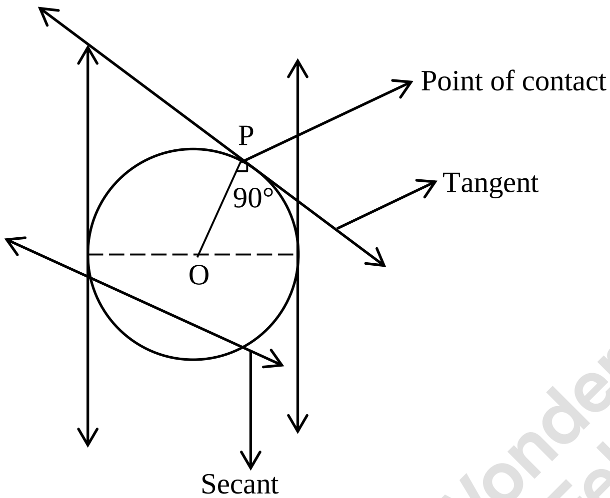
Exercise – 10.1

1. Fill in the blanks
   (i) The common point of tangent and the circle is called point of contact.
   (ii) A circle may have two parallel tangents.
   (iii) A tangent to a circle intersects it in one point.
   (iv) A line intersecting a circle in two points A called a secant.
   (v) The angle between tangent at a point P on circle and radius through the point is 90°.

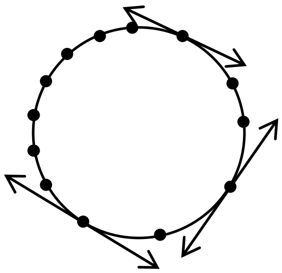
   Sol:



2. How many tangents can a circle have?

   Sol:

   Tangent: A line intersecting circle in one point is called a tangent.

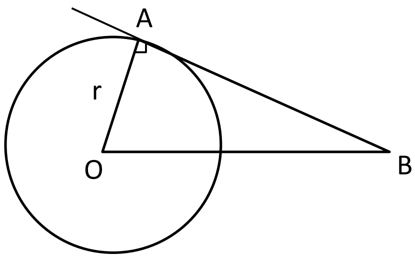


As there are infinite number of points on the circle a circle has many (infinite) tangents.

3. O is the center of a circle of radius 8cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB

   Sol:

   Consider a circle with center O and radius OA = 8cm = r, AB = 15 cm.



(AB) tangent is drawn at A (point of contact)

At point of contact, we know that radius and tangent are perpendicular.

In ΔOAB, ∠OAB = 90°, By Pythagoras theorem

$OB^2 = OA^2 + AB^2$

$OB = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{229} = 17\ cm$

$\therefore OB = 17\ cm$

4. If the tangent at point P to the circle with center O cuts a line through O at Q such that PQ = 24cm and OQ = 25 cm. Find the radius of circle

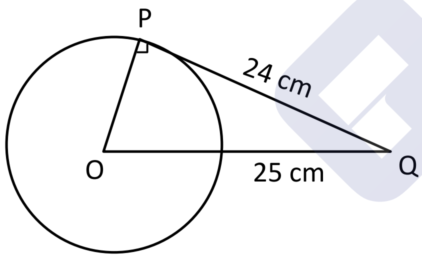
Sol:

Given,

PQ = 24 cm

OQ = 25 cm

OP = radius = ?



P is point of contact, At point of contact, tangent and radius are perpendicular to each other

∴ ΔPOQ is right angled triangle ∠OPQ = 90°

By Pythagoras theorem,

$PQ^2 + OP^2 = OQ^2$

$\Rightarrow 24^2 + OP^2 = 25^2$

$\Rightarrow OP = \sqrt{25^2 - 24^2} = \sqrt{625 - 576}$

$= \sqrt{49} = 7cm$

$\therefore OP = radius = 7cm$

Exercise – 10.2

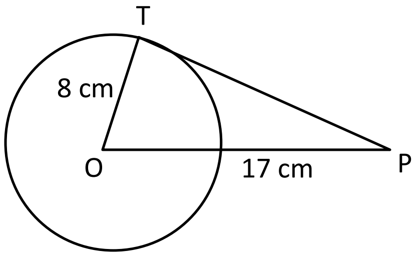
1. If PT is a tangent at T to a circle whose center is O and OP = 17 cm, OT = 8 cm. Find the length of tangent segment PT.

