Ex 11.1

Q1(i)

We have,

$$\sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \sin\frac{\pi}{6} \qquad \left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$$

$\Rightarrow$ the general solution is

$$\theta = n\pi + (-1)^n \frac{\pi}{6};\ n \in z \qquad \left[\because \text{if } \sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha\right]$$

Q1(ii)

We have,

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\left(\pi + \frac{\pi}{6}\right)$$

$$\Rightarrow \quad \cos\theta = \cos\frac{7\pi}{6} \qquad \left[\because \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}\right]$$

$\therefore$ the general solution is

$$\theta = 2n\pi \pm \frac{7\pi}{6},\ n \in z$$

Q1(iii)

$$\operatorname{cosec}\theta = -\sqrt{2}$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = -\sqrt{2}$$

$$\Rightarrow \quad \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\theta = \sin\left(\pi + \frac{\pi}{4}\right)$$

$$\Rightarrow \quad \sin\theta = \sin\frac{5\pi}{4} \text{ or } \sin\theta = \sin\left(-\frac{\pi}{4}\right)$$

$$\because \sin(-\theta) = -\sin\theta.$$

$$\therefore \theta = n\pi + (-1)^{n+1}\frac{\pi}{4}.\ n \in z$$

Q1(iv)

We have,

$$\sec\theta = \sqrt{2}$$

$$\Rightarrow \quad \frac{1}{\cos\theta} = \sqrt{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}} \qquad \Rightarrow \cos\theta = \cos\frac{\pi}{4}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}, n \in z$$

Q1(v)

We have,

$$\tan\theta = \frac{-1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \tan\theta = \tan\left(-\frac{\pi}{6}\right) \qquad \left[\because \tan(-\theta) = -\tan\theta\right]$$

$$\Rightarrow \quad \theta = n\pi + \left(-\frac{\pi}{6}\right), n \in z$$

$$\text{or } \theta = n\pi - \frac{\pi}{6}, n \in z$$

Q1(vi)

We have,

$$\sqrt{3}\sec\theta = 2$$

$$\Rightarrow \quad \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \theta = 2n\pi \pm \frac{\pi}{6}, n \in z$$

Q2(i)

We have,

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin 2\theta = \sin\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad 2\theta = n\pi + (-1)^n \frac{\pi}{3}, n \in z$$

$$\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}, n \in z$$

Q2(ii)

We have,

$$\cos 3\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos 3\theta = \cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad 3\theta = 2n\pi \pm \frac{\pi}{3}, n \in z$$

$$\Rightarrow \quad \theta = 2n\frac{\pi}{3} \pm \frac{\pi}{9}, n \in z$$

Q2(iii)

$$\sin 9\theta = \sin \theta$$

$$\sin 9\theta - \sin \theta = 0$$

Apply sin A - sin B formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin 9\theta - \sin \theta = 2\cos 5\theta \sin 4\theta = 0$$

$$\cos 5\theta \sin 4\theta = 0$$

$$\Rightarrow \cos 5\theta = 0\ (or)\ \sin 4\theta = 0$$

$$5\theta = \frac{(2n+1)\pi}{2} (or) 4\theta = n\pi$$

$$\theta = \left\{\frac{(2n+1)\pi}{10}\right\} (or) \theta = \left\{\frac{n\pi}{4}\right\} where\ n \in Z$$

Q2(iv)

We have,

$\sin 2\theta = \cos 3\theta$

$\Rightarrow \quad \cos 3\theta = \sin 2\theta$

$\Rightarrow \quad \cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right) \qquad \left[\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right]$

$\Rightarrow \quad 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right), n \in z$

$\Rightarrow$ either

$5\theta = 2n\pi + \frac{\pi}{2}, n \in z$ or $\theta = 2n\pi - \frac{\pi}{2}, n \in z$

$\Rightarrow \quad 5\theta = (4n+1)\frac{\pi}{2}, n \in z$ or $\theta = (4n-1)\frac{\pi}{2}$

$\Rightarrow \quad \theta = (4n+1)\frac{\pi}{10}, n \in z$ or $\theta(4n-1)\frac{\pi}{2}, n \in z$

