

Exercise – 12.1

- Find the area of a triangle whose sides are respectively 150 cm, 120 cm and 200 cm.

Sol:

The triangle whose sides are

$$a = 150 \text{ cm}$$

$$b = 120 \text{ cm}$$

$$c = 200 \text{ cm}$$

$$\text{The area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Here $1s$ = semi perimeter of triangle

$$2s = a + b + c$$

$$S = \frac{a+b+c}{2} = \frac{150+200+120}{2} = 235 \text{ cm}$$

$$\therefore \text{area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{235(235-150)(235-200)(235-120)}$$

$$= \sqrt{235(85)(35)(115)} \text{ cm}^2$$

$$= 8966.56 \text{ cm}^2$$

- Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm.

Sol:

The triangle whose sides are $a = 9 \text{ cm}$, $b = 12 \text{ cm}$ and $c = 15 \text{ cm}$

$$\text{The area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Here $1s$ = semi-perimeter of a triangle

$$2s = a + b + c$$

$$S = \frac{a+b+c}{2} = \frac{9+12+15}{2} = \frac{36}{2} = 18 \text{ cm}$$

$$\therefore \text{area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)} = \sqrt{18(9)(6)(3)}$$

$$= \sqrt{18 \text{ cm} \times 3 \text{ cm} \times 54 \text{ cm}^2} = 54 \text{ cm}^2.$$

- Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42cm.

Sol:

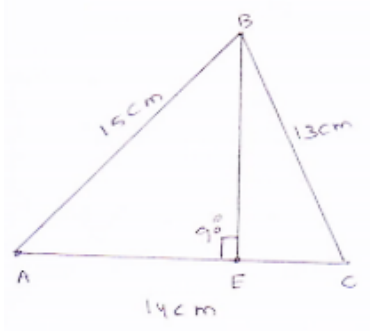
$$21\sqrt{11} \text{ cm}^2$$



4. In a $\triangle ABC$, $AB = 15$ cm, $BC = 13$ cm and $AC = 14$ cm. Find the area of $\triangle ABC$ and hence its altitude on AC .

Sol:

The triangle sides are



Let $a = AB = 15$ cm, $BC = 13$ cm = b .
 $c = AC = 14$ cm say.

Now,

$$2s = a + b + c$$

$$\Rightarrow S = \frac{1}{2}(a + b + c)$$

$$\Rightarrow s = \left(\frac{15+13+14}{2}\right) \text{ cm}$$

$$\Rightarrow s = 21 \text{ cm}$$

$$\therefore \text{area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-15)(21-14)(21-13)} \text{ cm}^2$$

$$= \sqrt{21 \times 6 \times 8 \times 7} \text{ cm}^2$$

$$= 84 \text{ cm}^2$$

Let BE be perpendicular (\perp^{er}) to AC

Now, area of triangle = 84 cm^2

$$\Rightarrow \frac{1}{2} \times BE \times AC = 84$$

$$\Rightarrow BE = \frac{84 \times 2}{AC}$$

$$\Rightarrow BE = \frac{168}{14} = 12 \text{ cm}$$

\therefore Length of altitude on AC is 12 cm.

5. The perimeter of a triangular field is 540 m and its sides are in the ratio $25 : 17 : 12$. Find the area of the triangle.

Sol:

The sides of a triangle are in the ratio $25 : 17 : 12$



Let the sides of a triangle are $a = 25x$, $b = 17x$ and $c = 12x$ say.

$$\text{Perimeter} = 25x + 17x + 12x = 54x \text{ cm}$$

$$\Rightarrow 25x + 17x + 12x = 540 \text{ cm}$$

$$\Rightarrow 54x = 540 \text{ cm}$$

$$\Rightarrow x = \frac{540}{54}$$

$$\Rightarrow x = 10 \text{ cm}$$

\therefore The sides of a triangle are $a = 250 \text{ cm}$, $b = 170 \text{ cm}$ and $c = 120 \text{ cm}$

$$\text{Now, Semi perimeter } s = \frac{a+b+c}{2}$$

$$= \frac{540}{2} = 270 \text{ cm}$$

$$\therefore \text{The area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270(270-250)(270-170)(270-120)}$$

$$= \sqrt{270(20)(100)(150)}$$

$$= \sqrt{(9000)(9000)}$$

$$= 9000 \text{ cm}^2$$

\therefore The area of triangle = 900 cm^2 .