Sol:

OT = radius = 8cm

OP = 17cm

PT = length of tangent = ?



T is point of contact. We know that at point of contact tangent and radius are perpendicular.

∴ OTP is right angled triangle ∠OTP = 90°, from Pythagoras theorem $OT^2 + PT^2 = OP^2$

$8^2 + PT^2 = 17^2$

PT $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

= $\sqrt{225} = 15cm$

∴ PT = length of tangent = 15 cm.

2. Find the length of a tangent drawn to a circle with radius 5cm, from a point 13 cm from the center of the circle.

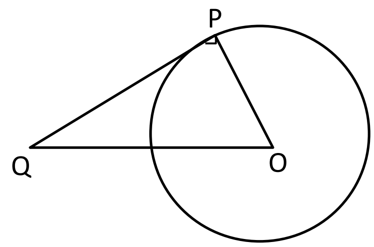
Sol:

Consider a circle with center O.

OP = radius = 5 cm.

A tangent is drawn at point P, such that line through O intersects it at Q, OB = 13cm.

Length of tangent PQ = ?



A + P, we know that tangent and radius are perpendicular.

ΔOPQ is right angled triangle, ∠OPQ = 90°

$By\ pythagoras\ theorem, OQ^2 = OP^2 + PQ^2$

$\Rightarrow 13^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

$\Rightarrow$ PQ = $\sqrt{144} = 12cm$

Length of tangent = 12 cm

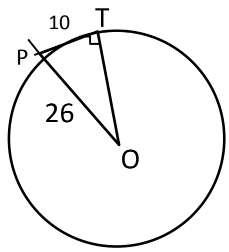
3. A point P is 26 cm away from O of circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Sol:

Given OP = 26 cm

PT = length of tangent = 10cm

radius = OT = ?



At point of contact, radius and tangent are perpendicular ∠OTP = 90°, ΔOTP is right angled triangle.

By Pythagoras theorem, $OP^2 = OT^2 + PT^2$

$26^2 = OT^2 + 10^2$

$OT^k = \left(\sqrt{676 - 100}\right)^k$

$OT = \sqrt{576}$

= 24 cm

OT = length of tangent = 24 cm

4. If from any point on the common chord of two intersecting circles, tangents be drawn to circles, prove that they are equal.

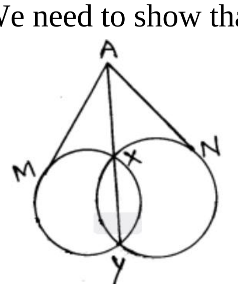
Sol:

Let the two circles intersect at points X and Y.

XY is the common chord.

Suppose 'A' is a point on the common chord and AM and AN be the tangents drawn A to the circle

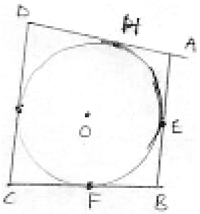
We need to show that AM = AN.



In order to prove the above relation, following property will be used.

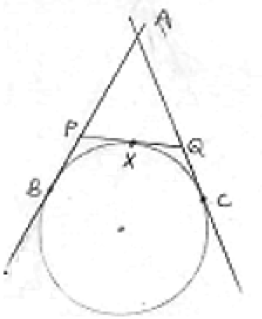
"Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$"
Now AM is the tangent and AXY is a secant ∴ $AM^2 = AX \times AY \ldots..(i)$
AN is a tangent and AXY is a secant ∴ $AN^2 = AX \times AY \ldots..(ii)$
From (i) & (ii), we have $AM^2 = AN^2$
∴ AM = AN

5. If the quadrilateral sides touch the circle prove that sum of pair of opposite sides is equal to the sum of other pair.
Sol:
Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



We know that
The tangents drawn from same external points to the circle are equal in length.
1. Consider tangents from point A [AM ⊥ AE]
   AH = AE …. (i)
2. From point B [EB & BF]
   BF = EB …. (ii)
3. From point C [CF & GC]
   FC = CG …. (iii)
4. From point D [DG & DH]
   DH = DG …. (iv)
   Adding (i), (ii), (iii), & (iv)
   (AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]
   ⇒ (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)
   ⇒ AD + BC = AB + DC [from fig.]
   Sum of one pair of opposite sides is equal to other.