Q2(v)

We have,

$\tan\theta + \cot 2\theta = 0$

$\tan\theta = -\cot 2\theta$

$\Rightarrow \quad \cot 2\theta = -\tan\theta$

$\Rightarrow \quad \tan 2\theta = -\cot\theta$

$\Rightarrow \quad \tan 2\theta = -\tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow \quad \tan 2\theta = \tan\left(\theta - \frac{\pi}{2}\right)$

$\Rightarrow \quad 2\theta = n\pi + \left(\theta - \frac{\pi}{2}\right), n \in z$

$\Rightarrow \quad \theta = n\pi - \frac{\pi}{2}, n \in z$

Q2(vi)

We have,

$$\tan 3\theta = \cot \theta$$

$$\Rightarrow \quad \tan 3\theta = \tan\left(\frac{\pi}{2} - \theta\right) \quad \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta\right]$$

$$\Rightarrow \quad 3\theta = n\pi + \frac{\pi}{2} - \theta, n \in z$$

$$\Rightarrow \quad 4\theta = n\pi + \frac{\pi}{2}, n \in z$$

$$\Rightarrow \quad \theta = \frac{n\pi}{4} + \frac{\pi}{8}, n \in z$$

Q2(vii)

We have,

$$\tan 2\theta . \tan \theta = 1$$

$$\Rightarrow \quad \tan 2\theta = \frac{1}{\tan \theta}$$

$$\Rightarrow \quad \tan 2\theta = \cot \theta$$

$$\Rightarrow \quad \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\Rightarrow \quad 2\theta = n\pi + \frac{\pi}{2} - \theta, n \in z$$

$$\Rightarrow \quad 3\theta = n\pi + \frac{\pi}{2}, n \in z$$

$$\Rightarrow \quad \theta = \frac{n\pi}{3} + \frac{\pi}{6}, n \in z$$

Q2(viii)

$$\tan m\theta + \cot n\theta = 0$$

$$\sin m\theta \sin n\theta + \cos m\theta \cos n\theta = 0$$

$$\cos(m-n)\theta = 0$$

$$(m-n)\theta = \left(\frac{2k+1}{2}\right)\pi$$

$$\theta = \left(\frac{2k+1}{2(m-n)}\right)\pi, \ k \in Z$$

Q2(ix)

We have,

$$\tan p\theta = \cot q\theta$$

$$\Rightarrow \quad \tan p\theta = \tan\left(\frac{\pi}{2} - q\theta\right)$$

$$\Rightarrow \quad p\theta = n\pi \pm \left(\frac{\pi}{2} - q\theta\right), n \in z$$

$$\Rightarrow \quad (p+q)\theta = n\pi + \frac{\pi}{2}, n \in z$$

$$\Rightarrow \quad (p+q)\theta = (2n+1)\frac{\pi}{2}, n \in z$$

$$\Rightarrow \quad \theta = \frac{(2n+1)}{(p+q)}\frac{\pi}{2}, n \in z$$

Q2(x)

$$\sin 2x + \cos x = 0$$

$$2\sin x \cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

$$\cos x = 0 \ or \ 2\sin x + 1 = 0$$

$$x = (4m-1)\frac{\pi}{2} or \sin x = \frac{-1}{2}$$

$$x = (4m-1)\frac{\pi}{2} or \ x = (4n-1)\frac{\pi}{6}, \ m, n \in Z$$

Q2(xi)

We have,

$$\sin\theta = \tan\theta$$

$$\Rightarrow \quad \sin\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \quad \sin\theta = \frac{\sin\theta}{\cos\theta} = 0$$

$$\Rightarrow \quad \sin\theta(\cos\theta - 1) = 0$$

$$\Rightarrow \quad \text{either } \sin\theta = 0 \quad \text{or } \cos\theta - 1 = 0$$

$$\Rightarrow \quad \theta = n\pi, n \in z \quad \text{or } \cos\theta = 1$$

$$\Rightarrow \cos\theta = \cos 0^\circ$$

$$\theta = 2m\pi, \ m \in z$$

Thus,

$$\theta = n\pi n \in z \quad \text{or } \theta = 2m\pi, m \in z$$

Q2(xii)

