Ex 9.1

Continuity Ex 9.1 Q1

We have to check the continuity of function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{-h}{|-h|} = \lim_{h\to 0}\frac{-h}{h} = -1$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{h}{|h|} = 1$$

Thus, $\text{LHL} \neq \text{R.H.L}$

So, the given function in discontinuous and the discontinuity is of first kind.

Continuity Ex 9.1 Q2

We have, to check the continuity at $x = 3$.

$$\text{L.H.L} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\frac{(3-h)^2-(3-h)-6}{(3-h)-3} = \lim_{h\to 0}\frac{h^2-5h}{-h} = \lim_{h\to 0} -h+5 = 5$$

$$\text{R.H.L} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\frac{(3+h)^2-(3+h)-6}{(3+h)-3} = \lim_{h\to 0}\frac{h^2+5h}{h} = \lim_{h\to 0} h+5 = 5$$

$f(3) = 5$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 5$

So, The function is continuous at $x = 3$

Continuity Ex 9.1 Q3

We have, to check the continuity of the function at $x = 3$.

$$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\frac{(3-h)^2-9}{(3-h)-3} = \lim_{h\to 0}\frac{h^2-6h}{-h} = \lim_{h\to 0} -h+6 = 6$$

$$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\frac{(3+h)^2-9}{(3+h)-3} = \lim_{h\to 0}\frac{h^2+6h}{h} = \lim_{h\to 0} h+6 = 6$$

$f(3) = 6$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 6$

So, the given function is continuous at $x = 3$.

Continuity Ex 9.1 Q4

We want, to check the continuity of the function at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{(1-h)^2-1}{(1-h)-1} = \lim_{h\to 0}\frac{h^2-2h}{-h} = \lim_{h\to 0} -h+2 = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0}\frac{h^2+2h}{h} = \lim_{h\to 0} h+2 = 2$$

$f(1) = 2$

we find that $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

Continuity Ex 9.1 Q5

We have, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 3(-h)}{-h} = \lim_{h\to 0}\frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sin 3h}{h} = 3$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$

$\Rightarrow$ Function is discontinuous at $x = 0$. It is removable discontinuty.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} e^{1/-h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} e^{1/h} = e^{\infty} = \infty$$

So, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuty of I^{st} kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos(-h)}{(-h)^2}$$
$$= \lim_{h \to 0} \frac{1-\cos h}{h^2} \quad [\because \cos(-\theta) = \cos\theta]$$
$$= \lim_{h \to 0} \frac{2\sin^2\frac{h}{2}}{h^2} \quad \left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$$
$$= \lim_{h \to 0} 2.\left(\frac{\sin\frac{h}{2}}{h}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{1-\cos h}{h^2} = \lim_{h \to 0} \frac{2\sin^2\frac{h}{2}}{h^2} = \lim_{h \to 0} 2\left(\frac{\sin^2\frac{h}{2}}{h}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$
Hence, the function is discontinuous at $x = 0$
This is removable discontinuity.

Continuity Ex 9.1 Q8
We want, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h - |-h|}{2} = \lim_{h \to 0} \frac{-h-h}{2} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h - (|h|)}{2} = 0$$

$f(0) = 2$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$
Hence, The function is discontinuous at $x = 0$
This is removable discontinuity.

Continuity Ex 9.1 Q9
We want, to check the continuty of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} \frac{|a-h-a|}{a-h-a} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} \frac{|a+h-a|}{a+h-a} = \lim_{h \to 0} \frac{h}{h} = 1$$

Thus, $\text{LHL} \neq \text{RHL}$
Hence, function is discontinuous at $x = a$. And the discontinuty is of first kind.

Continuity Ex 9.1 Q10(i)
We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} |-h| \cos\left(\frac{1}{-h}\right) = \lim_{h \to 0} h\cos\left(\frac{1}{h}\right) = 0$$
$$\text{RHL} = \lim_{x \to 0^+} = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} |h| \cos\left(\frac{1}{h}\right) = 0$$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$
Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} (a-h-a)\sin\left(\frac{1}{a-h-a}\right) = \lim_{h\to 0} -h\sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} (a+h-a)\sin\left(\frac{1}{a+h-a}\right) = \lim_{h\to 0} h\sin\left(\frac{1}{h}\right) = 0$$

$$f(a) = 0$$

Thus, LHL $\neq$ RHL = $f(a) = 0$

Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1+2(-h))} = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1-2h)} = DNE$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{h}-1}{\log(1+2h)} = DNE$$

Thus, Both LHL and RHL do not exist

$\therefore$ Function is discontinuous and the discontinutiy is of II^{nd} kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{1-(1-h)^n}{1-(1-h)} = \lim_{h\to 0}\frac{1-\left[1-nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{h}$$

$$= \lim_{h\to 0} n - \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{1-(1+h)^n}{1-(1+h)} = \lim_{h\to 0}\frac{1-\left[1+nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{-h}$$

$$= \lim_{h\to 0} n + \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$f(1) = n-1$

Thus, LHL = RHL $\neq f(1)$
Hence, function is discontinuous at $x = 1$
This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{\left|(1-h)^2-1\right|}{(1-h)-1} = \lim_{h\to 0}\frac{\left|h^2-2h\right|}{-h} = \lim_{h\to 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{\left|(1+h)^2-1\right|}{1+h-1} = \lim_{h\to 0}\frac{h^2+2h}{h} = 2$$

$f(1) = 2$

$\therefore$ LHL = RHL = $f(1) = 2$

Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{2(|-h|)+(-h)^2}{-h} = \lim_{h\to 0}\frac{2h+h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{2\times|h|+h^2}{h} = 2$$

Thus, LHL $\neq$ RHL
Function is not continuous at $x = 0$
This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q11

We want to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} 1+(1-h)^2 = \lim_{h\to 0} 1+1-2h+h^2 = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} 2-(1+h) = 1$$

LHL $\neq$ RHL
Hence, the function is discontinuous at $x = 1$
This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q12

We want to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin(3\times(-h))}{\tan(2\times(-h))} = \lim_{h\to 0}\frac{-\sin 3h}{-\tan 2h} = \lim_{h\to 0}\frac{\frac{\sin 3h}{3h}\times 3h}{\frac{\tan 2h}{2h}\times 2h} = \frac{3}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\log(1+3h)}{e^{2h}-1} = \lim_{h\to 0}\frac{\frac{\log(1+3h)}{3h}\times 3h}{\frac{e^{2h}-1}{2h}\times 2h} = \frac{3}{2}$$

$f(0) = \frac{3}{2}$

Thus, $\text{LHL} = \text{RHL} = f(0) = \frac{3}{2}$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q13

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 2(-h) - |-h| = \lim_{h\to 0} -2h - h = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 2h - |h| = 0$$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q14