6. If AB, AC, PQ are tangents in Fig. and AB = 5cm find the perimeter of ΔAPQ.
Sol:
Perimeter of ΔAPQ, (P) = AP + AQ + PQ
= AP + AQ + (PX + QX)



We know that

The two tangents drawn from external point to the circle are equal in length from point A,

AB = AC = 5 cm

From point P, PX = PB

From point Q, QX = QC

Perimeter (P) = AP + AQ + (PB + QC)

= (AP + PB) + (AQ + QC)

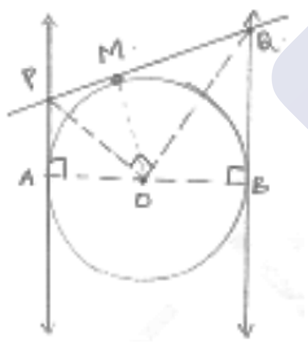
= AB + AC = 5 + 5

= 10 cms.

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at center.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that ∠POQ = 90°.

From fig. it is clear that ABQP is a quadrilateral

∠A + ∠B = 90° + 90° = 180° [At point of contact tangent & radius are perpendicular]

∠A + ∠B + ∠P + ∠Q = 360° [Angle sum property]

∠P + ∠Q = 360°−180° = 180° …..(i)

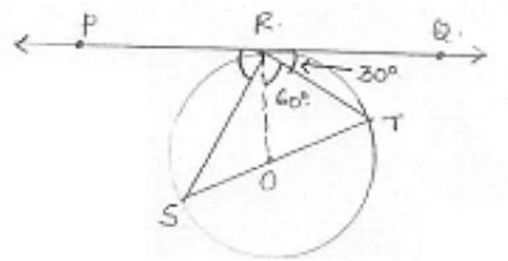
At P & Q ∠APO = ∠OPQ = $\frac{1}{2}\angle P$

∠BQO = ∠PQO = $\frac{1}{2}\angle Q$ in (i)

2∠OPQ + 2 ∠PQO = 180°

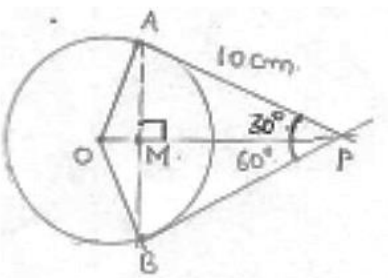
∠OPQ + ∠PQO = 90° …. (ii)
In ΔOPQ, ∠OPQ + ∠PQO + ∠POQ = 180° [Angle sum property]
90° + ∠POQ = 180° [from (ii)]
∠POQ = 180° − 90° = 90°
∴ ∠POQ = 90°

8. In Fig below, PQ is tangent at point R of the circle with center O. If ∠TRQ = 30°. Find ∠PRS.



Sol:
Given ∠TRQ = 30°.
At point R, OR ⊥ RQ.
∠ORQ = 90°
⇒ ∠TRQ + ∠ORT = 90°
⇒ ∠ORT = 90° − 30° = 60°
ST is diameter, ∠SRT = 90° [∵ Angle in semicircle = 90°]
∠ORT + ∠ SRO = 90°
∠SRO + ∠PRS = 90°
∠PRS = 90° − 30° = 60°

9. If PA and PB are tangents from an outside point P. such that PA = 10 cm and ∠APB = 60°. Find the length of chord AB.
Sol:
AP = 10 cm ∠APB = 60°
Represented in the figure
We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point ∠APO = ∠OPB = $\frac{1}{2}$ × 60° = 30°

The chord AB will be bisected perpendicularly
∴ AB = 2AM
In ΔAMP,

$$\sin 30° = \frac{opp.side}{hypotenuse} = \frac{AM}{AP}$$

AM = AP sin 30°

$$= \frac{AP}{2} = \frac{10}{2} = 5cm$$

AP = 2 AM = 10 cm ---- Method (i)

