Ex 11.1

Differentiation Ex 11.1 Q1

Let $f(x) = e^{-x}$

$\Rightarrow f(x+h) = e^{-(x+h)}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{e^{-(x+h)}-e^{-x}}{h}$$
$$= \lim_{h\to 0}\frac{e^{-x}\times e^{-h}-e^{-x}}{h}$$
$$= \lim_{h\to 0} e^{-x}\left\{\frac{\left(e^{-h}-1\right)}{-h}\right\}\times(-1)$$
$$= -e^{-x} \qquad \left[\text{Since, } \lim_{h\to 0}\frac{e^{h}-1}{h} = 1\right]$$

So,
$$\frac{d}{dx}\left(e^{-x}\right) = -e^{-x}$$

Differentiation Ex 11.1 Q2

Let $f(x) = e^{3x}$

$\Rightarrow f(x+h) = e^{3(x+h)}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{e^{3(x+h)}-e^{3x}}{h}$$
$$= \lim_{h\to 0}\frac{e^{3x}e^{3h}-e^{3x}}{h}$$
$$= \lim_{h\to 0} e^{3x}\left\{\frac{\left(e^{3h}-1\right)}{3h}\right\}\times 3$$
$$= 3e^{3x} \qquad \left[\text{Since, } \lim_{x\to 0}\frac{e^{x}-1}{x} = 1\right]$$

Hence,
$$\frac{d}{dx}\left(e^{3x}\right) = 3e^{3x}$$

Differentiation Ex 11.1 Q3

Let $\quad f(x) = e^{ax+b}$

$\Rightarrow \quad f(x+h) = e^{a(x+h)+b}$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{a(x+h)+b} - e^{(ax+b)}}{h}$$

$$= \lim_{h\to 0} \frac{e^{ax+b}e^{ax} - e^{ax+b}}{h}$$

$$= \lim_{h\to 0} e^{ax+b}\left\{\frac{\left(e^{ah} - 1\right)}{ah}\right\} \times a$$

$$= ae^{ax+b} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1\right]$$

So,

$$\frac{d}{dx}\left(e^{ax+b}\right) = ae^{ax+b}$$

Differentiation Ex 11.1 Q4

Let $\quad f(x) = e^{\cos x}$

$\Rightarrow \quad f(x+h) = e^{\cos(x+h)}$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\cos(x+h)} - e^{\cos x}}{h}$$

$$= \lim_{h\to 0} e^{\cos x}\left[\frac{e^{\cos(x+h)-\cos x} - 1}{h}\right]$$

$$= \lim_{h\to 0} e^{\cos x}\left[\frac{e^{\cos(x+h)-\cos x} - 1}{\cos(x+h) - \cos x}\right] \times \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} e^{\cos x} \times \left(\frac{\cos(x+h) - \cos x}{h}\right) \qquad \left[\text{Since, } \lim_{h\to 0} \frac{e^x - 1}{x} = 1\right]$$

$$= \lim_{h\to 0} e^{\cos x} \times \left(\frac{-2\sin\frac{x+h+x}{2} \times \sin\frac{x+h-x}{2}}{h}\right) \qquad \left[\text{Since, } \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= e^{\cos x} \lim_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)}{2} \times \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= e^{\cos x} \lim_{h\to 0} -2\sin\left(\frac{2x+h}{2}\right) \times \frac{1}{2} \qquad \left[\text{since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= e^{\cos x}\left(-\sin x\right)$$

$$= -\sin x e^{\cos x}$$

Hence,

$$\frac{d}{dx}\left(e^{\cos x}\right) = -\sin x e^{\cos x}$$

Differentiation Ex 11.1 Q5

Let $f(x) = e^{\sqrt{2x}}$

$\Rightarrow$ $f(x+h) = e^{\sqrt{2(x+h)}}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{2x}}\frac{e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1}{h}$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\left(\frac{\left(e^{\sqrt{2(x+h)}-\sqrt{2x}}-1\right)}{\sqrt{2(x+h)}-\sqrt{2x}}\right)\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}\right)$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h} \qquad \left[\text{Since, } \lim_{h\to 0}\frac{e^h-1}{h}=1\right]$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}\times\frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}} \qquad [\text{Rationalizing the numerator}]$$

$$= de^{\sqrt{2x}}\lim_{h\to 0}\frac{2(x+h)-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\frac{2x+2h-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\frac{2h}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}}\lim_{h\to 0}\frac{2}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{2x}}\right) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Differentiation Ex 11.1 Q6

Let $f(x) = \log\cos x$

$\Rightarrow$ $f(x+h) = \log\cos(x+h)$

$$\therefore \quad \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\log\cos(x+h)-\log\cos x}{h}$$

$$= \lim_{h\to 0}\frac{\log\frac{\cos(x+h)}{\cos x}}{h} \qquad \left[\text{Since, } \log A - \log B = \log\frac{A}{B}\right]$$

$$= \lim_{h\to 0}\frac{\log\left\{1+\frac{\cos(x+h)}{\cos x}-1\right\}}{h}$$

$$= \lim_{h\to 0}\frac{\log\left\{1+\frac{\cos(x+h)}{\cos x}\right\}}{\left(\frac{\cos(x+h)}{\cos x}\right)h\times\left(\frac{\cos x}{\cos(x+h)-\cos x}\right)}$$

$$= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{\cos x\times h} \qquad \left[\text{Since, } \lim_{x\to 0}\frac{\log(1+x)}{x} = 1\right]$$

$$= \lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{\cos x\times h}$$

$$= -2\lim_{h\to 0}\frac{\sin\left(\frac{2x+h}{2}\right)\times\left(\sin\frac{h}{2}\right)}{2\cos x\left(\frac{h}{2}\right)}$$

$$= \frac{-2\sin x}{2\cos x} \qquad \left[\text{Since, } \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= -\tan x$$

So,

$$\frac{d}{dx}(\log\cos x) = -\tan x$$

Differentiation Ex 11.1 Q7

Let $f(x)=e^{\sqrt{\cot x}}$

$\Rightarrow$ $f(x+h)=e^{\sqrt{\cot(x+h)}}$

$$\frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{e^{\sqrt{\cot(x+h)}}-e^{\sqrt{\cot x}}}{h}$$

$$=\lim_{h\to 0}\frac{e^{\sqrt{\cot x}}\left(e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}}-1\right)}{h}$$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\left(\frac{e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}}-1}{\sqrt{\cot(x+h)}-\sqrt{\cot x}}\right)\times\left(\frac{\sqrt{\cot(x+h)}-\sqrt{\cot x}}{h}\right)$$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\frac{\left(\sqrt{\cot(x+h)}-\sqrt{\cot x}\right)}{h}\times\frac{\sqrt{\cot(x+h)}+\sqrt{\cot x}}{\sqrt{\cot(x+h)}+\sqrt{\cot x}}$$

$\left[\text{Since, } \lim_{x\to 0}\frac{e^x-1}{x}=1 \text{ and rationalizing numerator}\right]$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\frac{\frac{\cot(x+h)\cot x+1}{\cot(x+h-x)}}{h\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$\left[\text{Since, } \cot(A-B)=\frac{\cot A\cot B+1}{\cot A-\cot B}\right]$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\frac{\cot(x+h)\cot x+1}{\text{cothx}\, h\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$$=e^{\sqrt{\cot x}}\lim_{h\to 0}\frac{\left(\cot(x+h)\cot x+1\right)}{\left(\frac{h}{\tanh}\right)\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$$=\frac{e^{\sqrt{\cot x}}\times\left(\cot^2 x+1\right)}{2\sqrt{\cot x}}$$

$\left[\text{Since, } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$

$$=\frac{e^{\sqrt{\cot x}}\times\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

$\left[\text{Since, } \left(1+\cot^2 x\right)=\operatorname{cosec}^2 x\right]$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right)=\frac{e^{\sqrt{\cot x}}\times\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.1 Q8

Let $f(x)=x^2e^x$

$\Rightarrow$ $f(x+h)=(x+h)^2e^{(x+h)}$

$$\therefore\quad \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2e^{(x+h)}-x^2e^x}{h}$$

$$=\lim_{h\to 0}\left(\frac{x^2e^{(x+h)}-x^2e^x}{h}+\frac{2xhe^{(x+h)}}{h}+\frac{h^2e^{(x+h)}}{h}\right)$$

$$=\lim_{h\to 0}\left(\frac{x^2e^x\left(e^{(x+h)-x}-1\right)}{h}+2xe^{(x+h)}+he^{(x+h)}\right)$$

$$=\lim_{h\to 0}\left[x^2e^x\frac{\left(e^h-1\right)}{h}+2xe^{(x+h)}+he(x+h)\right]$$

$$=x^2e^x+2xe^x+0\times e^x$$

$\left[\text{Since, } \lim_{x\to 0}\frac{e^x-1}{x}=1\right]$

So,

$$\frac{d}{dx}\left(x^2e^x\right)=e^x\left(x^2+2x\right)$$

Differentiation Ex 11.1 Q9

Let $f(x) = \log \operatorname{cosec} x$

$\Rightarrow$ $f(x+h) = \log \operatorname{cosec}(x+h)$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log \operatorname{cosec}(x+h) - \log \operatorname{cosec} x}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{\operatorname{cosec}(x+h)}{\operatorname{cosec} x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(1 + \left(\frac{\sin x}{\sin(x+h)} - 1\right)\right)}{h}$$

$$= \lim_{h\to 0} \left\{\frac{\log\left(1 + \left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)\right)}{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}\right\} \frac{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)h}$$

$$\left[\text{Since, } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)(-2)}\left\{\frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}\right\} \quad \left[\text{Since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\cot x$$

So,

$$\frac{d}{dx}(\log \operatorname{cosec} x) = -\cot x.$$

Differentiation Ex 11.1 Q10

Let $f(x) = \sin^{-1}(2x+3)$

$\Rightarrow$ $f(x+h) = \sin^{-1}(2(x+h)+3)$

$\Rightarrow$ $f(x+h) = \sin^{-1}(2x+2h+3)$

$$\therefore \quad \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}(2x+2h+3)-\sin^{-1}(2x+3)}{h}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}\left[(2x+2h+3)\sqrt{1-(2x+3)^2}-(2x+3)\sqrt{1-(2x+2h+3)^2}\right]}{h}$$

$$\left[\text{Since, } \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]\right]$$

$$= \lim_{h\to 0}\frac{\sin^{-1}z}{z}\times\frac{z}{h}$$

Where, $z = (2x+2h+3)\sqrt{1-(2x+3)^2}-(2x+3)\sqrt{1-(2x+2h+3)^2}$ and $\lim_{h\to 0}\frac{\sin^{-1}h}{h} = 1$

$$= \lim_{h\to 0}\frac{z}{h}$$

$$= \lim_{h\to 0}\frac{(2x+2h+3)\sqrt{1-(2x+3)^2}-(2x+3)\sqrt{1-(2x+2h+3)^2}}{h}$$

$$= \lim_{h\to 0}\frac{(2x+2h+3)^2-(2x+3)^2-(2x+3)^2\left(1-(2x+2h+3)^2\right)}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

[Since, rationalizing numerator]

$$= \lim_{h\to 0}\frac{\left[(2x+3)^2+4h^2+4h(2x+3)\right]\left(1-(2x+3)^2\right)-(2x+3)^2\left[1-(2x+3)^2-4h^2-4h(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h\to 0}\frac{\left[(2x+3)^2+4h^2+4h(2x+3)-(2x+3)^4-4h^2(2x+3)^2-4h(2x+3)^3-(2x+3)^2+(2x+3)^4+4h^2(2x+3)^2+4h(2x+3)^3\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h\to 0}\frac{4h\left[h+(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \frac{4(2x+3)}{(2x+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{4(2x+3)}{2(2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{2}{\sqrt{1-(2x+3)^2}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}(2x+3)\right) = \frac{2}{\sqrt{1-(2x+3)^2}}$$

Ex 11.2

Differentiation Ex 11.2 Q1

Let,

$$y = \sin(3x+5)$$

Differentiate y with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin(3x+5)\right)$$
$$= \cos(3x+5)\frac{d}{dx}(3x+5) \qquad \text{[using chain rule]}$$
$$= \cos(3x+5)\times[3(1)+0]$$
$$= 3\cos(3x+5)$$

So,

$$\frac{d}{dx}\left(\sin(3x+5)\right) = 3\cos(3x+5).$$

Differentiation Ex 11.2 Q2

Let,

$$y = \tan^2 x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 2\tan x\frac{d}{dx}(\tan x) \qquad \text{[using chain rule]}$$
$$= 2\tan x \times \sec^2 x$$

So,

$$\frac{d}{dx} = \left(\tan^2 x\right) = 2\tan x \sec^2 x.$$

Differentiation Ex 11.2 Q3

Let,

$$y = \tan(x° + 45°)$$
$$y = \tan\left\{(x° + 45°)\frac{\pi}{180°}\right\}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left\{(x° + 45°)\frac{\pi}{180°}\right\}$$
$$= \sec^2\left\{(x° + 45°)\frac{\pi}{180°}\right\} \times \frac{d}{dx}(x° + 45°)\frac{\pi}{180°} \quad \text{[Using chain rule]}$$
$$= \frac{\pi}{180°}\sec^2(x° + 45°)$$

So,

$$\frac{d}{dx} = (\tan(x° + 45°)) = \frac{\pi}{180°}\sec^2(x° + 45°).$$

Differentiation Ex 11.2 Q4

Let,

$$y = \sin(\log x)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\log x)$$
$$= \cos(\log x)\frac{d}{dx}(\log x) \quad \text{[Using chain rule]}$$
$$= \frac{1}{x}\cos(\log x)$$

So,

$$\frac{d}{dx} = (\sin(\log x)) = \frac{1}{x}\cos(\log x).$$

Differentiation Ex 11.2 Q5

Let,

$$y = e^{\sin\sqrt{x}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin\sqrt{x}}\right)$$
$$= e^{\sin\sqrt{x}}\frac{d}{dx}\left(\sin\sqrt{x}\right) \quad \text{[Using chain rule]}$$
$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x}\frac{d}{dx}\sqrt{x} \quad \text{[Using chain rule]}$$
$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}$$

So,

$$\frac{d}{dx} = \left(e^{\sin\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}.$$

Differentiation Ex 11.2 Q6

Let,

$$y = e^{\tan x}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan x}\right)$$
$$= e^{\tan x}\frac{d}{dx}(\tan x) \quad \text{[Using chain rule]}$$
$$= e^{\tan x} \times \sec^2 x$$

So,

$$\frac{d}{dx} = \left(e^{\tan x}\right) = \sec^2 x \times e^{\tan x}.$$

Differentiation Ex 11.2 Q7

Let,

$$y = \sin^2(2x+1)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2(2x+1)\right]$$

$$= 2\sin(2x+1)\frac{d}{dx}\sin(2x+1) \qquad \text{[Using chain rule]}$$

$$= 2\sin(2x+1)\cos(2x+1)\frac{d}{dx}(2x+1) \qquad \text{[Using chain rule]}$$

$$= 4\sin(2x+1)\cos(2x+1)$$

$$= 2\sin 2(2x+1) \qquad \left[\text{Since, } \sin^2 A = 2\sin A\cos A\right]$$

$$= 2\sin(4x+2)$$

So,

$$\frac{d}{dx}\left(\sin^2(2x+1)\right) = 2\sin(4x+2).$$

Differentiation Ex 11.2 Q8

Let,

$$y = \log_7(2x-3)$$

$$\Rightarrow \quad y = \frac{\log(2x-3)}{\log 7} \qquad \left[\text{Since, } \log_a{}^b = \frac{\log b}{\log a}\right]$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\log 7}\frac{d}{dx}\left(\log(2x-3)\right)$$

$$= \frac{1}{\log 7}\times\frac{1}{(2x-3)}\frac{d}{dx}(2x-3) \qquad \text{[Using chain rule]}$$

$$= \frac{2}{(2x-3)\log 7}$$

Hence,

$$\frac{d}{dx}\left(\log_7(2x-3)\right) = \frac{2}{(2x-3)\log 7}.$$

Differentiation Ex 11.2 Q9

Let,

$$y = \tan 5x^\circ$$

$$\Rightarrow \quad y = \tan\left(5x^\circ \times \frac{\pi}{180^\circ}\right)$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(5x^\circ \times \frac{\pi}{180^\circ}\right)$$

$$= \sec^2 x\left(5x^\circ \times \frac{\pi}{180^\circ}\right)\frac{d}{dx}\left(5x^\circ\frac{\pi}{180^\circ}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{5\pi}{180^\circ}\right)\sec^2\left(5x^\circ\frac{\pi}{180^\circ}\right)$$

$$= \frac{5\pi}{180^\circ}\sec^2(5x^\circ)$$

Hence,

$$\frac{d}{dx}\left(\tan(5x^\circ)\right) = \frac{5\pi}{180^\circ}\sec^2(5x^\circ).$$

Differentiation Ex 11.2 Q10

Let,

$$y = 2^{x^3}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(2^{x^3}\right)$$
$$= 2^{x^3} \times \log_2 \frac{d}{dx}\left(x^3\right) \qquad \text{[Using chain rule]}$$
$$= 3x^2 \times 2^{x^3} \times \log_2$$

So,

$$\frac{d}{dx}\left(2^{x^3}\right) = 3x^2 \times 2^{x^3} \log_2 .$$

Differentiation Ex 11.2 Q11

Let, $y = 3^{e^x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{e^x}\right)$$
$$= 3^{e^x} \log 3 \frac{d}{dx}\left(e^x\right) \qquad \text{[Using chain rule]}$$
$$= e^x \times 3^{e^x} \log 3$$

So,

$$\frac{d}{dx}\left(3^{e^x}\right) = e^x \times 3^{e^x} \log 3.$$

Differentiation Ex 11.2 Q12

Let $\quad y = \log_x 3$

$$\Rightarrow \quad y = \frac{\log 3}{\log x} \qquad \left[\text{Since, } \log_a{}^b = \frac{\log b}{\log a}\right]$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log 3}{\log x}\right)$$
$$= \log 3 \frac{d}{dx}(\log x)^{-1}$$
$$= \log 3 \times \left[-1(\log x)^{-2}\right]\frac{d}{dx}(\log x) \qquad \text{[Using chain rule]}$$
$$= -\frac{\log 3}{(\log x)^2} \times \frac{1}{x}$$
$$= -\left(\frac{\log 3}{\log x}\right)^2 \times \frac{1}{x} \times \frac{1}{\log 3}$$
$$= -\frac{1}{x \log 3 (\log_3 x)^2} \qquad \left[\text{Since, } \frac{\log b}{\log a} = loa_a{}^b\right]$$

So,

$$\frac{d}{dx}(\log_x 3) = -\frac{1}{x \log 3 (\log_3 x)^2}.$$

Differentiation Ex 11.2 Q13

Let $y = 3^{x^2+2x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x^2+2x}\right)$$
$$= 3^{x^2+2x} \times \log 3 \frac{d}{dx}\left(x^2 + 2x\right) \quad \text{[Using chain rule]}$$
$$= (2x + 2)\log 3 \times 3^{x^2+2x}$$

So,

$$\frac{d}{dx}\left(3^{x^2+2x}\right) = (2x + 2)\log 3 \times 3^{x^2+2x}.$$

Differentiation Ex 11.2 Q14

Let $\quad y = \sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$

$\Rightarrow \quad y = \left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$$
$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}-1} \times \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right) \quad \text{[Using chain rule]}$$
$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{-1}{2}} \times \left\{\frac{\left(a^2 + x^2\right)\frac{d}{dx}\left(a^2 - x^2\right) - \left(a^2 - x^2\right)\frac{d}{dx}\left(a^2 + x^2\right)}{\left(a^2 + x^2\right)^2}\right\} \quad \text{[Using chain rule]}$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2x\left(a^2 + x^2\right) - 2x\left(a^2 - x^2\right)}{\left(a^2 + x^2\right)^2}\right\}$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{\left(a^2 + x^2\right)^2}\right\}$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left(\frac{-4xa^2}{\left(a^2 + x^2\right)^2}\right)$$
$$= \frac{-2xa^2}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}\right) = \frac{-2a^2x}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q15

Let $y = 3^{x\log x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x\log x}\right)$$

$$= 3^{x\log x} \times \log 3 \frac{d}{dx}(x\log x) \qquad \text{[Using chain rule]}$$

$$= 3^{x\log x} \times \log 3\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] \qquad \text{[Using chain rule]}$$

$$= 3^{x\log x} \times \log 3\left[\frac{x}{x} + \log x\right]$$

$$= 3^{x\log x}(1+\log x)\times\log 3$$

So,

$$\frac{d}{dx}\left(3^{x}\log x\right) = \log 3 \times 3^{x\log x}(1+\log x).$$

Differentiation Ex 11.2 Q16

Let $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$= \frac{1}{2}\left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}}\left[\frac{(1-\sin x)(\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2}\right]$$

$$= \frac{1}{2}\frac{(1-\sin x)^{\frac{1}{2}}}{(1+\sin x)^{\frac{1}{2}}}\left[\frac{\cos x - \cos x\sin x + \cos x + \sin x\cos x}{(1-\sin x)^2}\right]$$

$$= \frac{1}{2}\times\frac{2\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\sqrt{1-\sin x}\,(1-\sin x)}$$

$$= \frac{\cos x}{\sqrt{1-\sin^2 x}\,(1-\sin x)}$$

$$= \frac{\cos x}{\cos x(1-\sin x)} \qquad \left[\text{Using } 1-\sin^2 x = \cos^2 x\right]$$

$$= \frac{1}{(1-\sin x)}\times\frac{(1+\sin x)}{(1+\sin x)}$$

$$= \frac{(1+\sin x)}{\left(1-\sin^2 x\right)}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

Thus, $\frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$\Rightarrow \frac{dy}{dx} = \sec^2 x + \tan x\sec x$

$\Rightarrow \frac{dy}{dx} = \sec x[\tan x + \sec x]$

Differentiation Ex 11.2 Q17

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$

$y = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{\left(1+x^2\right)\frac{d}{dx}\left(1-x^2\right)-\left(1-x^2\right)\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}\right] \quad \text{[Using quotient rule]}$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{\left(1+x^2\right)(-2x)-\left(1-x^2\right)(2x)}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{-2x-2x^3-2x+2x^3}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\frac{-4x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) = \frac{-2x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q18

Let $y = (\log\sin x)^2$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\log\sin x)^2$$

$$= 2(\log\sin x)\frac{d}{dx}(\log\sin x) \quad \text{[Using chain rule]}$$

$$= 2(\log\sin x)\times\frac{1}{\sin x}\frac{d}{dx}(\log x)$$

$$= 2(\log\sin x)\times\frac{1}{\sin x}\times\frac{1}{x}$$

$$= \frac{2\log\sin x}{x\sin x}$$

So,

$$\frac{d}{dx}(\log\sin x)^2 = \frac{2\log\sin x}{x\sin x}$$

Differentiation Ex 11.2 Q19

Let $y = \sqrt{\frac{1+x}{1-x}}$

$\Rightarrow \quad y = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+x}{1-x}\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{-1}{2}}\left[\frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right] \quad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{1-x+1+x}{(1-x)^2}\right]$$

$$= \frac{1}{2}\frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}\times\frac{2}{(1-x)^2}$$

$$= \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

Differentiation Ex 11.2 Q20

Let $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\left(\frac{1+x^2}{1-x^2}\right)\right)$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\frac{d}{dx}\left(\frac{1+x^2}{1-x^2}\right) \quad \text{[Using chain rule]}$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{\left(1-x^2\right)\frac{d}{dx}\left(1+x^2\right)-\left(1+x^2\right)\frac{d}{dx}\left(1-x^2\right)}{(1-x)^2}\right] \quad \text{[Using quotient rule]}$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{\left(1-x^2\right)(2x)-\left(1+x^2\right)(-2x)}{\left(1-x^2\right)^2}\right]$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{2x-2x^3+2x+2x^3}{\left(1-x^2\right)^2}\right]$$

$$= \frac{4x}{\left(1-x^2\right)^2}\cos\left(\frac{1+x^2}{1-x^2}\right)$$

So,

$$\frac{d}{dx}\left(\sin\left(\frac{1+x^2}{1-x^2}\right)\right) = \frac{4x}{\left(1-x^2\right)^2}\cos\left(\frac{1+x^2}{1-x^2}\right).$$

Differentiation Ex 11.2 Q21

Let $y = e^{3x}\cos 2x$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(e^{3x}\cos 2x\right)\\ &= e^{3x}\times\frac{d}{dx}(\cos 2x)+\cos 2x\frac{d}{dx}\left(e^{3x}\right) && \text{[Using product rule]}\\ &= e^{3x}\times(-\sin 2x)\frac{d}{dx}(2x)+\cos 2x e^{3x}\frac{d}{dx}(3x) && \text{[Using chain rule]}\\ &= -2e^{3x}\sin 2x+3e^{3x}\cos 2x\\ &= e^{3x}(3\cos 2x-2\sin 2x)\end{aligned}$$

so,

$$\frac{d}{dx}\left(e^{3x}\cos 2x\right)=e^{3x}(3\cos 2x-2\sin 2x).$$

Differentiation Ex 11.2 Q22

Let $y = \sin(\log\sin x)$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\sin(\log\sin x)\\ &= \cos(\log\sin x)\frac{d}{dx}(\log\sin x) && \text{[Using chain rule]}\\ &= \cos(\log\sin x)\times\frac{1}{\sin x}\frac{d}{dx}0\sin x\\ &= \cos(\log\sin x)\frac{\cos x}{\sin x}\\ &= \cos(\log s\sin x)\times\cot x\end{aligned}$$

Hence,

$$\frac{d}{dx}\left(\sin(\log\sin x)\right)=\cos(\log\sin x)\times\cot x.$$

Differentiation Ex 11.2 Q23

Let $y = e^{\tan 3x}$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(e^{\tan 3x}\right)\\ &= e^{\tan 3x}\frac{d}{dx}(\tan 3x) && \text{[Using chain rule]}\\ &= e^{\tan 3x}\times\sec^2 3x\times\frac{d}{dx}(3x)\\ &=\end{aligned}$$

So,

$$\frac{d}{dx}\left(e^{\tan 3x}\right)=3e^{\tan 3x}\times\sec^2 3x$$

Differentiation Ex 11.2 Q24

Let $y = e^{\sqrt{\cot x}}$

$\Rightarrow$ $y = e^{(\cot x)^{\frac{1}{2}}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{(\cot x)^{\frac{1}{2}}}\right)$$

$$= e^{(\cot x)^{\frac{1}{2}}} \times \frac{d}{dx}(\cot x)^{\frac{1}{2}} \qquad \text{[Using chain rule]}$$

$$= e^{\sqrt{\cot x}} \times \frac{1}{2}(\cot x)^{\frac{1}{2}-1}\frac{d}{dx}(\cot x)$$

$$= -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.2 Q25

Let $y = \log\left(\frac{\sin x}{1+\cos x}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \frac{1}{\left(\frac{\sin x}{1+\cos x}\right)} \times \frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{1+\cos x}{\sin x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\left(\frac{\sin x}{1+\cos x}\right)\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q26

Let $y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\Rightarrow \quad y = \log\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}$

$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right)$ $\qquad$ [Using $\log a^b = b\log a$]

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right)\right\}$$

$$= \frac{1}{2}\times\frac{1}{\left(\frac{1-\cos x}{1+\cos x}\right)}\times\frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{\sin x+\sin x\cos x+\sin x-\sin x\cos x}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{2\sin x}{(1+\cos x)^2}\right]$$

$$= \frac{\sin x}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin x}{1-\cos^2 x}$$

$$= \frac{\sin x}{\sin^2 x} \qquad \left[\text{Sicne } 1-\cos^2 x = \sin^2 x\right]$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q27