$\cos(2x) = -\sin(3x)$

$= -\cos(\frac{\pi}{2} - 3x)$

$= \cos(\frac{\pi}{2} + 3x)$

$\Rightarrow 2n\pi + 2x = \frac{\pi}{2} + 3x$

$x = (4m-1)\frac{\pi}{2}, m \in Z$

or

$\Rightarrow 2n\pi - 2x = \frac{\pi}{2} + 3x$

$x = (4n-1)\frac{\pi}{10}, n \in Z$

Q3(i)

We have,

$\sin^2\theta - \cos\theta = \frac{1}{4}$

$\Rightarrow \quad 1 - \cos^2\theta - \cos\theta = \frac{1}{4} \qquad [\because \sin^2\theta = 1 - \cos^2\theta]$

$\Rightarrow \quad \cos^2\theta + \cos\theta - \frac{3}{4} = 0$

$\Rightarrow \quad 4\cos^2\theta + 4\cos\theta - 3 = 0$

$\Rightarrow \quad 4\cos^2\theta + 6\cos\theta - 2\cos\theta - 3 = 0 \qquad$ [factorize it]

$\Rightarrow \quad 2\cos\theta(2\cos\theta + 3) - 1(\cos\theta + 3) = 0$

$\Rightarrow \quad (2\cos\theta - 1)(2\cos\theta + 3) = 0$

$\Rightarrow$ either

$2\cos\theta - 1 = 0 \qquad$ or $\qquad 2\cos\theta + 3 = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \qquad$ or $\qquad \cos\theta = -\frac{3}{2} \qquad$ [This is not possible as $-1 < \cos\theta < 1$]

$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \quad \theta = 2n\pi \pm \frac{\pi}{3}, n \in z$

Q3(ii)

We have,

$$2\cos^2\theta - 5\cos\theta + 2 = 0$$

$\Rightarrow \quad 2\cos^2\theta - 4\cos\theta - \cos\theta + 2 = 0 \qquad$ [use factorization]

$\Rightarrow \quad 2\cos\theta(\cos\theta - 2) - 1(\cos\theta - 2) = 0$

$\Rightarrow \quad (2\cos\theta - 1)(\cos\theta - 2) = 0$

$\Rightarrow$ either

$2\cos\theta - 1 = 0 \qquad$ or $\quad \cos\theta - 2 = 0$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2} \qquad$ or $\quad \cos\theta = 2$

$\Rightarrow \quad \cos\theta = \cos\dfrac{\pi}{3} \qquad$ [This is not possible as $-1 < \cos\theta < 1$]

$\Rightarrow \quad \theta = 2n\pi \pm \dfrac{\pi}{3}, n \in z$

Thus,

$$\theta = 2n\pi \pm \frac{\pi}{3}, n \in z$$

Q3(iii)

We have,

$$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$$

$\Rightarrow \quad 2(1 - \cos^2 x) + \sqrt{3}\cos x + 1 = 0$

$\Rightarrow \quad 2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

factorise it, we get,

$\Rightarrow \quad 2\cos^2 x - 2\sqrt{3}\cos x + \sqrt{3}\cos x - 3 = 0$

$\Rightarrow \quad 2\cos x(\cos x - \sqrt{3}) + \sqrt{3}(\cos x - \sqrt{3}) = 0$

$\Rightarrow \quad (2\cos x + \sqrt{3})(\cos x - \sqrt{3}) = 0$

$\Rightarrow$ either

$\cos x = -\dfrac{\sqrt{3}}{2} \qquad$ or $\quad \cos x = \sqrt{3} \qquad$ [This is not possible as $-1 < \cos x < 1$]

$\Rightarrow \quad \cos x = \cos\left(\pi - \dfrac{\pi}{6}\right)$

$\Rightarrow \quad \cos x = \cos\dfrac{5\pi}{6}$

$\Rightarrow \quad x = 2n\pi \pm \dfrac{5\pi}{6}, n \in z$

Q3(iv)