In ΔAMP, ∠AMP = 90°, ∠APM = 30°

∠AMP + ∠APM + ∠MAP = 180°

$$90° + 30° + \angle MAP = 180°$$

$$\angle MAP = 180°$$

In $\Delta PAB, \angle MAP = \angle BAP = 60°, \angle APB = 60°$

We also get, $\angle PBA = 60°$

∴ΔPAB is equilateral triangle

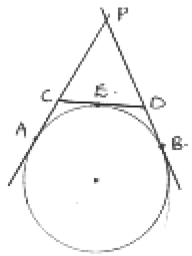
$AB = AP = 10$ cm. -----Method (ii)

10. From an external point P, tangents PA and PB are drawn to the circle with centre O. If CD is the tangent to the circle at point E and PA = 14 cm. Find the perimeter of ABCD.

Sol:

PA = 14 cm

Perimeter of ΔPCD = PC + PD + CD = PC + PD + CE + ED



We know that

The two tangents drawn from external point to the circle are equal in length.

From point P, PA = PB = 14cm

From point C, CE = CA

From point D, DB = ED

Perimeter = PC + PD + CA +DB

= (PC + CA) + (PD + DB)

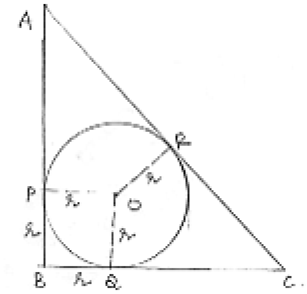
= PA + PB = 14 + 14 = 28 cm.

11. In the fig. ABC is right triangle right angled at B such that BC = 6cm and AB = 8cm. Find the radius of its in circle.

Sol:

BC = 6cm AB = 8cm

As ABC is right angled triangle



By Pythagoras theorem

$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$

$AC = 10\ cm$

Consider BQOP ∠B = 90°,

∠BPO = ∠OQB = 90° [At point of contact, radius is perpendicular to tangent]

All the angles = 90° & adjacent sides are equal

∴ BQOP is square BP = BQ = r

We know that

The tangents drawn from any external point are equal in length.

AP = AR = AB – PB = 8 – r

QC = RC = BC – BQ = 6 – r

AC = AR + RC ⇒ 10 = 8 – r + 6 – r

⇒ 10 = 14 – 2r

⇒ 2r = 4

⇒ Radius = 2cm

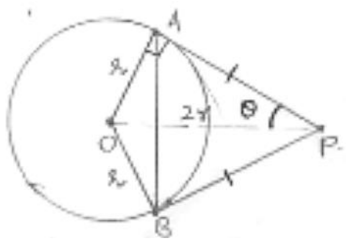
12. From a point P, two tangents PA and PB are drawn to a circle with center O. If OP = diameter of the circle shows that ΔAPB is equilateral.

Sol:

OP = 2r

Tangents drawn from external point to the circle are equal in length

PA = PB



At point of contact, tangent is perpendicular to radius.

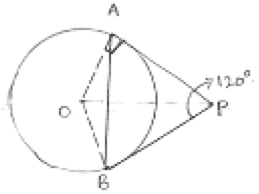
In ΔAOP, sin $\theta = \frac{opp.side}{hypotenuse} = \frac{r}{2r} = \frac{1}{2}$

$\theta = 30°$

∠APB = 20 = 60°, as PA = PB ∠BAP = ∠ABP = x.
In ΔPAB, by angle sum property
∠APB + ∠BAP + ∠ABP = 180°
2x = 120° ⇒ x = 60°
In this triangle all angles are equal to 60°
∴ ΔAPB is equilateral.

13. Two tangent segments PA and PB are drawn to a circle with center O such that ∠APB = 120°. Prove that OP = 2AP

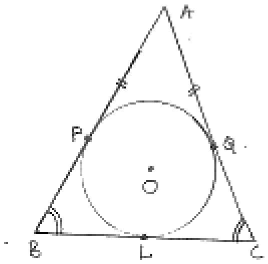
Sol:
A + P
OP bisects ∠APB
∠APO = ∠OPB = $\frac{1}{2}\angle APB = \frac{1}{2} \times 120° = 60°$



At point A
OA ⊥ AP, ∠OAP = 90°
In ΔPDA, cos 60° = $\frac{AP}{DP}$
$\frac{1}{2} = \frac{AP}{DP} \Rightarrow DP = 2AP$

14. If ΔABC is isosceles with AB = AC and C (0, 2) is the in circle of the ΔABC touching BC at L, prove that L, bisects BC.