Let $y = \tan\left(e^{\sin x}\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan e^{\sin x}\right]$$

$$= \sec^2\left(e^{\sin x}\right)\frac{d}{dx}\left(e^{\sin x}\right) \qquad \text{[Using chain rule]}$$

$$= \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\frac{d}{dx}(\sin x)$$

$$= \cos x\sec^2\left(e^{\sin x}\right)\times e^{\sin x}$$

So,

$$\frac{d}{dx}\left(\tan e^{\sin x}\right) = \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\cos x.$$

Differentiation Ex 11.2 Q28

Let $y = \log\left(x + \sqrt{x^2+1}\right)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(x + \sqrt{x^2+1}\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}}\frac{d}{dx}\left(x + \left(x^2+1\right)^{\frac{1}{2}}\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2+1\right)\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2\sqrt{x^2+1}} \times 2x\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\sqrt{x^2+1}}$$

So,

$$\frac{d}{dx}\left(\log\left(x+\sqrt{x^2+1}\right)\right) = \frac{1}{\sqrt{x^2+1}}.$$

Differentiation Ex 11.2 Q29

Let $y = \frac{e^x \log x}{x^2}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{x^2\frac{d}{dx}\left(e^x \log x\right) - \left(e^x \log x\right)\frac{d}{dx}\left(x^2\right)}{\left(x^2\right)^2} \quad \text{[Using quotient rule]}$$

$$= \frac{x^2\left[e^x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(e^x\right)\right] - e^x \log x \times 2x}{x^4} \quad \text{[Using product rule]}$$

$$= \frac{x^2\left[\frac{e^x}{x} + e^x \log x\right] - 2xe^x \log x}{x^4}$$

$$= \frac{\frac{x^2e^x\left(1+x\log x\right)}{x} - 2xe^x \log x}{x^4}$$

$$= \frac{xe^x\left[1 + x\log x - 2\log x\right]}{x^4}$$

$$= \frac{xe^x}{x^3}\left[\frac{1}{x} + \frac{x\log x}{x} - \frac{2\log x}{x}\right]$$

$$= e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right]$$

So,

$$\frac{d}{dx}\left[\frac{e^x \log x}{x^2}\right] = e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right].$$

Differentiation Ex 11.2 Q30

Let $y = \log(\operatorname{cosec}x - \cot x)$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\log(\operatorname{cosec}x - \cot x)\\
&= \frac{1}{(\operatorname{cosec}x - \cot x)}\frac{d}{dx}(\operatorname{cosec}x - \cot x) \qquad \text{[Using chain rule]}\\
&= \frac{1}{(\operatorname{cosec}x - \cot x)}\times\left(-\operatorname{cosec}x\cot x + \operatorname{cosec}^2 x\right)\\
&= \frac{\operatorname{cosec}x(\operatorname{cosec}x - \cot x)}{(\operatorname{cosec}x - \cot x)}\\
&= \operatorname{cosec}x\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log(\operatorname{cosec}x - \cot x)\right) = \operatorname{cosec}x.$$

Differentiation Ex 11.2 Q31

Let $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right]\\
&= \left[\frac{\left(e^{2x}-e^{-2x}\right)\frac{d}{dx}\left(e^{2x}+e^{-2x}\right)-\left(e^{2x}+e^{-2x}\right)\frac{d}{dx}\left(e^{2x}-e^{-2x}\right)}{\left(e^{2x}-e^{-2x}\right)^2}\right] \qquad \left[\begin{array}{l}\text{Using quotient rule}\\ \text{and chain rule}\end{array}\right]\\
&= \frac{\left(e^{2x}-e^{-2x}\right)\left[e^{2x}\frac{d}{dx}(2x)+e^{-2x}\frac{d}{dx}(-2x)\right]-\left(e^{2x}+e^{-2x}\right)\left[\begin{array}{l}e^{2x}\frac{d}{dx}(2x)-\\ e^{-2x}\frac{d}{dx}(-2x)\end{array}\right]}{\left(e^{2x}-e^{-2x}\right)^2}\\
&= \frac{\left(e^{2x}-e^{-2x}\right)\left(2e^{2x}-2e^{-2x}\right)-\left(e^{2x}+e^{-2x}\right)\left(2e^{2x}+2e^{-2x}\right)}{\left(e^{2x}-e^{-2x}\right)^2}.\\
&= \frac{2\left(e^{2x}-e^{-2x}\right)^2-2\left(e^{2x}+e^{-2x}\right)^2}{\left(e^{2x}-e^{-2x}\right)^2}\\
&= \frac{2\left[e^{4x}+e^{-4x}-2e^{2x}e^{-2x}-e^{4x}-e^{-4x}-2e^{2x}e^{-2x}\right]}{\left(e^{2x}-e^{-2x}\right)^2}\\
&= \frac{-8}{\left(e^{2x}-e^{-2x}\right)^2}\end{aligned}$$

So,

$$\frac{d}{dx}\left(\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right) = \frac{-8}{\left(e^{2x}-e^{-2x}\right)^2}.$$

Differentiation Ex 11.2 Q32

Let $y = \log\left(\frac{x^2+x+1}{x^2-x+1}\right)$

Differentiating with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{x^2+x+1}{x^2-x+1}\right)\right]$$

$$= \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right) \quad \text{[Using chain rule and quotient rule]}$$

$$= \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right)-\left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}\right]$$

$$= \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{\left(x^2-x+1\right)(2x+1)-\left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)^2}\right]$$

$$= \frac{\left(x^2-x+1\right)}{\left(x^2+x+1\right)}\left[\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{\left(x^2-x+1\right)^2}\right]$$

$$= \frac{-4x^2+2x^2+2}{\left(x^2+x+1\right)\left(x^2-x+1\right)}$$

$$= \frac{-2\left(x^2-1\right)}{x^4+1+2x^2-x^2}$$

$$= \frac{-2\left(x^2-1\right)}{x^4+x^2+1}$$

So,

$$\frac{d}{dx}\left(\log\frac{x^2+x+1}{x^2-x+1}\right) = \frac{-2\left(x^2-1\right)}{x^4+x^2+1}$$

Differentiation Ex 11.2 Q33

Let $y = \tan^{-1}\left(e^x\right)$

Differentiate it with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}e^x\right)$$

$$= \frac{1}{1+\left(e^x\right)^2}\frac{d}{dx}\left(e^x\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{1+e^{2x}} \times e^x$$

$$= \frac{e^x}{1+e^{2x}}$$

So,

$$\frac{d}{dx}\left(\tan^{-1}e^x\right) = \frac{e^x}{1+e^{2x}}.$$

Differentiation Ex 11.2 Q34

Let $y = e^{\sin^{-1} 2x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} 2x}\right)$$
$$= e^{\sin^{-1} 2x} \times \frac{d}{dx}\left(\sin^{-1} 2x\right) \qquad \text{[Using chain rule]}$$
$$= e^{\sin^{-1} 2x} \times \frac{1}{\sqrt{1-(2x)^2}} \frac{d}{dx}(2x)$$
$$= \frac{2e^{\sin-1}2x}{\sqrt{1-4x^2}}$$

So,

$$\frac{d}{dx}\left(e^{\sin-1}2x\right) = \frac{2e \sin^{-1} 2x}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.2 Q35

Let $y = \sin\left(2\sin^{-1} x\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right)$$
$$= \cos\left(2\sin^{-1} x\right) \frac{d}{dx}\left(2\sin^{-1} x\right) \qquad \text{[Using chain rule]}$$
$$= \cos\left(2\sin^{-1} x\right) \times 2 \frac{1}{\sqrt{1-x^2}}$$
$$= \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1-x^2}}$$

So,

$$\frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right) = \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.2 Q36

Let $y = e^{\tan^{-1}\sqrt{x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan^{-1}}\sqrt{x}\right)$$
$$= e^{\tan^{-1}\sqrt{x}} \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) \qquad \text{[Using chain rule]}$$
$$= e^{\tan^{-1}\sqrt{x}} \times \frac{1}{1+\left(\sqrt{x}\right)^2} \frac{d}{dx}\left(\sqrt{x}\right)$$
$$= \frac{e^{\tan^{-1}\sqrt{x}}}{1+x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{e^{\tan-1}\sqrt{x}}{2\sqrt{x}\,(1+x)}$$

So,

$$\frac{d}{dx}\left(e^{\tan-1}\sqrt{x}\right) = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.2 Q37

Let $y = \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}$

$\Rightarrow \quad y = \left(\tan^{-1}\left(\frac{x}{2}\right)\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{2}\right)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\tan^{-1}\frac{x}{2}\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{\frac{-1}{2}} \times \frac{1}{1+\left(\frac{x}{2}\right)^2} \times \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$= \frac{4}{4\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}\left(4+x^2\right)}$$

$$= \frac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}\right) = \frac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}}.$$

Differentiation Ex 11.2 Q38

Let $y = \log\left(\tan^{-1}x\right)$

Differentiate with respect to x, .

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\tan^{-1}x\right)$$

$$= \frac{1}{\tan^{-1}x} \times \frac{d}{dx}\left(\tan^{-1}x\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{\left(1+x^2\right)\tan^{-1}x}$$

So,

$$\frac{d}{dx}\left(\log\tan^{-1}x\right) = \frac{1}{\left(1+x^2\right)\tan^{-1}x}.$$

Differentiation Ex 11.2 Q39

Let $y = \frac{2^x \cos x}{\left(x^2+3\right)^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2^x \cos x}{\left(x^2+3\right)^2}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2 \frac{d}{dx}\left(2^x \cos x\right) - \left(2^x \cos x\right)\frac{d}{dx}\left(x^2+3\right)^2}{\left[\left(x^2+3\right)^2\right]^2}\right]$$

[Using quotient rule, product rule and chain rule]

$$= \left[\frac{\left(x^2+3\right)^2\left[2^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx} 2^x\right] - \left(2^x \cos x\right)2\left(x^2+3\right)\frac{d}{dx}\left(x^2+3\right)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2\left[-2^x \sin x + \cos x\, 2^x \log 2\right] - 2\left(2^x \cos x\right)\left(x^2+3\right)(2x)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{2^x\left(x^2+3\right)\left[\left(x^2+3\right)\{\cos x \log 2 - \sin x\}\right] - 4x \cos x}{\left(x^2+3\right)^4}\right]$$

$$= \frac{2^x}{\left(x^2 3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2+3\right)}\right]$$

So,

$$\frac{d}{dx}\left(\frac{2^x \cos x}{\left(x^2+3\right)^2}\right) = \frac{2^x}{\left(x^2+3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2+3\right)}\right].$$

Differentiation Ex 11.2 Q40

Let $y = x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x \sin 2x + 5^x + k^k + \left(\tan^6 x\right)\right]$$

$$= \frac{d}{dx}(x \sin 2x) + \frac{d}{dx}\left(5^x\right) + \frac{d}{dx}\left(k^k\right) + \frac{d}{dx}\left(\tan^6 x\right)$$

$$= \left[x \frac{d}{dx}(\sin 2x) + \sin 2x \frac{d}{dx}(x)\right] + 5^x \log 5 + 0 + 6 \tan^5 x \frac{d}{dx}(\tan x)$$

[Using product rule and chain rule]

$$= \left[x \cos 2x \frac{d}{dx}(2x) + \sin 2x\right] + 5^x \log 5 + 6 \tan^5 x \sec^2 x$$

$$= 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x$$

so,

$$\frac{d}{dx}\left(x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3\right) = 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x.$$

Differentiation Ex 11.2 Q41

Let $y = \log(3x+2) - x^2\log(2x-1)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+2) - x^2\log(2x-1)\right]$$
$$= \frac{d}{dx}\log(3x+2) - \frac{d}{dx}\left(x^2\log(2x-1)\right)$$
$$= \frac{1}{(3x+2)}\frac{d}{dx}(3x+2) - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}\left(x^2\right)\right]$$

[Using product rule and chain rule]

$$= \frac{3}{3x+2} - \left[x^2 \times \frac{1}{(2x-1)}\frac{d}{dx}(2x-1) + \log(2x-1)\times 2x\right]$$
$$= \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1)$$

So,

$$\frac{d}{dx}\left(\log(3x+2) - x^2\log(2x-1)\right) = \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1).$$

Differentiation Ex 11.2 Q42

Let $y = \frac{3x^2\sin x}{\sqrt{7-x^2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^2\sin x}{\left(7-x^2\right)^{\frac{1}{2}}}\right)$$
$$= \frac{\left(7-x^2\right)^{\frac{1}{2}} \times \frac{d}{dx}\left(3x^2\sin x\right) - 3x^2\sin x\,\frac{d}{dx}\left(7-x^2\right)^{\frac{1}{2}}}{\left[\left(7-x^2\right)^{\frac{1}{2}}\right]^2}$$

[Using quotient rule, chain and product rule]

$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3 \times \left[x^2\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x^2\right] - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}\frac{d}{dx}\left(7-x^2\right)}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} 3\left(x^2\cos x + 2x\sin x\right) - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}(-2x)}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3\left(x^2\cos x + 2x\sin x\right)}{\left(7-x^2\right)} + \frac{3x^3\sin x\left(7-x^2\right)^{\frac{-1}{2}}}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{\left(7-x^2\right)}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right]$$

So,

$$\frac{d}{dx}\left(\frac{3x^2\sin x}{\sqrt{7-x^2}}\right) = \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{7-x^2}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right].$$

Differentiation Ex 11.2 Q43

Let $y = \sin^2[\log(2x+3)]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2(\log(2x+3))\right]$$

$$= 2\sin(\log(2x+3))\frac{d}{dx}\sin(\log(2x+3)) \quad \text{Using chain rule}$$

$$= 2\sin(\log(2x+3))\cos(\log(2x+3))\frac{d}{dx}\log(2x+3)$$

$$= \sin(2\log(2x+3)) \times \frac{1}{(2x+3)}\frac{d}{dx}(2x+3)$$

$$\left[\text{Since, } 2\sin A\cos A = \sin^2 A\right]$$

$$= \sin(2\log(2x+3)) \times \frac{2}{(2x+3)}$$

So,

$$\frac{d}{dx}\left(\sin^2\log(2x+3)\right) = \sin(2\log(2x+3)) \times \frac{2}{(2x+3)}.$$

Differentiation Ex 11.2 Q44

Let $y = e^x \log\sin 2x$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^x \log\sin 2x\right]$$

$$= e^x\frac{d}{dx}\log\sin 2x + \log\sin 2x\frac{d}{dx}\left(e^x\right)$$

[Using product rule and chain rule]

$$= e^x\frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \log\sin 2x\left(e^x\right)$$

$$= \frac{e^x}{\sin 2x}\cos 2x\frac{d}{dx}(2x) + e^x\log\sin 2x$$

$$= \frac{2\cos 2xe^x}{\sin 2x} + e^x\log\sin 2x$$

$$= e^x(2\cot 2x + \log\sin 2x)$$

so,

$$\frac{d}{dx}\left(e^x\log\sin 2x\right) = e^x(2\cot 2x + \log\sin 2x).$$

Differentiation Ex 11.2 Q45

Let $y = \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$

$\Rightarrow \quad y = \frac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}\right]$$

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}-\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}}{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}^2}\right]$$

[Using quotient rule and chian rule]

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\left[\frac{1}{2}\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)+\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)+\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$-\left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{1}{2}\left[\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)-\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}+\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right] - \left[\frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}-\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{x\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}+\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right] - \left[\frac{x\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}-\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right]$$

$$= \left[\frac{x\left(x^2+1-x^2+1\right)-x\left(x^2-1-x^2-1\right)}{2\left[x^2-\sqrt{x^4-1}\right]\sqrt{x^4-1}}\right]$$

$$= \left[\frac{4x}{2\left(x^2-\sqrt{x^4-1}\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{1\times\left(x^2+\sqrt{x^4-1}\right)}{\left(x^2-\sqrt{x^4-1}\sqrt{x^4-1}\times\left(x^2+\sqrt{x^4-1}\right)\right)}\right]$$

Multiplying and divide by $\left(x^2+\sqrt{x^4-1}\right)$,

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\left(x^4-x^4+1\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\sqrt{x^4-1}}\right]$$

$$= \frac{2x^3}{\sqrt{x^4-1}}+2x$$

So,

$$\frac{d}{dx}\left[\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}\right] = \frac{2x^3}{\sqrt{x^4-1}}+2x.$$

Differentiation Ex 11.2 Q46

Let $y = \log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$$

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}\frac{d}{dx}\left[x + 2 + \left(x^2 + 4x + 1\right)^{\frac{1}{2}}\right]$$

[using chain rule]

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]} \times \left[1 + 0 + \frac{1}{2}\left(x^2 + 4x + 1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2 + 4x + 1\right)\right]$$

$$= \frac{1 + \frac{(2x + 4)}{2\left(\sqrt{x^2 + 4x + 1}\right)}}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}$$

$$= \frac{\sqrt{x^2 + 4x + 1} + x + 2}{\left[x + 2 + \sqrt{x^2 + 4x + 1}\right] \times \sqrt{x^2 + 4x + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 4x + 1}}$$

So,

$$\frac{d}{dx}\left[\log\left\{x + 2 + \sqrt{x^2 + 4x + 1}\right\}\right] = \frac{1}{\sqrt{x^2 + 4x + 1}}.$$

Differentiation Ex 11.2 Q47

Let $y = \left(\sin^{-1} x^4\right)^4$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1} x^4\right)^4$$

$$= 4\left(\sin^{-1} x^4\right)\frac{d}{dx}\left(\sin^{-1} x^4\right) \qquad \text{[Using chain rule]}$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{1}{\sqrt{1 - \left(x^4\right)^2}}\frac{d}{dx}\left(x^4\right)$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{4x^3}{\sqrt{1 - x^8}}$$

$$= \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1} x^4\right) = \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}.$$

Differentiation Ex 11.2 Q48

Let $y = \sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \frac{d}{dx}\left(\frac{x}{\sqrt{x^2+a^2}}\right) \quad \text{[Using chain rule and quotient rule]}$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \left[\frac{(x^2+a^2)^{\frac{1}{2}}\frac{d}{dx}(x) - \frac{d}{dx}(x^2+a^2)^{\frac{1}{2}}}{\left[(x^2+a^2)^{\frac{1}{2}}\right]^2}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{\sqrt{x^2+a^2-x^2}}\left[\frac{\sqrt{x^2+a^2} - x \times \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}(x^2+a^2)}{(x^2+a^2)}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a(x^2+a^2)}\left[\sqrt{x^2+a^2} - \frac{x}{2\sqrt{x^2+a^2}} \times 2x\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a(x^2+a^2)}\left[\frac{x^2+a^2-x^2}{\sqrt{x^2+a^2}}\right]$$

$$= \frac{a^2}{a(x^2+a^2)}$$

$$= \frac{a}{(a^2+x^2)}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}\right) = \frac{a}{a^2+x^2}$$

Differentiation Ex 11.2 Q49

Consider

$$y = \frac{e^x \sin x}{(x^2+2)^3}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{(x^2+2)^3\frac{d}{dx}(e^x \sin x) - e^x \sin x \frac{d}{dx}(x^2+2)^3}{\left[(x^2+2)^3\right]^2}$$

$$= \frac{(x^2+2)^3\left[e^x \cos x + \sin x e^x\right] - e^x \sin x 3(x^2+2)^2(2x)}{(x^2+2)^6}$$

$$= \frac{(x^2+2)^3\left[e^x \cos x + \sin x e^x\right] - 6xe^x \sin x (x^2+2)^2}{(x^2+2)^6}$$

$$= \frac{(x^2+2)^2\left[(x^2+2)(e^x \cos x + \sin x e^x) - 6xe^x \sin x\right]}{(x^2+2)^6}$$

$$= \frac{x^2 e^x \cos x + x^2 \sin x e^x + 2e^x \cos x + 2\sin x e^x - 6xe^x \sin x}{(x^2+2)^4}$$

$$= \frac{e^x \sin x}{(x^2+2)^3} + \frac{e^x \cos x}{(x^2+2)^3} - \frac{6xe^x \sin x}{(x^2+2)^4}$$

Therefore,

$$\frac{dy}{dx} = \frac{e^x \sin x}{(x^2+2)^3} + \frac{e^x \cos x}{(x^2+2)^3} - \frac{6xe^x \sin x}{(x^2+2)^4}$$

Differentiation Ex 11.2 Q50

Consider

$$y = 3e^{-3x}\log(1+x)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = 3\frac{d}{dx}\left[e^{-3x}\log(1+x)\right]$$

$$\frac{dy}{dx} = 3\left(e^{-3x}\frac{1}{1+x} + \log(1+x)\left(-3e^{-3x}\right)\right)$$

$$= 3\left(\frac{e^{-3x}}{1+x} - 3e^{-3x}\log(1+x)\right)$$

$$= 3e^{-3x}\left(\frac{1}{1+x} - 3\log(1+x)\right)$$

Differentiation Ex 11.2 Q51

Consider

$$y = \frac{x^2+2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\sqrt{\cos x}\frac{d}{dx}(x^2+2) - (x^2+2)\frac{d}{dx}\sqrt{\cos x}}{\left(\sqrt{\cos x}\right)^2}$$

$$= \frac{2x\sqrt{\cos x} - (x^2+2)\left(-\frac{1}{2}\frac{\sin x}{\sqrt{\cos x}}\right)}{\cos x}$$

$$= \frac{2x\sqrt{\cos x} + \frac{(x^2+2)\sin x}{2\sqrt{\cos x}}}{\cos x}$$

$$= \frac{4x\cos x + (x^2+2)\sin x}{2(\cos x)^{\frac{3}{2}}}$$

$$= \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{(x^2+2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Therefore,

$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{(x^2+2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Differentiation Ex 11.2 Q52

Consider

$$y = \frac{x^2(1-x^2)^3}{\cos 2x}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\cos 2x\frac{d}{dx}x^2(1-x^2)^3 - x^2(1-x^2)^3\frac{d}{dx}\cos 2x}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[x^2\frac{d}{dx}(1-x^2)^3 + (1-x^2)^3\frac{d}{dx}x^2 - x^2(1-x^2)^3(-2\sin 2x)\right]}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[-6x^3(1-x^2)^2 + (1-x^2)^3 2x + 2x^2(1-x^2)^3\sin 2x\right]}{\cos^2 2x}$$

$$= \frac{2x(1-x^2)^3}{\cos 2x} - \frac{6x^3(1-x^2)^2}{\cos 2x} + \frac{2x^2(1-x^2)^3\sin 2x}{\cos^2 2x}$$

$$= 2x(1-x^2)\sec 2x\left\{1-4x^2+x(1-x^2)\tan 2x\right\}$$

Therefore,

$$\frac{dy}{dx} = 2x(1-x^2)\sec 2x\left\{1-4x^2+x(1-x^2)\tan 2x\right\}$$

Differentiation Ex 11.2 Q53

Consider

$$y = \log(3x+2) - x^2\log(2x-1)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+2) - x^2\log(2x-1)\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}x^2\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left(\frac{2x^2}{2x-1} + 2x\log(2x-1)\right)$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Therefore,

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Differentiation Ex 11.2 Q54

Consider

$y = e^{ax} \sec x \tan 2x$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{ax} \sec x \tan 2x\right)$$
$$= e^{ax}\frac{d}{dx}\sec x \tan 2x + \sec x \tan 2x \frac{d}{dx}e^{ax}$$
$$= e^{ax}\left[\sec x \tan x \tan 2x + \left(2 + 2\tan^2 2x\right)\sec x\right] + ae^{ax}\sec x \tan 2x$$
$$= e^{ax}\left[\sec x \tan x \tan 2x + 2\sec x + 2\tan^2 2x \sec x\right] + ae^{ax}\sec x \tan 2x$$
$$= ae^{ax}\sec x \tan 2x + e^{ax}\sec x \tan x \tan 2x + e^{ax}\sec x\left(2 + 2\tan^2 2x\right)$$
$$\frac{dy}{dx} = e^{ax}\sec x\left\{a\tan 2x + \tan x \tan 2x + 2\sec^2 2x\right\}$$

Therefore,

$$\frac{dy}{dx} = e^{ax}\sec x\left\{a\tan 2x + \tan x \tan 2x + 2\sec^2 2x\right\}$$

Differentiation Ex 11.2 Q55

Consider

$y = \log\left(\cos x^2\right)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\cos x^2\right)$$
$$= \frac{-2x\sin x^2}{\cos x^2}$$
$$\frac{dy}{dx} = -2x\tan x^2$$

Therefore,

$$\frac{dy}{dx} = -2x\tan x^2$$

Differentiation Ex 11.2 Q56

Consider

$y = \cos\left(\log x\right)^2$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\cos\left(\log x\right)^2$$
$$= -\sin\left(\log x\right)^2\frac{d}{dx}\left(\log x\right)^2$$
$$= -\sin\left(\log x\right)^2\frac{2\log x}{x}$$
$$\frac{dy}{dx} = \frac{-2\log x\sin\left(\log x\right)^2}{x}$$

Therefore,

$$\frac{dy}{dx} = \frac{-2\log x\sin\left(\log x\right)^2}{x}$$

Differentiation Ex 11.2 Q57

Consider

$y = \log\sqrt{\frac{x-1}{x+1}}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$y = \log\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$
$$y = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$$
$$y = \frac{1}{2}\left[\log\left(x-1\right) - \log\left(x+1\right)\right]$$
$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\log\left(x-1\right) - \frac{d}{dx}\log\left(x+1\right)\right]$$
$$= \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$
$$= \frac{1}{2}\left(\frac{2}{x^2-1}\right)$$
$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Differentiation Ex 11.2 Q58

Here $y = \log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\frac{d}{dx}\left(\sqrt{x-1} - \sqrt{x+1}\right)$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{d}{dx}\sqrt{x-1} - \frac{d}{dx}\sqrt{x+1}\right]$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{1}{2}(x-1)^{-\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}\right]$$

$$= \frac{1}{2}\frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x+1}}\right)$$

$$= \frac{1}{2}\frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left(\frac{-\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$= \frac{-1}{2}\left(\frac{1}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Therefore,

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Differentiation Ex 11.2 Q59

Here $y = \sqrt{x+1} + \sqrt{x-1}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{x+1} + \frac{d}{dx}\sqrt{x-1}$$

$$= \frac{1}{2}(x+1)^{-\frac{1}{2}} + \frac{1}{2}(x-1)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{x-1} + \sqrt{x+1}}{\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{\left(\sqrt{x^2-1}\right)}\right)$$

$$\sqrt{x^2-1}\frac{dy}{dx} = \frac{1}{2}y$$

Differentiation Ex 11.2 Q60

Here $y = \frac{x}{x+2}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{x+2}\right)$$

$$= \frac{(x+2)\frac{dx}{dx} - x\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{x+2-x}{(x+2)^2}$$

$$= \frac{x+2}{(x+2)^2} - \frac{x}{(x+2)^2}$$

$$= \frac{1}{x+2} - \frac{xy^2}{x^2} \qquad \left[\text{Since } x+2 = \frac{x}{y}\right]$$

$$= \frac{y}{x} - \frac{y^2}{x}$$

$$\frac{dy}{dx} = \frac{1}{x}y(1-y)$$

$$x\frac{dy}{dx} = (1-y)y$$

Differentiation Ex 11.2 Q61

Here $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\log\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{x^{\frac{1}{2}}+x^{-\frac{1}{2}}}\frac{d}{dx}\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{\sqrt{x}+\frac{1}{\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}x^{-1}\right)$$

$$=\frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{1}{\sqrt{x}}-\frac{1}{x\sqrt{x}}\right)$$

$$=\frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{x-1}{x\sqrt{x}}\right)$$

$$\frac{dy}{dx}=\frac{x-1}{2x(x+1)}$$

Differentiation Ex 11.2 Q62

Given, $y=\sqrt{\frac{1+e^x}{1-e^x}}$

Differentiate with respect to x,

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\frac{1+e^x}{1-e^x}}\right)$$

$$=\frac{1}{2\sqrt{\left(\frac{1+e^x}{1-e^x}\right)}}\times\frac{d}{dx}\left(\frac{1+e^x}{1-e^x}\right)$$ [Using chain rule, quotient rule]

$$=\frac{1}{2}\times\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\frac{d}{dx}\left(1+e^x\right)-\left(1+e^x\right)\frac{d}{dx}\left(1-e^x\right)\right)}{\left(1-e^x\right)^2}\right]$$

$$=\frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\right)e^x+\left(1+e^x\right)e^x}{\left(1-e^x\right)2}\right]$$

$$=\frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\times\left[\frac{2e^x}{\left(1-e^x\right)^2}\right]$$

$$=\frac{e^x}{\sqrt{\left(1+e^x\right)}\sqrt{\left(1-e^x\right)}}\frac{1}{\left(1-e^x\right)}$$

$$\frac{dy}{dx}=\frac{e^x}{\left(1-e^x\right)\sqrt{1-e^{2x}}}.$$

Differentiation Ex 11.2 Q63

Given, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$
$$= \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$
$$= \frac{1}{2\sqrt{x}} + \left(-\frac{1}{2} \times x^{-\frac{1}{2}-1}\right)$$
$$= \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$
$$\frac{dy}{dx} = \frac{x-1}{2x\sqrt{x}}$$
$$2x\frac{dy}{dx} = \frac{x-1}{\sqrt{x}}$$
$$\Rightarrow \quad 2x\frac{dy}{dx} = \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$
$$\Rightarrow \quad 2x\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}.$$