We have,

$$4\sin^2\theta - 8\cos\theta + 1 = 0$$

$\Rightarrow$ $4\left(1-\cos^2\theta\right) - 8\cos\theta + 1 = 0$

$\Rightarrow$ $4\cos^2\theta + 8\cos\theta - 5 = 0$

factorise it, we get,

$\Rightarrow$ $4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

$\Rightarrow$ $2\cos\theta\left(2\cos\theta + 5\right) - 1\left(2\cos\theta + 5\right) = 0$

$\Rightarrow$ $\left(2\cos\theta - 1\right)\left(2\cos\theta + 5\right) = 0$

either $2\cos\theta - 1 = 0$ or $2\cos\theta + 5 = 0$

$\Rightarrow$ $\cos\theta = \frac{1}{2}$ or $\cos\theta = -\frac{5}{2}$ [This is not possible as $-1 < \cos\theta < 1$]

$\Rightarrow$ $\cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow$ $\theta = 2n\pi \pm \frac{\pi}{3}, n \in z$

Q3(v)

We have,

$$\tan^2 x + \left(1-\sqrt{3}\right)\tan x - \sqrt{3} = 0$$

$\Rightarrow$ $\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$

$\Rightarrow$ $\tan x\left(\tan x + 1\right) - \sqrt{3}\left(\tan x + 1\right) = 0$

$\Rightarrow$ $\left(\tan x - \sqrt{3}\right)\left(\tan x + 1\right) = 0$

$\Rightarrow$ either

$\tan x = \sqrt{3}$ or $\tan x = -1$

$\Rightarrow$ $\tan x = \tan\frac{\pi}{3}$ or $\tan x = -\tan\frac{\pi}{4}$

$\Rightarrow$ $x = n\pi + \frac{\pi}{3}, n \in z$ or $x = m\pi - \frac{\pi}{4}, m \in z$

$\therefore$ $x = n\pi + \frac{\pi}{3}$ or $m\pi - \frac{\pi}{4}, n, m \in z$

Q3(vi)

$$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$$

$$\sqrt{3}\cos^2\theta - 2\sin\theta\cos\theta - \sqrt{3}\sin^2\theta = 0 \quad (\text{Dividing by } \sqrt{3})$$

$$\sqrt{3}\cos^2\theta + \sin\theta\cos\theta - 3\sin\theta\cos\theta - \sqrt{3}\sin^2\theta = 0$$

$$\cos\theta(\sqrt{3}\cos\theta + \sin\theta) - \sqrt{3}\sin\theta(\sqrt{3}\cos\theta + \sin\theta) = 0$$

$$(\sqrt{3}\cos\theta + \sin\theta)(\cos\theta - \sqrt{3}\sin\theta) = 0$$

$$\sqrt{3}\cos\theta + \sin\theta = 0 \quad \text{or} \quad \cos\theta - \sqrt{3}\sin\theta = 0$$

$$\tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3} \quad \text{or} \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$$

$$\theta = n\pi - \frac{\pi}{3} \quad \text{or} \quad \theta = m\pi + \frac{\pi}{6}$$

$n, m \in Z$

Q3(vii)

We have,

$$\cos 4\theta = \cos 2\theta$$

$\Rightarrow \quad \cos 4\theta - \cos 2\theta = 0$

$\Rightarrow \quad 2\sin\theta . \sin 3\theta = 0$

$\Rightarrow \quad$ either

$\sin\theta = 0 \quad$ or $\quad \sin 3\theta = 0$

$\Rightarrow \quad \theta = n\pi, n \in z \quad$ or $\quad 3\theta = m\pi, m \in$

Thus,

$$\theta = n\pi \text{ or } m\frac{\pi}{3}, n, m \in z$$

Q4(i)

$$\cos\theta + \cos 2\theta + \cos 3\theta = 0$$

$\Rightarrow \quad \cos 2\theta + 2\cos 2\theta.\cos\theta = 0 \qquad [\because \cos\theta + \cos 3\theta = 2\cos 2\theta.\cos\theta]$

