$

441 = (n – 1) × 9

n – 1 = $\frac{449}{9} = 49$

n = 50.

$\therefore S_{50} = \frac{50}{2}\{108 + 549\}$ $\left(\therefore S_n = \frac{n}{2}(10 + 1)\right)$

= 25 × 657

= 16425

∴ Sum of all integers between 100 and 550, which are divisible by 9 is 16425.

19. In an A.P., if the first term is 22, the common difference is — 4 and the sum to n terms is 64, find n.

Sol:

Given, a = 22, d = −4, $S_n = 64$

$S_n = \frac{n}{2}(2a + (n-1)d)$

64 = $\frac{n}{2} \times (2.22 + (n-1) - 4)$

64 = n(24 – 2n)

64 = 2n (12 – n)

$12n - n^2 = \frac{64}{2} = 32$

$n^2 - 12n + 32 = 0$
$(n - 4)(n - 8) = 0$
∴ n = 4 or 8

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?

Sol:
Given, $a_5 = 30, a_{12} = 65$
⟹ 30 = a + (5 – 1)d
30 = a + 4d .....(i)
⟹ 65 = a + (12 – 1)d
65 = a + 11d .....(ii)
(ii) – (i) ⟹ a + 11d = 65
a + 4d = 30
0 + 7d = 35
d = $\frac{35}{7}$ = 5
put d = 5 in ...(i) ⟹ 80 = a + 4 (5)
a = 30 – 20 = 10
$S_{20} = \frac{20}{2}(2(10) + (20 - 1)5)$ $\left(\therefore S_n = \frac{n}{2}(20 + (n - 1)d)\right)$
= 10[20 + 95]
= 10 × 115
= 1150
∴ Sum of first 20 terms $S_{20} = 1150$

21. Find the sum of the first
(i) 11 terms of the A.P : 2, 6, 10, 14, ...
(ii) 13 terms of the A.P : — 6, 0, 6, 12,....
(iii) 51 terms of the A.P : whose second term is 2 and fourth term is 8.

Sol:
(i)
Given AP, 2; 6, 10, 14, ...
a = 2, d = 4, $S_n = S_{11} = \frac{11}{2}(2.2 + (11 - 1).4)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n - 1)d)\right)$
$= \frac{11}{2}(4 + 40)$
$= \frac{11}{2} \times 49$
$\therefore S_{11} = 242$
(ii)
Given AP $-6, 0, 6, 12, \ldots \ldots$

a = −6, d = 6, $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = S_{13} = \frac{13}{2}(2 \times -6 + (13-1) \times 6)$

$= \frac{13}{2}(-12 + 72)$

$= \frac{13}{2} \times 60$

$= 390$

$\therefore S_{13} = 890$

(iii)

Given, $a_2 = 2$ and $a_4 = 8$

a + d = 2 ….(i) a + 3d = 8 ….(ii)

(ii) – (i) ⟹ a + 3d = 8

a + d = 2

2d = 6

d = 3

put d = 3 in ….(i) ⟹ a + d = 2

a + 3 = 2

a = −1

$S_{51} = \frac{51}{2}(2 \times -1 + (51-1) \times 3)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{51}{2}(-2 + 50 \times 3)$

$= \frac{51}{2} \times 148$

= 3774.

$\therefore S_n = 3774$

22. Find the sum of

(i) the first 15 multiples of 8

(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) all 3 — digit natural numbers which are divisible by 13.

(iv) all 3-digit natural numbers, which are multiples of 11.

Sol:

The first 15 multiples of 8 are 8, 16, 24, ……

a = 8, d = 8, n = 15

$S_{15} = \frac{15}{2}(28 + (15-1) \times 8)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{15}{2}(16 + 112)$

$= \frac{15}{2} \times 128$

$= 960$

$\therefore Sum\ of\ first\ 15\ multiples\ of\ 8\ is\ 960.$

Given, $a_2 = 2$ *and* $a_4 = 8$
a + d = 2 ….(i) a + 3d = 8 …..(ii)
(ii) – (i) ⟹ a + 3d = 8
a + d = 2
2d = 6
d = 3
Put d = 3 in ….(i) ⟹ a + d = 2
a + 3 = 2
a = −1

$S_{51} = \frac{51}{2}(2 \times -1 + (51 - 1 \times 3))$ $\left(\because S_n = \frac{n}{2}(20 + (n-1)d)\right)$

$= \frac{51}{2}(-2 + 50 \times 3)$

$= \frac{51}{2} \times 148$

$= 3774$

$= 44550$

∴ Sum of all 3 – digit natural numbers which are multiples of 11 is 44550.