Sol:
Given ΔABC is isosceles AB = AC



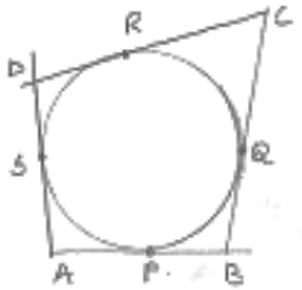
We know that
The tangents from external point to circle are equal in length
From point A, AP = AQ
But AB = AC ⇒ AP + PB = AQ + QC
⇒ PB = PC …. (i)

From B, PB = BL; ….(ii) from C, CL = CQ …..(iii)
From (i), (ii) & (iii)
BL = CL
∴ L bisects BC.

15. In fig. a circle touches all the four sides of quadrilateral ABCD with AB = 6cm, BC = 7cm, CD = 4cm. Find AD.

Sol:

We know that the tangents drawn from any external point to circle are equal in length.

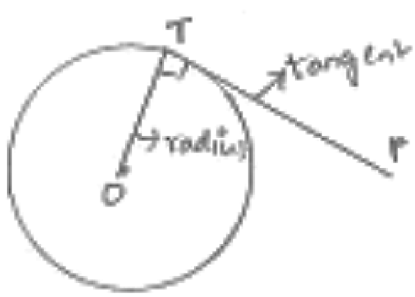


From A ⟶ AS = AP ….(i)
From B ⟶ QB = BP …. (ii)
From C ⟶ QC = RC …..(iii)
From D ⟶ DS = DR …. (iv)
Adding (i), (ii), (iii) & (iv)
(AS + QB + QC + DS) = (AB + BP + RC + OR)
(AS + DS) + (QB + QC) = (AP + BP) + (RC + DR)
AD + BC = AB + CD
⇒ AD + 7 = 6 + 4 AD = 3cm
⇒ AD = 10 – 7 = 3cm

16. Prove that the perpendicular at the point of contact to a circle passes through the centre of the circle.

Sol:

We know that



The at point of contact, the tangent is perpendicular to the radius. Radius is line from center to point on circle. Therefore, perpendicular to tangent will pass through center of circle.

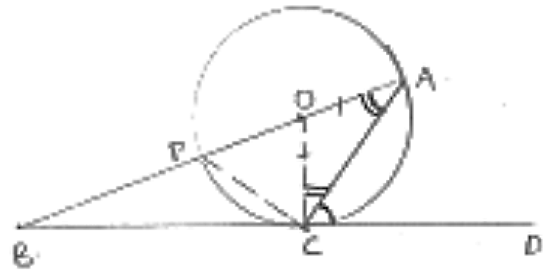
17. In fig.. O is the center of the circle and BCD is tangent to it at C. Prove that ∠BAC + ∠ACD = 90°

Sol:

Given

O is center of circle

BCD is tangent.



Required to prove: ∠BAC + ∠ACD = 90°

Proof: OA = OC [radius]

In ΔOAC, angles opposite to equal sides are equal.

∠OAC = ∠OCA …. (i)

∠OCD = 90° [tangent is radius are perpendicular at point of contact]

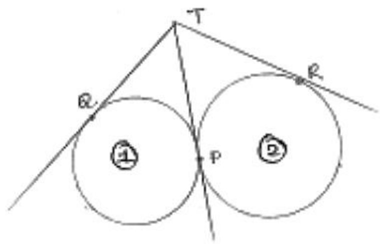
∠ACD + ∠OCA = 90°

∠ACD + ∠OAC = 90° [∵ ∠OAC = ∠BAC]

∠ACD + ∠BAC = 90° ⟶ Hence proved

18. Two circles touch externally at a point P. from a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and E respectively. Prove that TQ = TR.