Differentiation Ex 11.2 Q64

Given, $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$= \left[\frac{\sqrt{1-x^2}\frac{d}{dx}\left(x\sin^{-1}x\right) - \left(x\sin^{-1}x\right)\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}\right]$$

[Using quotient rule, product rule, chain rule]

$$= \left[\frac{\sqrt{1-x^2}\left\{x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right\} - \left(x\sin^{-1}x\right)\frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)}{\left(1-x^2\right)}\right]$$
$$= \left[\frac{\sqrt{1-x^2}\left\{\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right\} - \frac{x\sin^{-1}x\,(-2x)}{2\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$
$$= \left[\frac{x + \sqrt{1-x^2}\sin^{-1}x + \frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$
$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{\sqrt{1-x^2}\sin^{-1}x}{1} + \frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\left(1-x^2\right)\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1}x - x^2\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{y}{x} \qquad \left\{\text{Since, given } y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right\}.$$

Differentiation Ex 11.2 Q65

Given, $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$$
$$= \left[\frac{\left(e^x + e^{-x}\right)\frac{d}{dx}\left(e^x - e^{-x}\right) - \left(e^x - e^{-x}\right)\frac{d}{dx}\left(e^x + e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

[Using quotient rule and chain rule]

$$= \left[\frac{\left(e^x + e^{-x}\right)\left[e^x - e^{-x}\frac{d}{dx}(-x)\right) - \left(e^x - e^{-x}\right)\left(e^x + e^{-x}\frac{d}{dx}(-x)\right)\right]}{\left(e^x + e^{-x}\right)^2}\right]$$
$$= \left[\frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$
$$= \left[\frac{e^{2x} + e^{-2x} + 2e^x \times e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{\left(e^x + e^{-x}\right)2}\right]$$
$$\frac{dy}{dx} = \left[\frac{4}{\left(e^x + e^{-x}\right)2}\right] \qquad \text{---(i)}$$

Now,

$$1 - y^2 = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$
$$= 1 - \frac{\left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$
$$= \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$
$$= \frac{4}{\left(e^x + e^{-x}\right)2}$$

Differentiation Ex 11.2 Q66

Given, $y = (x-1)\log(x-1) - (x+1)\log(x+1)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x-1)\log(x-1) - (x+1)\log(x+1)\right]$$
$$= \left[(x-1)\frac{d}{dx}\log(x-1) + \log(x-1)\frac{d}{dx}(x-1)\right] -$$
$$\left[(x+1)\frac{d}{dx}\log(x+1) + \log(x+1)\frac{d}{dx}(x+1)\right]$$

[Using product rule, chain rule]

$$= \left[(x-1)\times\frac{1}{(x-1)}\frac{d}{dx}(x-1) + \log(x-1)\times(1)\right] -$$
$$\left[(x+1)\frac{1}{(x+1)}\times\frac{d}{dx}(x+1) + \log(x+1)(1)\right]$$
$$= \left[(1) + \log(x-1)\right] - \left[1 + \log(x+1)\right]$$
$$= \log(x-1) - \log(x+1)$$

$$\frac{dy}{dx} = \log\frac{(x-1)}{(x+1)} \qquad \left[\text{Since, } \log\left(\frac{a}{b}\right) = \log a - \log b\right].$$

Differentiation Ex 11.2 Q67

Given, $y = e^x \cos x$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \cos x\right)$$
$$= e^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}e^x \quad \text{[Using product rule]}$$
$$= e^x(-\sin x) + e^x \cos x$$
$$= e^x(\cos x - \sin x)$$
$$= \sqrt{2}e^x\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right) \quad \text{[Multiplying and dividing by } \sqrt{2}\text{]}$$
$$= \sqrt{2}e^x\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)$$

$$\frac{dy}{dx} = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right).$$

Differentiation Ex 11.2 Q68

Given, $y = \frac{1}{2}\log\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{2\sin^2 x}{2\cos^2 x}\right) \quad \left[\text{Since, } 1-\cos 2x = 2\sin^2 x, \; 1+co2x = 2\cos^2 x\right]$$
$$\Rightarrow \quad y = \frac{1}{2}\log\left(\tan^2 x\right)$$
$$\Rightarrow \quad y = \frac{2}{2}\log\tan x \quad \left[\text{Since, } \log a^b = b\log a\right]$$
$$\Rightarrow \quad y = \log\tan x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = (\log\tan x)$$
$$= \frac{1}{\tan x} \times \frac{d}{dx}(\tan x) \quad \text{[Using chain rule]}$$
$$= \frac{\sec^2 x}{\tan x}$$
$$= \frac{1}{\cos^2 x \times \frac{\sin x}{\cos x}}$$
$$= \frac{1}{\sin x\cos x}$$
$$= \frac{2}{2\sin x\cos x}$$
$$= \frac{2}{\sin 2x} \quad \left[\text{Since, } \frac{1}{\sin x = \cos ecx}\right]$$

So,

$$\frac{dy}{dx} = 2\operatorname{cosec} 2x.$$

Differentiation Ex 11.2 Q69

Here, $y = x\sin^{-1}x + \sqrt{1-x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$$
$$= \frac{d}{dx}\left(x\sin^{-1}x\right) + \frac{d}{dx}\left(\sqrt{1-x^2}\right)$$
$$= \left[x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right] + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)$$

[Using product rule and chain rule]

$$= \left[\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right] - \frac{2x}{2\sqrt{1-x^2}}$$
$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$
$$= \sin^{-1}x$$

So,

$$\frac{dy}{dx} = \sin^{-1}x.$$

Differentiation Ex 11.2 Q70

Here, $y = \sqrt{x^2+a^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x^2+a^2}\right)$$
$$= \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}\left(x^2+a^2\right) \quad \text{[Using chain rule]}$$
$$= \frac{1}{2\sqrt{x^2+a^2}}\times(2x)$$
$$= \frac{x}{\sqrt{x^2+a^2}}$$
$$\frac{dy}{dx} = \frac{x}{y} \quad \left[\text{Since } \sqrt{x^2+a^2} = y\right]$$
$$\Rightarrow \quad y\frac{dy}{dx} = x$$
$$\Rightarrow \quad y\frac{dy}{dx} - x = 0.$$

Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{-x}\right)$$
$$= \frac{d}{dx}e^x + \frac{d}{dx}e^{-x}$$
$$= e^x + e^{-x}\frac{d}{dx}(-x) \quad \text{[Using chian rule]}$$
$$= e^x + e^{-x}(-1)$$
$$= \left(e^x - e^{-x}\right)$$
$$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4e^x \times e^{-x}} \quad \left[\text{Since } (a-b) = \sqrt{(a+b)^2 - 4ab}\right]$$
$$= \sqrt{y^2 - 4} \quad \left[\text{Since } e^x + e^{-x} = y\right]$$

So,

$$\frac{dy}{dx} = \sqrt{y^2-4}.$$

Differentiation Ex 11.2 Q72

Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx}\left(a^2 - x^2\right) \quad \text{[Using chain rule]}$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y} \quad \left[\text{Since } \sqrt{a^2 - x^2} = y\right]$$

$$\Rightarrow \quad y\frac{dy}{dx} = -x$$

$$y\frac{dy}{dx} + x = 0$$

Differentiation Ex 11.2 Q73

Here, $xy = 4$

$$\Rightarrow \quad y = \frac{4}{x}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{4}{x}\right)$$

$$= 4\frac{d}{dx}(x^{-1})$$

$$= 4\left(-1 \times x^{-1-1}\right)$$

$$= 4\left(-\frac{1}{x^2}\right)$$

$$= \frac{-4}{x^2}$$

$$= -\frac{4}{\left(\frac{x}{y}\right)^2} \quad \left[\text{Since } x = \frac{4}{y}\right]$$

$$= -\frac{4y^2}{16}$$

$$\frac{dy}{dx} = -\frac{y^2}{4}$$

$$\Rightarrow \quad 4\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} = 3y^2 - 4y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} + 4y^2 = 3y^2$$

$$\Rightarrow \quad 4\left(\frac{dy}{dx} + y^2\right) = 3y^2$$

Dividing both the sides by x,

$$\Rightarrow \quad \frac{4}{x}\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x} \quad \left[\text{Since } \frac{4}{x} = y\right]$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{y}$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = 3y.$$

Differentiation Ex 11.2 Q74

$$\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}=\sqrt{a^2-x^2}$$

$$\text{LHS} = \frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}$$

$$= \frac{d}{dx}\left(\frac{x}{2}\sqrt{a^2-x^2}\right)+\frac{d}{dx}\left(\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)$$

$$= \frac{1}{2}\left[x\frac{d}{dx}\sqrt{a^2-x^2}+\sqrt{a^2-x^2}\frac{d}{dx}(x)\right]+\frac{a^2}{2}\times\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\times\frac{d}{dx}\left(\frac{x}{a}\right)$$

[Using product rule, chain rule]

$$= \frac{1}{2}\left[x\times\frac{1}{2\sqrt{a^2-x^2}}\frac{d}{dx}\left(a^2-x^2\right)+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\times\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\times\left(\frac{1}{a}\right)$$

$$= \frac{1}{2}\left[\frac{x(-2x)}{2\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\frac{a}{\sqrt{a^2-x^2}}\times\left(\frac{1}{a}\right)$$

$$= \frac{1}{2}\left[\frac{-2x^2+2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{1}{2}\left[\frac{2\left(a^2-2x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{a^2-2x^2}{2\sqrt{a^2-x^2}}+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{a^2-2x^2+a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{2a^2-2x^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}$$

$$= \frac{\left(a^2-x^2\right)}{\sqrt{a^2-x^2}}$$

$$= \sqrt{a^2-x^2}$$

= RHS

Ex 11.3

Differentiation Ex 11.3 Q1

Let $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$

Put $x = \cos\theta$

$$y = \cos^{-1}\left\{2\cos\theta\sqrt{1-\cos^2\theta}\right\}$$

$$= \cos^{-1}\left\{2\cos\theta\sin\theta\right\}$$

$$y = \cos^{-1}\left\{\sin 2\theta\right\} \qquad \left[\text{Since } \sin 2\theta = 2\sin\theta\cos\theta, \sin^2\theta + \cos^2\theta = 1\right]$$

$$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \theta\right)\right] \qquad \text{---(i)}$$

Now,

$$\frac{1}{\sqrt{2}} < x < 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} < \cos\theta < 1$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow \quad 0 > -2\theta > -\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > 0$$

Hence, from equation (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta, \text{if } \theta \in [0, \pi]\right]$$

$$y = \frac{\pi}{2} - 2\cos^{-1}x \qquad \left[\text{Since } x = \cos\theta\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - 2\frac{d}{dx}\left(\cos^{-1}x\right)$$

$$= 0 - 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q2

Let $y = \cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}$

Put $x = \cos 2\theta$

$$y = \cos^{-1}\left\{\sqrt{\frac{1+\cos 2\theta}{2}}\right\}$$

$$= \cos^{-1}\left\{\sqrt{\frac{2\cos^2\theta}{2}}\right\}$$

$$y = \cos^{-1}\left\{\cos\theta\right\} \qquad \text{---(i)}$$

Here, $-1 < x < 1$

$$\rightarrow \quad -1 < \cos 2\theta < 1$$

$$\Rightarrow \quad 0 < 2\theta < \pi$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta \text{ if } \theta \in [0, \pi]\right]$$

$$y = \frac{1}{2}\cos^{-1}x \qquad \left[\text{Since } x = \cos 2\theta\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q3

Let $y = \sin^{-1}\left\{\sqrt{\frac{1-x}{2}}\right\}$

Let $x = \cos 2\theta$

$y = \sin^{-1}\left\{\sqrt{\left(\frac{1-\cos 2\theta}{2}\right)}\right\}$

$= \sin^{-1}\left\{\sqrt{\frac{2\sin^2\theta}{2}}\right\}$

$y = \sin^{-1}(\sin\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \cos 2\theta < 1$

$\Rightarrow$ $0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow$ $0 < \theta < \frac{\pi}{4}$

so, from equation (i),

$y = \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \frac{1}{2}\cos^{-1} x$ $[\text{Since, } x = \cos 2\theta]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q4

Let $y = \sin^{-1}\left\{\sqrt{1-x^2}\right\}$

Let $x = \cos\theta$

$y = \sin^{-1}\left\{\sqrt{1-\cos^2\theta}\right\}$

$y = \sin^{-1}(\sin\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \cos\theta < 1$

$\Rightarrow$ $0 < \theta < \frac{\pi}{2}$

From equatoin (i),

$y = \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \cos^{-1} x$ $[\text{Since } x = \cos\theta]$

Differentiating with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q5

Let $y = \tan^{-1}\left\{\dfrac{x}{\sqrt{a^2 - x^2}}\right\}$

Let $x = a\sin\theta$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2\left(1 - \sin^2\theta\right)}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a\cos\theta}\right\}$$

$$y = \tan^{-1}(\tan\theta) \qquad \text{---(i)}$$

Here, $-a < x < a$

$\Rightarrow \quad -1 < \dfrac{x}{a} < 1$

$\Rightarrow \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

From equatoin (i),

$$y = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \sin^{-1}\left(\frac{x}{a}\right) \qquad [\text{Since } x = a\sin\theta]$$

Differentiating it with respect to x,

Using chain rule,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q6

Let $y = \sin^{-1}\left\{\dfrac{x}{\sqrt{x^2 + a^2}}\right\}$

Put $x = a\tan\theta$

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2 + \tan^2\theta + a^2}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2\left(\tan^2\theta + 1\right)}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{a\sec\theta}\right\}$$

$$= \sin^{-1}\{\sin\theta\}$$

$$= \theta$$

$$y = \tan^{-1}\left(\frac{x}{a}\right) \qquad [x = a\tan\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a^2}{a^2 + x^2} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q7

Let $y = \sin^{-1}\left\{2x^2 - 1\right\}$

Let $x = \cos\theta$

$y = \sin^{-1}\left(2\cos^2\theta - 1\right)$

$= \sin^{-1}(\cos 2\theta)$

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$ ---(i)

Here, $0 < x < 1$

$\Rightarrow 0 < \cos\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$

$\Rightarrow 0 < 2\theta < \pi$

$\Rightarrow 0 > -2\theta > -\pi$

$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > -\frac{\pi}{2}$

So, from equatoin (i),

$y = \frac{\pi}{2} - 2\theta$ $\left[\text{Since, } \sin^{-1}(\cos\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \frac{\pi}{2} - 2\cos^{-1}x$ $[\text{Since } x = \cos\theta]$

$\frac{dy}{dx} = 0 - 2\frac{d}{dx}\left(\cos^{-1}x\right)$

$= -2\left(-\frac{1}{\sqrt{1 - x^2}}\right)$

$\frac{dy}{dx} = \frac{2}{\sqrt{1 - x^2}}.$

Differentiation Ex 11.3 Q8

Let $y = \sin^{-1}\left\{1 - 2x^2\right\}$

Let $x = \sin\theta$, So,

$y = \sin^{-1}\left(1 - 2\sin^2\theta\right)$

$= \sin^{-1}(\cos 2\theta)$

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$ ---(i)

Here, $0 < x < 1$

$\Rightarrow 0 < \sin\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$

$\Rightarrow 0 < 2\theta < \pi$

$\Rightarrow 0 > -2\theta > -\pi$

$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \frac{\pi}{2} - \pi$

$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \left(-\frac{\pi}{2}\right)$

So, from equatoin (i),

$y = \frac{\pi}{2} - 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \frac{\pi}{2} - 2\sin^{-1}x$ $[\text{Since } x = \sin\theta]$

Differentiating with respect to x,

$\frac{dy}{dx} = 0 - 2\left(\frac{1}{\sqrt{1 - x^2}}\right)$

$\frac{dy}{dx} = -\frac{2}{\sqrt{1 - x^2}}.$

Differentiation Ex 11.3 Q9

Let $y = \cos^{-1}\left\{\frac{x}{\sqrt{x^2+a^2}}\right\}$

Put $x = a\cot\theta$,

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{\sqrt{a^2\cot^2\theta + a^2}}\right\}$$
$$= \cos^{-1}\left\{\frac{a\cot\theta}{a\,\text{cosec}\theta}\right\}$$
$$= \cos^{-1}\left\{\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\right\}$$
$$= \cos^{-1}(\cos\theta)$$
$$= \theta$$
$$y = \cot^{-1}\left(\frac{x}{a}\right) \qquad [\text{Since, } a\cot\theta = x]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{-1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$
$$= \frac{-a^2}{a^2+x^2}\times\left(\frac{1}{a}\right)$$
$$\frac{dy}{dx} = \frac{-a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q10

Let $y = \sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}$

$$= \sin^{-1}\left\{\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x \times \left(\frac{1}{\sqrt{2}}\right)\right\}$$
$$= \sin^{-1}\left\{\sin x\cos\frac{\pi}{4} + \cos x \times \sin\frac{\pi}{4}\right\}$$
$$y = \sin^{-1}\left\{\sin\left(x+\frac{\pi}{4}\right)\right\}$$

Here, $\frac{-3\pi}{4} < x < \frac{\pi}{4}$

$\Rightarrow \left(\frac{-3\pi}{4}+\frac{\pi}{4}\right)$ $\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[\frac{-\pi}{2},\frac{\pi}{2}\right]\right]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 1+0$$
$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.3 Q11

Let $y = \cos^{-1}\left\{\frac{\cos x + \sin x}{\sqrt{2}}\right\}$

$y = \cos^{-1}\left\{\cos x\left(\frac{1}{\sqrt{2}}\right) + \sin x\left(\frac{1}{\sqrt{2}}\right)\right\}$

$= \cos^{-1}\left\{\cos x \cos\left(\frac{\pi}{4}\right) + \sin x \sin x\left(\frac{\pi}{4}\right)\right\}$

$y = \cos^{-1}\left[\cos\left(x - \frac{\pi}{4}\right)\right]$ ---(i)

Here, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$\Rightarrow \left(-\frac{\pi}{4} - \frac{\pi}{4}\right) < \left(x - \frac{\pi}{4}\right) < \left(\frac{\pi}{4} - \frac{\pi}{4}\right)$

$\Rightarrow -\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < 0$

So, from equation (i),

$y = -\left(x - \frac{\pi}{4}\right)$ $\left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\right]$

$y = -x + \frac{\pi}{4}$

Differentiating it with respect to x,

$\frac{dy}{dx} = -1.$

Differentiation Ex 11.3 Q12

Let $y = \tan^{-1}\left\{\frac{x}{1 + \sqrt{1 - x^2}}\right\}$

Put $x = \sin\theta$, so

$y = \tan^{-1}\left\{\frac{\sin\theta}{1 + \sqrt{1 - \sin^2\theta}}\right\}$

$= \tan^{-1}\left\{\frac{\sin\theta}{1 + \cos\theta}\right\}$

$= \tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$

$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$ ---(i)

Here, $-1 < x < 1$

$\Rightarrow -1 < \sin\theta < 1$

$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, from equation (i),

$y = \frac{\theta}{2}$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \frac{1}{2}\sin^{-1}x$ $[\text{Since, } x = \sin\theta]$

Differentiating it with respect to x,

$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}.$

Differentiation Ex 11.3 Q13

Let $y = \tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}$

Put $x = a\sin\theta$, so

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2-a^2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2\left(1-\sin^2\theta\right)}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a+a\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a\left(1+\cos\theta\right)}\right\}$$

$$= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) \qquad \text{---(i)}$$

Here, $-a < x < a$

$\Rightarrow \quad -1 < \frac{x}{a} < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, from equation (i),

$$y = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2} + \sin^{-1}\left(\frac{x}{a}\right) \qquad \left[\text{Since, } x = a\sin\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{2\sqrt{a^2-x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2-x^2}}.$$

Differentiation Ex 11.3 Q14

Let $y = \sin^{-1}\left\{\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$

Put $x = \sin\theta$, so

$$= \sin^{-1}\left\{\frac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right) + \cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\} \quad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad \left(-\frac{\pi}{2} + \frac{\pi}{4}\right) < \left(\frac{\pi}{4} + \theta\right) < \frac{3\pi}{4}$

So, from equation (i),

$y = \theta + \frac{\pi}{4}$ $\quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ as } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = \sin^{-1}x + \frac{\pi}{4}$ $\quad [\text{Since, } \sin\theta = x]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} + 0$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.3 Q15

Let $y = \cos^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}$

Put $x = \sin\theta$, so

$$y = \cos^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right)+\cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \cos^{-1}\left\{\sin\theta\times\sin\frac{\pi}{4}+\cos\theta\times\cos\frac{\pi}{4}\right\}$$

$$y = \cos^{-1}\left\{\cos\left(\theta-\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{2}+\frac{\pi}{4} < \left(\theta-\frac{\pi}{4}\right) < \frac{\pi}{2}-\frac{\pi}{4}$

$\Rightarrow \quad \left(-\frac{3\pi}{4}\right) < \left(\theta-\frac{\pi}{4}\right) < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$y = -\left(\theta-\frac{\pi}{4}\right) \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta\in[-\pi,0]\right]$$

$$y = -\theta+\frac{\pi}{4}$$

$$y = -\sin^{-1}x+\frac{\pi}{4} \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}+0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q16

Let $y = \tan^{-1}\left\{\frac{4x}{1-4x^2}\right\}$

Put $2x = \tan\theta$, so

$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$

$y = \tan^{-1}\{\tan 2\theta\}$ ---(i)

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow$ $-1 < 2x < 1$

$\Rightarrow$ $-1 < \tan\theta < 1$

$\Rightarrow$ $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow$ $-\frac{\pi}{2} < (2\theta) < \frac{\pi}{2}$

So, from equation (i),

$y = 2\theta$ $\left[\text{Sicne, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$y = 2\tan^{-1}(2x)$ $[\text{Since, } 2x = \tan\theta]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+(2x)^2}\right)\frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

Let $y = \tan^{-1}\left\{\frac{2^{x+1}}{1-4^x}\right\}$

Put $2^x = \tan\theta$, so,

$= \tan^{-1}\left\{\frac{2^x \times 2}{1-(2^x)^2}\right\}$

$= \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$

$y = \tan^{-1}\{\tan(2\theta)\}$ ---(i)

Here, $-\infty < x < 0$

$\Rightarrow$ $2^{-\infty} < 2^x < 2^0$

$\Rightarrow$ $0 < 2^x < 1$

$\Rightarrow$ $0 < \theta < \frac{\pi}{4}$

$\Rightarrow$ $0 < (2\theta) < \frac{\pi}{2}$

From equatoin (i),

$y = 2\theta$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$y = 2\tan^{-1}(2^x)$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+(2^x)^2}\frac{d}{dx}(2^x)$$

$$= \frac{2\times 2^x \log 2}{1+4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1}\log 2}{1+4^x}.$$

Differentiation Ex 11.3 Q18

Let $y = \tan^{-1}\left\{\frac{2a^x}{1-a^{2x}}\right\}$

Put $a^x = \tan\theta$,

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\{\tan(2\theta)\} \qquad \text{---(i)}$$

Here, $-\infty < x < 0$

$\Rightarrow a^{-\infty} < a^x < 2^0$

$\Rightarrow 0 < \tan\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$

$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}(a^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+(a^x)^2}\frac{d}{dx}(a^x)$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1+a^{2x}}.$$

Differentiation Ex 11.3 Q19

Let $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$

Put $x = \cos 2\theta$, so,

$$= \sin^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right\}$$

$$= \sin^{-1}\left\{\cos\theta\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\cos\theta\sin\left(\frac{\pi}{4}\right)+\cos\frac{\pi}{4}\sin\theta\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} < \left(\theta+\frac{\pi}{4}\right) < \frac{\pi}{2}$

So, from equatoin (i),

$$y = \theta + \frac{\pi}{4} \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$$

$$y = \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)+0$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

Let $y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Put $ax = \tan\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2}\tan^{-1}(ax)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2}\times\left(\frac{1}{1+(ax)^2}\right)\frac{d}{dx}(ax)$$

$$\frac{dy}{dx} = \frac{1}{2\left(1+a^2x^2\right)}(a)$$

$$\frac{dy}{dx} = \frac{a}{2\left(1+a^2x^2\right)}.$$

Differentiation Ex 11.3 Q21

Let $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

This function is defined for all real numbers where $\cos x \neq 1$

i.e at all odd multiples of π

$$f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right] = \frac{x}{2}$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$

Differentiation Ex 11.3 Q22

Let $y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $x = \cot\theta$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta}}\right)$$

$$= \sin^{-1}(\sin\theta)$$

$$= \theta$$

$$y = \cot^{-1}x \qquad [\text{Since, } \cot\theta = x]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\left(1+x^2\right)}.$$

Differentiation Ex 11.3 Q23

Let $y = \cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

Put $x^n = \tan\theta$, so,

$$y = \cos^{-1}\left(\frac{1-\left(x^n\right)^2}{1+\left(x^n\right)^2}\right)$$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}\left(\cos 2\theta\right) \quad ---(i)$$

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < x^n < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < (2\theta) < \pi$

So, from equation (i),

$$y = 2\theta \quad \left[\text{Sicne, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0,\pi]\right]$$

$$y = 2\tan^{-1}\left(x^n\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+\left(x^n\right)^2}\right)\frac{d}{dx}\left(x^n\right)$$

$$= \frac{2}{1+x^{2n}} \times \left(nx^{n-1}\right)$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1+x^{2n}}.$$

Differentiation Ex 11.3 Q24

Let $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$$= \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \quad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$y = \frac{\pi}{2} \quad \left[\text{Since, } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

Let $y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$

$$y = \tan^{-1}a + \tan^{-1}x \quad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}a\right) + \frac{d}{dx}\left(\tan^{-}x\right)$$

$$= 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q26

Let $y = \tan^{-1}\left(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}}\right)$

$y = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{a}$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) + \frac{d}{dx}\left(\tan^{-1}\sqrt{a}\right)$$
$$= \frac{1}{1+\left(\sqrt{x}\right)^2}\frac{d}{dx}\left(\sqrt{x}\right) + 0$$
$$= \left(\frac{1}{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.3 Q27

Let $y = \tan^{-1}\left[\frac{a+b\tan x}{b-a\tan x}\right]$

$$= \tan^{-1}\left[\frac{\frac{a+b\tan x}{b}}{\frac{b-a\tan x}{b}}\right]$$
$$= \tan^{-1}\left[\frac{\frac{a}{b}+\tan x}{1+\frac{a}{b}\tan x}\right]$$
$$= \tan^{-1}\left[\frac{\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}{1-\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}\right]$$
$$= \tan^{-1}\left[\tan\left(\tan^{-1}\frac{a}{b}+x.\right)\right]$$
$$y = \tan^{-1}\left(\frac{a}{b}\right)+x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 0+1$$
$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

Let $y = \tan^{-1}\left(\frac{a+bx}{b-ax}\right)$

$$= \tan^{-1}\left(\frac{\frac{a+bx}{b}}{\frac{b-ax}{b}}\right)$$
$$= \tan^{-}\left(\frac{\frac{a}{b}+\frac{bx}{b}}{\frac{b}{a}-\frac{ax}{b}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{a}{b}+x}{1-\left(\frac{a}{b}\right)x}\right)$$

$y = \tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}x$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0+\frac{1}{1+x^2}$$
$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q29

Let $y = \tan^{-1}\left(\frac{x-a}{x+a}\right)$

$$= \tan^{-1}\left(\frac{\frac{x-a}{x}}{\frac{x+a}{x}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x}{x}-\frac{a}{x}}{\frac{x}{x}+\frac{a}{x}}\right)$$

$$= \tan^{-1}\left(\frac{1-\frac{a}{x}}{1+1\times\frac{a}{x}}\right)$$

$$y = \tan^{-1}(1) - \tan^{-1}\left(\frac{a}{x}\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \frac{1}{1+\left(\frac{a}{x}\right)^2}\frac{d}{dx}\left(\frac{a}{x}\right)$$

$$= -\frac{x^2}{x^2+a^2}\left(\frac{-a}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q30

Let $y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$

$$= \tan^{-1}\left(\frac{3x-2x}{1+(3x)(2x)}\right)$$

$$y = \tan^{-1}3x - \tan^{-1}2x \qquad \left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) - \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) - \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} - \frac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q31

Let $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$= \tan^{-1}\left(\frac{3x+2x}{1-(3x)(2x)}\right)$$

$$y = \tan^{-1}(3x) + \tan^{-1}(2x) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) + \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) + \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q32

Let $y = \tan^{-1}\left[\frac{\cos x + \sin x}{\cos x - \sin x}\right]$

$$= \tan^{-1}\left[\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{1 + \tan x}{1 - \tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + x\right)\right]$$

$$y = \frac{\pi}{4} + x$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q33

Let $y = \tan^{-1}\left[\frac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - (ax)^{\frac{1}{3}}}\right]$

$$y = \tan^{-1}\left(x^{\frac{1}{3}}\right) + \tan^{-1}\left(a^{\frac{1}{3}}\right) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(x^{\frac{1}{3}}\right)^2} \times \frac{d}{dx}\left(x^{\frac{1}{3}}\right) + 0$$

$$= \frac{\left(\frac{1}{3} \times x^{\frac{1}{3}-1}\right)}{1 + x^{\frac{2}{3}}}$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}.$$

Differentiation Ex 11.3 Q34

Let $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

To find the domain, we need to find all x such that

$-1 \le \frac{2^{x+1}}{1+4^x} \le 1$

Since the quantity in the middle is always positive, we need to find all x such that $\frac{2^{x+1}}{1+4^x} \le 1$

i.e all x such that $2^{x+1} \le 1+4^x$

We may rewrite as $2 \le \frac{1}{2^x} + 2^x$, which is true for all x

Hence the function is defined at all real numbers.