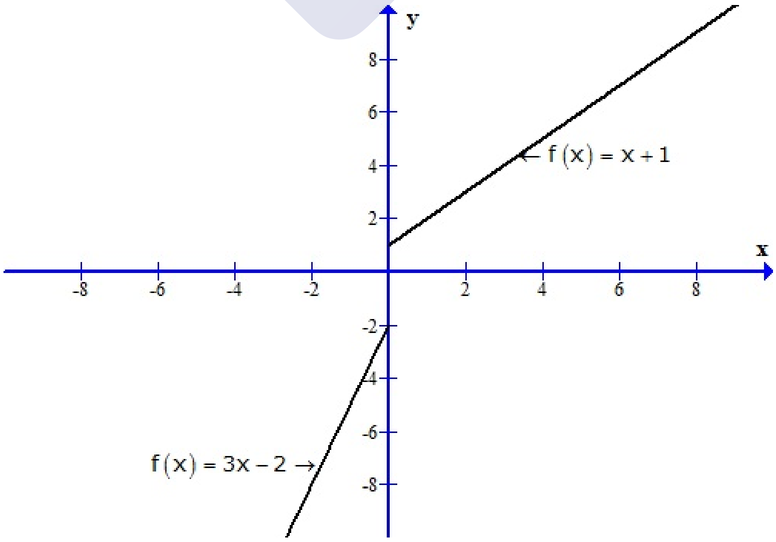
We want to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 3(-h) - 2 = \lim_{h\to 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h + 1 = 1 = 0$$

$\text{LHL} \neq \text{RHL}$

So, the function is discontinuous



Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} -(-h) = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h = 0$$

$$f(0) = 1$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

$$\text{LHL} = \lim_{x\to \frac{1}{2}^-} f(x) = \lim_{h\to 0} f\left(\frac{1}{2}-h\right) = \lim_{h\to 0} \frac{1}{2} - h = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to \left(\frac{1}{2}\right)^+} = \lim_{h\to 0} f\left(\frac{1}{2}+h\right) = \lim_{h\to 0} 1 - \left(\frac{1}{2}+h\right) = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

Thus, $\text{LHL} = \text{RHL} = f\left(\frac{1}{2}\right) = \frac{1}{2}$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 2(-h) - 1 = -1$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 2h + 1 = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$

$\text{LHL} = \text{RHL} = f(1) \ldots\ldots (1)$

Now, $$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h\to 0} \frac{h^2 - 2h}{-h} = 2$$

$$f(1) = k$$

From (1), $\text{LHL} = f(1)$

$\therefore\ 2 = k$

Continuity Ex 9.1 Q19

We have that the function is continuous at $x = 1$

$\therefore\ \text{LHL} = \text{RHL} = f(1) \quad \ldots (1)$

Now,

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} \frac{(1-h)^2 - 3(1-h) + 2}{(1-h) - 1} = \lim_{h\to 0} \frac{h^2 + h}{-h} = \lim_{h\to 0} -h - 1 = -1$$

$$f(1) = k$$

From (1), we get,

$k = -1$

Continuity Ex 9.1 Q20

We know that a function is continuous at 0 if

LHL = RHL = $f(0)$ ....(1)

Now,

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 5(-h)}{3(-h)} = \lim_{h\to 0}\frac{-\sin 5h}{-3h} = \lim_{h\to 0}\frac{\sin 5h}{5h}\times\frac{5h}{3h} = \frac{5}{3}$$

$f(0) = k$

Thus, from (1),

$k = \frac{5}{3}$

Continuity Ex 9.1 Q21

The given function is $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x\to 2^-}(kx^2) = \lim_{x\to 2^+}(3) = 4k$$
$$\Rightarrow k\times 2^2 = 3 = 4k$$
$$\Rightarrow 4k = 3 = 4k$$
$$\Rightarrow 4k = 3$$
$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of k is $\frac{3}{4}$.

Continuity Ex 9.1 Q22

We have given that the function is continuous at $x = 0$

So, LHL = RHL = $f(0)$.....(1)

Now,

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 2(-h)}{5(-h)} = \lim_{h\to 0}\frac{-\sin 2h}{-5h} = \lim_{h\to 0}\frac{\sin 2h}{2h}\times\frac{2h}{5h} = \frac{2}{5}$$

$f(0) = k$

Using (1), $k = \frac{2}{5}$

Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

LHL = RHL = $f(2)$.....(1)

Now,

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} a(2-h)+5 = 2a+5$$

$$f(2) = 2a+5$$

$$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} 2+h-1 = 1$$

$\therefore$ Using (1),

$2a+5 = 1 \Rightarrow a = -2$

Continuity Ex 9.1 Q24

We have, at $x = 0$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-h}{|-h|+2(-h)^2} = \lim_{h\to 0} \frac{-h}{h+2h^2} = \lim_{h\to 0} \frac{-1}{1+2h} = -1$$

$$f(0) = k$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{h}{|h|+2h^2} = \lim_{h\to 0} \frac{1}{1+2h} = 1$$

Since, LHL $\neq$ RHL, function will remain discontinuous at $x = 0$, regardless the choice of k.

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, L.H.Limit = R.H.Limit.

$$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^+} f(x) = \lim_{x\to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x\to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x\to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

$LHL = RHL = f(0) \dots (1)$

$f(0) = c$

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h\to 0} \frac{-\sin(ah+h) - \sinh}{-h}$$

$$= \lim_{h\to 0} \frac{\sin(a+h)h}{h} + \lim_{h\to 0} \frac{\sinh}{h}$$

$$= a + 1 + 1 = a + 2$$

$$RHL = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}}$$

$$= \lim_{h\to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h+bh^2} + \sqrt{h}}{\sqrt{h+bh^2} + \sqrt{h}}$$

$$= \lim_{h\to 0} \frac{h+bh^2-h}{bh^{\frac{3}{2}}\left(\sqrt{h+bh^2} + \sqrt{h}\right)} = \lim_{h\to 0} \frac{bh^2}{bh^2\left(\sqrt{1+bh} + 1\right)} = \frac{1}{2}$$

$\therefore$ from (1),

$a + 2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$

$c = \frac{1}{2}$ and

$b \in R - \{0\}$

Hence, $a = \frac{-3}{2}$, $b \in R - \{0\}$, $c = \frac{1}{2}$

Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$\therefore LHL = RHL = f(0) \dots\dots (1)$

$f(0) = \frac{1}{2}$

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{1-\cos k(-h)}{-h\sin(-h)} = \lim_{h\to 0} \frac{1-\cos kh}{+h\sinh}$$

$$= \lim_{h\to 0} \frac{2\sin^2 \frac{kh}{2}}{h.2\sin\frac{h}{2}.\cos\frac{h}{2}}$$

$$= \lim_{h\to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \times \frac{\frac{k^2h^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{h}{2}} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}}$$

$$= \frac{k^2}{2}$$

$\therefore$ Using (1) we get,

$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$\therefore$ LHL = RHL = $f(4)$....(1)