23. Find the sum:
(i) 2 + 4 + 6 + …… + 200
(ii) 3 + 11 + 19 + …..+ 803
(iii) 34 + 32 + 30 + ….. + 10
(iv) 25 + 28 + 31 + ….. + 100

Sol:

(i) 2 + 4 + 6 + …… + 200
a = 2, d = 4 – 2 = 2, l = 200 = a_n

$\therefore S_n = \frac{n}{2}(a + l)$ *and* $a_n = a + (n-1)d$

200 = 2 + (n – 1)2
198 = (n – 1)2

n – 1 = $\frac{198}{2} = 99$

n = 100

$S_n = \frac{100}{2}(2 + 200)$

$= 50 \times 202$

$= 10100$

(ii) 3 + 11 + 19 + …..+ 803
a = 3, d = 11 – 3 = 8, l = a_n = 803
803 = 3+ (n – 1)8

$\frac{800}{8} = n - 1$

$n = 101$

$S_n = \frac{101}{2}(3 + 803)$

$= \frac{101}{2} \times 806$

= 504

$S_n = 504$

(iii) 34 + 32 + 30 + ….. + 10

a = 34, d = −2, l = a_n = 10

10 = 34 + (n – 1) × 2

+24 = 2 (n – 1)

n – 1 = 12

n = 13

$\therefore S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13}{2} \times 44$

$= 286$

(iv) 25 + 28 + 31 + ….. + 100

a = 25, d = 8, l = a_n = 100

100 = 25 + (n – 1) × 3

75 = (n – 1) × 3

n – 1 = 25

n = 26

24. Find the sum of the first 15 terms of each of the following sequences having n^{th} term as

(i) $a_n = 3 + 4n$

(ii) $b_n = 5 + 2n$

(iii) $Y_n = 9 – 5n$

Sol:

(i) Given $a_n = 3 + 4n$

Put n = 1, $a_1 = 3 + 4(1) = 7$

Put n = 15, $a_{15} = 3 + 4(15) = 63 = 8$

Sum of 15 terms, $S_{15} = \frac{15}{2}(7 + 63)$ $\left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times 70$

$\therefore S_{15} = 525$

(ii) Given $b_n = 5 + 2n$

Put n = 1, $b_1 = 5 + 2(1) = 7$

Put n = 15, $b_{15} = 5 + 2(15) = 35 = l$

Sum of 15 terms , $S_{15} = \frac{15}{2}(7 + 35)$ $\left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times 42$

$= 315$

$\therefore S_{15} = 315$

(iii) Given, $Y_n = 9 – 5n$

Put n = 1, $y_1 = 9 - 5.1 = -4$

Put n = 15, $y_{15} 9 - 5.15 = 9 - 75 = \ -66 = (l)$

$\therefore S_{15} = \frac{15}{2}(-4 - 66) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times -70$

$= \ -465$

$\therefore S_{15} = -465$

25. Find the sum of first 20 terms of the sequence whose n[th] term is a = An + B.

Sol:

Given, n[th] term $a_n = A_n + B$

Put n = 1, $a_1 = A + B$

Put n = 20, $a_{20} = 20\ A + B = (l)$

$\therefore S_{20} = \frac{20}{2}(A + B + 20A + B) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

= 10 (21A + 2B)

= 210A + 20B

$\therefore S_n = 210A + 2B$

26. Find the sum of the first 25 terms of an A.P. whose n[th] term is given by $a_n = 2 - 3n$.

Sol:

Given, n[th] term $a_n = 2 - 3n$

Put n = 1, $a_1 = 2 - 3.1 = -1$

Put n = 25, $a_{15} = l = 2 - 3.15 = -43$

$\therefore 25 = \frac{25}{2}\ (-1 - 43) = \frac{25}{2}(-44) = -925$

$\therefore S_{25} = -925$

27. Find the sum of the first 25 terms of an A.P. whose n[th] term is given by $a_n = 7 - 3n$.

Sol:

Given, $a_n = 7 - 3n$

Put n = 1, $a_1 = 7 - 3.1 = 4$

Put n = 25, $a_{25} = l = 7 - 3.25 = -68$

$\therefore S_{25} = \frac{25}{2}(4 - 68) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{25}{2} \times -64$

$= -800$

$\therefore S_{25} = -800$

28. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Sol:

Given, $a_2 = 14 \Longrightarrow a + d = 14 \ldots..(i)$

$a_3 = 18 \ \Longrightarrow a + 2d = 18 \ldots..(ii)$

(ii) – (i) ⟹ a + 2d = 18

a + d = 14

0 + d = 4

Put d = 4 is (i) a + 4 = 4

a = 10

$\therefore S_{50} = \frac{51}{2}\{2.10 + (51 - 1) \times 4\} \quad \left(S_n = \frac{n}{2}\{2a + (n - 1)d\}\right)$