Putting $2^x = \tan\theta$

$$f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x.2}{1+(2^x)^2}\right)$$

$$= \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}(2^x)$$

Thus, $f'(x) = 2.\frac{1}{1+(2^x)^2}\frac{d}{dx}(2^x)$

$$= \frac{2}{1+4^x}.(2^x)\log 2 = \frac{2^{x+1}\log 2}{1+4^x}$$

Differentiation Ex 11.3 Q35

Let $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put, $x = \tan\theta$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow 0 < \tan\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$

$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$

So, from eqauation (i),

$y = 2\theta + 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right];\ \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$y = 4\theta$

$y = 4\tan^{-1} x$ [Since, $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}.$$

Differentiation Ex 11.3 Q36

Here, $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\sec\theta}\right) + \cos^{-1}\left(\frac{1}{\sec\theta}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\right) + \cos^{-1}(\cos\theta)$$

$$y = \sin^{-1}(\sin\theta) + \cos^{-1}(\cos\theta) \quad \text{---(i)}$$

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < \tan\theta < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from eqation (i),

$$y = \theta + \theta \quad \left[\begin{array}{l}\text{Since, } \sin^{-1}(\sin\theta)\theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1}x \quad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Differentiation Ex 11.3 Q37

Let $f(x) = \cos^{-1}(\sin x)$

We observe that this function is defined for all real numbers.

$$f(x) = \cos^{-1}(\sin x)$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$

Let $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan\theta$, so,

$$y = \cot^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \cot^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \cot^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{\pi}{4} + \theta\right)\right]$$

$$= \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \tan^{-1}x \quad [\text{Since } x = \tan\theta]$$

Differentiating it with respect do x,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q38

Let $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ ...(1)

Then, $\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

$$= \frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x)-(1-\sin x)}$$

$$= \frac{2+2\sqrt{1-\sin^2 x}}{2\sin x}$$

$$= \frac{1+\cos x}{\sin x}$$

$$= \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \cot\frac{x}{2}$$

Therefore, equation (1) becomes

$$y = \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

Differentiation Ex 11.3 Q39

Here, $y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \sec^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$

$y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$,

$y = \tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \tan^{-1}(\tan 2\theta) + \cos^{-1}(\cos 2\theta)$ ---(i)

Here, $< x < \infty$

$\Rightarrow$ $0 < \tan\theta < \infty$

$\Rightarrow$ $0 < \theta < \dfrac{\pi}{2}$

$\Rightarrow$ $0 < 2\theta < \pi$

So, from equation (i),

$y = 2\theta + 2\theta$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \text{ and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$y = 4\theta$

$y = 4\tan^{-1} x$ [Using $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ $\left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)\right]$

$y = \frac{\pi}{2}$ $\left[\text{Since, } \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}\right]$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0$$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2\tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$$y = \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\tan^2\theta}\right]$$

$$= \sin\left[2\tan^{-1}(\tan\theta)\right]$$

$$= \sin(2\theta)$$

$$= \sin\left[2\times\frac{1}{2}\cos^{-1}x\right] \qquad [\text{Since, } x = \cos 2\theta]$$

$$= \sin\left(\sin^{-1}\sqrt{1-x^2}\right)$$

$$y = \sqrt{1-x^2}$$

Differentiating with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right).$$

Differentiation Ex 11.3 Q42

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right) \quad \text{---(i)}$$

Here, $0 < x < \frac{1}{2}$

$\Rightarrow \quad 0 < 2x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

and

$\Rightarrow \quad 0 > -\theta > -\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2}-\theta\right) > 0$

So, from equation (i),

$$y = \theta + 2\left(\frac{\pi}{2}-\theta\right) \quad \left[\text{Since, } \cos^{-1}(\cos(\theta)) = \theta, \text{ if } \theta \in [0,\pi]\right]$$
$$= \theta + \pi - 2\theta$$
$$y = \pi - \theta$$
$$y = \pi - \cos^{-1}(2x) \quad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \left[\frac{-1}{\sqrt{1-(2x)^2}}\right]\frac{d}{dx}(2x)$$
$$= \frac{1}{\sqrt{1-4x^2}}(2)$$
$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.3 Q43

Here, $\frac{d}{dx}\left[\tan^{-1}(a+bx)\right] = 1 \text{ at } x = 0$

So, using chain rule,

$$\left[\left\{\frac{1}{1+(a+bx)^2}\right\}\frac{d}{dx}(a+bx)\right]_{x=0} = 1$$
$$\left[\frac{1}{1+(a+bx)^2}\times(b)\right]_{x=0} = 1$$

$\Rightarrow \quad \frac{b}{1+(a+0)^2} = 1$

$\Rightarrow \quad b = 1 + a^2.$

Differentiation Ex 11.3 Q44

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so,

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$
$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right) \quad ---(i)$$

Now, $-\frac{1}{2} < x < 0$

$\Rightarrow \quad -1 < 2x < 0$

$\Rightarrow \quad -1 < \cos\theta < 0$

$\Rightarrow \quad \frac{\pi}{2} < \theta < \pi$

And

$\Rightarrow \quad -\frac{\pi}{2} > -\theta > -\pi$

$\Rightarrow \quad \left(\frac{\pi}{2}-\frac{\pi}{2}\right) > \left(\frac{\pi}{2}-\theta\right) > \left(\frac{\pi}{2}-\pi\right)$

$\Rightarrow \quad 0 > \left(\frac{\pi}{2}-\theta\right) > -\frac{\pi}{2}$

So, from equation (i),

$$y = \theta + 2\left[-\left(\frac{\pi}{2}-\theta\right)\right] \quad \left[\text{Since, } \cos^{-1}\cos(\theta) = \theta, \text{if } \theta \in [0,\pi];\ \cos^{-1}\cos(\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\right]$$
$$y = \theta - 2\times\frac{\pi}{2} + 2\theta$$
$$y = -\pi + 3\theta$$
$$y = -\pi + 3\cos^{-1}(2x) \quad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 + 3\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$
$$= \frac{-3}{\sqrt{1-4x^2}}(2)$$
$$\frac{dy}{dx} = -\frac{6}{\sqrt{1-4x^2}}$$

Differentiation Ex 11.3 Q45

Here, $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta$, so

$$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}(\cos\theta - \sin\theta)}{\sqrt{2}(\cos\theta + \sin\theta)}\right)$$

$$= \tan^{(1)}\left(\frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}}\right)$$

[Dividing numerator and denominator by $\cos\theta$]

$$= \tan^{-}1\left(\frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4} + \tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \frac{\pi}{4} - \theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \qquad [\text{Using } x = \cos 2\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 - \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q46

Here, $y = \cos^{-1}\left\{\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}}\right\}$

Let $x = \cos\theta$, so,

$$y = \cos^{-1}\left\{\frac{2\cos\theta - 3\sqrt{1-\cos^2\theta}}{\sqrt{13}}\right\}$$

$$= \cos^{-1}\left\{\frac{2}{\sqrt{13}}\cos\theta - \frac{3}{\sqrt{13}}\sin\theta\right\}$$

Let $\cos\phi = \frac{2}{\sqrt{13}}$

$\Rightarrow$ $\sin\phi = \sqrt{1-\cos^2\phi}$

$$= \sqrt{1-\left(\frac{2}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{13-4}{13}}$$

$$= \sqrt{\frac{9}{13}}$$

$$\sin\phi = \frac{3}{\sqrt{13}}$$

So,

$$y = \cos^{-1}\{\cos\phi\cos\theta - \sin\phi\sin\theta\}$$

$$= \cos^{-1}[\cos(\theta+\phi)]$$

$$y = \phi + \theta$$

$$y = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}x \qquad \left[\text{Since, } x = \cos\theta, \cos\phi = \frac{2}{\sqrt{13}}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$y = \sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times 2^x \times 3^x}{1+(6^2)^x}\right\}$$

$$y = \sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\} \ldots.(1)$$

Substituting $6^x = \tan\theta$ in the above equation, we get,

$$y = \sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times \tan\theta}{1+\tan^2\theta}\right\}$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}(6^x)$$

Differentiating the above function with respect to x, we have,

$$\frac{d}{dx}\left[\sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}\right] = \frac{d}{dx}\left[2\tan^{-1}(6^x)\right]$$

$$= 2 \times \frac{1}{1+(6^x)^2} \times 6^x \log 6$$

$$= \frac{2 \times 6^x \log 6}{1+6^{2x}}$$

Ex 11.4

Differentiation Ex 11.4 Q1

Given,

$$xy = c^2$$

Differentiate with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \quad x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

Differentiation Ex 11.4 Q2

Here, $y^3 - 3xy^2 = x^3 + 3x^2y$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}y^2\frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow \quad 3\frac{dy}{dx}(y^2 - 2xy - x^2) = 3(x^2 + 2xy + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

Differentiation Ex 11.4 Q3

Here, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiate it with respect to x,

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$$\Rightarrow \frac{2}{3}x^{\left(\frac{2}{3}-1\right)} + \frac{2}{3}y^{\left(\frac{2}{3}-1\right)}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}} \times \frac{3}{2y^{\frac{-1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Differentiation Ex 11.4 Q4

Given, $4x+3y=\log(4x-3y)$

Differentiating with respect to x,

$$\frac{d}{dx}(4x)+\frac{d}{dx}(3y)=\frac{d}{dx}\left(\log(4x-3y)\right)$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\frac{d}{dx}(4x-3y) \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\left(4-3\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{4}{(4x-3y)}-\frac{3}{(4x-3y)}\frac{dy}{dx}$$

$$\Rightarrow \quad 3\frac{dy}{dx}+\frac{3}{(4x-3y)}\frac{dy}{dx}=\frac{4}{(4x-3y)}-4$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left(1+\frac{1}{(4x-3y)}\right)=4\left(\frac{1}{(4x-3y)}-1\right)$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left[\frac{4x-3y+1}{(4x-3y)}\right]=4\left[\frac{1-4x+3y}{(4x-3y)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{4}{3}\left[\frac{1-4x+3y}{(4x-3y)}\right]\left[\frac{4x-3y}{4x-3y+1}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{4}{3}\left(\frac{1-4x+3y}{4x-3y+1}\right)$$

Differentiation Ex 11.4 Q5

Given,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{x^2}{a^2}\right)+\frac{d}{dx}\left(\frac{y^2}{b^2}\right)=0$$

$$\Rightarrow \quad \frac{1}{a^2}(2x)+\frac{1}{b^2}(2y)\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{2x}{a^2}\right)\left(\frac{b^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

Differentiation Ex 11.4 Q6

Given,

$$x^5+y^5=5xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(y^5\right)=\frac{d}{dx}(5xy)$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\frac{dy}{dx}(x)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\,(1)\right]$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5x\frac{dy}{dx}+5y$$

$$\Rightarrow \quad 5y^4\frac{dy}{dx}-5x\frac{dy}{dx}=5y-5x^4$$

$$\Rightarrow \quad 5\frac{dy}{dx}\left(y^4-x\right)=5\left(y-x^4\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{5\left(y-x^4\right)}{5\left(y^4-x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y-x^4}{y^4-x}$$

Differentiation Ex 11.4 Q7

Given,

$$(x+y)^2 = 2axy$$

Differentiating with respect to x,

$$\Rightarrow \frac{d}{dx}(x+y)^2 = \frac{d}{dx}(2axy)$$

$$\Rightarrow 2(x+y)\frac{d}{dx}(x+y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] \quad \text{[Using chain rule and product rule]}$$

$$\Rightarrow 2(x+y)\left[1+\frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow 2(x+y) + 2(x+y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$$

$$\Rightarrow \frac{dy}{dx}\left[2(x+y) - 2ax\right] = 2ay - 2(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2[ay - x - y]}{2[x + y - ax]}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{ay - x - y}{x + y - ax}\right)$$

Differentiation Ex 11.4 Q8

Given,

$$\left(x^2+y^2\right)^2 = xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\left(x^2+y^2\right)^2\right) = \frac{d}{dx}(xy)$$

$$\Rightarrow 2\left(x^2+y^2\right)\frac{d}{dx}\left(x^2+y^2\right) = x\frac{dy}{dx} + y\frac{d}{dx}(x) \quad \text{[Using chain rule and product rule]}$$

$$\Rightarrow 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y(1)$$

$$\Rightarrow 4x\left(x^2+y^2\right) + 4y\left(x^2+y^2\right)\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow 4y\left(x^2+y^2\right)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}\left[4yx^2 + 4y^3 - x\right] = y - 4x^3 - 4xy^2$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x}\right)$$

Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}\left(x^2+y^2\right) = a$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\tan^{-1}\left(x^2+y^2\right)\right) = \frac{d}{dx}(a)$$

$$\Rightarrow \frac{1}{1+\left(x^2+y^2\right)^2} \times \frac{d}{dx}\left(x^2+y^2\right) = 0 \quad \text{[Using chain rule]}$$

$$\Rightarrow \left[\frac{1}{1+\left(x^2+y^2\right)^2}\right]\left(2x+2y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \left\{\frac{2x}{1+\left(x^2+y^2\right)^2}\right\} + \left\{\frac{2y}{1+\left(x^2+y^2\right)^2}\right\}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2y}{1+\left(x^2+y^2\right)^2}\frac{dy}{dx} = -\frac{2x}{1+\left(x^2+y^2\right)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{2x}{1+\left(x^2+y^2\right)^2}\right)\left(\frac{1+\left(x^2+y^2\right)^2}{2y}\right)$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)$$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y} = \log\left(\frac{x}{y}\right)$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\log\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$\Rightarrow \quad e^{(x-y)}\left(1-\frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{y}{xy} - \frac{x}{xy}\frac{dy}{dx}$$

$$\Rightarrow \quad e^{(x-y)} - e(x-y)\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1}{y} - \frac{e^{(x-y)}}{1}\right] = \frac{1}{x} - \frac{e^{(x-y)}}{1}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right] = \frac{\left(1-xe^{(x-y)}\right)}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left[\frac{1-xe^{(x-y)}}{1-ye^{(x-y)}}\right]$$

$$= \frac{-y}{-x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\sin xy + \frac{d}{dx}\cos(x+y) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \cos xy\frac{d}{dx}(xy) - \sin(x+y)\frac{d}{dx}(x+y) = 0 \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] - \sin(x+y)\left[1+\frac{dy}{dx}\right] = 0$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y(1)\right] - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx} + y\cos(xy) - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad [x\cos(xy) + \sin(x+y)]\frac{dy}{dx} = [\sin(x+y) - y\cos xy]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{\sin(x+y) - y\cos xy}{x\cos xy + \sin(x+y)}\right]$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$$

Let $x=\sin A, y=\sin B$, so

$\Rightarrow \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B}=a(\sin A-\sin B)$

$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$ $\left[\text{Since } (1-\sin^2\theta)=\cos^2\theta\right]$

$\Rightarrow a=\dfrac{\cos A+\cos B}{\sin A-\sin B}$

$\Rightarrow a=\dfrac{2\cos\frac{A+B}{2}\times\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\times\sin\frac{A-B}{2}}$ $\left[\begin{array}{l}\text{Since, } \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$

$\Rightarrow a=\cot\left(\dfrac{A-B}{2}\right)$

$\Rightarrow \cot^{-1}a=\dfrac{A-B}{2}$

$\Rightarrow 2\cot^{-1}a=A-B$

$\Rightarrow 2\cot^{-1}a=\sin^{-1}x-\sin^{-1}y$ $[\text{Since } x=\sin A, y=\sin B]$

Differentiating with respect to x,

$$\frac{d}{dx}\left(2\cot^{-1}a\right)=\frac{d}{dx}\left(\sin^{-1}x\right)-\frac{d}{dx}\left(\sin^{-1}y\right)$$

$\Rightarrow 0=\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}$

$\Rightarrow \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$$

Let $x=\sin A, y=\sin B$

$\Rightarrow \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$

$\Rightarrow \sin B\cos A+\sin A\cos B=1$ $\left[\begin{array}{l}\text{since } 1-\sin^2\theta=\cos^2\theta \text{ and}\\ \sin(x+y)=\sin x\cos y+\cos x\sin y\end{array}\right]$

$\Rightarrow \sin(A+B)=1$

$\Rightarrow A+B=\sin^{-1}(1)$

$\Rightarrow \sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2}$ $[\text{Since } x=\sin A, y=\sin B]$

Differentiating with respect to x,

$\Rightarrow \dfrac{d}{dx}\left(\sin^{-1}x\right)+\dfrac{d}{dx}\left(\sin^{-1}y\right)=\dfrac{d}{dx}\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \dfrac{1}{\sqrt{1-x^2}}+\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=0$

$\Rightarrow \dfrac{dy}{dx}=-\sqrt{\dfrac{1-y^2}{1-x^2}}$

Differentiation Ex 11.4 Q14

Here,

$$xy = 1 \qquad ---(i)$$

Differentiating with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x} \qquad \left[\text{Put } x = \frac{1}{y} \text{ from equation (i)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} + y^2 = 0$$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(xy^2\right) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{d}{dx}\left(y^2\right) + y^2\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2x}$$

Put $x = \frac{1}{y^2}$ from equation (i)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow \quad 2\frac{dy}{dx} = -y^3$$

$$2\frac{dy}{dx} + y^3 = 0$$

Differentiation Ex 11.4 Q16

Given,

$$x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$\Rightarrow \quad x\sqrt{1+y}=-y\sqrt{1+x}$$

Squaring both the sides,

$$\Rightarrow \quad \left(x\sqrt{1+y}\right)^2=\left(-y\sqrt{1+x}\right)^2$$

$$\Rightarrow \quad x^2(1+y)=y^2(1+x)$$

$$\Rightarrow \quad x^2+x^2y=y^2+y^2x$$

$$\Rightarrow \quad x^2-y^2=y^2x-x^2y$$

$$\Rightarrow \quad (x-y)(x+y)=xy(y-x)$$

$$\Rightarrow \quad (x+y)=-xy$$

$$\Rightarrow \quad y+xy=-x$$

$$\Rightarrow \quad y(1+x)=-x$$

$$\Rightarrow \quad y=\frac{-x}{(1+x)}$$

Differentiating with respect to x using quotient rule,

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-(1+x)\frac{d}{dx}(x)+(-x)\frac{d}{dx}(x+1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-(1+x)(1)+x(1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-1-x+x}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-1}{(1+x)^2}$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx}=-1$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx}+1=0$$

Differentiation Ex 11.4 Q17

Here,

$$\log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad \log\left(x^2+y^2\right)^{\frac{1}{2}}=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\log\left(x^2+y^2\right)=\tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{2}\frac{d}{dx}\log\left(x^2+y^2\right)=\frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times\left(\frac{1}{x^2+y^2}\right)\frac{d}{dx}\left(x^2+y^2\right)=\frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x+2y\frac{dy}{dx}\right]=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\times 2\left(x+y\frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right]$$

$$\Rightarrow \quad x+y\frac{dy}{dx}=x\frac{dy}{dx}-y$$

$$\Rightarrow \quad y\frac{dy}{dx}-x\frac{dy}{dx}=-y-x$$

$$\Rightarrow \quad \frac{dy}{dx}(y-x)=-(y+x)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-(y+x)}{y-x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{x+y}{x-y}$$

Differentiation Ex 11.4 Q18

Here,

$$\sec\left(\frac{x+y}{x-y}\right) = a$$

$$\Rightarrow \quad \frac{x+y}{x-y} = \sec^{-1}(a)$$

Differentiating with respect to x,

$$\Rightarrow \quad \left[\frac{(x-y)\frac{d}{dx}(x+y)-(x+y)\frac{d}{dx}(x-y)}{(x-y)^2}\right] = 0 \qquad \text{[Using quotient rule]}$$

$$\Rightarrow \quad (x-y)\left(1+\frac{dy}{dx}\right)-(x+y)\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad (x-y)+(x-y)\frac{dy}{dx}-(x+y)+(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[x-y+x+y] = x+y-x+y$$

$$\Rightarrow \quad \frac{dy}{dx}(2x) = 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.4 Q19

Here,

$$\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \tan a$$

$$\Rightarrow \quad x^2-y^2 = \tan a\left(x^2+y^2\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(x^2-y^2\right) = \tan a\frac{d}{dx}\left(x^2+y^2\right)$$

$$\Rightarrow \quad \left(2x-2y\frac{dy}{dx}\right) = \tan a\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 2x-2y\frac{dy}{dx} = 2x\tan a+2t\tan a\frac{dy}{dx}$$

$$\Rightarrow \quad 2y\tan a\frac{dy}{dx}+2y\frac{dy}{dx} = 2x-2x\tan a$$

$$\Rightarrow \quad 2y\frac{dy}{dx}(1+\tan a) = 2x(1-\tan a)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}\left(\frac{1-\tan a}{1+\tan a}\right)$$

Differentiation Ex 11.4 Q20

Here,

$$xy\log(x+y)=1$$

Differentiating it with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[xy\log(x+y)\right]=\frac{d}{dx}(1)$$

$$\Rightarrow \quad xy\frac{d}{dx}\log(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)\frac{d}{dx}(x)=0$$

[Using chain rule and product rule]

$$\Rightarrow \quad xy\times\left(\frac{1}{x+y}\right)\frac{d}{dx}(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)(1)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\frac{dy}{dx}+\left(\frac{xy}{x+y}\right)+x\left(\frac{1}{xy}\right)\frac{dy}{dx}+y\left(\frac{1}{xy}\right)=0$$

$$\left[\text{Sicne from equation (i) } \log(x+y)=\frac{1}{xy}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{xy}{x+y}+\frac{1}{y}\right]=-\left[\frac{1}{x}+\frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2+x+y}{(x+y)y}\right]=-\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx}=-\left(\frac{x+y+x^2y}{x(x+y)}\right)\left(\frac{y(x+y)}{xy^2+x+y}\right)$$

$$=-\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

So,

$$\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x^2y+x+y}{xy^2+x+y}\right)$$

Differentiation Ex 11.4 Q21

Here,

$$y=x\sin(a+y) \quad \text{---(i)}$$

Differentiating with respect to y,

$$\Rightarrow \quad \frac{dy}{dx}=\frac{d}{dx}\left[x\sin(a+y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=x\frac{d}{dx}\sin(a+y)+\sin(a+y)\frac{d}{dx}(x) \quad \text{[Using product rule, chain ruel]}$$

$$\Rightarrow \quad \frac{dy}{dx}=x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)$$

$$=x\cos(a+y)\left(0+\frac{dy}{dx}\right)+\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx}\left(1-x\cos(a+y)\right)=\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin(a+y)}{1-x\cos(a+y)}$$

Put x from equation (i), $x=\frac{y}{\sin(a+y)}$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin(a+y)}{1-\frac{y}{\sin(a+y)}\cos(a+y)}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y)-y\cos(a+y)}$$

Differentiation Ex 11.4 Q22

Here,

$$x\sin(a+y)+\sin a\cos(a+y)=0 \qquad \text{---(i)}$$

Differentiating with respect to x,

$$\Rightarrow \frac{d}{dx}\left[x\sin(a+y)\right]+\frac{d}{dx}\left[\sin a\cos(a+y)\right]=0$$

$$\Rightarrow \left[x\frac{d}{dx}\sin(a+y)+\sin(a+y)\frac{d}{dx}(x)\right]+\sin a\frac{d}{dx}\cos(a+y)=0$$

[Using product rule and chain rule]

$$\Rightarrow \left[x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)\right]+\sin a\left[-\sin(a+y)\frac{d}{dx}(a+y)\right]=0$$

$$\Rightarrow \left[x\cos(a+y)\left(0+\frac{dy}{dx}\right)+\sin(a+y)\right]-\sin a\sin(a+y)\left(0+\frac{dy}{dx}\right)=0$$

$$\Rightarrow x\cos(a+y)\frac{dy}{dx}+\sin(a+y)-\sin a\sin(a+y)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}\left[x\cos(a+y)-\sin a\sin(a+y)\right]=-\sin(a+y)$$

Put $x=-\sin a\frac{\cos(a+y)}{\sin(a+y)}$ from equation (i),

$$\Rightarrow \frac{dy}{dx}\left[-\sin a\frac{\cos^2(a+y)}{\sin(a+y)}-\sin a\sin(a+y)\right]=-\sin(a+y)$$

$$\Rightarrow -\frac{dy}{dx}\left[\frac{\sin a\cos^2(a+y)+\sin a\sin^2(a+y)}{\sin(a+y)}\right]=-\sin(a+y)$$

$$\Rightarrow \frac{dy}{dx}=\sin(a+y)^2\left[\frac{\sin(a+y)}{\sin a\left(\cos^2(a+y)+\sin^2(a+y)\right)}\right]$$