$f(4) = a + b$.....(A)

$$\text{LHL} = \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0}\frac{(4-h)-4}{|(4-h)-4|} + a = \lim_{h\to 0}\frac{-h}{h} + a = a - 1 \quad ....(B)$$

$$\text{RHL} = \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0}\frac{(4+h)-4}{|(4+h)-4|} + b = \lim_{h\to 0}\frac{h}{h} + b = b + 1 \quad ....(C)$$

$\therefore$ from (1)

$a - 1 = b + 1 \Rightarrow a - b = 2$ .....(D)

from (A) and (B)

$a + b = a - 1 \Rightarrow b = -1$

from (A) and (C)

$a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$.....(1)

$f(0) = k$

$$\text{LHL} = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 2(0-h)}{-h} = \lim_{h\to 0}\frac{-\sin 2h}{-h} = 2$$

$\therefore$ using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

LHL = RHL = $f(0)$....(1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\log\left(1-\frac{h}{a}\right) - \log\left(1+\frac{h}{b}\right)}{(-h)} = \lim_{h\to 0}\frac{\log\left(1+\left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right)\times a} + \frac{\log\left(1+\frac{h}{b}\right)}{h}$$

$$= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

from (1),

$$f(0) = \frac{a+b}{ab}$$

Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$\therefore$ LHL = RHL = $f(2)$ ..... (1)

Now,

$f(2) = k$ .... (A)

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0}\frac{2^{(2-h)+2}-16}{4^{(2-h)}-16} = \lim_{h\to 0}\frac{2^{4-h}-16}{4^{2-h}-16}$$

$$= \lim_{h\to 0}\frac{2^4.2^{-h}-16}{4^2.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16.2^{-h}-16}{16.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16\left(2^{-h}-1\right)}{16\left(4^{-h}-1\right)}$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}\right)-1^2} \qquad \left[\because 2^{-2h} = \left(2^{-h}\right)^2 = 4^{-h}\right]$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}-1\right)\left(2^{-h}+1\right)} = \frac{1}{2} \quad \text{......(B)}$$

$\therefore$ Using (1) from (A) & (B)

$k = \frac{1}{2}$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

LHL = RHL = value of the function at $x = \pi$ ..... (1)

$$\text{LHL} = \lim_{x\to \pi^-} f(x) = \lim_{h\to 0} f(\pi-h) = \lim_{h\to 0}\frac{1-\cos 7(\pi-h-\pi)}{5\left((\pi-h)-\pi\right)^2} = \lim_{h\to 0}\frac{1-\cos 7h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{7}{2}h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2}{5}\left(\frac{\sin\frac{7}{2}h}{\frac{7}{2}h}\right)^2 \times \left(\frac{7}{2}\right)^2$$

$$= \frac{2}{5}\times\frac{49}{4} = \frac{49}{10} \quad \text{....(B)}$$

Thus, using (1) we get,

$f(\pi) = \frac{49}{10}$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ .... (1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{2(-h)+3\sin(-h)}{3(-h)+2\sin(-h)} = \lim_{h\to 0}\frac{-2h-3\sin h}{-3h-2\sin h}$$

$$= \lim_{h\to 0}\frac{\frac{2h+3\sin h}{h}}{\frac{3h+2\sin h}{h}}$$

$$= \lim_{h\to 0}\frac{2+3\frac{\sin h}{h}}{3+2\frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \quad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

Using (1) we get,

$f(0) = 1$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.

$\text{LHL} = \text{RHL} = f(0) \ldots (1)$

$f(0) = k \ldots (A)$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos 4(-h)}{8(-h)^2} = \lim_{h \to 0} \frac{1-\cos 4h}{8h^2} = \lim_{h \to 0} \frac{2\sin^2 2h}{8h^2} = \lim_{h \to 0} \left(\frac{\sin 2h}{2h}\right)^2 = 1$$

Thus, using (1) we get,

$k = 1$

Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if

$\text{LHL} = \text{RHL} = f(0) \ldots (1)$

$f(0) = 8 \ldots (A)$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos 2k(-h)}{(-h)^2} = \lim_{h \to 0} \frac{1-\cos 2kh}{h^2} = \lim_{h \to 0} \frac{2\sin^2 kh}{h^2}$$

$$= \lim_{h \to 0} 2\left(\frac{\sin kh}{kh}\right)^2 . k^2$$

$$= 2k^2$$

Thus, using (1) we get,

$2k^2 = 8 \quad \Rightarrow k^2 = 4 \quad \Rightarrow k = \pm 2$

Hence, $k = \pm 2$

Let $x - 1 = y$

$\Rightarrow x = y + 1$

Thus,

$$\lim_{x\to 1}(x-1)\tan\frac{\pi x}{2} = \lim_{y\to 0} y\tan\frac{\pi(y+1)}{2}$$

$$= \lim_{y\to 0} y\tan\left(\frac{\pi y}{2}+\frac{\pi}{2}\right)$$

$$= -\lim_{y\to 0} y\cot\frac{\pi y}{2}$$

$$= -\lim_{y\to 0} y\frac{\cos\frac{\pi y}{2}}{\sin\frac{\pi y}{2}}$$

$$= -\lim_{y\to 0} y\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi}{2}}}$$

$$= -\lim_{y\to 0}\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi y}{2}}}$$

$$= -\lim_{y\to 0}\frac{2}{\pi}\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}}$$

$$= -\frac{2}{\pi}\lim_{y\to 0}\cos\frac{\pi y}{2}$$

$$= -\frac{2}{\pi}$$

Since the function is continuous, L.H.Limit = R.H.Limit

Thus, $k = -\frac{2}{\pi}$

Since the function is continuous at every point, therefore

$$LHL = RHL = f(0)$$

Now

$$f(0) = \cos 0$$
$$= 1$$

Again

$$LHL = \lim_{x\to 0} k\left(x^2 - 2x\right)$$
$$= \lim_{h\to 0^-} k\left(h^2 - 2h\right)$$
$$= 0$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL = RHL = f(\pi)$$

Now

$$f(\pi) = k\pi + 1$$

Again

$$RHL = \lim_{x\to\pi^+}\cos x$$
$$= \lim_{h\to 0^+}\cos(\pi - h)$$
$$= -\lim_{h\to 0^+}\cosh$$
$$= -1$$

Therefore we can write

$$k\pi + 1 = -1$$
$$k = -\frac{2}{\pi}$$

We are given that function is continuous at $x=5$.