Sol:



Let the circles be represented by (i) & (ii) respectively

TQ, TP are tangents to (i)

TP, TR are tangents to (ii)

We know that

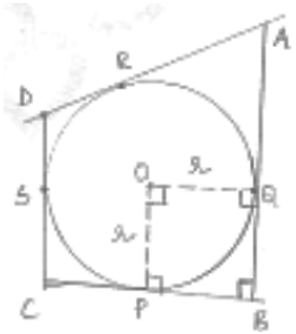
The tangents drawn from external point to the circle will be equal in length.

For circle (i), TQ = TP …. (i)

For circle (ii), TP = TR …. (ii)
From (i) & (ii) TQ = TR

19. In the fig. a circle is inscribed in a quadrilateral ABCD in which ∠B = 90° if AD = 23cm, AB = 29cm and DS = 5cm, find the radius of the circle.

Sol:
Given AD = 23 cm
AB = 29 cm
∠B = 90°
DS = 5cm

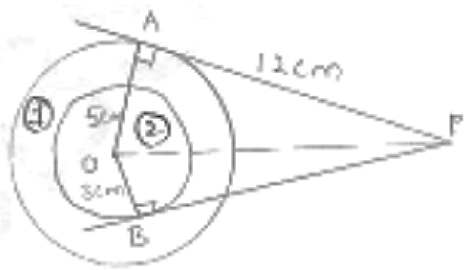


From fig in quadrilateral POQB
∠OPB = ∠OQB = 90° = ∠B = ∠POQ
and PO = OQ. ∴ POQB is a square PB = BQ = r
We know that
Tangents drawn from external point to circle are equal in length.
We know that
Tangents drawn from external point to circle are equal in length.
From A, AR = AQ …. (i)
From B, PB = QB …. (ii)
From C, PC = CS …. (iii)
From D, DR = DS …. (iv)
(i) + (ii) + (iv) ⇒ AR + DB + DR = AQ + QB + DS
⇒ (AR + DR) + r = (AQ + QB) + DS
AD + r = AB + DS
⇒ 23 + r = 29 + 5
⇒ r = 34 – 23 = 11 cm
∴ radius = 11 cm

20. In fig. there are two concentric circles with Centre O of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles if AP = 12cm, find the tangent length of BP.

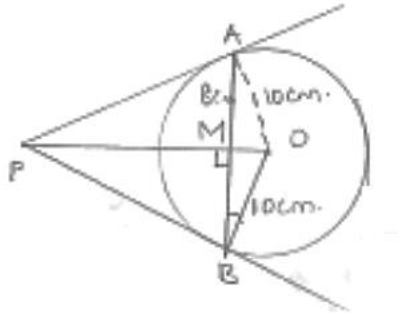
Sol:

Given



OA = 5 cm
OB = 3 cm
AP = 12 cm
BP = ?
We know that
At the point of contact, radius is perpendicular to tangent.
For circle 1, ΔOAP is right triangle
By Pythagoras theorem, $OP^2 = OA^2 + AP^2$
$\Rightarrow OP^2 = 5^2 + 12^2 = 25 + 144$
= 169
⇒ OP = $\sqrt{169} = 13\ cm$
For circle 2, ΔOBP is right triangle by Pythagoras theorem,
$OP^2 = OB^2 + BP^2$
$13^2 = 3^2 + BP^2$
$BP^2 = 169 - 9 = 160$
$BP = \sqrt{160} = 4\sqrt{10}\ cm$

21. In fig. AB is chord of length 16cm of a circle of radius 10cm. The tangents at A and B intersect at a point P. Find the length of PA.
Sol:
Given length of chord AB = 16cm.
Radius OB = OA = 10 cm.



Let line through Centre to point from where tangents are drawn be intersecting chord AB at M. we know that the line joining Centre to point from where tangents are drawn be intersecting chord AB at M. we know that

The line joining Centre to point from where tangents are drawn bisects the chord joining the points on the circle where tangents intersects the circle.