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a} \qquad \left[\text{Since } \sin^2\theta+\cos^2\theta=1\right]$$

Differentiation Ex 11.4 Q23

Here,

$$y=x\sin y$$

Differentiating with respect to x,

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(x\sin y)$$

$$\Rightarrow \frac{dy}{dx}=x\frac{d}{dx}(\sin y)+\sin y\frac{d}{dx}(x) \qquad \text{[Using product rule]}$$

$$\Rightarrow \frac{dy}{dx}=x\cos\frac{dy}{dx}+\sin y(1)$$

$$\Rightarrow \frac{dy}{dx}(1-x\cos y)=\sin y$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin y}{1-x\cos y}$$

Differentiation Ex 11.4 Q24

Here,

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(y\sqrt{x^2+1}\right) = \frac{d}{dx}\log\left(\sqrt{x^2+1}-x\right) \qquad \text{[Using product rule and chain rule]}$$

$$\Rightarrow \quad y\frac{d}{dx}\left(\sqrt{x^2+1}\right)+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\frac{d}{dx}\left(\sqrt{x^2+1}-x\right)$$

$$\Rightarrow \quad y\frac{1}{2\sqrt{x^2+1}}\times\frac{d}{dx}\left(x^2+1\right)+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}\left(x^2+1\right)-1\right]$$

$$\Rightarrow \quad \frac{2xy}{2\sqrt{x^2+1}}+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\left[\frac{2x}{2\sqrt{x^2+1}}-1\right]$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1}-x}\right]\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}}-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx} = -(1+xy)$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx}+1+xy = 0$$

Differentiation Ex 11.4 Q25

Here,

$$y = [\log_{\cos x}\sin x][\log_{\sin x}\cos x]^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = [\log_{\cos x}\sin x][\log_{\cos x}\sin x]+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Since, } \log_a a = (\log_b a)^{-1}\right]$$

$$y = \left[\frac{\log\sin x}{\log\cos x}\right]^2+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Sicne, } \log_a b = \frac{\log b}{\log a}\right]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\log\sin x}{\log\cos x}\right]^2+\frac{d}{dx}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right)$$

$$= 2\left[\frac{\log\sin x}{\log\cos x}\right]\frac{d}{dx}\left(\frac{\log\sin x}{\log\cos x}\right)+\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\frac{d}{dx}\left[\frac{2x}{1+x^2}\right]$$

$$\frac{dy}{dx} = 2\left[\frac{\log\sin x}{\log\cos x}\right]\left(\frac{(\log\cos x)\frac{d}{dx}(\log\sin x)-\log\sin x\frac{d}{dx}(\log\cos x)}{(\log\cos x)^2}\right)+$$

$$\text{[Using chain rule, quotient rule]}\quad\left(\frac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\times\frac{1}{\sin x}\frac{d}{dx}(\sin x)-\log\sin x\times\frac{1}{\cos x}\frac{d}{dx}(\cos x)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\left(\frac{\cos x}{\sin x}\right)+\log\sin x\left(\frac{\sin x}{\cos x}\right)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{1+x^2}{\sqrt{(1-x^2)^2}}\right)\left(\frac{2+2x^2-4x^2}{(1+x^2)^2}\right)$$

$$\frac{dy}{dx} = 2\frac{\log\sin x}{(\log\cos x)^3}(\cot x\log\cos x+\tan x\log\sin x)+\frac{2}{1+x^2}$$

Put $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = 2\left(\frac{\log\sin\frac{\pi}{4}}{\left(\log\cos\frac{\pi}{4}\right)^3}\right)\left(\cot\frac{\pi}{4}\log\cos\frac{\pi}{4}+\tan\frac{\pi}{4}\log\sin\frac{\pi}{4}\right)+2\left(\frac{1}{1+\left(\frac{\pi}{4}\right)^2}\right)$$

$$= 2\left(\frac{1}{\left(\log\frac{1}{\sqrt{2}}\right)^2}\right)\left(1\times\log\frac{1}{\sqrt{2}}+1\times\log\frac{1}{\sqrt{2}}\right)+2\left(\frac{16}{16+\pi}\right)$$

$$= 2\times\frac{2\log\left(\frac{1}{\sqrt{2}}\right)}{\left(\log\left(\frac{1}{\sqrt{2}}\right)\right)}+\frac{32}{16+\pi^2}$$

$$= 4\frac{1}{\log\left(\frac{1}{\sqrt{2}}\right)}+\frac{32}{16+\pi^2}$$

$$= 4\frac{1}{-\frac{1}{2}\log^2}+\frac{32}{16+\pi^2}$$

$$= -\frac{8}{\log 2}+\frac{32}{16+\pi^2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 8\left[\frac{4}{16+\pi^2}-\frac{1}{\log 2}\right]$$

Differentiation Ex 11.4 Q26

Here,

$$\sin(xy)+\frac{y}{x} = x^2-y^2$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(\sin xy)+\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}\left(x^2\right)-\frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \quad \cos(xy)\frac{d}{dx}(xy)+\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right] = 2x-2y\frac{dy}{dx} \qquad \left[\begin{matrix}\text{Using chain rule, quotient ruel,}\\ \text{product rule}\end{matrix}\right]$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]+\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right] = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx}+y(1)\right]+\frac{1}{x^2}\left(x\frac{dy}{dx}-y\right) = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx}+y\cos(xy)+\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2} = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x\cos(xy)+\frac{1}{x}+2y\right] = \frac{y}{x^2}-y\cos(xy)+2x$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{x^2\cos(xy)+1+2xy}{x}\right] = \frac{1}{x^2}\left(y-x^2y\cos xy+2x^3\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x^3+y-x^2y\cos(xy)}{x\left(x^2\cos xy+1+2xy\right)}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x}+\sqrt{y-x}=c$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(\sqrt{y+x}\right)+\frac{d}{dx}\sqrt{y-x}=\frac{d}{dx}(c)$$

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\frac{d}{dx}(y+x)+\frac{1}{2\sqrt{y-x}}\frac{d}{dx}(y-x)=0$$

Using chain rule

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\left[\frac{dy}{dx}+1\right]+\frac{1}{2\sqrt{y-x}}\left[\frac{dy}{dx}-1\right]=0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right)+\frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right)=\frac{1}{2\sqrt{y-x}}-\frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \quad \frac{dy}{dx}\times\frac{1}{2}\left[\frac{1}{\sqrt{y+x}}+\frac{1}{\sqrt{y-x}}\right]=\frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{\sqrt{y-x}+\sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}}\right]=\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{y+x}}\times\frac{\left(\sqrt{y+x}-\sqrt{y-x}\right)}{\left(\sqrt{y+x}-\sqrt{y-x}\right)}$$

[rationalizing the denominator]

$$\Rightarrow \quad \frac{dy}{dx}=\frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y-2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y}{2x}-\frac{2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2-x^2}{x^2}}$$

$$\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y)+\tan(x-y)=1$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\tan(x+y)+\frac{d}{dx}\tan(x-y)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \sec^2(x+y)\frac{d}{dx}(x+y)+\sec^2(x-y)\frac{d}{dx}(x-y)=0 \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \sec^2(x+y)\left[1+\frac{dy}{dx}\right]+\sec^2(x-y)\left[1-\frac{dy}{dx}\right]=0$$

$$\Rightarrow \quad \sec^2(x+y)\frac{dy}{dx}-\sec^2(x-y)\frac{dy}{dx}=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\sec^2(x+y)-\sec^2(x-y)\right]=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x-y)-\sec^2(x+y)}$$

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}e^y = \frac{d}{dx}\left(e^{x+y}\right)$$

$$\Rightarrow \quad e^x + e^y\frac{dy}{dx} = e^{x+y}\frac{d}{dx}(x+y)$$

$$\Rightarrow \quad e^x + e^y\frac{dy}{dx} = e^{x+y}\left[1+\frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^y\frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x\left(e^y-1\right)}{e^y\left(1-e^x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^x\left(e^y-1\right)}{e^y\left(e^x-1\right)}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x\cos(a+y)$

$$\therefore \frac{d}{dx}[\cos y] = \frac{d}{dx}[x\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y)\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}[\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y) + x\cdot[-\sin(a+y)]\frac{dy}{dx}$$

$$\Rightarrow [x\sin(a+y) - \sin y]\frac{dy}{dx} = \cos(a+y) \quad \ldots(1)$$

Since $\cos y = x\cos(a+y)$, $x = \dfrac{\cos y}{\cos(a+y)}$

Then, equation (1) reduces to

$$\left[\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow [\cos y\cdot\sin(a+y) - \sin y\cdot\cos(a+y)]\cdot\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+b)}{\sin a}$$

Hence, proved.

Ex 11.5

Differentiation Ex 11.5 Q1

Let $y = x^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$\Rightarrow \log y = \log x^{\frac{1}{x}}$

$\Rightarrow \log y = \frac{1}{x}\log x$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{-1}\right)$ [Using product rule]

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\times\frac{1}{x} + (\log x)\times\left(-\frac{1}{x^2}\right)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{(1-\log x)}{x^2}$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{1-\log x}{x^2}\right]$

Put the value of y from equation (i),

$$\frac{dy}{dx} = x^{\frac{1}{x}}\left[\frac{1-\log x}{x}\right]$$

Differentiation Ex 11.5 Q2

Let $y = x^{\sin x}$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\sin x}$

$\log y = \sin x \log x$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\frac{1}{y}\frac{dy}{dx} = \sin x\frac{d}{dx}\log x + \log x\frac{d}{dx}\sin x$ [Using product rule]

$\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log x(\cos x)$

$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$

Put the value of y,

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Differentiation Ex 11.5 Q3

Let $y = (1+\cos x)^x$ ---(i)

Taking log on both the sides,

$$\log y = \log(1+\cos x)^x$$
$$\log y = x\log(1+\cos x)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\frac{d}{dx}(x) \quad \text{[Using product rule and chain rule]}$$
$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{(1+\cos x)}\frac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{(1+\cos x)}(0-\sin x) + \log(1+\cos x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log(1+\cos x) - \frac{x somx}{(1+\cos x)}$$
$$\frac{dy}{dx} = y\left[\log(1+\cos x) - \frac{x\sin x}{1+\cos x}\right]$$
$$\frac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}\right] \quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q4

Let $y = x^{\cos^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\cos^{-1} x}$$
$$\log y = \cos^{-1} x \log x \quad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1} x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\cos^{-1} x\right)$$
$$= \cos^{-1} x\left(\frac{1}{x}\right) + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$
$$\frac{dy}{dx} = y\left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right]$$
$$\frac{dy}{dx} = x^{\cos^{-1} x}\left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right] \quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q5

Let $y = (\log x)^x$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^x$$
$$\log y = x\log(\log x) \quad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(\log x) + \log\log x\frac{d}{dx}(x)$$
$$= x\frac{1}{\log x}\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$= \frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\log x} + \log\log x$$
$$\frac{dy}{dx} = y\left[\frac{1}{\log x} + \log\log x\right]$$
$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q6

Let $y = (\log x)^{\cos x}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^{\cos x}$$
$$\log y = \cos x \log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x\frac{d}{dx}\log(\log x) + \log\log x\frac{d}{dx}(\cos x)$$
$$= \frac{\cos x}{\log x}\frac{d}{dx}(\log x) + \log\log x \times (-\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\log x} \times \left(\frac{1}{x}\right) - \sin x \log\log x$$
$$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \sin x\log\log x\right]$$
$$\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log\log x\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q7

Let $y = (\sin x)^{\cos x}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\cos x}$$
$$\log y = \cos x \log \sin x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}\cos x$$
$$= \cos x\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x(-\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\sin x}(\cos x) - \sin x\log\sin x$$
$$\frac{dy}{dx} = y\left[\cos x\cot x - \sin x\log\sin x\right]$$
$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x\cot x - \sin x\log\sin x\right]$$

Differentiation Ex 11.5 Q8

Let $y = e^{x\log x}$

$\Rightarrow$ $y = e^{\log x^x}$ $\left[\text{Since, } \log a^b = b\log a\right]$

$\Rightarrow$ $y = x^x$ ---(i) $\left[\text{Since, } e^{\log a} = a\right]$

Taking log both the sides,

$$\log y = \log x^x$$
$$\log y = x\log x$$

Differentiating with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x(1)$$
$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$$
$$\frac{dy}{dx} = y\left[1 + \log x\right]$$
$$\frac{dy}{dx} = x^x(1 + \log x) \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q9

Let $y = (\sin x)^{\log x}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\log x}$$
$$\log y = \log x \log(\sin x) \qquad [\text{Using } \log a^b = b \log a]$$

Differentiating with respect to x, using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\log x)$$
$$= \log x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\left(\frac{1}{x}\right)$$
$$= \frac{\log x}{\sin x} \times \cos x + \frac{\log \sin x}{x}$$
$$\frac{1}{y}\frac{dy}{dx} = \log x \cot x + \frac{\log \sin x}{x}$$
$$\frac{dy}{dx} = y\left[\log x \cot x + \frac{\log \sin x}{x}\right]$$
$$\frac{dy}{dx} = (\sin x)^{\log x}\left[\log x \cot x + \frac{\log \sin x}{x}\right] \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q10

Let $y = 10^{\log \sin x}$ ---(i)

Taking log on both the sides,

$$\log y = \log 10^{\log \sin x}$$
$$\log y = \log \sin x \log 10 \qquad [\text{Since, } \log a^b = b \log a]$$

Differentiating with respect to x, using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log 10 \frac{d}{dx}(\log \sin x)$$
$$= \log 10\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log 10\left(\frac{1}{\sin x}\right)(\cos x)$$
$$\frac{dy}{dx} = y[\log 10 \cot x]$$
$$\frac{dy}{dx} = 10^{\log \sin x}[\log 10 \times \cot x] \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q11

Let $y = (\log x)^{\log x}$

Taking logarithm on both the sides, we obtain

$$\log y = \log x \cdot \log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}[\log x \cdot \log(\log x)]$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log(\log x) \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}[\log(\log x)]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\log(\log x) \cdot \frac{1}{x} + \log x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)\right]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x}\log(\log x) + \frac{1}{x}\right]$$
$$\therefore \frac{dy}{dx} = (\log x)^{\log x}\left[\frac{1}{x} + \frac{\log(\log x)}{x}\right]$$

Differentiation Ex 11.5 Q12

Let $\quad y = 10^{(10x)}$ ---(i)

Taking log on both the siedes,

$$\log y = \log 10^{(10x)}$$
$$\log y = 10^x \log 10$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \log 10 \times 10^x \log 10$$
$$\frac{1}{y}\frac{dy}{dx} = 10^x \times (\log 10)^2$$
$$\frac{dy}{dx} = 10^{(10^x)} \times 10^x (\log 10)^2 \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q13

Let $\quad y = \sin x^x$

$\Rightarrow \quad \sin^{-1} y = x^x$

Taking log on both the siedes,

$$\log\left(\sin^{-1} y\right) = \log x^x$$
$$\log\left(\sin^{-1} y\right) = x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{1}{\sin^{-1} y}\frac{dy}{dx} = \left(\sin^{-1} y\right) = x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$\frac{1}{\sin^{1} y} \times \left(\frac{1}{\sqrt{1 - y^2}}\right)\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{dy}{dx} = \sin^{-1} y \sqrt{1 - y^2}\,(1 + \log x)$$
$$= \sin^{-1} \left(\sin x^x\right)\sqrt{1 - \left(\sin x^x\right)^2}\,(1 + \log x)$$
$$= x^x \sqrt{\cos^2 x^x}\,(1 + \log x) \qquad \text{[Using equation (i)]}$$

$$\frac{dy}{dx} = x^x \cos x^x\,(1 + \log x)$$

Differentiation Ex 11.5 Q14

Let $\quad y = \left(\sin^{-1} x\right)^x$

Taking log on both the sides,

$$\log y = \log\left(\sin^{-1} x\right)^x$$
$$\log y = x \log\left(\sin^{-1} x\right) \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x \frac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \frac{d}{dx}(x)$$
$$= x \times \frac{1}{\sin^{-1} x}\frac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x\,(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1 - x^2}}\right) + \log \sin^{-1} x$$
$$\frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1 - x^2}\right)}\right]$$
$$\frac{dy}{dx} = \left(\sin^{-1} x\right)^2\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1 - x^2}}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q15

Let $y = x^{\sin^{-1}x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin^{-1}x}$$
$$\log y = \sin^{-1}x \log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1}x\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}\left(\sin^{-1}x\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1}x\left(\frac{1}{x}\right) + (\log x)\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{dy}{dx} = y\left[\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$$

$$\frac{dy}{dx} = x^{\sin^{-1}x}\left[\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q16

Let $y = (\tan x)^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\tan x)^{\frac{1}{x}}$$
$$\log y = \frac{1}{x}\log(\tan x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}\log(\tan x) + \log(\tan x)\frac{d}{dx}\left(\frac{1}{x}\right)$$
$$= \frac{1}{x}\times\frac{1}{\tan x}\frac{d}{dx}(\tan x) + \log(\tan x)\left(-\frac{1}{x^2}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x\tan x}\left(\sec^2 x\right) - \frac{\log(\tan x)}{x^2}$$

$$\frac{dy}{dx} = y\left[\frac{\sec^2 x}{x\tan x} - \frac{\log(\tan x)}{x^2}\right]$$
$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\frac{\sec^2 x}{x\tan x} - \frac{\log\tan x}{x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q17

Let $y = x^{\tan^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\tan^{-1} x}$$

$$\log y = \tan^{-1} x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x \left(\frac{1}{x}\right) + \log x \left(\frac{1}{1+x^2}\right)$$

$$\frac{dy}{dx} = y\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right]$$

$$\frac{dy}{dx} = x^{\tan^{-1} x}\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(i)

Let $y = x^x \sqrt{x}$ ---(i)

Taking log on both the sides,

$$\log y = \log\left(x^x \sqrt{x}\right)$$

$$= \log x^x + \log x^{\frac{1}{2}} \qquad \left[\text{Since, } \log^{(ab)} = \log a + \log b\right]$$

$$\log y = x \log x + \frac{1}{2}\log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) + \frac{1}{2}\frac{d}{dx}(\log x)$$

$$= x\left(\frac{1}{x}\right) + \log x (1) + \frac{1}{2}\left(\frac{1}{x}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x + \frac{1}{2x}$$

$$\frac{dy}{dx} = y\left(1 + \log x + \frac{1}{2x}\right)$$

$$\frac{dy}{dx} = x^x\sqrt{x}\left(1 + \log x + \frac{1}{2x}\right) \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(ii)

Let $\quad y = x^{(\sin x - \cos x)} + \left(\frac{x^2-1}{x^2+1}\right)$

$$y = e^{\log x^{(\sin x - \cos x)}} + \left(\frac{x^2-1}{x^2+1}\right)$$

$$y = e^{(\sin x - \cos x)\log x} + \left(\frac{x^2-1}{x^2+1}\right) \qquad \left[\text{Since, } e^{\log a} = a,\ \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{(\sin x - \cos x)\log x}\right] + \frac{d}{dx}\left[\frac{x^2-1}{x^2+1}\right]$$

$$= e^{(\sin x - \cos x)\log x}\frac{d}{dx}\{(\sin x - \cos x)\log x\} + \left[\frac{\left(x^2+1\right)\frac{d}{dx}\left(x^2-1\right) - \left(x^2-1\right)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}\right]$$

$$= e^{\log x^{(\sin x - \cos x)}}\left[(\sin x - \cos x)\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(\sin x - \cos x)\right] + \left[\frac{\left(x^2+1\right)(2x) - \left(x^2-1\right)(2x)}{\left(x^2+1\right)^2}\right]$$

$$= x^{(\sin x - \cos x)}\left[(\sin x - \cos x)\left(\frac{1}{x}\right) + \log x\,(\sin x + \cos x)\right] + \left[\frac{2x^3 + 2x - 2x^3 + 2x}{\left(x^2+1\right)^2}\right]$$

$$\frac{dy}{dx} = x^{(\sin x - \cos x)}\left[\frac{(\sin x - \cos x)}{x} + \log x\,(\sin x + \cos x)\right] + \frac{4x}{\left(x^2+1\right)^2}$$

Differentiation Ex 11.5 Q18(iii)

Let $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

Also, let $u = x^{x\cos x}$ and $v = \dfrac{x^2+1}{x^2-1}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = x^{x\cos x}$

$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$

$\Rightarrow \log u = x\cos x\log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\cdot\cos x\cdot\log x + x\cdot\frac{d}{dx}(\cos x)\cdot\log x + x\cos x\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1\cdot\cos x\cdot\log x + x\cdot(-\sin x)\log x + x\cos x\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left(\cos x\log x - x\sin x\log x + \cos x\right)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1 + \log x) - x\sin x\log x\right] \qquad \ldots(2)$$

$v = \dfrac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log\left(x^2+1\right) - \log\left(x^2-1\right)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{2x}{x^2+1}-\frac{2x}{x^2-1}$$
$$\Rightarrow \frac{dv}{dx}=v\left[\frac{2x\left(x^2-1\right)-2x\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$
$$\Rightarrow \frac{dv}{dx}=\frac{x^2+1}{x^2-1}\times\left[\frac{-4x}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$
$$\Rightarrow \frac{dv}{dx}=\frac{-4x}{\left(x^2-1\right)^2} \quad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=x^{x\cos x}\left[\cos x\left(1+\log x\right)-x\sin x\log x\right]-\frac{4x}{\left(x^2-1\right)^2}$$

Differentiation Ex 11.5 Q18(iv)

Let $y=\left(x\cos x\right)^x+\left(x\sin x\right)^{\frac{1}{x}}$

Also, let $u=\left(x\cos x\right)^x$ and $v=\left(x\sin x\right)^{\frac{1}{x}}$

$\therefore y=u+v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \quad ...(1)$$

$$u=\left(x\cos x\right)^x$$
$$\Rightarrow \log u=\log\left(x\cos x\right)^x$$
$$\Rightarrow \log u=x\log\left(x\cos x\right)$$
$$\Rightarrow \log u=x\left[\log x+\log\cos x\right]$$
$$\Rightarrow \log u=x\log x+x\log\cos x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}\left(x\log x\right)+\frac{d}{dx}\left(x\log\cos x\right)$$
$$\Rightarrow \frac{du}{dx}=u\left[\left\{\log x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log x)\right\}+\left\{\log\cos x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x\cdot 1+x\cdot\frac{1}{x}\right)+\left\{\log\cos x\cdot 1+x\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x+1\right)+\left\{\log\cos x+\frac{x}{\cos x}\cdot(-\sin x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(1+\log x\right)+\left(\log\cos x-x\tan x\right)\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\left(\log x+\log\cos x\right)\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\log\left(x\cos x\right)\right] \quad ...(2)$$

$$v=(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v=\log(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v=\frac{1}{x}\log(x\sin x)$$

$$\Rightarrow \log v=\frac{1}{x}(\log x+\log\sin x)$$

$$\Rightarrow \log v=\frac{1}{x}\log x+\frac{1}{x}\log\sin x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{x}\log x\right)+\frac{d}{dx}\left[\frac{1}{x}\log(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}(\log x)\right]+\left[\log(\sin x)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}\{\log(\sin x)\}\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{x}\right]+\left[\log(\sin x)\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\frac{1}{x^2}(1-\log x)+\left[-\frac{\log(\sin x)}{x^2}+\frac{1}{x\sin x}\cdot\cos x\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2}+\frac{-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right] \quad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]+(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

Differentiation Ex 11.5 Q18(v)

Let $y=\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

Also, let $u=\left(x+\frac{1}{x}\right)^{x}$ and $v=x^{\left(1+\frac{1}{x}\right)}$

$\therefore y=u+v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad \ldots(1)$$

Then, $u=\left(x+\frac{1}{x}\right)^{x}$

$$\Rightarrow \log u=\log\left(x+\frac{1}{x}\right)^{x}$$

$$\Rightarrow \log u=x\log\left(x+\frac{1}{x}\right)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\cdot\frac{du}{dx}=\frac{d}{dx}(x)\times\log\left(x+\frac{1}{x}\right)+x\times\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx}=1\times\log\left(x+\frac{1}{x}\right)+x\times\frac{1}{\left(x+\frac{1}{x}\right)}\cdot\frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx}=u\left[\log\left(x+\frac{1}{x}\right)+\frac{x}{\left(x+\frac{1}{x}\right)}\times\left(1-\frac{1}{x^2}\right)\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\log\left(x+\frac{1}{x}\right)+\frac{\left(x-\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]$$

$$v = x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log v = \log\left[x^{\left(1+\frac{1}{x}\right)}\right]$$

$$\Rightarrow \log v = \left(1+\frac{1}{x}\right)\log x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx} = \left[\frac{d}{dx}\left(1+\frac{1}{x}\right)\right]\times\log x + \left(1+\frac{1}{x}\right)\cdot\frac{d}{dx}\log x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(-\frac{1}{x^2}\right)\log x + \left(1+\frac{1}{x}\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x + x + 1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right) \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right)$$

Differentiation Ex 11.5 Q18(vi)

Let $y = e^{\sin x} + (\tan x)^x$

$y = e^{\sin x} + e^{\log(\tan x)^x}$

$y = e^{\sin x} + e^{x\log(\tan x)}$ $\qquad \left[\text{Since, } \log a^b = b\log a, e^{\log a} = a\right]$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin x}\right) + \frac{d}{dx}\left(e^{x\log(\tan x)}\right)$$

$$= e^{\sin x}\frac{d}{dx}(\sin x) + e^{x\log(\tan x)}\times\frac{d}{dx}(x\log\tan x)$$

$$= e^{\sin x}(\cos x) + e^{\log(\tan x)^x}\left[x\frac{d}{dx}\log\tan x + \log\tan x\frac{d}{dx}(x)\right]$$

$$= e^{\sin x}(\cos x) + (\tan x)^x\left[\frac{x}{\tan x}\frac{d}{dx}(\tan x) + \log\tan x\,(1)\right]$$

$$\frac{dy}{dx} = \cos x\, e^{\sin x} + (\tan x)^x\left[\frac{x}{\tan x}\left(\sec^2 x\right) + \log\tan x\right]$$

Differentiation Ex 11.5 Q18(vii)

Let $y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$

$y = e^{\log(\cos x)^{x}} + e^{\log(\sin)^{\frac{1}{x}}}$ $\left[\text{Since, } \log a^{b} = b\log a, e^{\log a} = a\right]$

$y = e^{x\log(\cos x)} + e^{\frac{1}{x}\log\sin x}$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}e^{x\log\cos x} + \frac{d}{dx}e^{\frac{1}{x}\log\sin x}$$

$$= e^{x\log\cos x} \times \frac{d}{dx}(x\log x) + e^{\frac{1}{x}\log\sin x}\frac{d}{dx}\left(\frac{1}{x}\log\sin x\right)$$

$$= e^{\log(\cos x)} \times \left[x\frac{d}{dx}\log\cos x + \log\cos x \times \frac{d}{dx}(x)\right] + e^{\log(\sin x)^{\frac{1}{x}}} \times \left[\frac{1}{x}\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= (\cos x)^{x}\left[x \times \left(\frac{1}{\cos x}\right)\frac{d}{dx}\cos x + \log\cos x\,(1)\right] + (\sin)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x} \times \frac{d}{dx}(\sin x) + \log\sin x\left(-\frac{1}{x^{2}}\right)\right]$$

$$= (\cos x)^{x}\left[x\left(\frac{1}{\cos x}\right)(-\sin x) + \log\cos x\right] + (\sin x)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x}(\cos x) - \frac{1}{x^{2}}\log\sin x\right]$$

$$\frac{dy}{dx} = (\cos x)^{x}\left[\log\cos x - x\tan x\right] + (\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x} - \frac{1}{x^{2}}\log\sin x\right]$$

Differentiation Ex 11.5 Q18(viii)