$\therefore$ LHL = RHL = $f(5)$....(1)

$f(5)=5k+1$

LHL = $\lim_{x\to5^+} f(x)=\lim_{h\to0} f(5+h)=\lim_{h\to0} 3(5+h)-5=10$

Thus, using (1), we get,

$5k+1=10$

$5k=9$

$k=\frac{9}{5}$

We know that the function will be continuous at $x=5$. if

LHL = RHL = $f(5)$.....(1)

$f(5)=k$

LHL = $\lim_{x\to5^-} f(x)=\lim_{h\to0} f(5-h)=\lim_{h\to0}\frac{(5-h)^2-25}{(5-h)-5}=\lim_{h\to0}\frac{h^2-10h}{-h}=\lim_{h\to0}-h+10=10$

Thus, using (1), we get,

$k=10$

We know that a function will be continuous at $x=1$. if

LHL = RHL = $f(1)$ ....(1)

$f(1)=k.1^2=k$

LHL = $\lim_{x\to1^-} f(x)=\lim_{h\to0} f(1-h)=\lim_{h\to0} 4=4$

Thus, using (1), we get,

$k=4$

We know that a function will be continuous at $x=0$. if

LHL = RHL = $f(0)$.....(1)

$f(0)=k(0+2)=2k$

LHL = $\lim_{x\to0^+} f(x)=\lim_{h\to0} f(0+h)=\lim_{h\to0} 3(h)+1=1$

Thus, using (1), we get,

$2k=1$

$k=\frac{1}{2}$

Continuity Ex 9.1 Q37

It is given that the function is continuous at $x = 3$ and at $x = 5$

$\therefore$ LHL = RHL = $f(3)$.....(1) and
LHL = RHL = $f(5)$.....(2)

Now,
$f(3) = 1$
$\text{RHL} = \lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3+h) = \lim_{h \to 0} a(3+h) + b = 3a + b$

Thus, using (1), we get,
$3a + b = 1$....(3)

$f(5) = 7$
$\text{LHL} = \lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} a(5-h) + b = 5a + b$

Thus, using (2), we get
$5a + b = 7$ .....(4)

Now, solving (3) and (4) we get,
$a = 3$ and $b = -8$

Continuity Ex 9.1 Q38
We want to discuss the continuity of the function at $x = 1$

We need to prove that

LHL = RHL = $f(1)$

$f(1) = \frac{1^2}{2} = \frac{1}{2}$

$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2}{2} = \frac{1}{2}$

$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$

Thus, LHL = RHL = $f(1) = \frac{1}{2}$
Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q39
We want to discuss the continuity at $x = 0$ and $x = 1$

Now,
$f(0) = 1$
$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} |-h| + |-h-1| = 1.$
$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} |h| + |h-1| = 1$

$\because$ LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$f(1) = 1$
$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} |1-h| + |1-h-1| = 1$
$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} |1+h| + |1+h-1| = 1$

$\because$ LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$f(-1) = |-1-1| + |-1+1| = 2$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(-1-h) = \lim_{h \to 0} |-1-h-1| + |-1-h+1| = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(-1+h) = \lim_{h \to 0} |-1+h-1| + |-1+h+1| = 2$

Thus, LHL = RHL = $f(-1) = 2$
Hence, function is continuous at $x = -1$

For $x = 1$
$f(1) = |1-1| + |1+1| = 2$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} |1-h-1| + |1-h+1| = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} |1+h-1| + |1+h+1| = 2$

Thus, LHL = RHL = $f(1) = 2$
Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since $f(x)$ is continuous at $x=0$, L.H.Limit = R.H.Limit.
Thus, we have

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{x\to 0^-} a\sin\frac{\pi}{2}(x+1)=\lim_{x\to 0^+}\frac{\tan x-\sin x}{x^3}$$

$$\Rightarrow a\times 1=\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{x}\left(\frac{1}{\cos x}-1\right)}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{x}\left(\frac{1-\cos x}{\cos x}\right)}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\frac{1}{\cos x}\times\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\Rightarrow a=1\times 1\times\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{1-\cos x}{x^2}\times\frac{1+\cos x}{1+\cos x}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$\Rightarrow a=1\times\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$\Rightarrow a=1\times\frac{1}{1+1}$$

$$\Rightarrow a=\frac{1}{2}$$

Continuity Ex 9.1 Q41

It is given that function is continuous at $x=0$. then,
LHL = RHL = $f(0)$ .... (1)

Now,
$f(0)=2\cdot 0+k=k$

$$\text{LHL}=\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0} -2(-h)^2+k=k$$

$$\text{RHL}=\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0} 2\left(h^2\right)+k=k$$

Thus, the function will be continuous for any $k\in R$.

Continuity Ex 9.1 Q42

The given function f is $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$

If f is continuous at $x=0$, then

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$
$$\Rightarrow \lim_{x\to 0^-}\lambda(x^2-2x)=\lim_{x\to 0^+}(4x+1)=\lambda(0^2-2\times 0)$$
$$\Rightarrow \lambda(0^2-2\times 0)=4\times 0+1=0$$
$$\Rightarrow 0=1=0, \text{ which is not possible}$$

Therefore, there is no value of λ for which f is continuous at $x=0$

At $x=1$,

$f(1)=4x+1=4\times 1+1=5$

$$\lim_{x\to 1}(4x+1)=4\times 1+1=5$$
$$\therefore \lim_{x\to 1} f(x)=f(1)$$

Therefore, for any values of λ, f is continuous at $x=1$

Continuity Ex 9.1 Q43

The function will be continuous at $x=2$
if LHL = RHL = $f(2)$ ....(1)

Now,
$f(2)=k$

LHL = $\lim_{x\to 2^-} f(x)=\lim_{h\to 0} f(2-h)=\lim_{h\to 0} 2(2-h)+1=5.$

Thus, using (1) we get,
$k=5$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(1)

Now,

$f\left(\frac{\pi}{2}\right) = a$

$$\text{LHL} = \lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2}-h\right) = \lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{(1-\cos h)\left(1+\cos^2 h+\cos h\right)}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}\left(1+\cos^2 h+\cos h\right)}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{2\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{h^2}{4}.\left(1+\cos^2 h+\cos h\right)}{3\left(\frac{\sin h}{h}\right)^2.h^2}$$

$$= \lim_{h\to 0}\frac{2.\frac{1}{4}\left(1+\cos^2 h+\cos h\right)}{3} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2}+h\right) = \lim_{h\to 0}\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2} = \lim_{h\to 0}\frac{b(1-\cos h)}{(\pi-\pi-2h)^2}$$

$$= \lim_{h\to 0}\frac{b.2\sin^2\frac{h}{2}}{(2h)^2}$$

$$= \lim_{h\to 0}\frac{b}{2}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4}$$

$$= \lim_{h\to 0}\frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$a = \frac{1}{2}$

And

$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$

Thus, $a = \frac{1}{2}$ and $b = 4$

Continuity Ex 9.1 Q45

It is given that the function is continuous at $x = 0$, then

LHL = RHL = $f(0)$.....(1)

Now,

$f(0) = k$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{h}{|h|} = 1....(B)$$

Thus, using (1) we get,

$k = 1$

Continuity Ex 9.1 Q46

Since the function is continuous at $x = 3$, therefore

$$LHL = RHL = f(3)$$

Now

$$\begin{aligned} RHL &= \lim_{x \to 3^+} f(x) \\ &= \lim_{h \to 0} f(3+h) \\ &= \lim_{h \to 0} b(3+h)+3 \\ &= \lim_{h \to 0} 3b+3h+3 \\ &= 3b+3 \end{aligned}$$

Again

$$\begin{aligned} f(3) &= a(3)+1 \\ &= 3a+1 \end{aligned}$$

Thus we can write

$$f(3) = RHL$$

$$3a+1 = 3b+3$$

$$3a-3b = 2$$

Ex 9.2

Chapter 9 Continuity Ex 9.2 Q1

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function x both are everywhere continuous.