6. The perimeter of a triangle is 300 m. If its sides are in the ratio 3 : 5 : 7. Find the area of the triangle.

Sol:

Given that

The perimeter of a triangle = 300 m

The sides of a triangle in the ratio 3 : 5 : 7

Let $3x$, $5x$, $7x$ be the sides of the triangle

$$\text{Perimeter} \Rightarrow 2s = a + b + c$$

$$\Rightarrow 3x + 5x + 7x = 300$$

$$\Rightarrow 15x = 300$$

$$\Rightarrow x = 20 \text{ m}$$

The triangle sides are $a = 3x$

$$= 3(20) \text{ m} = 60 \text{ m}$$

$$b = 5x = 5(20) \text{ m} = 100 \text{ m}$$

$$c = 7x = 140 \text{ m}$$

$$\text{Semi perimeter } s = \frac{a+b+c}{2}$$

$$= \frac{300}{2} \text{ m}$$

$$= 150 \text{ m}$$

$$\therefore \text{The area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$



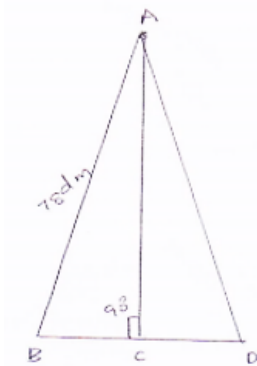
$$\begin{aligned}
 &= \sqrt{150(150 - 60)(150 - 100)(150 - 140)} \\
 &= \sqrt{150 \times 10 \times 90 \times 50} \\
 &= \sqrt{1500 \times 1500 \times 3} \text{ cm}^2 \\
 \therefore \Delta \text{le Area} &= 1500\sqrt{3} \text{ cm}^2
 \end{aligned}$$

7. The perimeter of a triangular field is 240 dm. If two of its sides are 78 dm and 50 dm, find the length of the perpendicular on the side of length 50 dm from the opposite vertex.

Sol:

ABC be the triangle, Here $a = 78 \text{ dm} = AB$,

$BC = b = 50 \text{ dm}$



Now, perimeter = 240 dm

$$\Rightarrow AB + BC + CA = 240 \text{ dm}$$

$$\Rightarrow AC = 240 - BC - AB$$

$$\Rightarrow AC = 112 \text{ dm}$$

Now, $2s = AB + BC + CA$

$$\Rightarrow 2s = 240$$

$$\Rightarrow s = 120 \text{ dm}$$

\therefore Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$ by heron's formula

$$= \sqrt{120(120 - 78)(120 - 50)(120 - 112)}$$

$$= \sqrt{120 \times 42 \times 70 \times 8}$$

$$= 1680 \text{ dm}^2$$

Let AD be perpendicular on BC

$$\text{Area of } \triangle ABC = \frac{1}{2} \times AD \times BC \text{ (area of triangle} = \frac{1}{2} \times b \times h)$$

$$= \frac{1}{2} \times AD \times BC = 1680$$

$$\Rightarrow AD = \frac{2 \times 1680}{50} = 67.2 \text{ dm}$$



8. A triangle has sides 35 cm, 54 cm and 61 cm long. Find its area. Also, find the smallest of its altitudes.

Sol:

The sides of a triangle are $a = 35$ cm, $b = 54$ cm and $c = 61$ cm

Now, perimeter $a + b + c = 25$

$$\Rightarrow S = \frac{1}{2}(35 + 54 + 61)$$

$$\Rightarrow s = 75 \text{ cm}$$

By using heron's formula

$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{75(75-35)(75-54)(75-61)}$$

$$= \sqrt{75(40)(21)(14)} = 939.14 \text{ cm}^2$$

\therefore The altitude will be a smallest when the side corresponding to it is longest Here, longest side is 61 cm

$$[\therefore \text{Area of } \triangle = \frac{1}{2} \times b \times h] = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\therefore \frac{1}{2} \times h \times 61 = 939.14$$

$$\Rightarrow h = \frac{939.14 \times 2}{61} = 30.79 \text{ cm}$$

Hence the length of the smallest altitude is 30.79 cm

9. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle and the height corresponding to the longest side.