Let $y = x^{x^2-3} + (x-3)^{x^2}$

Also, let $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$

$\therefore y = u + v$

Differentiating both sides with respect to x, we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = x^{x^2-3}$

$\therefore \log u = \log\left(x^{x^2-3}\right)$

$\log u = \left(x^2-3\right)\log x$

Differentiating with respect to x, we obtain

$$\frac{1}{u}\cdot\frac{du}{dx} = \log x\cdot\frac{d}{dx}\left(x^2-3\right) + \left(x^2-3\right)\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log x\cdot 2x + \left(x^2-3\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{x^2-3}\cdot\left[\frac{x^2-3}{x} + 2x\log x\right]$$

Also,

$v = (x-3)^{x^2}$

$\therefore \log v = \log(x-3)^{x^2}$

$\Rightarrow \log v = x^2\log(x-3)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx}=\log(x-3)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}[\log(x-3)]$$
$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\log(x-3)\cdot 2x+x^2\cdot\frac{1}{x-3}\cdot\frac{d}{dx}(x-3)$$
$$\Rightarrow\frac{dv}{dx}=v\left[2x\log(x-3)+\frac{x^2}{x-3}\cdot 1\right]$$
$$\Rightarrow\frac{dv}{dx}=(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Substituting the expressions of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx}=x^{x^2-3}\left[\frac{x^2-3}{x}+2x\log x\right]+(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Differentiation Ex 11.5 Q19

Here,

$$y=e^x+10^x+x^x$$
$$=e^x+10^x+e^{\log x^x}\qquad\left[\text{Since, } e^{\log_e a}=a, \log a^b=b\log a\right]$$
$$y=e^x+10^x+e^{x\log x}$$

Differentiating it with respect to x using product rule, chain rule,

$$\frac{dy}{dx}=\frac{d}{dx}(e^x)+\frac{d}{dx}(10^x)+\frac{d}{dx}(e^{x\log x})$$
$$=e^x+10^x\log 10+e^{x\log x}\frac{d}{dx}(x\log x)$$
$$=e^x+10^x\log 10+e^{x\log x}\left[x\times\frac{d}{dx}(\log x)+\log x\frac{d}{dx}(x)\right]$$
$$=e^x+10^x\log 10+e^{\log x^x}\left[x\left(\frac{1}{x}\right)+\log x(1)\right]$$
$$=e^x+10^x\log 10+x^x[1+\log x]$$
$$=e^x+10^x\log 10+x^x[\log e+\log x]\qquad[\text{Since, } \log_e e=1]$$

$$\frac{dy}{dx}=e^x+10^x\log 10+x^x(\log ex)\qquad[\text{Sicne } \log A+\log B=\log AB]$$

Differentiation Ex 11.5 Q20

Here,

$$y=x^n+n^x+x^x+n^n$$
$$y=x^n+n^x+e^{\log x^x}+n^n\qquad\left[\text{Since, } e^{\log_e a}=a \text{ and } \log a^b=b\log a\right]$$
$$y=x^n+n^x+e^{x\log x}+n^n$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}(x^n)+\frac{d}{dx}(n^x)+\frac{d}{dx}(e^{x\log x})+\frac{d}{dx}(n^n)$$
$$=nx^{n-1}+n^x\log n+e^{\log x^x}\left[a\frac{d}{dx}\log x+\log x\frac{d}{dx}(1)\right]$$
$$=nx^{n-1}+n^x\log n+x^x\left[x\left(\frac{1}{x}\right)+\log x\right]$$
$$=nx^{n-1}+n^x\log n+x^x[1+\log x]$$
$$=nx^{n-1}+n^x\log n+x^x[\log e+\log x]\qquad[\text{Since, } \log_e e=1 \text{ and } \log A+\log B=\log(AB)]$$

$$\frac{dy}{dx}=nx^{n-1}+n^x\log n+x^x\log(ex)$$

Differentiation Ex 11.5 Q21

Here,

$$y = \frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}} \quad \text{---(i)}$$

$$y = \frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log\left[\frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-1)^{\frac{1}{2}}}\right]$$

$$= \log\left(x^2-1\right)^3 + \log(2x-1) - \log(x-3)^{\frac{1}{2}} - \log(4x-1)^{\frac{1}{2}}$$

$$\left[\text{Since, } \log(AB) = \log A + \log B, \log\left(\frac{A}{B}\right) = \log A - \log B\right]$$

$$= 3\log\left(x^2-1\right) + \log(2x-1) - \frac{1}{2}\log(x-3) - \frac{1}{2}\log(4x-1)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = 3\frac{d}{dx}\log\left(x^2-1\right) + \frac{d}{dx}\log(2x-1) - \frac{1}{2}\frac{d}{dx}\log(x-3) - \frac{1}{2}\log(4x-1)$$

$$= 3\left(\frac{1}{x^2-1}\right)\frac{d}{dx}\left(x^2-1\right) + \frac{1}{(2x-1)}\frac{d}{dx}(2x-1) - \frac{1}{2}\left(\frac{1}{x-3}\right)\frac{d}{dx}(x-3) - \frac{1}{2}\frac{1}{(4x-1)}\frac{d}{dx}(4x-1)$$

$$= 3\left(\frac{1}{x^2-1}\right)(2x) + \frac{1}{(2x-1)}(2) - \frac{1}{2}\left(\frac{1}{x-3}\right)(1) - \frac{1}{2}\left(\frac{1}{4x-1}\right)(4)$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{6x}{x^2-1} + \frac{2}{2x-1} - \frac{1}{2(x-3)} - \frac{2}{4x-1}\right]$$

$$\frac{dy}{dx} = y\left[\frac{6x}{x^2-1} + \frac{2}{2x-1} - \frac{1}{2(x-3)} - \frac{2}{4x-1}\right]$$

$$\frac{dy}{dx} = \frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}}\left[\frac{6x}{x^2-1} + \frac{2}{2x-1} - \frac{1}{2(x-3)} - \frac{2}{4x-1}\right] \quad \text{[Using equaiton (i)]}$$

Differentiation Ex 11.5 Q22

Here,

$$y = \frac{e^{ax}\sec^x x\log x}{\sqrt{1-2x}} \quad \text{---(i)}$$

$$\Rightarrow \quad y = \frac{e^{ax} \times \sec^x x \log x}{(1-2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log e^{ax} + \log^{\sec x} + \log\log x - \frac{1}{2}\log(1-2x) \quad \left[\begin{array}{l}\text{Since, } \log\left(\frac{A}{B}\right) = \log A - \log B,\\ \log(AB) = \log A + \log B\end{array}\right]$$

$$\log y = ax + \log^{\sec x} + \log\log x - \frac{1}{2}\log(1-2x) \quad \left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(ax) + \frac{d}{dx}(\log\sec x) + \frac{d}{dx}(\log\log x) - \frac{1}{2}\log(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx} = a + \frac{1}{\sec x}\frac{d}{dx}(\sec x) + \frac{1}{\log x}\frac{d}{dx}(\log x) - \frac{1}{2}\left(\frac{1}{1-2x}\right)\frac{d}{dx}(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx} = a + \frac{\sec x\tan x}{\sec x} + \frac{1}{(\log x)}\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{1}{1-2x}\right)(-2)$$

$$\frac{dy}{dx} = y\left[a + \tan x + \frac{1}{x\log x} + \frac{1}{1-2x}\right]$$

$$\frac{dy}{dx} = \frac{e^{ax}\sec x\log x}{\sqrt{1-2x}}\left[a + \tan x + \frac{1}{x\log x} + \frac{1}{1-2x}\right] \quad \text{[Using equaton (i)]}$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log e^{3x} + \log \sin 4x + \log 2^x \qquad [\text{Since, } \log(AB) = \log A + \log B]$$
$$\log y = 3x \log e + \log \sin 4x + x \log 2 \qquad [\text{Since, } \log_e e = 1, \log a^b = b \log a]$$
$$\log y = 3x + \log \sin 4x + x \log 2$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(\log \sin 4x) + \frac{d}{dx}(x \log 2)$$
$$= 3 + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x) + \log 2 (1)$$
$$= 3 + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x) + \log 2$$
$$= 3 + \cot x (4) + \log 2$$
$$\frac{1}{y}\frac{dy}{dx} = 3 + 4\cot 4x + \log 2$$
$$\frac{dy}{dx} = y[3 + 4\cot 4x + \log 2]$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x [3 + 4\cot 4x + \log 2]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log(\sin x \sin 2x \sin 3x \sin 4x)$$
$$\log y = \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log \sin x + \frac{d}{dx}\log \sin 2x + \frac{d}{dx}\log \sin 3x + \frac{d}{dx}\log \sin 4x$$
$$= \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \frac{1}{\sin 3x}\frac{d}{dx}(\sin 3x) + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x)$$
$$= \frac{1}{\sin x}(\cos x) + \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) + \frac{1}{\sin 3x}(\cos 3x)\frac{d}{dx}(3x) + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x)$$
$$\frac{1}{y}\frac{dy}{dx} = [\cot x + \cot 2x (2) + \cot 3x (3) + \cot 4x (4)]$$
$$\frac{dy}{dx} = y[\cot x + 2\cot 2x + 3\cot x3x + 4\cot 4x]$$

$$\frac{dy}{d} = (\sin x \sin 2x \sin 3x \sin 4x)[\cot x + 2\cot 2x + 3\cot x3x + 4\cot 4x] \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q25

Let $y = x^{\sin x} + (\sin x)^x$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^x$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \ldots(1)$$

$u = x^{\sin x}$

$\Rightarrow \log u = \log\left(x^{\sin x}\right)$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\sin x) \cdot \log x + \sin x \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\cos x \log x + \sin x \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] \quad \ldots(2)$$

$v = (\sin x)^x$

$\Rightarrow \log v = \log(\sin x)^x$

$\Rightarrow \log v = x \log(\sin x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}(x) \times \log(\sin x) + x \times \frac{d}{dx}[\log(\sin x)]$$

$$\Rightarrow \frac{dv}{dx} = v\left[\log(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + \frac{x}{\sin x} \cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + x \cot x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + x \cot x\right] \quad \ldots(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + (\sin x)^x \left[\log \sin x + x \cot x\right]$$

Differentiation Ex 11.5 Q26

Here,

$$y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$$
$$y = e^{\log(\sin x)^{\cos x}} + e^{\log(\cos x)^{\sin x}}$$
$$y = e^{\cos x \log \sin x} + e^{\sin x \log \cos x} \qquad \left[\text{Since, } \log_e e = 1 \text{ and } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log \sin x}\right) + \frac{d}{dx}\left(e^{\sin x \log \cos x}\right)$$
$$= e^{\cos x \log \sin x}\frac{d}{dx}(\cos x \log \sin x) + e^{\sin x \log \cos x}\frac{d}{dx}(\sin x \log \cos x)$$
$$= e^{\log(\sin x)^{\cos x}}\left[\cos x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\cos x)\right] + e^{\log(\cos x)^{\sin x}}\left[\sin x \frac{d}{dx}\log \cos x + \log \cos x \frac{d}{dx}(\sin x)\right]$$
$$= (\sin)^{\cos x}\left[\cos x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \times (-\sin x)\right] + (\cos x)^{\sin x}\left[\sin x\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x\,(\cos x)\right]$$
$$= (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\tan x\,(-\sin x) + \cos x \log \cos x\right]$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right]$$

Differentiation Ex 11.5 Q27

Here,

$$y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$$
$$y = e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$
$$y = e^{\cot x \log \tan x} + e^{\tan x \log(\cot x)}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cot x \log \tan x}\right) + \frac{d}{dx}\left(e^{\tan x \log \cot x}\right)$$
$$= e^{\cot x \log \tan x}\frac{d}{dx}(\cot x \log \tan x) + e^{\tan x \log \cot x}\frac{d}{dx}(\tan x \log \cot x)$$
$$= e^{\log(\tan x)^{\cot x}}\left[\cot x \frac{d}{dx}\log \tan x + \log \tan x \frac{d}{dx}\cot x\right] + e^{\log(\cot x)^{\tan x}}\begin{bmatrix}\tan x \frac{d}{dx}\log \cot x + \\ \log \cot x \frac{d}{dx}(\tan x)\end{bmatrix}$$
$$= (\tan x)^{\cot x}\left[\cot x \times \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(-\operatorname{cosec}^2 x\right)\right] + (\cot x)^{\tan x}\begin{bmatrix}\tan x\left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x) \\ + \log \cot x\left(\sec^2 x\right)\end{bmatrix}$$
$$= \tan x^{\cot x}\left[(1)\left(\sec^2 x\right) - \operatorname{cosec}^2 x \log \tan x\right] + (\cot)^{\tan x}\left[(1)\left(-\operatorname{cosec}^2 x\right) + \sec^2 x \log \cot x\right]$$

$$\frac{dy}{dx} = (\tan)^{\cot x}\left[\sec^{2x} - \operatorname{cosec}^2 x \log \tan x\right] + (\cot x)^{\tan x}\left[\sec^2 x \log \cot x - \operatorname{cosec}^2 x\right]$$

Differentiation Ex 11.5 Q28

Here,

$$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$
$$= e^{\log(\sin x)^x} + \sin^{-1}\sqrt{x}$$
$$y = e^{x \text{ lgo } \sin x} + \sin^{-1}\sqrt{x} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \log \sin x}\right) + \frac{d}{dx}\sin^{-1}\left(\sqrt{x}\right)$$
$$= e^{x \log \sin x}\frac{d}{dx}(x \log \sin x) + \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$
$$= e^{\log(\sin x)^x}\left[x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(x) + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}\right]$$
$$= (\sin x)^x\left[x \times \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x\,(1)\right] + \frac{1}{2\sqrt{x - x^2}}$$
$$= (\sin x)^x\left[\frac{x}{\sin x}(\cos x) + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

$$\frac{dy}{dx} = (\sin x)^x\left[x \cot x + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

Differentiation Ex 11.5 Q29

Here,

$$y = x^{\cos x} + (\sin x)^{\tan x}$$
$$y = e^{\log x^{\cos x}} + e^{\log(\sin x)^{\tan x}} \qquad \left[\text{Since, } e^{\log_e a} = a \text{ and } \log a^b = b\log a\right]$$
$$y = e^{\cos x \log x} + e^{\tan x \log \sin x}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log x}\right) + \frac{d}{dx}\left(e^{\tan x \log \sin x}\right)$$
$$= e^{\cos x \log x}\frac{d}{dx}(\cos x \log x) + e^{\tan x \log \sin x} \times \frac{d}{dx}(\tan x \log \sin x)$$
$$= e^{\log x^{\cos x}}\left[\cos x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\cos x)\right] + e^{\log(\sin x)^{\tan x}}\left[\tan x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\tan x)\right]$$
$$= x^{\cos x}\left[\cos x\left(\frac{1}{x}\right) + \log x(-\sin x)\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\left(\sec^2 x\right)\right]$$
$$= x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)(\cos x) + \sec^2 x \log \sin x\right]$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[1 + \sec^2 x \log \sin x\right]$$

Here,

$$y = x^x + (\sin x)^x$$
$$= e^{\log x^x} + e^{\log(\sin x)^x}$$
$$y = e^{x\log x} + e^{x\log \sin x} \qquad \left[\text{Using } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log \sin x}\right)$$
$$= e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log \sin x}\frac{d}{dx}(x\log \sin x)$$
$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log(\sin x)^x}\left[x\frac{d}{dx}(\log \sin x) + \log \sin x\frac{d}{dx}(x)\right]$$
$$= x^x\left[x\left(\frac{1}{x}\right) + \log x(1)\right] + (\sin x)^x\left[x \times \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x(1)\right]$$
$$= x^x[1 + \log x] + (\sin x)^x\left[x\left(\frac{1}{\sin x}\right)(\cos x) + \log \sin x\right]$$

$$\frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x[x\cot x + \log \sin x]$$

Differentiation Ex 11.5 Q30

Here,

$$y = (\tan x)^{\log x} + \cos^2\left(\frac{\pi}{4}\right)$$
$$y = e^{\log(\tan x)^{\log x}} + \cos^2\left(\frac{\pi}{4}\right)$$
$$y = e^{\log x \log \tan x} + \cos x^2\left(\frac{\pi}{4}\right) \qquad \left[\text{Since, } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating it using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\log x \log \tan x}\right) + \frac{d}{dx}\cos^2\left(\frac{\pi}{4}\right)$$
$$= e^{\log x \log \tan x}\frac{d}{dx}(\log x \log \tan x) + 0$$
$$= e^{\log(\tan x)^{\log x}}\left[\log x\frac{d}{dx}(\log \tan x) + \log \tan x\frac{d}{dx}(\log x)\right]$$
$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(\frac{1}{x}\right)\right]$$
$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)\left(\sec^2 x\right) + \frac{\log \tan x}{x}\right]$$

$$\frac{dy}{dx} = (\tan x)^{\log x}\left[\log x\left(\frac{\sec^2 x}{\tan x}\right) + \frac{\log \tan x}{x}\right]$$

Differentiation Ex 11.5 Q31

Here,

$$y = x^x + x^{\frac{1}{x}}$$

$$= e^{\log x^x} + e^{\log x^{\frac{1}{x}}}$$

$$y = e^{x\log x} + e^{\left(\frac{1}{x}\log x\right)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{\frac{1}{x}\log x}\right)$$

$$= e^{x\log x} + \frac{d}{dx}(x\log x) + e^{\frac{1}{x}\log x}\frac{d}{dx}\left(\frac{1}{x}\log x\right)$$

$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log x^{\frac{1}{x}}}\left[\frac{1}{x}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + x^{\frac{1}{x}}\left[\left(\frac{1}{x}\right)\left(\frac{1}{x}\right) + \log x\left(-\frac{1}{x^2}\right)\right]$$

$$= x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{1}{x^2}\log x\right)$$

$$\frac{dy}{dx} = x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\frac{(1 - \log x)}{x^2}$$

Differentiation Ex 11.5 Q32

Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = (\log x)^x$

$$\Rightarrow \log u = \log\left[(\log x)^x\right]$$

$$\Rightarrow \log u = x\log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\times\log(\log x) + x\cdot\frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1\times\log(\log x) + x\cdot\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x}\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{\log(\log x)\cdot\log x + 1}{\log x}\right]$$

$$\frac{1}{v}\cdot\frac{dv}{dx}=\frac{d}{dx}\left[(\log x)^2\right]$$
$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx}=2(\log x)\cdot\frac{d}{dx}(\log x)$$
$$\Rightarrow \frac{dv}{dx}=2v(\log x)\cdot\frac{1}{x}$$
$$\Rightarrow \frac{dv}{dx}=2x^{\log x}\frac{\log x}{x}$$
$$\Rightarrow \frac{dv}{dx}=2x^{\log x-1}\cdot\log x \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(\log x)^{x-1}\left[1+\log x.\log(\log x)\right]+2x^{\log x-1}\cdot\log x$$

Differentiation Ex 11.5 Q33

Here,

$$x^{13}y^{7}=(x+y)^{20}$$

Taking log on both the sides,

$$\log\left(x^{13}y^{7}\right)=\log(x+y)^{20}$$
$$13\log x+7\log y=20\log(x+y) \qquad \left[\text{Since, } \log(AB)=\log A+\log B, \log a^b=b\log a\right]$$

Differentiating it with respect to x using chain rule,

$$13\frac{d}{dx}(\log x)+7\frac{d}{dx}(\log y)=20\frac{d}{dx}\log(x+y)$$
$$\frac{13}{x}+\frac{7}{y}\frac{dy}{dx}=\frac{20}{x+y}\frac{d}{dx}(x+y)$$
$$\frac{13}{x}+\frac{7}{y}\frac{dy}{dx}=\frac{20}{(x+y)}\left[1+\frac{dy}{dx}\right]$$
$$\frac{7}{y}\frac{dy}{dx}-\frac{20}{(x+y)}=\frac{20}{(x+y)}-\frac{13}{x}$$
$$\frac{dy}{dx}\left[\frac{7}{y}-\frac{20}{(x+y)}\right]=\frac{20}{(x+y)}-\frac{13}{x}$$
$$\frac{dy}{dx}\left[\frac{7(x+y)-20y}{y(x+y)}\right]=\left[\frac{20x-13(x+y)}{x(x+y)}\right]$$
$$\frac{dy}{dx}=\left[\frac{20x-13x-13y}{x(x+y)}\right]\left(\frac{y(x+y)}{7x+7y-20y}\right)$$
$$=\frac{y}{x}\left(\frac{7x-13y}{7x-13y}\right)$$

$$\frac{dy}{dx}=\frac{y}{x}$$

Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^{9} = \left(x^2 + y\right)^{17}$$

Taking log on both the sides,

$$\log\left(x^{16} \times y^{9}\right) = \log\left(x^2 + y\right)^{17} \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b\log a\right]$$

$$16\log x + 9\log y = 17\log\left(x^2 + y\right)$$

Differentiating it with respect to x using chain rule,

$$16\frac{d}{dx}(\log x) + 9\frac{d}{dx}(\log y) = 17\frac{d}{dx}\log\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = 17\frac{1}{\left(x^2 + y\right)}\frac{d}{dx}\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{17}{x^2 + y}\left[2x + \frac{dy}{dx}\right]$$

$$\frac{9}{y}\frac{dy}{dx} - \frac{17}{\left(x^2 + y\right)}\frac{dy}{dx} = \left(\frac{34x}{x^2 + y}\right) - \frac{16}{x}$$

$$\frac{dy}{dx}\left[\frac{9}{y} - \frac{17}{\left(x^2 + y\right)}\right] = \frac{34x^2 - 16x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx}\left[\frac{9x^2 + 9y - 17y}{y\left(x^2 + y\right)}\right] = \frac{18x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{2\left(9x^2 - 8y\right)}{9x^2 - 8y}\right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x\frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin\left(x^x\right) \quad \text{---(i)}$$

Let $u = x^x$ ---(ii)

Taking log on both the sides,

$$\log u = \log x^x$$
$$\log u = x \log x$$

Differentiating it with respect to x,

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\log x)$$
$$= x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{1}{u}\frac{du}{dx} = 1 + \log x$$
$$\frac{du}{dx} = u\left(1 + \log x\right)$$
$$\frac{du}{dx} = x^x\left(1 + \log x\right) \quad \text{---(iii) [Using equation (ii)]}$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\sin u)$$
$$= \cos u\frac{du}{dx}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos\left(x^x\right) \times x^x\left(1 + \log x\right)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^x = 1$$
$$e^{\log x^x} + e^{\log y^x} = 1$$
$$e^{x\log x} + e^{x\log y} = 1 \quad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$$
$$e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log y}\frac{d}{dx}(x\log y) = 0$$
$$e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$$
$$x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1)\right] = 0$$
$$x^x\left[1 + \log x\right] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$
$$y^x \times \frac{x}{y}\frac{dy}{dx} = -\left[x^x\left(1 + \log x\right) + y^x\log y\right]$$
$$\left(xy^{x-1}\right)\frac{dy}{dx} = -\left[x^x\left(1 + \log x\right) + y^x\log y\right]$$
$$\frac{dy}{dx} = -\left[\frac{x^x\left(1 + \log x\right) + y^x\log y}{xy^{x-1}}\right]$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log\left(x^y \times y^x\right) = \log(1)$$

$$y = \log x + x\log y = \log 1 \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(y\log x) + \frac{d}{dx}(x\log y) = \frac{d}{dx}(\log 1)$$

$$\left[y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}\right] + \left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$$

$$\left[y\left(\frac{1}{x}\right) + \log x\frac{dy}{dx}\right] + \left[x\left(\frac{1}{y}\frac{dy}{dx}\right) + \log y\,(1)\right] = 0$$

$$\frac{y}{x} + \log x\frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

$$\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = -\left[\log y + \frac{y}{x}\right]$$

$$\frac{dy}{dx}\left[\frac{y\log x + x}{y}\right] = -\left[\frac{x\log y + y}{x}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left[\frac{x\log y + y}{y\log x + x}\right]$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x+y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{(x+y)}}$$

$$e^{y\log x} + e^{x\log y} = e^{(x+y)\log(x+y)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^{y\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}e^{(x+y)\log(x+y)}$$

$$\Rightarrow \quad e^{y\log x}\left[y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}\right] + e^{x\log y}\left[x\frac{d}{dx}\log y + \log y\frac{d}{dx}(x)\right] = e^{(x+y)\log(x+y)}\frac{d}{dx}\left[(x+y)\log(x+y)\right]$$

$$\Rightarrow \quad e^{\log x^y}\left[y\left(\frac{1}{x}\right) + \log x\frac{dy}{dx}\right] + e^{\log x}\left[\frac{x}{y}\frac{dy}{dx} + \log y\,(1)\right] = e^{\log(x+y)^{(x+y)}}\left[(x+y)\frac{d}{dx}\log(x+y) + \log(x+y)\frac{d}{dx}(x+y)\right]$$

$$\Rightarrow \quad x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = (x+y)^{(x+y)}\left[(x+y)\frac{1}{(x+y)}\frac{d}{dx}(x+y) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^y \times \frac{y}{x} + x^y\log x\frac{dy}{dx} + y^x \times \frac{x}{y}\frac{dy}{dx} + y^x\log y = (x+y)^{(x+y)}\left[1 \times \left(1 + \frac{dy}{dx}\right) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^{y-1} \times y + x^y\log x\frac{dy}{dx} + y^{x-1} \times x\frac{dy}{dx} + y^x\log y = (x+y)^{(x+y)} + (x+y)^{(x+y)}\frac{dy}{dx} + (x+y)^{(x+y)}\log(x+y) + (x+y)^{(x+y)}\log(x+y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x^y\log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)\right] = (x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x\log y$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{(x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x\log y}{x^y\log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)}\right]$$

Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking log on both the side,

$$\log\left(x^m y^n\right) = \log(1)$$
$$m\log x + n\log y = \log(1)$$

Differentiating it with respect to x,

$$\frac{dy}{dx}(m\log x) + \frac{d}{dx}(n\log y) = \frac{d}{dx}(\log(1))$$
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{m}{x} \times \frac{y}{n}$$

$$\frac{dy}{dx} = -\frac{my}{nx}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking log on both the sides,

$$\log y^x = \log e^{(y-x)} \qquad \left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1\right]$$
$$x\log y = (y-x)\log e$$
$$x\log y = y - x \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y-x)$$
$$\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = \frac{dy}{dx} - 1$$
$$x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1) = \frac{dy}{dx} - 1$$
$$\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -1 - \log y$$
$$\frac{dy}{dx}\left(\frac{y}{(1+\log y)\,y}\right) = -(1+\log y) \qquad \left[\text{Since, from equation (i), } x = \frac{y}{(1+\log y)}\right]$$

$$\frac{dy}{dx}\left[\frac{1-1-\log y}{(1+\log y)}\right] = -(1+\log y)$$
$$\frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(\cos y)^x \qquad [\text{Using } \log a^b = b\log a]$$

$$y\log(\sin x) = x\log(\cos y)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}[y\log\sin x] = \frac{d}{dx}[x\log\cos y]$$

$$y\frac{d}{dx}(\log\sin x) + \log\sin x\frac{dy}{dx} = x\frac{dy}{dx}\log\cos y + \log\cos y\frac{d}{dx}(x)$$

$$y\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}\frac{d}{dx}(\cos y) + \log\cos y\,(1)$$

$$\frac{y}{\sin x}(\cos x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$

$$y\cot x + \log\sin x\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log\cos y$$

$$\frac{dy}{dx}(\log\sin x + x\tan y) = \log\cos y - y\cot x$$

$$\frac{dy}{dx} = \frac{\log\cos y - y\cot x}{\log\sin x + x\tan y}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\log(\cos x)^y = \log(\tan y)^x$$

$$y\log\cos x = x\log\tan y \qquad [\text{Since, } \log a^b = b\log a]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{d}{dx}(y\log\cos x) = \frac{d}{dx}(x\log\tan y)$$

$$\left(y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{d}{dx}\log\tan y + \log\tan y\frac{d}{dx}(x)\right)$$

$$\left(y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{1}{\tan y}\frac{d}{dx}(\tan y) + \log\tan y\,(1)\right)$$

$$\left(\frac{y}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx}\right) = \left(\frac{x}{\tan y}\left(\sec^2 y\right)\right)\frac{dy}{dx} + \log\tan y - y\tan x + \log\cos x\frac{dy}{dx}$$

$$= \left(\sec y\,\mathrm{cosec}\,y \times x\frac{dy}{dx} + \log\tan y\right)$$

$$\frac{dy}{dx}[\log\cos x - x\sec y\,\mathrm{cosec}\,y] = \log\tan y + y\tan x$$

$$\frac{dy}{dx} = \left[\frac{\log\tan y + y\tan x}{\log\cos x - x\sec y\,\mathrm{cosec}\,y}\right]$$