$\Rightarrow \quad \cos 2\theta(1 + 2\cos\theta) = 0$

either

$\cos 2\theta = 0$ or $1 + 2\cos\theta = 0$

$\Rightarrow \quad 2\theta = (2n+1)\frac{\pi}{4}, n \in z$ or $\cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{4}, n \in z$ or $\cos\theta = +\cos\left(\pi - \frac{\pi}{3}\right)$

or $\cos\theta = \cos 2\frac{\pi}{3}$

or $\theta = 2n\pi \pm \frac{2\pi}{3}, n \in z$

Thus,

$\theta = (2n+1)\frac{\pi}{4},$ or $\left(2n\pi \pm \frac{2\pi}{3}\right), n \in z$

Q4(ii)

$\cos\theta + \cos 3\theta - \cos 2\theta = 0$

$\Rightarrow \quad 2\cos 2\theta.\cos\theta - \cos 2\theta = 0$

$\Rightarrow \quad \cos 2\theta(2\cos\theta - 1) = 0$

either

$\cos 2\theta = 0$ or $2\cos\theta = 1$

$\Rightarrow \quad 2\theta = (2n+1)\frac{\pi}{2}, n \in z$ or $\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{4}, n \in z$ or $\theta = 2m\pi \pm \frac{\pi}{3}, m \in z$

Q4(iii)

$\sin\theta + \sin 5\theta = \sin 3\theta$

$\Rightarrow \quad 2\sin 3\theta.\cos 2\theta - \sin 3\theta = 0 \quad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2}\right]$

$\Rightarrow \quad \sin 3\theta\left[2\cos 2\theta - 1\right] = 0$

$\Rightarrow$ either

$\sin 3\theta = 0 \quad$ or $\quad 2\cos 2\theta - 1 = 0$

$\Rightarrow \quad 3\theta = n\pi, n \in z \quad$ or $\quad \cos 2\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{n\pi}{3}, n \in z \quad$ or $\quad 2\theta = 2m\pi \pm \frac{\pi}{3}, m \in z$

or $\quad \theta = m\pi \pm \frac{\pi}{6}$

Thus,

$\theta = \frac{n\pi}{3} \quad$ or $m\pi \pm \frac{\pi}{6}, n, m \in z$

Q4(iv)

We have,

$\cos\theta.\cos 2\theta.\cos 3\theta = \frac{1}{4}$

$\Rightarrow \quad 2\cos\theta.\cos 3\theta.\cos 2\theta = \frac{1}{2}$

$\Rightarrow \quad (\cos 4\theta + \cos 2\theta)\cos 2\theta = \frac{1}{2}$

$\Rightarrow \quad (2\cos^2 2\theta - 1 + \cos 2\theta)\cos 2\theta = \frac{1}{2}$

$\Rightarrow \quad 2\cos^3 2\theta + \cos^2 2\theta - \cos 2\theta = \frac{1}{2}$

$\Rightarrow \quad 4\cos^2 2\theta + 2\cos^2 2\theta - 2\cos 2\theta - 1 = 0$

$\Rightarrow \quad 2\cos^2 2\theta(2\cos\theta + 1) - 1(2\cos 2\theta + 1) = 0$

$\Rightarrow \quad (2\cos^2 2\theta - 1)(2\cos 2\theta + 1) = 0$

either

$2\cos^2 2\theta - 1 = 0 \quad$ or $\Rightarrow 2\cos 2\theta + 1 = 0$

$\Rightarrow \quad \cos 4\theta = 0 \quad$ or $\Rightarrow \cos 2\theta = -\frac{1}{2}$

$\Rightarrow \quad 4\theta = (2n+1)\frac{\pi}{2} \quad$ or $\Rightarrow \cos 2\theta = \cos 2\frac{\pi}{3}$

$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{8} \quad$ or $\Rightarrow 2\theta = 2m\pi \pm 2\frac{\pi}{3}$

$\Rightarrow \theta = m\pi \pm \frac{\pi}{3}$

Thus,

$\theta = (2n+1)\frac{\pi}{8} \quad$ or $\theta = m\pi \pm \frac{\pi}{3}, m, n \in z$

Q4(v)

We have,

$$\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$$

$\Rightarrow \quad \cos\theta - \cos 2\theta = \sin 2\theta - \sin\theta$

$\Rightarrow \quad 2\sin\frac{3\theta}{2} . \sin\frac{\theta}{2} = 2\cos\frac{3\theta}{2} . \sin\frac{\theta}{2}$