AM = MB = $\frac{1}{2}(AB) = \frac{1}{2} \times 16 = 8cm$

Consider ΔOAM from fig. ∠AMO = 90°

By Pythagoras theorem, $OA^2 = AM^2 + OM^2$

$10^2 = 8^2 + OM^2$

$OM = \sqrt{100 - 64} = \sqrt{36} = 6cm$

In ΔAMP, ∠AMP = 90° by Pythagoras theorem $AP^2 = AM^2 + PM^2$

$AP^2 = 8^2 + (OP - OM)^2$

$PA^2 = 64 + (OP - 6)^2$

$(OP - 6)^2 = -64 + PA^2$ ….(i)

$In\ \Delta APO, \angle PAO = 90°$ [At point of contact, radius is perpendicular to tangent]

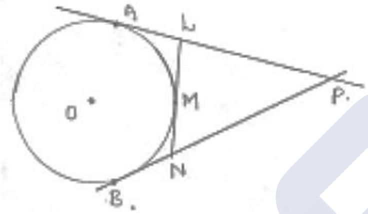
$OP^2 = OA^2 + PA^2$ [Pythagoras theorem]

$PA^2 = OP^2 - 10^2$

$= OP^2 - 100$ ….. (ii)

22. In figure PA and PB are tangents from an external point P to the circle with centre O. LN touches the circle at M. Prove that PL + LM = PN + MN

Sol:



Given

O is Centre of circle

PA and PB are tangents

We know that

The tangents drawn from external point to the circle are equal in length.

From point P, PA = PB

⇒ PL + AL = PN + NB …. (i)

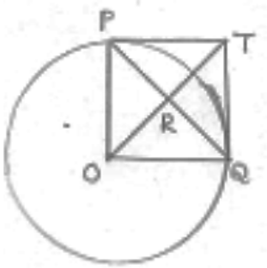
From point L & N, AL = LM and MN = NB } …. Substitute in (i)

PL + Lm = PN + MN

⇒ Hence proved.

23. In the fig. PO ⊥ QO. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.

Sol:



Given

PO ⊥ OQ

Consider quadrilateral OQTP.

∠POQ = 90°

∠OPT = ∠OQT = 90° [At point of contact, tangent and radius are perpendicular]

∴ ∠PTO = 90°

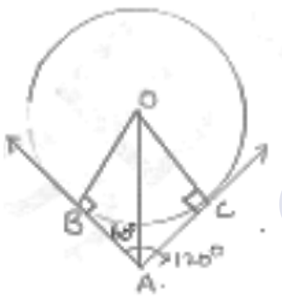
OP = OQ = radius

In this quadrilateral, all the angles are equal and pair of adjacent sides are equal.

∴ OQTP is a square.

24. In the fig two tangents AB and AC are drawn to a circle O such that ∠BAC = 120°. Prove that OA = 2AB.

Sol:



Consider Centre O for given circle

∠BAC = 120°

AB and AC are tangents

From the fig.

In ΔOBA, ∠OBA = 90° [radius perpendicular to tangent at point of contact]

∠OAB = ∠OAC = $\frac{1}{2}\angle BAC = \frac{1}{2} \times 120° = 60°$

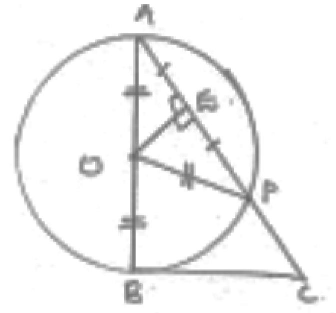
[Line joining Centre to external point from where tangents are drawn bisects angle formed by tangents at that external point1]

In ΔOBA, cos 60° = $\frac{AB}{OA}$

$\frac{1}{2} = \frac{AB}{OA} \Rightarrow OA = 2AB$

25. In the fig. BC is a tangent to the circle with Centre O. OE bisects AP. Prove that ΔAEO ~ ΔABC.

Sol:



Given

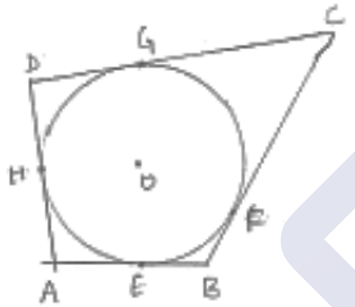
BC is tangent to circle

OE bisects AP, AE = EP

Consider ΔAOP

26. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4cm, 5cm and 7cm respectively. Determine the length of fourth side.

Sol:



Let us consider a quadrilateral ABCD, AB = 4cm, BC = 5 cm, CD = 7cm, CD as sides circumscribing circle with centre O. and intersecting at points E, F, G, H. as in fig.