Differentiation Ex 11.5 Q43

Here,

$$e^x + e^y = e^{x+y} \quad \text{---(i)}$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^y\right) = \frac{d}{dx}\left(e^{x+y}\right)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y}\frac{d}{dx}(x+y)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1+\frac{dy}{dx}\right]$$

$$e^y \frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y}\right) \quad \text{[Using equation (i)]}$$

$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

Differentiation Ex 11.5 Q44

Here,

$$e^y = y^x$$

Taking log on both the sides,

$$\log e^y = \log y^x$$

$$y \log e = x \log y \quad \left[\text{Since, } \log a^b = b \log a, \log_e e = 1\right]$$

$$y = x \log y \quad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \log y)$$

$$= x\frac{dy}{dx}(\log y) + \log y \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y\,(1)$$

$$\frac{dy}{dx}\left(1 - \frac{x}{y}\right) = \log y$$

$$\frac{dy}{dx}\left(\frac{y-x}{y}\right) = \log y$$

$$\frac{dy}{dx} = \frac{y \log y}{y - x}$$

$$\frac{dy}{dx} = \frac{y \log y}{\left(y - \frac{y}{\log y}\right)} \quad \text{[Since, using equation (i)]}$$

$$= \frac{y \log y \times \log y}{y \log y - y}$$

$$= \frac{y\,(\log y)^2}{y\,(\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$

$$e^{x+y} = x \qquad ---(i)$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(x)$$

$$e^{x+y}\frac{d}{dx}(x+y) = 1$$

$$x\left[1+\frac{dy}{dx}\right] = 1 \qquad \text{[Using equatoin (i)]}$$

$$1+\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1-x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x\sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx}$$

$$\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$

$$(1-x\cos(a+y))\frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1-x\cos(a+y))}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1-\frac{y}{\sin(a+y)}\cos(a+y)\right)} \qquad \left[\text{Since } \frac{y}{\sin(a+y)} = x\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x\sin(a+y) + \sin a\cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}\left[x\sin(a+y) + \sin a\cos(a+y)\right] = 0$$

$$x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx} + \sin a\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{d}{dx}\sin a = 0$$

$$x\cos(a+y)\left(0+\frac{dy}{dx}\right) + \sin(a+y) + \sin a\left(-\sin(a+y)\frac{dy}{dx}\right) + 0 = 0$$

$$\left[x\cos(a+y) - \sin a\sin(a+y)\right]\frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x\cos(a+y) - \sin a\sin(a+y)}$$

$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\frac{\sin a\cos(a+y)}{\sin(a+y)}\right)\cos(a+y) - \sin a\sin(a+y)} \qquad \left[\text{Since } x = -\frac{\sin a\cos(a+y)}{\sin(a+y)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\cos^2(a+y) + \sin a\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\left[\cos^2(a+y) + \sin^2(a+y)\right]}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \qquad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1\right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x+y)$$

$$y\log(\sin x) = \log(x+y) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx}\left(y\log(\sin x)\right) = \frac{d}{dx}\log(x+y)$$

$$y\frac{d}{dx}\log\sin x + \log\sin x\frac{dy}{dx} = \frac{1}{x+y}\frac{d}{dx}(x+y)$$

$$\frac{y}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)}\left[1+\frac{dy}{dx}\right]$$

$$\frac{y(\cos x)}{(\sin x)} + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)} + \frac{1}{(x+y)}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\log\sin x - \frac{1}{x+y}\right) = \frac{1}{(x+y)} - y\cot x$$

$$\frac{dy}{dx}\left(\frac{(x+y)\log\sin x - 1}{(x+y)}\right) = \left(\frac{1-y(x+y)\cot x}{x+y}\right)$$

$$\frac{dy}{dx} = \left(\frac{1-y(x+y)\cot x}{(x+y)\log\sin x - 1}\right)$$

Differentiation Ex 11.5 Q49

Here,

$$xy\log(x+y) = 1 \qquad \text{---(i)}$$

Differentiating with respect to x using chain rule, product rule,

$$\frac{dy}{dx}\left(xy\log(x+y)\right) = \frac{d}{dx}(1)$$

$$xy\frac{d}{dx}\log(x+y) + x\log(x+y)\frac{dy}{dx} + y\log(x+y)\frac{d}{dx}(x) = 0$$

$$\frac{xy}{(x+y)}\left(1+\frac{dy}{dx}\right) + x\log(x+y)\frac{dy}{dx} + y\log(x+y)(1) = 0$$

$$\left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right) + x\log(x+y)\frac{dy}{dx} + y\log(x+y) = 0$$

$$\left(\frac{xy}{x+y}\right)\frac{dy}{dx} + \frac{xy}{x+y} + x\left(\frac{1}{xy}\right)\frac{dy}{dx} + y\left(\frac{1}{xy}\right) = 0 \qquad \left[\text{Using equation (i)}\right]$$

$$\frac{dy}{dx}\left[\frac{xy}{x+y} + \frac{1}{y}\right] = -\left[\frac{1}{x} + \frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2 + x + y}{(x+y)y}\right] = -\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

Differentiation Ex 11.5 Q50

Here,

$$y = x \sin y \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \sin y)$$
$$= x\frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}(x)$$
$$= x \cos y \frac{dy}{dx} + \sin y\,(1)$$
$$\frac{dy}{dx} - x \cos y \frac{dy}{dx} = \sin y$$
$$\frac{dy}{dx}(1 - x\cos y) = \sin y$$
$$\frac{dy}{dx} = \frac{\sin y}{(1 - x\cos y)}$$

Put the value of $\sin y = \frac{y}{x}$ form equation (i),

$$\frac{dy}{dx} = \frac{y}{x(1 - x\cos y)}$$

Differentiation Ex 11.5 Q51

Here,

$$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$$

Differentiating with respect to x using product rule and chain rule,

$$\Rightarrow \quad f'(x) = (1+x)(1+x^2)\frac{d}{dx}(1+x^8) + (1+x)(1+x^2)(1+x^8)\frac{d}{dx}(1+x^4) + (1+x)(1+x^4)(1+x^8)\frac{d}{dx}(1+x^2) + (1+x^2)(1+x^4)(1+x^8)\frac{d}{dx}(1+x)$$

$$\Rightarrow \quad f'(x) = (1+x)(1+x^2)(1+x^4)8x^7 + (1+x)(1+x^2)(1+x^8)(4x^3) + (1+x)(1+x^4)(1+x^8)(2x) + (1+x^2)(1+x^4)(1+x^8)(1)$$

$$f'(1) = (1+1)(1+1)(8) + (1+1)(1+1)(1+1)(4) + (1+1)(1+1)(1+1)(2) + (1+1)(1+1)(1+1)$$

$$f'(1) = (2)(2)(2)(8) + (2)(2)(2)(4) + (2)(2)(2)(2) + (2)(2)(2)$$
$$= 64 + 32 + 16 + 8$$
$$= 120$$

So,

$$f'(1) = 120$$

Differentiation Ex 11.5 Q52

Here,

$$y = \log\left(\frac{x^2+x+}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\left\{\frac{1}{1+\left(\frac{\sqrt{3}x}{1-x^2}\right)^2}\right\}\frac{d}{dx}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right) - \left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}\right) + \frac{2}{\sqrt{3}}\left\{\frac{(1-x)^2}{1+x^4-2x^2+3x^2}\right\}\left\{\frac{\left(1-x^2\right)^2\frac{d}{dx}\left(\sqrt{3}x\right) - \sqrt{3}x\frac{d}{dx}(1-x)^2}{\left(1-x^2\right)^2}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)(2x+1) - \left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)}\right) + \frac{2}{\sqrt{3}}\left(\frac{\left(1-x^2\right)^2}{1+x^2+x^4}\right)\left(\frac{\left(1-x^2\right)\left(\sqrt{3}\right) - \sqrt{3}x(-2x)}{\left(1-x^2\right)^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{x^4+2x^2+1-x^2}\right) + \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}-\sqrt{3}x^2+2\sqrt{3}x^2}{1+x^2+x^4}\right)$$

$$= \left(\frac{-2x^2+2}{x^4+x^2+1}\right) + \frac{2\sqrt{3}\left(x^2+1\right)}{\sqrt{3}\left(1+x^2+x^4\right)}$$

$$= \frac{2\left(1-x^2\right)}{\left(x^4+x^2+1\right)} + \frac{2\left(x^2+1\right)}{1+x^2+x^4}$$

$$= \frac{2\left(1-x^2+x^2+1\right)}{1+x^2+x^4}$$

$$\frac{dy}{dx} = \frac{4}{1+x^2+x^4}$$

Differentiation Ex 11.5 Q53

Here,

$$y = (\sin x - \cos x)^{(\sin x - \cos x)}$$

Takig log on both the sides,

$$\Rightarrow \quad \log y = \log(\sin x - \cos x)^{(\sin x - \cos x)}$$

$$\Rightarrow \quad \log y = (\sin x - \cos x)\log(\sin x - \cos x)$$

Differentiating it with respect to x using product rule, chain rule,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\frac{d}{dx}(\sin x - \cos x) + (\sin x - \cos x)\frac{d}{dx}\log(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\times(\cos x + \sin x) + \frac{(\sin x - \cos x)}{(\sin x - \cos x)}\frac{d}{dx}(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\log(\sin x - \cos x) + (\cos x + \sin x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[(\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)\right]$$

Using equation (i),

$$\frac{dy}{dx} = (\sin x - \cos x)^{(\sin x - \cos x)}\left[(\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)\right]$$

Differentiation Ex 11.5 Q54

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\log(xy) = \log\left(e^{x-y}\right)$$
$$\Rightarrow \log x + \log y = (x-y)\log e$$
$$\Rightarrow \log x + \log y = (x-y)\times 1$$
$$\Rightarrow \log x + \log y = x - y$$

Differentiating both sides with respect to x, we obtain

$$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) = \frac{d}{dx}(x) - \frac{dy}{dx}$$
$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\Rightarrow \left(1 + \frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

Differentiation Ex 11.5 Q55

Given that $y^x + x^y + x^x = a^b$.

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

Therefore $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ...(1)

Now, $u = y^x$. Taking logrithm on both sides, we have

$$\log u = x \log y$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)$$
$$= x\frac{1}{y}\cdot\frac{dy}{dx} + \log y.1$$

So $\frac{du}{dx} = u\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$ ...(2)

Also $v = x^y$

Taking logarithm on both sides, we have

$$\log v = y \log x$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\cdot\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}$$
$$= y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

So $\frac{dv}{dx} = v\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$

$$= x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$ ...(3)

Again $w = x^x$

Takinglogarithm on both sides, we have

$$\log w = x \log x.$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{w}\cdot\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$$
$$= x.\frac{1}{x} + \log x\ .\ 1$$

i.e. $\frac{dw}{dx} = w(1 + \log x)$

$$= x^x(1 + \log x)$$ ...(4)

From (1), (2), (3), (4), we have

$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$

or $\left(x.y^{x-1} + x^y.\log x\right)\frac{dy}{dx} = -x^x(1 + \log x) - y.x^{y-1} - y^x \log y$

Therefore $\frac{dy}{dx} = \frac{-\left[y^x \log y + y.x^{y-1} + x^x(1 + \log x)\right]}{x.y^{x-1} + x^y \log x}$

Differentiation Ex 11.5 Q56

Here $(\cos x)^y = (\cos y)^x$

Taking log on both sides,

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y \log \cos x = x \log \cos y$$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}(y \log \cos x) = \frac{d}{dx}(x \log \cos y)$$

$$y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx} = x\frac{d}{dx}\log\cos y + \log\cos y\frac{d}{dx}x$$

$$y\frac{1}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx} = x\frac{1}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$

$$\left(\log\cos x + \frac{x\sin y}{\cos y}\right)\frac{dy}{dx} = \log\cos y + y\frac{\sin y}{\cos y}$$

$$(\log\cos x + x\tan y)\frac{dy}{dx} = \log\cos y + y\tan y$$

$$\frac{dy}{dx} = \frac{\log\cos y + y\tan y}{(\log\cos x + x\tan y)}$$

Differentiation Ex 11.5 Q57

Consider the given function,

$\cos y = x\cos(a+y)$, where $\cos a \neq \pm 1$

Differentiating both sides w.r.t. 'x' we get

$$-\sin y\frac{dy}{dx} = x\left(-\sin(a+y)\frac{dy}{dx}\right) + \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx}[x\sin(a+y) - \sin y] = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

Multiplying the numerator and the denominator by $\cos(a+y)$ on the R.H.S., we have,

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{x\cos(a+y)\sin(a+y) - \cos(a+y)\sin y}$$

$$= \frac{\cos^2(a+y)}{\cos y\sin(a+y) - \cos(a+y)\sin y} \quad [\because \cos y = x\cos(a+y), \text{ given function}]$$

$$= \frac{\cos^2(a+y)}{\sin[(a+y)-y]} = \frac{\cos^2(a+y)}{\sin a}$$

Differentiation Ex 11.5 Q58

Consider the given function, $(x-y)e^{\frac{x}{x-y}} = a$.

We need to prove that $y\frac{dy}{dx} + x = 2y$.

Differentiating the given equation w.r.t. 'x' we get

$$(x-y)\left[e^{\frac{x}{x-y}}\left(\frac{(x-y) - x\left(1-\frac{dy}{dx}\right)}{(x-y)^2}\right)\right] + e^{\frac{x}{x-y}}\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{(x-y) - x\left(1-\frac{dy}{dx}\right)}{(x-y)} + \left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \left(1-\frac{dy}{dx}\right)\left(1-\frac{x}{x-y}\right) + 1 = 0$$

$$\Rightarrow \left(1-\frac{dy}{dx}\right)\left(\frac{-y}{x-y}\right) + 1 = 0$$

$$\Rightarrow -y + y\frac{dy}{dx} + x - y = 0$$

$$\Rightarrow y\frac{dy}{dx} + x = 2y$$

Differentiation Ex 11.5 Q59

$$x = e^{x/y}$$

$$\log x = \frac{x}{y} \ldots\ldots..(i)$$

$$y = \frac{x}{\log x}$$

$$\frac{dy}{dx} = \frac{\log x \frac{d}{dx}(x) - x\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - x \times \frac{1}{x}}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{x}{y} - 1}{(\log x)^2} \ldots\ldots[\text{from (i)}]$$

$$\frac{dy}{dx} = \frac{x - y}{y(\log x)^2}$$

$$\frac{dy}{dx} = \frac{x - y}{x(\log x)} \ldots\ldots[\text{from (i)}]$$

Differentiation Ex 11.5 Q60

$$y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$$

$$y = e^{\tan x \log x} + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}$$

$$\frac{dy}{dx} = e^{\tan x \log x}\frac{d}{dx}(\tan x \log x) + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}\frac{d}{dx}\left(\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{1}{2} \times \frac{2}{x^2+1} \times (x)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{x}{x^2+1}\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \frac{x}{\sqrt{2(x^2+1)}}$$

Differentiation Ex 11.5 Q61

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)\left(\frac{1}{x} - \gamma\right)}$$

Using the theorem,

If $y = 1 + \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)}$ then,

$$\frac{dy}{dx} = \frac{y}{x}\left\{\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right\}$$

Here we have $\frac{1}{x}$ instead of x.

So using above theorem we get,

$$\frac{dy}{dx} = \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta}{\left(\frac{1}{x} - \beta\right)} + \frac{\gamma}{\left(\frac{1}{x} - \gamma\right)}$$

Ex 11.6

Differentiation Ex 11.6 Q1

Here,

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \text{ to } \infty}}}$$
$$y = \sqrt{x + y}$$

Squaring both the sides,

$$y^2 = x + y$$

Differentiating it with respect to x,

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$
$$\frac{dy}{dx}(2y - 1) = 1$$
$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Differentiation Ex 11.6 Q2

Here,

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \text{to } \infty}}}$$
$$y = \sqrt{\cos x + y}$$

squaring both the sides,

$$y^2 = \cos x + y$$

Differntiating it with respect to x,

$$2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$
$$\frac{dy}{dx}(2y - 1) = -\sin x$$
$$\frac{dy}{dx} = \frac{-\sin x}{(2y - 1)}$$
$$\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

Differentiation Ex 11.6 Q3

Here,

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\log x + y}$$

Squaring both sides,

$$y^2 = \log x + y$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \frac{1}{x}$$

Differentiation Ex 11.6 Q4

Here,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\tan x + y}$$

Squaring both the sides,

$$y^2 = \tan x + y$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

Differentiation Ex 11.6 Q5

Here,

$$y = (\sin x)^{(\sin x)^{(\sin x)^{\dots\infty}}}$$

$$\Rightarrow \quad y = (\sin x)^y$$

Taking log on both the sides,

$$\log y = \log(\sin x)^y$$
$$\log y = y(\log \sin x)$$

Differentiating it with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}(\log \sin x) + \log \sin x\frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} = y\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\frac{1}{y} - \log \sin x\right) = \frac{y}{\sin x}(\cot x)$$
$$\frac{dy}{dx}\left(\frac{1 - y\log \sin x}{y}\right) = y\cot x$$
$$\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y\log \sin x)}$$

Differentiation Ex 11.6 Q6

Here,

$$y = (\tan x)^{(\tan x)^{(\tan x)^{\dots\infty}}}$$
$$y = (\tan x)^y$$

Taking log on both the sides,

$$\log y = \log(\tan x)^y$$
$$\log y = y\log \tan x$$

Differentiating with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log \tan x + \log \tan\frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{\tan x}\frac{d}{dx}(\tan x) + \log \tan x\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\frac{1}{y} - \log \tan x\right) = \frac{y}{\tan x}\sec^2 x$$
$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} * \frac{y}{1 - y\log \tan\left(\frac{\pi}{4}\right)}$$
$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{y^2\left(\sqrt{2}\right)^2}{1(1 - y\log \tan 1)}$$
$$= \frac{2(1)^2}{(1-0)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2 \qquad \left\{\begin{array}{l}\text{since,} \\ (y)_{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\dots\infty}}} \\ \Rightarrow y = (1)^{\infty} \\ \Rightarrow y = 1\end{array}\right\}$$

Differentiation Ex 11.6 Q7

Here,

$$y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^x}$$
$$y = u + v + w$$
$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \quad \text{---(i)}$$

Were $u = e^{x^{e^x}}, v = x^{e^{e^x}}, w = e^{x^x}$

Now, $u = e^{x^{e^x}}$ ---(ii)

Taking log on both the sides,

$$\log x = \log e^{x^{e^x}}$$
$$\log x = x^{e^x} \log e$$
$$\log x = x^{e^x} \quad \text{---(iii)} \quad \left\{\begin{matrix}\text{since } \log e - 1, \\ \log a^b = b \log a\end{matrix}\right\}$$

Taking log on both the sides,

$$\log\log x = \log x^{e^x}$$
$$\log\log x = e^x \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log x}\frac{d}{dx}(\log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$$
$$\frac{1}{\log x}\frac{1}{4}\frac{du}{dx} = \frac{e^x}{x} + e^x \log x$$
$$\frac{du}{dx} = 4\log x\left[\frac{e^x}{x} + e^x \log x\right]$$
$$\frac{du}{dx} = e^{x^{e^x}} * x^{e^x}\left[\frac{e^x}{x} + e^x \log x\right] \quad \text{---(A)}$$

Using equqtion (ii) and (iii)

Now

$$v = x^{e^{e^x}} \quad \text{---(iv)}$$

Taking log on both the sides,

$$\log v = \log x^{e^{e^x}}$$
$$\log v = e^{e^x} \log x$$

Differentiating it with respect to x,

$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^{e^x}\right)$$
$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x}\frac{d}{dx}\left(e^x\right)$$
$$\frac{dv}{dx} = v[e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x} e^x]$$
$$\frac{dv}{dx} = x^{e^{e^x}} * e^{e^x}[\frac{1}{x} + e^x \log x] \quad \text{---(B)}$$

$\{$sinx using equation$(4)\}$

Now, $w = e^{x^x}$ ---(v)

Taking log on both the sides,

$$\log w = \log e^{x^x}$$
$$\log w = x^{x^x} \log e$$
$$\log w = x^{x^x} \quad \text{---(vi)}$$

Taking log on both the sides,

$$\log\log w = \log x^{x^x}$$
$$\log\log w = x^x \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log w}\frac{d}{dx}(\log w) = x^{e}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{e}\right)$$

$$\frac{1}{\log w}\left(\frac{1}{w}\right)\frac{dw}{dx} = x^{e}\left(\frac{1}{x}\right) + \log e x^{e-1}$$

$$\frac{dw}{dx} = w\log w[x^{e-1} + e\log x x^{e-1}]$$

$$\frac{dw}{dx} = e^{x^{e}} x^{x^{e}} x^{e-1}(1 + e\log x) \quad \text{---(C)} \quad \{\text{Using equation (v), (vi)}\}$$

Using equation (A), (B) and (C) in equation (i),

$$\frac{dy}{dx} = e^{x^{e^{x}}} x^{e^{x}}[\frac{e^{x}}{x} + e^{x}\log x] + x^{e^{e^{x}}} e^{e^{x}}[\frac{1}{x} + e^{x}\log x]$$

$$+e^{x^{e}} x^{x^{e}} x^{e-1}(1 + e\log x)$$

Differentiation Ex 11.6 Q8

Here,

$$y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots\infty}}}$$

$$y = (\cos x)^{y}$$

Taking log on both the sides,

$$\log y = \log(\cos x)^{y}$$

$$\log y = y\log(\cos x), \{\text{since } \log a^{b} = b\log a\}$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log(\cos x) + \log\cos x\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log\cos x\right) = \frac{y}{\cos x}(-\sin x)$$

$$\frac{dy}{dx}\left(\frac{1 - y\log\cos x}{y}\right) = -y\tan x$$

$$\frac{dy}{dx} = -\frac{y^{2}\tan x}{(1 - y\log\cos x)}$$

Ex 11.7

Differentiation Ex 11.7 Q1

Given that $x = at^2$, $y = 2at$

So, $\frac{dx}{dt} = \frac{d}{dt}(at^2) = 2at$

$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

Differentiation Ex 11.7 Q2

Here,

$x = a(\theta + \sin\theta)$

Differentiating it with respect to θ,

$\frac{dx}{d\theta} = a(1 + \cos\theta)$ ---(i)

And,

$y = a(1 - \cos\theta)$

Differentiating it with respect to θ,

$\frac{dy}{d\theta} = a(\theta + \sin\theta)$

$\frac{dy}{d\theta} = a\sin\theta$ ---(ii)

Using equation (i) and (ii),

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{a\sin\theta}{a(1 - \cos\theta)}$

$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$, $\left\{\text{Since, } 1 - \cos\theta = \frac{2\sin^{2\theta}}{2}, \ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta\right\}$

$= \frac{dy}{dx} = \frac{\tan\theta}{2}$

Differentiation Ex 11.7 Q3

Here $x = a\cos\theta$ and $y = b\sin\theta$

Then,

$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = -a\sin\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\sin\theta) = b\cos\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$

Differentiation Ex 11.7 Q4

Here,

$$x = ae^{\theta}(\sin\theta - \cos\theta)$$

Differentiating it with respcet to θ,

$$\frac{dx}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)\frac{d}{d\theta}(e^{\theta})\right]$$
$$= a\left[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}\right]$$
$$\frac{dx}{d\theta} = a\left[2e^{\theta}\sin\theta\right] \qquad ---(i)$$

And, $y = ae^{\theta}(\sin\theta + \cos\theta)$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta + \cos\theta) + (\sin\theta + \cos\theta)\frac{d}{d\theta}(e^{\theta})\right]$$
$$= a\left[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}\right]$$
$$\frac{dy}{d\theta} = a\left[2e^{\theta}\cos\theta\right] \qquad ---(ii)$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^{\theta}\cos\theta\right)}{a\left(2e^{\theta}\sin\theta\right)}$$
$$\frac{dy}{dx} = \cot\theta$$

Differentiation Ex 11.7 Q5

Here $x = b\sin^2\theta$ and $y = a\cos^2\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(b\sin^2\theta\right) = 2b\sin\theta\cos\theta$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\cos^2\theta\right) = -2a\cos\theta\sin\theta$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

Differentiation Ex 11.7 Q6

Here $x = a(1-\cos\theta)$ and $y = a(\theta + \sin\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(1-\cos\theta)\right] = a(\sin\theta)$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(\theta+\sin\theta)\right] = a(1+\cos\theta)$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{a(1+\cos\theta)}{a(\sin\theta)}\right|_{\theta=\frac{\pi}{2}} = \frac{a(1+0)}{a} = 1$$

Differentiation Ex 11.7 Q7

Here,

$$x = \frac{e^t + e^{-t}}{2}$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) + \frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left[e^t + e^{-t}\frac{d}{dt}(-t)\right]$$

$$\frac{dx}{dt} = \frac{1}{2}\left(e^t - e^{-t}\right) = y \qquad ---(i)$$

And, $y = \frac{e^t - e^{-t}}{2}$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) - \frac{d}{dt}e^{-t}\right]$$

$$= \frac{1}{2}\left[e^t - e^{-t}\frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left(e^t - e^{-t}(-1)\right)$$

$$\frac{dy}{dt} = \frac{1}{2}\left(e^t + e^{-t}\right) = x \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$$

$$\frac{dy}{dt} = \frac{x}{y}$$

Differentiation Ex 11.7 Q8

Here,

$$x = \frac{3at}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(3at) - 3at\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(3a) - 3at(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a+3at^2-6at^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a-3at^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{3a\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{3at^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at^2\right) - 3at^2\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(6at) - \left(3at^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{6at+6at^3-6at^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{6at}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{3a\left(1-t^2\right)}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

Differentiation Ex 11.7 Q9

The given equations are $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

Then, $\dfrac{dx}{d\theta} = a\left[\dfrac{d}{d\theta}\cos\theta + \dfrac{d}{d\theta}(\theta\sin\theta)\right] = a\left[-\sin\theta + \theta\dfrac{d}{d\theta}(\sin\theta) + \sin\theta\dfrac{d}{d\theta}(\theta)\right]$

$$= a[-\sin\theta + \theta\cos\theta + \sin\theta] = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\theta\cos\theta)\right] = a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}(\cos\theta) + \cos\theta\cdot\frac{d}{d\theta}(\theta)\right\}\right]$$

$$= a[\cos\theta + \theta\sin\theta - \cos\theta]$$

$$= a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

Differentiation Ex 11.7 Q10

Here,

$$x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$$

Differentiating it with respect to θ using product rule,

$$\frac{dx}{d\theta} = e^{\theta}\frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{\theta}\right)$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\frac{\theta^2 + 1}{\theta}\right)e^{\theta}$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$= e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dx}{d\theta} = \frac{e^{\theta}\left(\theta^3 + \theta^2 + \theta - 1\right)}{\theta^2} \qquad \text{---(i)}$$