So, the quotient function $\frac{\sin x}{x} = f(x)$ is continuous for $x < 0$

When $x > 0$, we have $f(x) = x + 1$, which being a polynomial, is continuous for $x > 0$

Let us now consider $x = 0$

LHL= $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$

$RHL = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$

$f(0) = 0 + 1 = 1$

Thus, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0) = 1$

$\therefore$ $f(x)$ is contiunuos at $x = 0$

Hence, $f(x)$ is contiunuos everywhere.

Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \frac{-x}{x} = -1 \ ; \ x < 0 \\ \frac{x}{|x|} = 1 \ ; \ x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$

hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h}{|-h|} = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h|} = 1$$

So, LHL $\neq$ RHL

Hence, function is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(i)

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$$

$f(1) = 4$

LHL = RHL $\neq f(1)$

Thus, function is not discontinuous at $x = 1$

Chapter 9 Continuity Ex 9.2 Q3(ii)

When $x \neq 2$, we have,

$$f(x) = \frac{x^4 - 16}{x - 2} = \frac{(x^2+4)(x^2-4)}{x-2} = \frac{(x^2+4)(x+2)(x-2)}{x-2} = f(x) = (x^2+4)(x+2)$$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{(2-h)^4 - 16}{(2-h) - 2}$$

$$= \lim_{h \to 0} \frac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} \frac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} 32 - 24h + 8h^2 - h^3 = 32$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{(2+h)^4 - 16}{(2+h) - 2} = \lim_{h \to 0} \frac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$$

$$= \lim_{h \to 0} 32 + 24h + 8h^2 + h^3$$

$$= 32$$

Also, $f(2) = 16$

Thus, LHL = RHL $\neq f(2)$

Hence, the function is discontinuous at $x = 2$

Chapter 9 Continuity Ex 9.2 Q3(iii)

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

$f(0) = 2 \times 0 + 3 = 3$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(iv)

When $x \neq 0$ $f(x) = \frac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function x are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \frac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin 3(-h)}{-h} = \lim_{h \to 0} \frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin 3h}{h} = 3$$

$f(0) = 4$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(v)

When $x \neq 0$, we have, $f(x) = \frac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function x is also continuous for $x < 0$ and $x > 0$.

$\therefore$ The quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\frac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} + \cos(-h) = \lim_{h \to 0} \frac{-\sin h}{-h} + \cos h = 1 + 1 = 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} + \cos h = 1 + 1 = 2$$

$f(0) = 5$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vi)

When $x \neq 0$, we have, $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$, Also the inverse trignometric function is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $\tan^{-1} x$ is also continuous for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{(-h)^4 + (-h)^3 + 2(-h)^2}{\tan^{-1}(-h)} = \lim_{h \to 0} \frac{h^4 - h^3 + 2h^2}{\tan^{-1} h} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h^4 + h^3 + 2h^2}{\tan^{-1} h} = 0$$

$f(0) = 10$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vii)

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{\log_e(1 + 2x)}$$

We know that e^x and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $\log_e(1 + 2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \dfrac{e^x - 1}{\log_e(1 + 2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{e^{-h} - 1}{\log_e(1 - 2h)} = \lim_{h \to 0} \frac{\dfrac{e^{-h} - 1}{-h}}{\dfrac{\log_e(1 - 2h)}{-2h} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{h} - 1}{\log_e(1 + 2h)} = \lim_{h \to 0} \frac{\dfrac{e^{h} - 1}{h}}{\dfrac{\log_e(1 + 2h)}{2h} \times 2} = \frac{1}{2}$$

$f(0) = 7$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(viii)

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on IR

(ii) Polynomial function are every where continuous.
So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$f(1) = |1-3| = |-2| = 2$
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} |x-3| = 2$
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right)$
$= \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{8}{4} = 2$

Since
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) = 2$

$\therefore$ $f(x)$ is continuous at x

Hence $f(x)$ has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q3(ix)

When $x < -3$,
$f(x) = |x| + 3$
We know that $|x|$ is continuous for $x < -3$
$\therefore$ $|x| + 3$ is continuous for $x < -3$

When $x > 3$
$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$
$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$
LHL $= \lim_{x\to -3^-} f(x) = \lim_{h\to 0} f(-3-h) = \lim_{h\to 0} |-3-h| + 3 = \lim_{h\to 0} |3+h| + 3 = 6$

RHL $= \lim_{x\to -3^+} f(x) = \lim_{h\to 0} f(-3+h) = \lim_{h\to 0} -2(-3+h) = 6$
$f(-3) = |-3| + 3 = 6$

Thus, LHL = RHL = $f(-3) = 6$
So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

LHL $= \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} -2(3-h) = -6$

Chapter 9 Continuity Ex 9.2 Q3(xi)

The given function is $f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\le x\le 1\\ 4x, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<0, \text{ then } f(c)=2c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<0$

Chapter 9 Continuity Ex 9.2 Q3(xii)

The given function f is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\ne 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c\ne 0, \text{ then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x\ne 0$

Chapter 9 Continuity Ex 9.2 Q3(xiii)

The given function f is $f(x)=\begin{cases}-2, & \text{if } x\le -1\\ 2x, & \text{if } -1<x\le 1\\ 2, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<-1, \text{ then } f(c)=-2 \text{ and } \lim_{x\to c} f(x)=\lim_{x\to c}(-2)=-2$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<-1$

Case II:

If $c=-1$, then $f(c)=f(-1)=-2$

The left hand limit of f at $x=-1$ is,

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-}(-2)=-2$$

The right hand limit of f at $x=-1$ is,

$$\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+}(2x)=2\times(-1)=-2$$

$$\therefore \lim_{x\to -1} f(x)=f(-1)$$

Therefore, f is continuous at $x=-1$

Case III:

If $-1<c<1$, then $f(c)=2c$

$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

If $c=1$, then $f(c)=f(1)=2\times 1=2$

The left hand limit of f at $x=1$ is,

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(2x)=2\times 1=2$$

The right hand limit of f at $x=1$ is,

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} 2=2$$

$$\therefore \lim_{x\to 1} f(x)=f(c)$$

Therefore, f is continuous at $x=2$

Case V:

If $c>1$, then $f(c)=2$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(2)=2$

$$\lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x>1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line

Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x=0$

$\therefore$ LHL = RHL = $f(0)$ ....(1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin(-2h)}{5(-h)} = \lim_{h\to 0}\frac{-\sin 2h}{-5h} = \lim_{h\to 0}\frac{\sin 2h}{2h}\times\frac{2h}{5h} = \frac{2}{5}$$

$f(0)=3k$

So, using (1) we get,

$$\frac{2}{5}=3k$$

$$k=\frac{2}{15}$$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} k(2-h)+5 \quad = 2k+5$

$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0}(2+h)-1 = 1$

Thus, using (1), we get,

$2k+5=1$

$k=-2$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

LHL = RHL = $f(0)$ .... (1)

$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} k\left((-h)^2+3(-h)\right) = \lim_{h\to 0} k\left(h^2-3h\right) = 0$

$f(0) = \cos 2\times 0 = \cos 0^\circ = 1$

LHL $\neq f(0)$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x=3$

$f(3)=2$ .....(A)

$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} a(3+h)+b \quad = 3a+b$.....(B)

$\therefore f(x)$ will be continuous at $x=3$ if $3a+b=2$.....(1)

Now, check the continuity at $x=5$

$f(5)=9$ ....(C)

$\text{LHL} = \lim_{x\to 5^-} f(x) = \lim_{h\to 0} f(5-h) = \lim_{h\to 0} a(5-h)+b = 5a+b$

$f(x)$ will be continuous at $x=5$ if $5a+b=9$....(2)

Solving (1) & (2), we get

$a=\frac{7}{2}$ and $b=\frac{-17}{2}$

Chapter 9 Continuity Ex 9.2 Q4(v)

It is given that the function is continuous

At $x=-1$

$f(-1)=4$

$\text{RHL} = \lim_{x\to -1^+} f(x) = \lim_{h\to 0} f(-1+h) = \lim_{h\to 0} a(-1+h)^2+b = a+b$

Since, $f(x)$ is continuous at $x=-1$

$\therefore a+b=4$ ....(A)

Now, at $x=0$,

$f(0)=\cos 0^\circ = 1$

$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} a(-h)^2+b = b$

Since, $f(x)$ is continuous at $x=0$

$\therefore f(0) = \text{LHL}$

$\Rightarrow b=1$

$\therefore$ from (A)

$a=3$

Thus, $a=3,\ b=1$

Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.
At $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sqrt{1-Ph} - \sqrt{1+Ph}}{-h} = \lim_{h \to 0} \frac{\left(\sqrt{1-Ph} - \sqrt{1+Ph}\right)}{-h} \times \frac{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}$$

$$= \lim_{h \to 0} \frac{(1-Ph)-(1+Ph)}{-h\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)} = \frac{2P}{2} = P$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$$P = \frac{-1}{2}$$

Chapter 9 Continuity Ex 9.2 Q4(vii)

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x \to 2^-} (5) = \lim_{x \to 2^+} (ax+b) = 5$$
$$\Rightarrow 5 = 2a+b = 5$$
$$\Rightarrow 2a+b = 5 \qquad \ldots(1)$$

Since f is continuous at $x = 10$, we obtain

$$\lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) = f(10)$$
$$\Rightarrow \lim_{x \to 10^-} (ax+b) = \lim_{x \to 10^+} (21) = 21$$
$$\Rightarrow 10a+b = 21 = 21$$
$$\Rightarrow 10a+b = 21 \qquad \ldots(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$a = 2$

By putting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$4 + b = 5$

$b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x=\frac{\pi}{2}$ therefore

LHL of $f(x)$ at $x=\frac{\pi}{2}$ is

$$=\lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$=\lim_{h\to 0} f\left(h-\frac{\pi}{2}\right)$$

$$=\lim_{h\to 0}\frac{k\cos\left(h-\frac{\pi}{2}\right)}{\pi-2\left(h-\frac{\pi}{2}\right)}$$

$$=\lim_{h\to 0}\frac{k\sinh}{2\pi-2h}$$

$$=\frac{k}{2}\lim_{h\to 0}\frac{\sin(\pi-h)}{(\pi-h)}$$

$$=\frac{k}{2}$$

Again

$$f\left(\frac{\pi}{2}\right)=3$$

Hence

$$LHL=f\left(\frac{\pi}{3}\right)$$

$$\frac{k}{2}=3$$

$$k=6$$

Chapter 9 Continuity Ex 9.2 Q5

We have given that $f(x)$ is continuous on $[0,\infty]$

$\therefore$ $f(x)$ is continuous at $x=1$ and $x=\sqrt{2}$

$\therefore$ At $x=1$, LHL = RHL = $f(1)$ .....(A)

$f(1)=a$ .....(1)

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{(1-h)^2}{a} = \frac{1}{a}$$

Using (A) we get,

$$a=\frac{1}{a} \quad \Rightarrow a^2=1 \Rightarrow \quad a=\pm 1$$

At $x=\sqrt{2}$ LHL = RHL = $f(\sqrt{2})$ .....(B)

$$f(\sqrt{2})=\frac{2b^2-4b}{(\sqrt{2})^2}=\frac{2b^2-4b}{2}=b^2-2b \qquad \text{......(2)}$$

$$\text{LHL} = \lim_{x\to\sqrt{2}^-} f(x) = \lim_{h\to 0} f(\sqrt{2}-h) = \lim_{h\to 0} a = a.$$

So, using (B), we get,

$b^2-2b=a$

For $a=1$, $\quad b^2-2b-1=0$

$$\Rightarrow b=\frac{2\pm\sqrt{4+4}}{2}=1\pm\sqrt{2}$$

For $a=-1$ $\quad b^2-2b+1=0$

$\Rightarrow (b-1)^2=0 \Rightarrow b=1$

Thus, $a=-1$, $b=1$ or $a=1$, $b=1\pm\sqrt{2}$

Chapter 9 Continuity Ex 9.2 Q6

Since, $f(x)$ is continuous on $[0,\pi]$

$f(x)$ is continuous at $x=\frac{\pi}{4}$ and $x=\frac{\pi}{2}$

At $x=\frac{\pi}{4}$,

$\text{LHL}=\text{RHL}=f\left(\frac{\pi}{4}\right).....(A)$

Now, $f\left(\frac{\pi}{4}\right)=2\frac{\pi}{4}.\cot\left(\frac{\pi}{4}\right)+b=\frac{\pi}{2}.1+b=\frac{\pi}{2}+b\ ....(1)$

$\text{LHL}=\lim_{x\to\frac{\pi}{4}^-}f(x)=\lim_{h\to0}f\left(\frac{\pi}{4}-h\right)=\lim_{h\to0}\left(\frac{\pi}{4}-h\right)+a\sqrt{2}\sin\left(\frac{\pi}{4}-h\right)=\frac{\pi}{4}+a\sqrt{2}.\frac{1}{\sqrt{2}}=\frac{\pi}{4}+a$