Sol:

Let the sides of a triangle are $3x$, $4x$ and $5x$.

Now, $a = 3x$, $b = 4x$ and $c = 5x$

The perimeter $2s = 144$

$$\Rightarrow 3x + 4x + 5x = 144 \quad [\therefore a + b + c = 2s]$$

$$\Rightarrow 12x = 144$$

$$\Rightarrow x = 12$$

$$\therefore \text{sides of triangle are } a = 3(x) = 36 \text{ cm}$$

$$b = 4(x) = 48 \text{ cm}$$

$$c = 5(x) = 60 \text{ cm}$$

$$\text{Now semi perimeter } s = \frac{1}{2}(a + b + c) = \frac{1}{2}(144) = 72 \text{ cm}$$

$$\text{By heron's formulas } \therefore \text{Area of } \triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{72(72-36)(72-48)(72-60)}$$

$$= 864 \text{ cm}^2$$

$$\text{Let } l \text{ be the altitude corresponding to longest side, } \therefore \frac{1}{2} \times 60 \times l = 864$$



$$\Rightarrow l = \frac{864 \times 2}{60}$$

$$\Rightarrow l = 28.8 \text{ cm}$$

Hence the altitude one corresponding long side = 28.8 cm

10. The perimeter of an isosceles triangle is 42 cm and its base is $\left(\frac{3}{2}\right)$ times each of the equal sides. Find the length of each side of the triangle, area of the triangle and the height of the triangle.

Sol:

Let 'x' be the measure of each equal sides

$$\therefore \text{Base} = \frac{3}{2}x$$

$$\therefore x + x + \frac{3}{2}x = 42 \quad [\because \text{Perimeter} = a + b + c = 42 \text{ cm}]$$

$$\Rightarrow \frac{7}{2}x = 42$$

$$\Rightarrow x = 12 \text{ cm}$$

$$\therefore \text{Sides are } a = x = 12 \text{ cm}$$

$$b = x = 12 \text{ cm}$$

$$c = x = \frac{3}{2}(12) \text{ cm} = 18 \text{ cm}$$

By heron's formulae

$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)} \text{ cm}^2$$

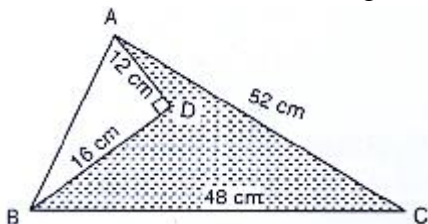
$$= \sqrt{21(9)(9)(21-18)} \text{ cm}^2$$

$$= \sqrt{(21)(9)(9)(3)} \text{ cm}^2$$

$$= 71.42 \text{ cm}^2$$

$$\therefore \text{Area of triangle} = 71.42 \text{ cm}^2$$

11. Find the area of the shaded region in Fig. Below.



Sol:

$$\text{Area of shaded region} = \text{Area of } \triangle ABC - \text{Area of } \triangle ADB$$