$\Rightarrow \quad 2\sin\frac{\theta}{2}\left(\sin\frac{3\theta}{2} - \cos\frac{3\theta}{2}\right) = 0$

$\Rightarrow \quad 2\sin\frac{\theta}{2}\left(\sin\frac{3\theta}{2} - \cos\frac{3\theta}{2} = 0\right)$

either

$\sin\frac{\theta}{2} = 0$ or $\sin\frac{3\theta}{2} - \cos\frac{3\theta}{2} = 0$

$\Rightarrow \quad \frac{\theta}{2} = n\pi, n \in$ or $\tan\frac{3\theta}{2} = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \quad \theta = 2n\pi, n \in z$ or $\frac{3\theta}{2} = n\pi + \frac{\pi}{4}$

or $\theta = 2n\frac{\pi}{3} + \frac{\pi}{3.2}, n \in z$

Thus,

$\Rightarrow \quad \theta = 2n\pi$ or $2n\frac{\pi}{3} + \frac{\pi}{6}, n \in$

Q4(vi)

We have,

$\sin\theta + \sin 2\theta + \sin 3\theta = 0$

$\Rightarrow \quad \sin 2\theta + 2\sin 2\theta . \cos\theta = 0$

$\Rightarrow \quad \sin 2\theta + (1 + 2\cos\theta) = 0$

$\Rightarrow$ either

$\sin 2\theta = 0$ or $1 + 2\cos\theta = 0$

$\Rightarrow \quad 2\theta = n\pi, n \in z$ or $\cos\theta = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \frac{n\pi}{2}, n \in z$ or $\theta = 2m\pi \pm \frac{2\pi}{3}, m \in z$

Thus,

$$\theta = \frac{n\pi}{2}, n \in z \text{ or } \theta = 2m\pi \pm \frac{2\pi}{3}, m \in z$$

Q4(vii)

Given , sinx + sin2x + sin3x + sin4x =0

(sin4x + sin 2x) + (sin3x + sinx) =0

Using , (sinA +sinB) formula =>

$$2\sin\left[\frac{(4x+2x)}{2}\right]\cos\left[\frac{4x-2x}{2}\right]+2\sin\left[\frac{(3x+x)}{2}\right]\cos\left[\frac{(3x-x)}{2}\right]=0$$

$$2\sin 3x\cos x+2\sin 2x\cos x=0$$

$$2\cos x(\sin 3x+\sin 2x)=0$$

$$2\cos x\left(2\sin\left[\frac{(3x+2x)}{2}\right]\cos\left[\frac{(3x-2x)}{2}\right]\right)=0$$

$$4\cos x\sin\frac{5x}{2}\cos\frac{x}{2}=0$$

$$\cos x=0;\ \sin\frac{5x}{2}=0;\ \cos\frac{x}{2}=0$$

$$x=\frac{(2n+1)\pi}{2};\ \frac{5x}{2}=m\pi;\ \frac{x}{2}=\frac{(2r+1)\pi}{2}$$

$$x=\frac{(2n+1)\pi}{2};\ x=\frac{2m\pi}{5};\ x=(2r+1)\pi,\ m,r,n\in Z$$

Q4(viii)

We have,

$$\sin 3\theta-\sin\theta=4\cos^2\theta-2$$

$$\Rightarrow\quad 2\cos 2\theta.\ \sin\theta=2\left(2\cos^2\theta-1\right)$$

$$\Rightarrow\quad 2\cos 2\theta.\ \sin\theta=2\cos 2\theta \qquad \left[\because \cos 2\theta=2\cos^2\theta-1\right]$$

$$\Rightarrow\quad 2\cos 2\theta\left(\sin\theta-1\right)=0$$

either

$$\cos 2\theta=0 \quad \text{or} \quad \sin\theta-1=0$$

$$\Rightarrow\quad 2\theta=(2n+1)\frac{\pi}{2},n\in z \quad \text{or} \quad \sin\theta=1=\sin\frac{\pi}{2}$$

$$\Rightarrow\quad \theta=(2n+1)\frac{\pi}{4},n\in z \quad \text{or} \quad \theta=m\pi+(-1)^m\frac{\pi}{2},m\in z$$