And, $y = e^{\theta}\left(\theta - \frac{1}{\theta}\right)$

Differentiating it with respect to θ using product rule and chain rule,

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}\frac{d}{d\theta}(-\theta)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right]$$

$$= e^{-\theta}\left[\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right]$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2}\right] \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dx} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - 2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(2) - 2t(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2-2t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{1-t^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{-4t}{\left(1+t^2\right)^2}\right] \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)}$$

$$= \frac{-2t}{1-t^2}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \left[\text{Sicne, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2}\right]$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{1+t^2}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{-1}{\sqrt{1-\frac{1}{\left(1+t^2\right)}}}\left\{\frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right\}\frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}(2t)$$

$$= \frac{-t}{\sqrt{t^2} \times \left(1+t^2\right)}$$

$$\frac{dx}{dt} = \frac{-1}{1+t^2} \qquad \text{---(i)}$$

Now, $y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-\frac{1}{\left(\sqrt{1+t^2}\right)^2}}} \times \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \left(\frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right)\frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{-1}{2\sqrt{t^2}\left(1+t^2\right)} \times (2t)$$

$$\frac{dy}{dt} = \frac{-1}{\left(1+t^2\right)} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{\left(1+t^2\right)} \times \frac{\left(1+t^2\right)}{-1}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q13

Here,

$$x = \frac{1-t^2}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \left(\frac{-4t}{\left(1+t^2\right)^2}\right) \qquad \text{---(i)}$$

And, $y = \dfrac{2t}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(2) - (2t)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q14

Here, x= 2cosθ – cos2θ

Differentiating it with respect to θ using chain rule,

$$\frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$
$$= -2\sin\theta + 2\sin 2\theta$$
$$\frac{dx}{d\theta} = 2(\sin 2\theta - \sin\theta) \qquad \text{---(i)}$$

And, $y = 2\sin\theta - \sin 2\theta$

Differentiating it with respect to θ using chain rule,

$$\frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta\frac{d}{d\theta}(2\theta)$$
$$= 2\cos\theta - \cos 2\theta(2)$$
$$= 2\cos\theta - 2\cos 2\theta$$
$$\frac{dy}{d\theta} = 2(\cos\theta - \cos 2\theta) \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$
$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$
$$\frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{\theta - 2\theta}{2}\right)}{2\cos\left(\frac{2\theta + \theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)}$$

$$\left[\begin{array}{l}\text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(\frac{-\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)}$$

$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(e^{\cos 2t}\right)$$
$$= e^{\cos 2t}\frac{d}{dt}(\cos 2t)$$
$$= e^{\cos 2t}(-\sin 2t)\frac{d}{dt}(2t)$$
$$= -\sin 2te^{\cos 2t}(2)$$
$$\frac{dx}{dt} = -2\sin 2te^{\cos 2t} \qquad \text{---(i)}$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(e^{\sin 2t}\right)$$
$$= e^{\sin 2t}\frac{d}{dt}(\sin 2t)$$
$$= e^{\sin 2t}(\cos 2t)\frac{d}{dt}(2t)$$
$$= e^{\sin 2t}(\cos 2t)(2)$$
$$\frac{dy}{dt} = 2\cos 2te^{\sin 2t} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2te^{\sin 2t}}{-2\sin 2te^{\cos 2t}}$$

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y} \qquad \left[\begin{array}{l}\text{Since, } x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ \qquad y = e^{\sin 2t} \Rightarrow \log y = \sin 2t\end{array}\right]$$

Differentiation Ex 11.7 Q16

Here,

$x = \cos t$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \qquad ---(i)$$

and, $y = \sin t$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\left(\frac{dy}{dx}\right) = -\cot\left(\frac{2\pi}{3}\right)$$

$$= -\cot\left(\pi - \frac{\pi}{3}\right)$$

$$= -\left[-\cot\left(\frac{\pi}{3}\right)\right]$$

$$= \cot\left(\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q17

Here,

$$x = a\left(t + \frac{1}{t}\right)$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = a\frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$= a\left(1 - \frac{1}{t^2}\right)$$

$$\frac{dx}{dt} = a\left(\frac{t^2 - 1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a\left(t - \frac{1}{t}\right)$

Differentiating it with respect to t,

$$\frac{dy}{dt} = a\frac{d}{dt}\left(t - \frac{1}{t}\right)$$

$$= a\left(1 + \frac{1}{t^2}\right)$$

$$\frac{dy}{dt} = a\left(\frac{t^2 + 1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = a\frac{\left(t^2 + 1\right)}{t^2} \times \frac{t^2}{a\left(t^2 - 1\right)}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad \left[\text{Since, } \frac{x}{y} = \frac{a\left(t^2 + 1\right)}{t} \times \frac{t}{a\left(t^2 - 1\right)} = \left(\frac{t^2 + 1}{t^2 - 1}\right)\right]$$

Differentiation Ex 11.7 Q18

Here,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

Put $\quad t = \tan\theta$

$$x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{\{1\}}(\sin 2\theta)$$

$$= 2\theta \qquad \left[\text{Since, } \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$

$$x = 2\left(\tan^{-1} t\right) \qquad [\text{Since, } t = \sin\theta]$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{2}{1+t^2} \qquad \text{---(i)}$$

Now,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

Put $\quad t = \tan\theta$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta) \qquad \left[\text{Sicne, } \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1} t \qquad [\text{Since, } t = \tan\theta]$$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{2}{1+t^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q19

The given equations are $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}\left[\frac{\sin^3 t}{\sqrt{\cos 2t}}\right]$$

$$= \frac{\sqrt{\cos 2t}\cdot\frac{d}{dt}(\sin^3 t) - \sin^3 t\cdot\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cdot\frac{d}{dt}(\sin t) - \sin^3 t\times\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}\cdot\sin^2 t\cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}}.(-2\sin 2t)}{\cos 2t}$$

$$= \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\frac{dy}{dt}=\frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$$

$$=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t\cdot\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\cdot\frac{d}{dt}\left(\cos t\right)-\cos^3 t.\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$=\frac{3\sqrt{\cos 2t}.\cos^2 t\left(-\sin t\right)-\cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}.\left(-2\sin 2t\right)}{\cos 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{\cos 2t\cdot\sqrt{\cos 2t}}$$

$$\therefore\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{3\cos 2t\sin^2 t\cos t+\sin^3 t\sin 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\left(2\sin t\cos t\right)}{3\cos 2t\sin^2 t\cos t+\sin^3 t\left(2\sin t\cos t\right)}$$

$$=\frac{\sin t\cos t\left[-3\cos 2t.\cos t+2\cos^3 t\right]}{\sin t\cos t\left[3\cos 2t\sin t+2\sin^3 t\right]}$$

$$=\frac{\left[-3\left(2\cos^2 t-1\right)\cos t+2\cos^3 t\right]}{\left[3\left(1-2\sin^2 t\right)\sin t+2\sin^3 t\right]}\qquad\left[\begin{array}{l}\cos 2t=\left(2\cos^2 t-1\right),\\ \cos 2t=\left(1-2\sin^2 t\right)\end{array}\right]$$

$$=\frac{-4\cos^3 t+3\cos t}{3\sin t-4\sin^3 t}$$

$$=\frac{-\cos 3t}{\sin 3t}\qquad\left[\begin{array}{l}\cos 3t=4\cos^3 t-3\cos t,\\ \sin 3t=3\sin t-4\sin^3 t\end{array}\right]$$

$$=-\cot 3t$$

Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^a$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\left(t + \frac{1}{t}\right)^a\right)$$

$$= a\left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{1-1}\left(1 - \frac{1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a^{\left(t+\frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(a^{\left(t+\frac{1}{t}\right)}\right)$$

$$= a^{\left(t+\frac{1}{t}\right)} \times \log a \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a\left(\frac{1+t^2}{1-t^2}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = a\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1+t^2\right) - \left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{\left(1-t^2\right)(2t) - \left(1+t^2\right)(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{2t - 2t^2 + 2t + 2t^3}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{4at}{\left(1-t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = 2\left[\frac{\left(1-t^2\right)\frac{d}{dt}(t) - t\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{\left(1-t^2\right)(1) - t(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{1-t^2+2t^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q22

It is given that, $y = 12(1-\cos t), x = 10(t-\sin t)$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left[10(t-\sin t)\right] = 10\cdot\frac{d}{dt}(t-\sin t) = 10(1-\cos t)$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[12(1-\cos t)\right] = 12\cdot\frac{d}{dt}(1-\cos t) = 12\cdot\left[0-(-\sin t)\right] = 12\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{12\sin t}{10(1-\cos t)} = \frac{12\cdot 2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}{10\cdot 2\sin^2\frac{t}{2}} = \frac{6}{5}\cot\frac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(\theta-\sin\theta)\right] = a(1-\cos\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(1+\cos\theta)\right] = a(-\sin\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{-a\sin\theta}{a(1-\cos\theta)}\right|_{\theta=\frac{\pi}{3}} = -\frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = -\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$x = a \sin 2t \ (1 + \cos 2t)$ and $y = b \cos 2t \ (1 - \cos 2t)$

Rewriting the above function, we have,

$x = a \sin 2t + \frac{a}{2} \sin 4t$

Differentiating the above function w.r.t. 't', we have,

$\frac{dx}{dt} = 2a \cos 2t + 2a \cos 4t \ldots(1)$

$y = b \cos 2t \ (1 - \cos 2t)$

$y = b \cos 2t - b \cos^2 2t$

$\frac{dy}{dt} = -2b \sin 2t + 2b \cos 2t \sin 2t = -2b \sin 2t + b \sin 4t \ldots(2)$

From (1) and (2),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2b \sin 2t + b \sin 4t}{2a \cos 2t + 2a \cos 4t}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\pi/4} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\pi/4} = \frac{-2b}{-2a} = \frac{b}{a}$$

Differentiation Ex 11.7 Q25

Consider the given functions,

$x = \cos t \left(3 - 2\cos^2 t\right)$

$x = 3\cos t - 2\cos^3 t$

$\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t \ldots(1)$

$y = \sin t \ \left(3 - 2\sin^2 t\right)$

$y = 3\sin t - 2\sin^3 t$

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t \cos t \ldots(2)$

$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right)$ ....[From equations (1) and (2)]

$$= \frac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t}$$

$$= \frac{3\cos t \left(1 - 2\sin^2 t\right)}{3\sin t \left(2\cos^2 t - 1\right)}$$

$$= \cot t \frac{\left(1 - 2\left(1 - \cos^2 t\right)\right)}{\left(2\cos^2 t - 1\right)}$$

$= \cot t$

$$\left.\frac{dy}{dx}\right|_{\pi/4} = \cot\frac{\pi}{4} = 1$$

Differentiation Ex 11.7 Q26

$$x = \frac{1 + \log t}{t^2}, \ y = \frac{3 + 2\log t}{t}$$

$$\frac{dx}{dt} = \frac{t^2\left(\frac{1}{t}\right) - (1 + \log t)(2t)}{t^4} = \frac{t - 2t - 2t\log t}{t^4} = \frac{-2\log t - 1}{t^3}$$

$$\frac{dy}{dt} = \frac{t\left(\frac{2}{t}\right) - (3 + 2\log t)(1)}{t^2} = \frac{2 - 3 - 2\log t}{t^2} = \frac{-2\log t - 1}{t^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2\log t - 1}{t^2}}{\frac{-2\log t - 1}{t^3}} = t$$

Differentiation Ex 11.7 Q27

$$x = 3\sin t - \sin 3t,\ y = 3\cos t - \cos 3t$$

$$\frac{dx}{dt} = 3\cos t - 3\cos 3t$$

$$\frac{dy}{dt} = -3\sin t + 3\sin 3t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t}$$

When $t = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{-3\sin\left(\frac{\pi}{3}\right) + 3\sin(\pi)}{3\cos\left(\frac{\pi}{3}\right) - 3\cos(\pi)} = \frac{-3 \times \frac{\sqrt{3}}{2} + 0}{3 \times \frac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q28

$$\sin x = \frac{2t}{1+t^2},\ \tan y = \frac{2t}{1-t^2}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1+t^2}\right) \text{ and } y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1 - \left(\frac{2t}{1+t^2}\right)^2}} \times \frac{2\left(1+t^2\right) - (2t)(2t)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt} = \frac{2}{\left(1+t^2\right)}$$

$$\frac{dy}{dt} = \frac{1}{\left(\frac{2t}{1-t^2}\right)^2 + 1} \times \frac{2\left(1-t^2\right) - (2t)(-2t)}{\left(1-t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{2}{\left(1+t^2\right)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{\left(1+t^2\right)}}{\frac{2}{\left(1+t^2\right)}} = 1$$

Ex 11.8

Differentiation Ex 11.8 Q1

Let $u = x^2,\ v = x^3$

Differentiating u with respect to x,

$$\frac{du}{dx} = 2x \qquad \text{---(i)}$$

Differentiating v with respect to x,

$$\frac{dv}{dx} = 3x^2 \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $u = \log\left(1+x^2\right)$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = \frac{1}{\left(1+x^2\right)}\frac{d}{dx}\left(1+x^2\right)$$

$$= \frac{1}{\left(1+x^2\right)}(2x)$$

$$\frac{du}{dx} = \frac{2x}{\left(1+x^2\right)} \qquad \text{---(i)}$$

Let $v = \tan^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{1+x^2} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{\left(1+x^2\right)} \times \frac{\left(1+x^2\right)}{1}$$

$$\frac{du}{dv} = 2x$$

Differentiation Ex 11.8 Q3

Let $u = (\log x)^x$

Taking log on both the sides,

$$\log u = \log(\log x)^x$$
$$\log u = x\log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x)$$
$$\frac{1}{u}\frac{du}{dx} = x\left(\frac{1}{\log x}\right)\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$\frac{du}{dx} = u\left[\frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x\right]$$
$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \qquad ---(i)$$

Again, let $v = \log x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x} \qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{(\log x)^x\left[\frac{1}{\log x} + \log\log x\right]}{\frac{1}{x}}$$
$$\frac{du}{dv} = \frac{(\log x)^x\left[\frac{1 + \log x \times \log\log x}{\log x}\right]}{\frac{1}{x}}$$

$$\frac{du}{dv} = (\log x)^{x-1}\;(1 + \log x \times \log\log x) \times x$$

Differentiation Ex 11.8 Q4(i)

Let $u = \sin^{-1}\sqrt{1-x^2}$

Put $x = \cos\theta$, so,

$u = \sin^{-1}\sqrt{1-\cos^2\theta}$

$u = \sin^{-1}(\sin\theta)$ ---(i)

And, $v = \cos^{-1}x$ ---(ii)

Now, $x \in (0,1)$

$\Rightarrow \cos\theta \in (0,1)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

So, from equation (i),

$u = \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$u = \cos^{-1}x$ $[\text{Since, } \cos\theta = x]$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$$ ---(ii)

From equation (ii),

$v = \cos^{-1}x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$$ ---(iv)

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q4(ii)

Let $u = \sin^{-1}\sqrt{1-x^2}$

Put $x = \cos\theta$, so,

$u = \sin^{-1}\sqrt{1-\cos^2\theta}$

$u = \sin^{-1}(\sin\theta)$ ---(i)

And, $v = \cos^{-1}x$ ---(ii)

Here,

$x \in (-1, 0)$

$\Rightarrow \cos\theta \in (-,10)$

$\Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

So, from equation (i),

$u = \pi - \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta, \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$

$u = \pi - \cos^{-1}x$ $[\text{Since, } x = \cos\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(v)}$$

And, from equation (ii),

$v = \cos^{-1}x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(vi)}$$

Dividing equation (v) by (vi)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = -1$$

Differentiation Ex 11.8 Q5(i)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$$u = \sin^{-1}\left(2\times\cos\theta\sqrt{1-\cos^2\theta}\right)$$
$$= \sin^{-1}(2\cos\theta\sin\theta)$$
$$u = \sin^{-1}(\sin 2\theta) \quad ---(i)$$

Let $v = \sqrt{1-4x^2}$ ---(ii)

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$
$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$
$$u = \pi - 2\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$
$$= \frac{2}{\sqrt{1-4x^2}}(2)$$
$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad ---(vi)$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$
$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad ---(vii)$$

Differentiating equation (ii) with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}\frac{d}{dx}\left(1-4x^2\right)$$
$$= \frac{1}{2\sqrt{1-4x^2}}(-8x)$$
$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \quad ---(iv)$$

Divide equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = -\frac{1}{x}$$

Differentiation Ex 11.8 Q5(ii)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$$u = \sin^{-1}\left(2\times\cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}(2\cos\theta\sin\theta)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

Let $v = \sqrt{1-4x^2}$ ---(ii)

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$$

$$\Rightarrow \quad 2x \in \left(\frac{1}{2\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \cos\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i)

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\cos^{-1}(2x) \quad [\text{Since, } 2x = \cos\theta]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \left(\frac{-2}{\sqrt{1-4x^2}}(2)\right)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \quad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q5(iii)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$u = \sin^{-1}\left(2\times\cos\theta\sqrt{1-\cos^2\theta}\right)$

$= \sin^{-1}(2\cos\theta\sin\theta)$

$u = \sin^{-1}(\sin 2\theta)$ ---(i)

Let $v = \sqrt{1-4x^2}$ ---(ii)

Here,

$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$\Rightarrow$ $2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow$ $\theta \in \left(\frac{3\pi}{4}, \pi\right)$

So, from equation (i),

$u = \pi - 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$

$u = \pi - 2\cos^{-1}(2x)$ [Since, $2x = \cos\theta$]

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \text{---(vii)}$$

Dividing equation (vi) by (vii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q6

Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\sin\theta}{2}}{\frac{\cos\theta}{2}}\right)$$

$$u = \tan^{-1}\left(\frac{\tan\theta}{2}\right) \quad \text{---(i)}$$

And,

Let $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$v = \sin^{-1}(\sin 2\theta) \quad \text{---(ii)}$$

Here,

$-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4} \quad$ ---(A)

So, from equation (i),

$u = \frac{\theta}{2}$ $\quad$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = \frac{1}{2}\tan^{-1}x$ $\quad$ [Since, $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = \frac{1}{2\left(1+x^2\right)} \quad \text{---(iii)}$$

Now, from equation (ii) and (A)

$v = 2\theta$ $\quad$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$v = 2\tan^{-1}x$ $\quad$ [Since, $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{dv}{dx} = 2\left(\frac{1}{1+x^2}\right) \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2\left(1+x^2\right)} \times \frac{(1+x^2)}{2}$$

$$\frac{du}{dv} = \frac{1}{4}$$

Differentiation Ex 11.8 Q7(i)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$
$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

And,

Let $v = \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$
$$= \sec^{-1}\left(\sec\theta\right)$$
$$= \cos^{-1}\left(\frac{1}{\frac{1}{\cos\theta}}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$v = \cos^{-1}\left(\cos\theta\right) \qquad \text{---(ii)}$$

Here,

$$x \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \sin\theta \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

Let $u = 2\sin^{-1}x$ $\quad [\text{Since}, x = \sin\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

And, from equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$v = \sin^{-1}x \qquad [\text{Since}, x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),3

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q7(ii)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \quad \text{---(i)}$$

And,

Let $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$= \sec^{-1}\left(\sec\theta\right)$$

$$= \cos^{-1}\left(\frac{1}{\sec\theta}\right) \quad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$v = \cos^{-1}\left(\cos\theta\right) \quad \text{---(ii)}$$

Here,

$$x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \sin\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\sin^{-1}x \quad \left[\text{Since, } x = \sin\theta\right]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right) \quad \text{---(iv)}$$

From equation (ii)

$$v = \theta \quad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$v = \sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(v)}$$

Dividing equation (iv) by (v),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q8

Let $u = (\cos x)^{\sin x}$

Taking on both the sides,

$$\log u = \log(\cos x)^{\sin x}$$
$$\log u = \sin x \log(\cos x)$$

Differentiating it with respect to x using product and chain rule,

$$\frac{1}{u}\frac{du}{dx} = \sin x \frac{d}{dx}(\log \cos x) + \log \cos x \frac{d}{dx}(\sin x)$$
$$\frac{1}{u}\frac{du}{dx} = \sin x\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x\,(\cos x)$$
$$\frac{du}{dx} = 4\left[(\tan x)\times(-\sin x) + \log\log x \times (\cos x)\right]$$
$$\frac{du}{dx} = (\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right] \qquad \text{---(i)}$$

Let $v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\log v = \log(\sin x)^{\cos x}$$
$$\log v = \cos x \log(\sin x)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\cos x)$$
$$\frac{1}{v}\frac{dv}{dx} = \cos x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\,(-\sin x)$$
$$\frac{dv}{dx} = v\left[\cot x\,(\cos x) - \sin x \log \sin x\right]$$
$$\frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x\,(\cos x) - \sin x \log \sin x\right] \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right]}{(\sin x)^{\cos x}\left[\cot x\,(\cos x) - \sin x \log \sin x\right]}$$

Differentiation Ex 11.8 Q9

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta$,

$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$u = \sin^{-1}(\sin 2\theta)$ ---(i)

Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$v = \cos^{-1}(\cos 2\theta)$ ---(ii)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \tan\theta < 1$

$\Rightarrow$ $0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = 2\tan^{-1}x$ $\left[\text{Since, } x = \tan\frac{\pi}{2}\right]$

Differentiating it with respect to x,

$\frac{du}{dx} = \frac{2}{1+x^2}$ ---(iii)

From equation (ii),

$v = 2\theta$ $\left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$v = 2\tan^{-1}x$ $[\text{Since, } x = \tan\theta]$

Differentiating it with respect to x,

$\frac{dv}{dx} = \frac{2}{1+x^2}$ ---(iv)

Differentiation Ex 11.8 Q10

Let $u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$

Put $ax = \tan\theta$

$$u = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta$$

$$u = \frac{\pi}{4}+\tan^{-1}(ax) \qquad [\text{Since, } \tan\theta = ax]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 0 + \frac{1}{1+(ax)^2}\frac{d}{dx}(ax)$$

$$\frac{du}{dx} = \frac{a}{1+a^2x^2} \qquad \text{---(i)}$$

Now,

Let $v = \sqrt{1+a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}}\frac{d}{dx}\left(1+a^2x^2\right)$$

$$= \frac{1}{2\sqrt{1+a^2x^2}}\left(2a^2x\right)$$

$$\frac{dv}{dx} = \frac{a^2x}{\sqrt{1+a^2x^2}} \qquad \text{---(ii)}$$

Differentiation Ex 11.8 Q11

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$,

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

Let $v = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$v = \tan^{-1}(\tan\theta) \quad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow$ $-\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow$ $\left(-\frac{\pi}{4}\right) < \theta < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \text{---(iii)}$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = \sin^{-1}x \quad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q12

Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$u = \tan^{-1}(\tan 2\theta) \quad ---(i)$$

Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \quad ---(ii)$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta$ $\quad \left[\text{Since, } tab^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = 2\tan^{-1}x$ $\quad [\text{Since, } x = \tan\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad ---(iii)$$

From equation (ii),

$v = \theta$ $\quad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$v = 2\tan^{-1}x$ $\quad [\text{Since, } x = \tan\theta]$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad ---(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

Let $u = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\tan\theta - 1}{\tan\theta + 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta\tan\frac{\pi}{4}}\right)$$

$$u = \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \qquad \text{---(i)}$$

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \tan\theta < \frac{1}{2}$

$\Rightarrow \quad -\tan^{-1}\left(\frac{1}{2}\right) < \theta < \tan^{-1}\left(\frac{1}{2}\right)$

So,

$u = \theta - \frac{\pi}{4}$ $\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = \tan^{-1}x - \frac{\pi}{4}$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \qquad \text{---(ii)}$$

And,

Let $v = \sin^{-1}\left(3x - 4x^3\right)$

Put $x = \sin\theta$, so

$$v = \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)$$

$$v = \sin^{-1}(\sin 3\theta) \qquad \text{---(iii)}$$

Now, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \sin\theta < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{6} < \theta < \frac{\pi}{6}$

So, from equation (iii),

$v = 3\theta$ $\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$v = 3\sin^{-1}x$ $\qquad [\text{Since, } x = \sin\theta]$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{3\left(1+x^2\right)}$$

Differentiation Ex 11.8 Q14

Let $u = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left(\frac{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}}{\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2} + \frac{2\sin x}{2}\frac{\cos x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\left(\frac{\cos x}{2}+\frac{\sin x}{2}\right)\left(\frac{\cos x}{2}-\frac{\sin x}{2}\right)}{\left(\frac{\cos x}{2}+\frac{\sin x}{2}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\cos x}{2}-\frac{\sin x}{2}}{\frac{\cos x}{2}+\frac{\sin x}{2}}\right)$$

$$= \tan^{-1}\left[\frac{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} - \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} + \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}\right]$$

$$= \tan^{-1}\left[\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\times\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{2}\right)$$

$$\frac{du}{dx} = -\frac{1}{2} \qquad \text{---(i)}$$

Let $v = \sec^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2}\times\frac{x\sqrt{x^2-1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$$

Differentiation Ex 11.8 Q15

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$u = \sin^{-1}(\sin 2\theta)$ ---(i)

Let $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$v = \tan^{-1}(\tan 2\theta)$ ---(ii)

Here, $-1 < x < 1$

$\Rightarrow -1 < \tan\theta < 1$

$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = 2\tan^{-1}x$

Differentiating it with respect to x,

$\frac{du}{dx} = \frac{2}{(1+x^2)}$ ---(iii)

From equation (ii),

$v = 2\theta$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$v = 2\tan^{-1}x$

Differentiating it with respect to x,

$\frac{dv}{dx} = \frac{2}{1+x^2}$ ---(iv)

Dividing equation (iii) by (iv),

$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$

$\frac{du}{dv} = 1$

Differentiation Ex 11.8 Q16

Let $u = \cos^{-1}\left(4x^3 - 3x\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$, so

$$u = \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$$

$$u = \cos^{-1}\left(\cos 3\theta\right) \qquad \text{---(i)}$$

Let $v = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cot\theta}\right)$$

$$v = \tan^{-1}\left(\tan\theta\right) \qquad \text{---(ii)}$$

Here,

$$\frac{1}{2} < x < 1$$

$$\Rightarrow \quad \frac{1}{2} < \cos\theta < 1$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{3}$$

So, from equation (i),

$$u = 3\theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$u = 3\cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$v = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-3}{\sqrt{1-x^2}}\right)\left(-\frac{\sqrt{1-x^2}}{1}\right)$$

$$\frac{du}{dv} = 3$$

Differentiation Ex 11.8 Q17

Let $u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Put $x = \cos\theta \Rightarrow \theta = \sin^{-1}x$, so

$$u = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$u = \tan^{-1}(\tan\theta) \qquad \text{---(i)}$$

And,

Let $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$v = \sin^{-1}(\sin 2\theta) \qquad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \sin^{-1}x$$

Differentiation Ex 11.8 Q18

Let $u = \sin^{-1}\left(\sqrt{1-x^2}\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And,

Let $v = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \cot^{-1}\left(\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right)$$

$$= \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$v = \cot^{-1}(\cot\theta) \qquad \text{---(ii)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \cos^{-1}x$$

Differentiation Ex 11.8 Q19

Let $u = \sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

Put $ax = \sin\theta \Rightarrow \theta = \sin^{-1}(ax)$

$$u = \sin^{-1}\left\{2\sin\theta\sqrt{1-\sin^2\theta}\right\}$$
$$= \sin^{-1}\{2\sin\theta\cos\theta\}$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

And,

Let $v = \sqrt{1-a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx}\left(1-a^2x^2\right)$$
$$= \left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}}\right)$$
$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \qquad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\sin^{-1} ax$$

Differentiation Ex 11.8 Q20

Let $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$$u = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4} - \tan\theta}{1 + \frac{\tan\pi}{4}\tan\theta}\right)$$

$$u = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) \qquad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \left(\frac{\pi}{4} - \theta\right) \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \tan^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = -\frac{1}{1+x^2} \qquad \text{---(ii)}$$

And,

Let $v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{\left(1+x^2\right)} \times \frac{\sqrt{1-x^2}}{-x}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}$$