We know that the tangents drawn from external point to the circle are equal in length.

From point A, AE = AH …. (i)

From point B, BE = BF ….. (ii)

From point C, GC = CE ….(iii)

From point D, GD = DH ….(iv)

(i) + (ii) + (iii) + (iv) ⇒ (AE + BE + GC + GD) = (AH + BF + CF + DH)

⇒ (AE + BE) + (GC + GD) = (AH + DH) + (BF + CF)

⇒ AB + CD = AD + BC

⇒ 4 + 7 = 5 + AD

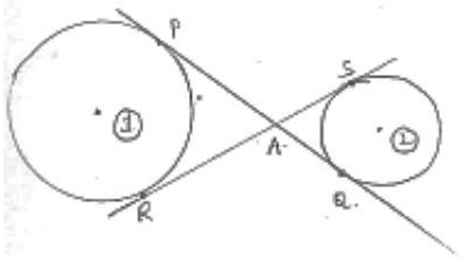
⇒ AD = 11 – 5 = 6 cm

Fourth side = 6 cm

27. In fig common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS.

Sol:

Consider



Two circles namely (i) & (ii) as shown with common tangents as PQ and RS.

We know that

The tangents from external point to the circle are equal in length.

From A to circle (i) AP = AR … (i)

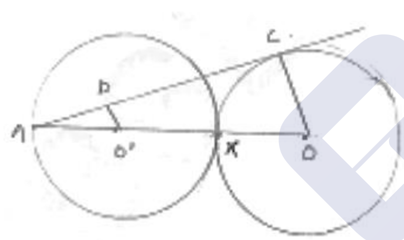
From A to circle (ii), AQ = AS …. (ii)

(i) + (ii) ⇒ AP + AQ = AR + RS

⇒ PQ = RS

28. Equal circles with centers O and O’ touch each other at X. OO’ produced to meet a circle with Centre O’ at A. AC is tangent to the circle whose Centre is a O’D is perpendicular to AC. Find the value of DO’/CO

Sol:



Given circles with centers O and O’

O’D ⊥ AC. Let radius = r

$O'A = O'X = OX = r$

In triangles, $\Delta AO'D$ and ΔAOC

∠A = ∠A [Common angle]

$\angle ADO' = \angle ACO = 90^\circ$ [O’D ⊥ AC and at point of contact C, radius ⊥ tangent]

By A · A similarity $\Delta AO'D \sim \Delta AOC$.

when two triangles are similar then their corresponding sides will be in proportion

By A.A similarity $\Delta AO'D \sim \Delta AOC$

When two triangles are similar then their corresponding sides will be in proportion

$$\frac{AO'}{AO} = \frac{DO'}{CO}$$

$$\Rightarrow \frac{DO'}{CO} = \frac{r}{r+r+r} = \frac{r}{3r} = \frac{1}{3}$$

$\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

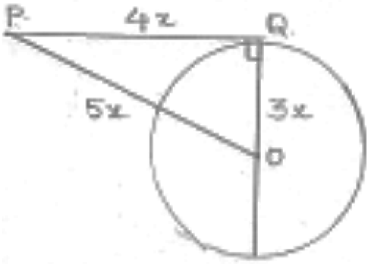
29. In figure OQ : PQ = 3 : 4 and perimeter of ΔPDQ = 60cm. determine PQ, QR and OP.

Sol:

Given OQ: PQ = 3 : 4

Let OQ = 3x PQ = 4x

OP = y



∠OQP = 90° [since at point of contact, tangent is perpendicular to radius]

In ΔOQP, by Pythagoras theorem

$OP^2 = OQ^2 + QP^2$

$\Rightarrow y^2 = (3x)^2 + (4x)^2$

$\Rightarrow y^2 = 9x^2 + 16x^2 = 25x^2$

$\Rightarrow y^2 = \sqrt{25x^2} = 5x$

Perimeter = OQ + PQ + OP = 3x + 4x + 5x = 12x

According to problem perimeter = 60

∴ 12x = 60

x = $\frac{60}{12} = 5cm$

OQ = $3 \times 5 = 15cm$

PQ = $4 \times 5 = 20\ cm$

OP = $5\ \times 5 = 25cm$