Thus,

$$\theta=(2n+1)\frac{\pi}{4},n\in z \quad \text{or} \quad m\pi+(-1)^m\frac{\pi}{2},m\in z$$

Q5(i)

$$\tan x + \tan 2x + \frac{(\tan x + \tan 2x)}{1 - \tan x . \tan 2x} = 0$$

$$\left[\tan x + \tan 2x\right]\left[1 + \frac{1}{1 - \tan x . \tan 2x}\right] = 0$$

$$\tan x + \tan 2x \, (2 - \tan x . \tan 2x) = 0$$

$$\tan x = \tan(-2x) \text{ or } \tan x . \tan 2x = 2$$

$$x = n\pi - 2x \text{ or } \tan x . \frac{2\tan x}{1 - \tan^2 x} = 2$$

$$3x = n\pi \text{ or } \frac{2\tan^2 x}{1 - \tan^2 x} = 2$$

$$3x = n\pi \; or \; 2\tan^2 x = 2 - 2\tan^2 x$$

$$3x = n\pi \; or \; 4\tan^2 x = 2$$

$$x = \frac{n\pi}{3} \; or \; \tan^2 x = 1/2$$

$$x = \frac{n\pi}{3} \text{ or } x = m\pi \pm \tan^{-1}\left(\frac{1}{\sqrt{2}}\right), \quad n, m \in Z$$

Q5(ii)

$$\tan\theta + \tan 2\theta = \tan(\theta + 2\theta)$$

$$\tan\theta + \tan 2\theta - \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = 0$$

$$\left[\tan\theta + \tan 2\theta\right]\left[1 - \frac{1}{1 - \tan\theta \tan 2\theta}\right] = 0$$

$$\left[\tan\theta + \tan 2\theta\right]\left[\frac{1 - \tan\theta \tan 2\theta - 1}{1 - \tan\theta \tan 2\theta}\right] = 0$$

$$\left[\tan\theta + \tan 2\theta\right]\left[\frac{-\tan\theta \tan 2\theta}{1 - \tan\theta \tan 2\theta}\right] = 0$$

$$\tan\theta = 0 \; or \; \tan 2\theta = 0 \; or \; \tan\theta + \tan 2\theta = 0$$

$$\theta = n\pi \; or \; \frac{n\pi}{2} \; or \; \tan\theta\left[\frac{1 - \tan^2\theta + 2}{1 - \tan^2\theta}\right] = 0$$

$$\theta = n\pi \; or \; \frac{n\pi}{2} \; or \; \tan\theta = \pm\sqrt{3}$$

$$\theta = m\pi \; or \; \frac{n\pi}{3} \quad m, n \in Z$$

Q5(iii)

We have,

$$\tan 3\theta + \tan\theta = 2\tan 2\theta$$

$\Rightarrow \tan 3\theta - \tan 2\theta = \tan 2\theta - \tan\theta$

$\Rightarrow \tan 3\theta - \tan 2\theta = \tan 2\theta - \tan\theta$

$\Rightarrow 2\sin^2\theta \sin 2\theta = 0$

$\Rightarrow$ either

$\sin\theta = 0$ or $\sin 2\theta = 0$

$\Rightarrow \theta = n\pi, n \in z$ or $2\theta = m\pi, m \in z$

$\Rightarrow \theta = n\pi, n \in z$ or $\theta = m\frac{\pi}{2}, m \in z$

Q6(i)

We have,

$$\sin\theta + \cos\theta = \sqrt{2}$$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 1$

$\Rightarrow \sin\frac{\pi}{4}\sin\theta + \cos\frac{\pi}{4}\cos\theta = 1 \qquad \left[\because \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos 0^\circ$

$\Rightarrow \theta - \frac{\pi}{4} = 2n\pi, n \in z$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{4}, n \in z$