$= \frac{51}{2}\{20 + 200\}$

$= \frac{51}{2} \times 220$

$= 5610$

$\therefore S_{51} = 5610$

29. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

Sol:

Given, $S_7 = 49$

$\frac{7}{2}(2a + (7 - 1)d) = 49 \quad \left(\therefore S_n = \frac{n}{2}\{2a + (n - 1)d\}\right)$

$\frac{7}{2}(2a + 6d) = 49$

$\frac{7}{2} \times 2(a + 3d) = 49$

$a + 3d = \frac{49}{7} = 7 \ldots.(i)$ and

$S_{17} = 289$

$\frac{17}{2}(2a + (17 - 1)d) = 289$

$\frac{17}{2} \times 2(a + 8d) = 289$

$a + 8d = \frac{289}{17} = 17$ …..(ii)

$Subtract\ (i) from\ (ii)$

a + 8d = 17

a + 3d = 7

5d = 10

d = 2

put d = 2, in (i) ⟹ a + 3 × 2 = 7

a = 1

$\therefore S_n = \frac{n}{2}\{2.1 + (n - 1).2\} \quad \left(\therefore S_n = \frac{n}{2}(2a + (n - 1)d)\right)$

$= n\{1 + n - 1\}$
$\therefore S_n = n^2.$

30. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol:

Given, a = 5, l = 45, Sum of terms = 400

$\therefore S_n = 400$

$\frac{n}{2}\{5 + 45\} = 400$

$\frac{n}{2} = 50 = 400$

$n = 40 \times \frac{2}{5}$

$\therefore n = 16$

16^{th} term is 45

$a_{16} = 45 \Longrightarrow 5 + (16 - 1) \times d = 45 = 15 \times d = 40$

d = $\frac{408}{15} = \frac{8}{3}$

$\therefore$ n = 16, d = $\frac{8}{3}$

31. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}$n. Find its 25^{th} term.

Sol:

Given, sum of n terms $S_n = \frac{3n^2}{2} + \frac{13}{2}n$

Let, $a_n = S_n - S_{n-1}$ ($\because$ Replace n by (n – 1) is S_n to get $S_{n-1} = \frac{3(n-1)^2}{2} + \frac{13}{2}(n-1)$)

$a_n = \frac{3n^2}{2} + \frac{13}{2}n - \frac{3(n-1)^2}{2} - \frac{13}{2}(n-1)$

$= \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{13}{2}\{n - (n-1)\}$

$= \frac{3}{2}\{n^2 - n^2 + 2n - 1\} + \frac{13}{2}\{1\}$

$= 3n + \frac{10}{2} = 3n + 5$

Put n = 25, $a_{25} = 3(25) + 5 = 75 + 5 = 80$

$\therefore 25^{th}$ term $a_{25} = 80$

32. Let there be an A.P. with first term 'a', common difference 'd'. If a denotes its nth term and S_n the sum of first n terms, find.

(i) n and S_n and if a = 5, d = 3 and a = 50
(ii) n and a, if a_n = 4, d = 2 and S_n = −14
(iii) d, if a = 3, n = 8 and S_n = 192
(iv) a, if a_n = 28, S_n = 144 and n = 9
(v) n and d, if a = 8, a = 62 and S_n = 210
(vi) n and a_n, if a = 2, d = 8 and S_n = 90

Sol:

(i) Given a = 5, d = 3, $a_n = 50$

$a_n = 50$

$a + (n - 1)d = 50$

5 + (n – 1)3 = 50

(n – 1)3 = 45

n – 1 = $\frac{45}{3} = 15$

n = 16

Some of n terms $S_n = \frac{n}{2}[a + l]$

$= \frac{16}{2}[5 + 50]$

$= 8 \times 55$

= 440

(ii) Given, $a_n = 4, d = 2, S_n = -14$

a + (n – 1).2 = 4 and $\frac{n}{2}[2a + (n - 1).2] = -14$

a + 2n = 6 $n[2a + 2n - 2] = -14$

(or)

$\frac{n}{2}[a + a_n] = -14$

$\frac{n}{2}[a + 4] = -14$

$n[6 - 2n + 4] = -28$

$n[10 - 2n] = -28$

$2n^2 - 10n - 28 = 0$

$2(n^2 - 5n - 14) = 0$

(n + 2) (n – 7) = 0

N = −2, n = 7

∵ n = −2 is not a natural number. So, n = 7.