Now in $\triangle ADB$

$$\Rightarrow AB^2 = AD^2 + BD^2 \quad \text{--(i)}$$

$$\Rightarrow \text{Given that } AD = 12 \text{ cm } BD = 16 \text{ cm}$$

Substituting the values of AD and BD in the equation (i), we get



$$AB^2 = 12^2 + 16^2$$

$$AB^2 = 144 + 256$$

$$AB = \sqrt{400}$$

$$AB = 20 \text{ cm}$$

$$\therefore \text{Area of triangle} = \frac{1}{2} \times AD \times BD$$

$$= \frac{1}{2} \times 12 \times 16$$

$$= 96 \text{ cm}^2$$

Now

$$\text{In } \triangle ABC, S = \frac{1}{2}(AB + BC + CA)$$

$$= \frac{1}{2} \times (52 + 48 + 20)$$

$$= \frac{1}{2}(120)$$

$$= 60 \text{ cm}$$

By using heron's formula

$$\text{We know that, Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{60(60-20)(60-48)(60-52)}$$

$$= \sqrt{60(40)(12)(8)}$$

$$= 480 \text{ cm}^2$$

$$= \text{Area of shaded region} = \text{Area of } \triangle ABC - \text{Area of } \triangle ADB$$

$$= (480 - 96) \text{ cm}^2$$

$$= 384 \text{ cm}^2$$

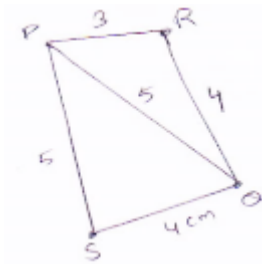
$$\therefore \text{Area of shaded region} = 384 \text{ cm}^2$$

Exercise – 12.2

- Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Sol:

For $\triangle PQR$



$$PQ^2 = QR^2 + RP^2$$



$$(5^2) = (3)^2 + (4)^2 [\because PR = 3 \text{ } QR = 4 \text{ and } PQ = 5]$$

So, ΔPQR is a right angled triangle. Right angle at point R.

$$\text{Area of } \Delta ABC = \frac{1}{2} \times QR \times RP$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6\text{cm}^2$$

For ΔQPS

$$\text{Perimeter} = 2s = AC + CD + DA = (5 + 4 + 5)\text{cm} = 14 \text{ cm}$$

$$S = 7 \text{ cm}$$

By Heron's formulae

$$\text{Area of } \Delta \text{le } \sqrt{s(s-a)(s-b)(s-c)} \text{ cm}^2$$

$$\begin{aligned} \text{Area of } \Delta \text{le PQS} &= \sqrt{7(7-5)(7-4)(7-3)}\text{cm}^2 \\ &= \sqrt{7 \times 2 \times 2 \times 3} \text{ cm}^2 \end{aligned}$$

$$= 2\sqrt{21}\text{cm}^2$$

$$= (2 \times 4.583)\text{cm}^2$$

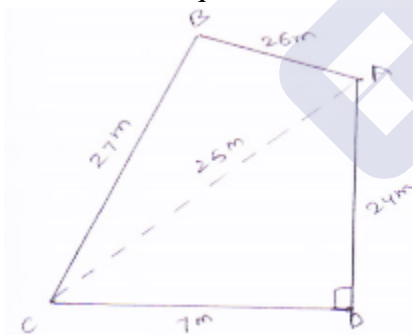
$$= 9.166 \text{ cm}^2$$

$$\text{Area of PQRS} = \text{Area of PQR} + \text{Area of } \Delta PQS = (6 + 9.166)\text{cm}^2 = 15.166\text{cm}^2$$

2. The sides of a quadrangular field, taken in order are 26 m, 27 m, 7m are 24 m respectively. The angle contained by the last two sides is a right angle. Find its area.

Sol:

The sides of a quadrilateral field taken order as $AB = 26\text{m}$



$$BC = 27 \text{ m}$$

$$CD = 7\text{m and } DA = 24 \text{ m}$$

Diagonal AC is joined

Now ΔADC

By applying Pythagoras theorem

$$\Rightarrow AC^2 = AD^2 + CD^2$$

$$\Rightarrow AC = \sqrt{AD^2 + CD^2}$$

$$\Rightarrow AC = \sqrt{24^2 + 7^2}$$



$$\Rightarrow AC = \sqrt{625} = 25 \text{ m}$$

Now area of $\triangle ABC$

$$\begin{aligned} S &= \frac{1}{2}(AB + BC + CA) = \frac{1}{2}(26 + 27 + 25) = \\ &= \frac{78}{2} = 39\text{m}. \end{aligned}$$