$\therefore \theta = (8n+1)\frac{\pi}{4}, n \in z$

Q6(ii)

$$\sqrt{3}\cos\theta + \sin\theta = 1$$

Divide both side by 2, we get

$$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{1}{2}$$

$\Rightarrow \cos\frac{\pi}{6}\cos\theta + \sin\frac{\pi}{6}\sin\theta = \frac{1}{2} \qquad \left[\because \sin\frac{\pi}{6} = \frac{1}{2}, \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow \cos\left(\theta - \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6} = 2n \pm \frac{\pi}{3}, n \in z$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}, n \in z$

$\Rightarrow \theta = (4n+1)\frac{\pi}{2}$ or $(12m-1)\frac{\pi}{6}, n, m \in z$

Q6(iii)

We have,

$$\sin\theta + \cos\theta = 1$$

divide both side by $\sqrt{2}$, we get,

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\frac{\pi}{4}\sin\theta + \cos\frac{\pi}{4}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in z$$

$$\Rightarrow \quad \theta = 2n\pi + \frac{\pi}{2} \quad \text{or} \quad 2n\pi, n \in z$$

Q6(iv)

We have,

$$\cos ec\theta = 1 + \cot\theta$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = 1 + \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad 1 = \sin\theta + \cos\theta$$

Divide both side by $\sqrt{2}$, we get,

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\frac{\pi}{4}\sin\theta + \cos\frac{\pi}{4}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in z$$

$$\therefore \theta\left(2n\pi + \frac{\pi}{2}\right) \quad \text{or} \quad 2n\pi, n \in z$$

Q6(v)

$$\left(\sqrt{3}-1\right)\cos\theta+\left(\sqrt{3}+1\right)\sin\theta=2$$

Divide on both sides by $2\sqrt{2}$

$$\frac{\left(\sqrt{3}-1\right)}{2\sqrt{2}}\cos\theta+\frac{\left(\sqrt{3}+1\right)}{2\sqrt{2}}\sin\theta=\frac{1}{\sqrt{2}}$$

$$\sin\left(\theta+\tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\right)=\sin\frac{\pi}{4}$$

$$\theta=2n\pi+\frac{\pi}{3}\ or\ 2n\pi-\frac{\pi}{6}\quad n\in Z$$

Q7(i)

$$\cot x+\tan x=2$$
$$2\sin x\cos x=1$$
$$\sin 2x=1$$
$$2x=\frac{(2n+1)}{2}\pi$$
$$x=\frac{(2n+1)}{4}\pi,n\in Z$$

Q7(ii)

$2\sin^2\theta = 3\cos\theta$

$2-2\cos^2\theta=3\cos\theta$

$2\cos^2\theta+3\cos\theta-2=0$

$2\cos^2\theta+4\cos\theta-\cos\theta-2=0$

$(\cos\theta+2)(2\cos\theta-1)=0$

$\cos\theta=-2$ or $\cos\theta=0.5$

$\cos\theta=-2$, never possible

$\cos\theta=0.5$, $\theta=60, 300$

Q7(iii)

$$\sec x \cos 5x + 1 = 0$$

$$\frac{\cos 5x + \cos x}{\cos x} = 0 \Rightarrow \cos x \neq 0$$

$$2\cos 3x \cos 2x = 0$$

$$\cos 3x = 0 \; or \; \cos 2x = 0$$

$$3x = \frac{\pi}{2} \; or \; 2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{\pi}{6}$$

Q7(iv)

$$2\sin^2\theta + 5 - 6 = 0$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\theta = n\pi \pm \frac{\pi}{4}$$

Q7(v)

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$$

$$(\sin x + \sin 3x) - 3\sin 2x - (\cos x + \cos 3x) + 3\cos 2x = 0$$

$$2\sin 2x\cos x - 3\sin 2x - 2\cos 2x\cos x + 3\cos 2x = 0$$

$$\sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$$

$$(2\cos x - 3)(\sin 2x - \cos 2x) = 0$$

$$\cos x = \frac{3}{2}\ or\ \sin 2x - \cos 2x = 0$$

$$but\ \cos x \in [-1,1] \Rightarrow \cos x \neq \frac{3}{2}$$

$$\sin 2x = \cos 2x$$

$$\tan 2x = 1$$

$$2x = n\pi + \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$