(iii) Given, a = 3, n = 8, S_n = 192.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$192 \times 2 = 8[6 + (8 - 1)d]$

$\frac{192 \times 2}{8} = 6 + 7d$

$48 = 6 + 7d$

7d = 42

d = 6

(iv) Given, $a_n = 28, S_n = 144, n = 9$

$S_n = \frac{n}{2}[a + l]$

144 = $\frac{9}{2}[a + 28]$

144 = $\frac{2}{9} = a + 28$

a + 28 = 32

a = 4

(v) Given, a = 8, 62 and S_n = 210

$S_n = \frac{n}{2}[a + l]$

$210 = \frac{n}{2}[8 + 62]$

210 × 2 = n[70]

$n = \frac{210 \times 2}{70} = 6$

a + (n – 1) d = 62

8 + (6 – 1) d = 62

5d = 54

d = 10.8

(vi) Given

a = 2, d = 8 and S_n = 90

$90 = \frac{n}{2}[4 + (n-1)8]$ $\left(\therefore S_n = \frac{n}{2}[2a + (n-1)d]\right)$

180 = n [4 + 8n – 8]

$8n^2 - 4n - 180 = 0$

$4(2n^2 - n - 45) = 0$

$2n^2 - n - 45 = 0$

(2n + 1) (n – 5) = 0

$\because n = -\frac{1}{2}$ is not a natural no. n = 5

$a_n = 2 + 4(8)$ $(\therefore a_n = a + (n-1)d)$

$a_n = 32$

33. A man saved Rs 16500 in ten years. In each year after the first he saved Rs 100 more than he did in the preceding year. How much did he save in the first year?

Sol:

Let ‘a’ be the money he saved in first year

⟹ First year he saved the money = Rs a

He saved Rs 100 more than, he did in preceding year.

⟹ Second year he saved the money = Rs (a + 100)

⟹ Third year he saved the money = Rs. (a + 2 (100))

So, the sequence is a, a + 100, a + 2(100), ……, This is in AP with common difference (d) = 100.

⟹ Sum of money he saved in 10 years $S_{10} = 16{,}500\ rupees$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + (10-1).100)$

16,500 = 5 (2a + 9 × 100)

2a + 900 = $\frac{16500}{5} = 3300$
2a = 2400
a = $\frac{2400}{2} = 1200$
∴ He saved the money in first year (a) = Rs. 1200

34. A man saved Rs 32 during the first year, Rs 36 in the second year and in this way he increases his savings by Rs 4 every year. Find in what time his saving will be Rs 200.
Sol:
Given
Saving in 1st yr (a_1) = Rs 32
Saving in 2nd yr (a_2) = Rs 36
Increase in salary every year (d) = Rs 4
Let in n years his saving will be Rs 200
$\Rightarrow S_n = 200$
$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 200$
$\Rightarrow \frac{n}{2}[64 + 4n - 4] = 200$
$\Rightarrow \frac{n}{2}[4n + 60] = 200$
$\Rightarrow 2n^2 + 30n = 200$
$\Rightarrow n^2 + 15 - 100 = 0$ [Divide by 2]
$\Rightarrow n^2 + 20n - 5n - 100 = 0$
⟹ n(n + 20) – 5(n + 20) = 0
⟹ (n + 20)(n – 5) = 0
If n + 20 = 0 or n – 5 = 0
n = −20 or n = 5 (Rejected as n cannot be negative)
∴ In 5 years his saving will be Rs 200

35. A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.
Sol:
Given
A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an A.P
i.e., sum of all 40 installments = Rs 3600
S_{40} = 3600
Let, the money he paid in first installment is a, and every year he paid with common difference = d
Then,
S40 = 3600 $(\therefore S_n = \frac{n}{2}[2a + (n-1)d])$

$\frac{40}{x}[2a+(40-1)d]=3600$

$2a+39d=\frac{3600}{20}=180 \dots \dots (i)$

$but,$

He died by leaving one third of the debt unpaid that means he paid remaining money in 30 installments.

∴ The money he paid in 30 installments = 3600 $-\frac{3600}{3}=3600-1200$

$\therefore S_{30}=2400$

$S_{30}=2400$

$=\frac{30}{2}[2a+(30-1)d]=2400 \quad \left(\therefore S_n=\frac{n}{2}(2a+(n-1)d)\right)$

2a + 2ad = $\frac{2400}{15}=160 \quad \dots .(ii)$

(i) – (ii) ⟹ 2a + 39d = 180

2a + 29d = 160

0 + 10 d = 20

d = $\frac{20}{10}=2$

put d = 2 in (ii) 2a + 29 (2) = 160

2a = 102

a = $\frac{102}{2}=51$

∴ The value of his first installment = 51.

Exercise – 9.1