Thus, using (A)

$\frac{1}{2}+b=\frac{\pi}{4}+a$

$a-b=\frac{\pi}{4}.....(B)$

At $x=\frac{\pi}{2}$

$\text{LHL}=\text{RHL}=f\left(\frac{\pi}{2}\right).....(C)$

Now, $f\left(\frac{\pi}{2}\right)=a\cos2.\frac{\pi}{2}-b\sin\frac{\pi}{2}=-a-b....(2)$

$\text{LHL}=\lim_{x\to\frac{\pi}{2}^-}f(x)=\lim_{h\to0}f\left(\frac{\pi}{2}-h\right)$

$=\lim_{h\to0}2\left(\frac{\pi}{2}-h\right)\cot\left(\frac{\pi}{2}-h\right)+b=\pi\times0+b=b$

using (C), we get,

$-a-b=b \quad\Rightarrow 2b=-a \quad\Rightarrow b=\frac{-a}{2}$

from (B), $a+\frac{a}{2}=\frac{\pi}{4}$

$\Rightarrow \quad \frac{3}{2}a=\frac{\pi}{4}$

$\Rightarrow \quad a=\frac{\pi}{6}$

and $b=\frac{-a}{2}=\frac{-\pi}{12}$

Thus, $a=\frac{\pi}{6},\quad b=\frac{-\pi}{12}$

Chapter 9 Continuity Ex 9.2 Q7

It is given that the $f(x)$ is continuous on $[0,8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

$$\text{LHL} = \text{RHL} = f(2) \ldots .(A)$$

$$f(2) = 3 \times 2 + 2 = 8 \ldots .(1)$$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$$

from (A)

$$4 + 2a + b = 8$$

$$2a + b = 4 \ldots .(B)$$

Now, At $x = 4$

$$\text{LHL} = \text{RHL} = f(4) \ldots .(C)$$

$$f(4) = 3 \times 4 + 2 = 14 \ldots .(2)$$

$$\text{RHL} = \lim_{x \to 4^+} f(x) = \lim_{h \to 0} f(4+h) = \lim_{h \to 0} 2a(4+h) + 5b = 8a + 5b$$

From (C), we get,

$$8a + 5b = 14 \ldots .(D)$$

Solving (B) and (D), we get,

$a = 3$ and $b = -2$

Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on $\left[0, \frac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \frac{\pi}{2}\right]$

Let us consider the point $x = \frac{\pi}{4}$,

We must have,

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{4}\right) \ldots .(A)$$

$$\text{LHL} = \lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \to 0} \frac{\tan h}{\tan 2h} \qquad \left[\because \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta\right]$$

$$= \lim_{h \to 0} \frac{\frac{\tan h}{h}}{\frac{\tan 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

Hence, $f(x)$ will be continuous on $\left[0, \frac{\pi}{2}\right]$ if $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

Chapter 9 Continuity Ex 9.2 Q9

When $x < 2$, we have
$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.
When $x > 2$, we have
$f(x) = \frac{3x}{2}$, which is again a polynomial of degree 1.
So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$
$f(2) = \frac{3 \times 2}{2} = 3$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) - 1 = 3$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{3(2+h)}{2} = 3$$

$\text{LHL} = \text{RHL} = f(2) = 3$

Thus, $f(x)$ is continuous at $x = 2$
Hence, $f(x)$ is continuous every where.

Chapter 9 Continuity Ex 9.2 Q10

Let $f(x) = \sin|x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g\, o\, h$, where $g(x) = |x|$ and $h(x) = \sin x$

$$\left[\because (goh)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)\right]$$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} (-x) = -c$
$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$
$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$$
$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$$
$$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = \sin x$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + k$

If $x \to c$, then $k \to 0$

$h(c) = \sin c$

$$h(c) = \sin c$$
$$\begin{aligned} \lim_{x \to c} h(x) = \lim_{x \to c} \sin x \\ &= \lim_{k \to 0} \sin(c + k) \\ &= \lim_{k \to 0} [\sin c \cos k + \cos c \sin k] \\ &= \lim_{k \to 0} (\sin c \cos k) + \lim_{h \to 0} (\cos c \sin k) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c \end{aligned}$$
$$\therefore \lim_{x \to c} h(x) = g(c)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\sin x) = |\sin x|$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q11

When $x < 0$, we have,

$f(x) = \frac{\sin x}{x}$

We know that the $\sin x$ and the identity function x are continuous for $x < 0$.

So, the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$, we have,

$f(x) = x + 1$, which is a polynomial of degree 1. So, $f(x)$ is continuous for $x > 0$

Now, consider the point $x = 0$.

$f(0) = 0 + 1 = 1$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} h + 1 = 1$$

Thus, LHL = RHL = $f(0) = 1$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere

Chapter 9 Continuity Ex 9.2 Q12

The given function is $g(x) = x - [x]$

It is evident that g is defined at all integral points.

Let n be an integer.

Then,

$g(n) = n - [n] = n - n = 0$

The left hand limit of f at $x = n$ is,

$$\lim_{x \to n^-} g(x) = \lim_{x \to n^-} (x - [x]) = \lim_{x \to n^-} (x) - \lim_{x \to n^-} [x] = n - (n-1) = 1$$

The right hand limit of f at $x = n$ is,

$$\lim_{x \to n^+} g(x) = \lim_{x \to n^+} (x - [x]) = \lim_{x \to n^+} (x) - \lim_{x \to n^+} [x] = n - n = 0$$

It is observed that the left and right hand limits of f at $x = n$ do not coincide.

Therefore, f is not continuous at $x = n$

Hence, g is discontinuous at all integral points

Chapter 9 Continuity Ex 9.2 Q13

It is known that if g and h are two continuous functions, then $g+h$, $g-h$, and $g.h$ are also continuous.

It has to proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$g(c) = \sin c$

$$\begin{aligned}\lim_{x\to c} g(x) &= \lim_{x\to c} \sin x \\ &= \lim_{h\to 0} \sin(c+h) \\ &= \lim_{h\to 0}[\sin c \cos h + \cos c \sin h] \\ &= \lim_{h\to 0}(\sin c \cos h) + \lim_{h\to 0}(\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$\therefore \lim_{x\to c} g(x) = g(c)$

Therefore, g is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c} \cos x \\ &= \lim_{h\to 0} \cos(c+h) \\ &= \lim_{h\to 0}[\cos c \cos h - \sin c \sin h] \\ &= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$\therefore \lim_{x\to c} h(x) = h(c)$

Therefore, h is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q14

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \to c$, then $h \to 0$

$$\begin{aligned}\lim_{x\to c} g(x) &= \lim_{x\to c} \cos x \\ &= \lim_{h\to 0} \cos(c+h) \\ &= \lim_{h\to 0}\left[\cos c \cos h - \sin c \sin h\right] \\ &= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$$

$$\therefore \lim_{x\to k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q15

The given function is $f(x) = |\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)\right]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim_{x\to c} g(x)=\lim_{x\to c}(-x)=-c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim_{x\to c} g(x)=\lim_{x\to c} x=c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x>0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, g is continuous at $x=0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x)=\cos x$

It is evident that $h(x)=\cos x$ is defined for every real number.