By using heron's formula

$$\begin{aligned} \text{Area } (\triangle ABC) &= \sqrt{S(S - AD)(S - BC)(S - CA)} \\ &= \sqrt{39(39 - 26)(39 - 21)(39 - 25)} \\ &= \sqrt{39 \times 14 \times 13 \times 12 \times 1} \\ &= 291.849 \text{ cm}^2 \end{aligned}$$

Now for area of $\triangle ADC$

$$\begin{aligned} S &= \frac{1}{2}(AD + CD + AC) \\ &= \frac{1}{2}(25 + 24 + 7) = 28\text{m} \end{aligned}$$

By using heron's formula

$$\begin{aligned} \therefore \text{Area of } \triangle ADC &= \sqrt{S(S - AD)(S - DC)(S - CA)} \\ &= \sqrt{28(28 - 24)(28 - 7)(28 - 25)} \\ &= 84\text{m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of rectangular field ABCD} &= \text{area of } \triangle ABC + \text{area of } \triangle ADC \\ &= 291.849 + 84 \\ &= 375.8\text{m}^2 \end{aligned}$$

3. The sides of a quadrilateral, taken in order are 5, 12, 14 and 15 meters respectively, and the angle contained by the first two sides is a right angle. Find its area.

Sol:

Given that sides of quadrilateral are $AB = 5 \text{ m}$, $BC = 12 \text{ m}$, $CD = 14 \text{ m}$ and $DA = 15 \text{ m}$

$AB = 5\text{m}$, $BC = 12\text{m}$, $CD = 14 \text{ m}$ and $DA = 15 \text{ m}$

Join AC

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times AB \times BC \left[\because \text{Area of } \triangle = \frac{1}{2}(3x + 1) \right] \\ &= \frac{1}{2} \times 5 \times 12 \\ &= 30 \text{ cm}^2 \end{aligned}$$

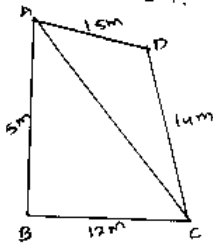
In $\triangle ABC$ By applying Pythagoras theorem.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC &= \sqrt{5^2 + 12^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ m} \end{aligned}$$



Now in $\triangle ADC$

Let $2s$ be the perimeter



$$\therefore 2s = (AD + DC + AC)$$

$$\Rightarrow S = \frac{1}{2}(15 + 14 + 13) = \frac{1}{2} \times 42 = 21m$$

By using Heron's formula

$$\therefore \text{Area of } \triangle ADC = \sqrt{S(S - AD)(S - DC)(S - AC)}$$

$$= \sqrt{21(21 - 15)(21 - 14)(21 - 13)}$$

$$= \sqrt{21 \times 6 \times 7 \times 8}$$

$$= 84m^2$$

$$\therefore \text{Area of quadrilateral } ABCD = \text{area of } (\triangle ABC) + \text{Area of } (\triangle ADC) = 30 + 84 = 114 m^2$$

4. A park, in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m and $AD = 8$ m. How much area does it occupy?

Sol:

Given sides of a quadrilateral are $AB = 9$, $BC = 12$, $CD = 5$, $DA = 8$

Let us join BD

In $\triangle BCD$ applying Pythagoras theorem.

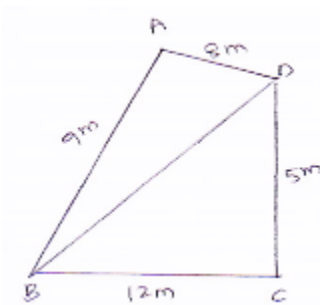
$$BD^2 = BC^2 + CD^2$$

$$= (12)^2 + (5)^2$$

$$= 144 + 25$$

$$= 169$$

$$BD = 13m$$



$$\text{Area of } \triangle BCD = \frac{1}{2} \times BC \times CD = \left[\frac{1}{2} \times 12 \times 5 \right] m^2 = 30 m^2$$