Let c be a real number. Put $x=c+h$

If $x\to c$, then $h\to 0$

$h(c)=\cos c$

$$\begin{aligned}\lim_{x\to c} h(x)&=\lim_{x\to c}\cos x\\ &=\lim_{h\to 0}\cos(c+h)\\ &=\lim_{h\to 0}[\cos c\cos h-\sin c\sin h]\\ &=\lim_{h\to 0}\cos c\cos h-\lim_{h\to 0}\sin c\sin h\\ &=\cos c\cos 0-\sin c\sin 0\\ &=\cos c\times 1-\sin c\times 0\\ &=\cos c\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=h(c)$$

Therefore, $h(x)=\cos x$ is a continuous function.

It is known that for real valued functions g and h, such that $(g\circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f\circ g)$ is continuous at c.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\cos x)=|\cos x|$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q16

The given function is $f(x)=|x|-|x+1|$

The two functions, g and h, are defined as

$g(x)=|x|$ and $h(x)=|x+1|$

Then, $f=g-h$

The continuity of g and h is examined first.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x>0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$\lim\limits_{x\to 0^-} g(x)=\lim\limits_{x\to 0^-}(-x)=0$

$\lim\limits_{x\to 0^+} g(x)=\lim\limits_{x\to 0^+}(x)=0$

$\therefore \lim\limits_{x\to 0^-} g(x)=\lim\limits_{x\to 0^+}(x)=g(0)$

Therefore, g is continuous at $x=0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x)=|x+1|$ can be written as

$$h(x)=\begin{cases}-(x+1), & \text{if, } x<-1\\ x+1, & \text{if } x\geq -1\end{cases}$$

Clearly, h is defined for every real number.

Let c be a real number.

Case I:

If $c < -1$, then $h(c) = -(c+1)$ and $\lim_{x\to c} h(x) = \lim_{x\to c}[-(x+1)] = -(c+1)$

$\therefore \lim_{x\to c} h(x) = h(c)$

Therefore, h is continuous at all points x, such that $x < -1$

Case II:

If $c > -1$, then $h(c) = c+1$ and $\lim_{x\to c} h(x) = \lim_{x\to c}(x+1) = c+1$

$\therefore \lim_{x\to c} h(x) = h(c)$

Therefore, h is continuous at all points x, such that $x > -1$

Case III:

If $c = -1$, then $h(c) = h(-1) = -1+1 = 0$

$\lim_{x\to -1^-} h(x) = \lim_{x\to -1^-}[-(x+1)] = -(-1+1) = 0$

$\lim_{x\to -1^+} h(x) = \lim_{x\to -1^+}(x+1) = (-1+1) = 0$

$\therefore \lim_{x\to -1^-} h(x) = \lim_{h\to -1^+} h(x) = h(-1)$

Therefore, h is continuous at $x = -1$

From the above three observations, it can be concluded that h is continuous at all points of the real line.

g and h are continuous functions. Therefore, $f = g - h$ is also a continuous function.

Therefore, f has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q17

He given function f is $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

If $c \neq 0$, then $f(c) = c^2 \sin\frac{1}{c}$

$\lim_{x\to c} f(x) = \lim_{x\to c}\left(x^2 \sin\frac{1}{x}\right) = \left(\lim_{x\to c} x^2\right)\left(\lim_{x\to c} \sin\frac{1}{x}\right) = c^2 \sin\frac{1}{c}$

$\therefore \lim_{x\to c} f(x) = f(c)$

Therefore, f is continuous at all points $x \neq 0$

Case II:

If $c = 0$, then $f(0) = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(x^2 \sin\frac{1}{x}\right) = \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right)$$

It is known that, $-1 \le \sin\frac{1}{x} \le 1,\ x \ne 0$

$$\Rightarrow -x^2 \le \sin\frac{1}{x} \le x^2$$

$$\Rightarrow \lim_{x\to 0}\left(-x^2\right) \le \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right) \le \lim_{x\to 0} x^2$$

$$\Rightarrow 0 \le \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right) \le 0$$

$$\Rightarrow \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right) = 0$$

$$\therefore \lim_{x\to 0^-} f(x) = 0$$

Similarly, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(x^2 \sin\frac{1}{x}\right) = \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right) = 0$

$$\therefore \lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Chapter 9 Continuity Ex 9.2 Q18

$$f(x) = \frac{1}{x+2}$$

$$\lim_{x\to -2^-} f(x) = \lim_{h\to 0}\frac{1}{-2-h+2} = \lim_{h\to 0} -\frac{1}{h} \to -\infty$$

$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0}\frac{1}{-2+h+2} = \lim_{h\to 0}\frac{1}{h} \to \infty$$

$\therefore$ $f(x)$ is discontinuous at $x = -2$

Let $g(x) = f(f(x)) = \dfrac{x+2}{2x+5}$

$$\lim_{x\to -\frac{5}{2}^-} g(x) = \lim_{h\to 0}\frac{-\frac{5}{2}-h+2}{-5-h+5} = \lim_{h\to 0} -\frac{-\frac{5}{2}-h+2}{h} \to -\infty$$

$$\lim_{x\to -\frac{5}{2}^+} f(x) = \lim_{h\to 0}\frac{-\frac{5}{2}+h+2}{-5+h+5} = \lim_{h\to 0}\frac{-\frac{5}{2}-h+2}{h} \to \infty$$

$\therefore$ $g(x)$ is discontinuous at $x = -\frac{5}{2}$

$\therefore$ $f(f(x))$ is discontinuous at $x = -\frac{5}{2}$

$\therefore$ $f(x)$ is discontinuous at $x = -2$ and $-\frac{5}{2}$.

Chapter 9 Continuity Ex 9.2 Q19

$f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x - 1}$

Clearly $t = \frac{1}{x - 1}$ is discontinuous at $x = 1$.

For $x \neq 1$, we have

$f(t) = \frac{1}{t^2 + t - 2} = \frac{1}{(t + 2)(t - 1)}$

This is discontinuous at $t = -2$ and $t = 1$

For $t = -2$, $t = \frac{1}{x - 1} \Rightarrow x = \frac{1}{2}$

For $t = 1$, $t = \frac{1}{x - 1} \Rightarrow x = 2$

Hence f is discontinuous at $x = \frac{1}{2}$, $x = 1$ and $x = 2$.