For $\triangle ABD$

$$S = \frac{\text{perimeter}}{2} = \frac{(9+8+13)}{2} = 15\text{cm}$$

By heron's formula $= \sqrt{s(s-a)(s-b)(s-c)}$

$$\text{Area of the triangle} = \sqrt{15(15-9)(15-8)(15-13)}\text{m}^2$$

$$= \sqrt{15(6)(7)(2)}\text{m}^2 = 6\sqrt{35}\text{m}^2 = 35.496\text{m}^2$$

Area of park = Area of $\triangle ABD + \triangle ABD + \text{Area of BCD}$

$$= 35.496 + 30\text{m}^2$$

$$= 65.5\text{m}^2 \text{ (approximately)}$$

5. Two parallel side of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm. Find the area of the trapezium.

Sol:

Given that two parallel sides of trapezium are $AB = 77$ and $CD = 60$ cm

Other sides are $BC = 26$ m and $AD = 25$ cm.

Join AE and CF

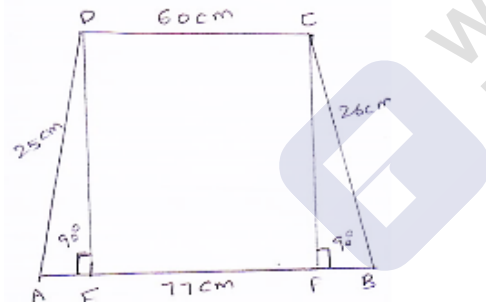
Now, $DE \perp AB$ and $CF \perp AB$

$$\therefore DC = EF = 60\text{ cm}$$

Let $AE = x$

$$\Rightarrow BF = 77 - 60 - x = 17 - x$$

$$\text{In } \triangle ADE, DE^2 = AD^2 - AE^2 = 25^2 - x^2 \quad [\because \text{Pythagoras theorem}]$$



$$\text{And in } \triangle BCF, CF^2 = BC^2 - BF^2 \quad [\because \text{By Pythagoras theorem}]$$

$$\Rightarrow CF = \sqrt{26^2 - (17 - x)^2}$$

$$\text{But } DE = CF \Rightarrow DE^2 = CF^2$$

$$\Rightarrow 25^2 - x^2 = 26^2 - (17 - x)^2$$

$$\Rightarrow 25^2 - x^2 = 26^2 - (289 + x^2 - 34x) \quad [\because (a - b)^2 = a^2 - 2ab + b^2]$$

$$\Rightarrow 625 - x^2 = 676 - 289 - x^2 + 34x$$

$$\Rightarrow 34x = 238$$

$$\Rightarrow x = 7$$

$$\therefore DE = \sqrt{25^2 - x^2} = \sqrt{625 - 7^2} = \sqrt{516} = 24\text{cm}$$

$$\therefore \text{Area of trapezium} = \frac{1}{2}(\text{sum of parallel sides}) \times \text{height} = \frac{1}{2}(60 + 77) \times 24 =$$

$$1644\text{cm}^2$$



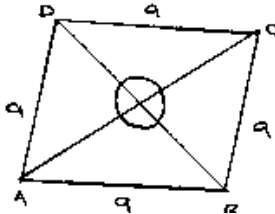
6. Find the area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m.

Sol:

Given that,

Perimeter of rhombus = 80m

Perimeter of rhombus = $4 \times \text{side}$



$$\Rightarrow 4a = 80$$

$$\Rightarrow a = 20\text{m}$$

Let $AC = 24\text{ m}$

$$\therefore OA = \frac{1}{2}AC = \frac{1}{2} \times 24 = 12\text{m}$$

In $\triangle AOB$

$$OB^2 = AB^2 - OA^2 \quad [\text{By using Pythagoras theorem}]$$

$$\Rightarrow OB = \sqrt{20^2 - 12^2}$$

$$= \sqrt{400 - 144}$$

$$= \sqrt{256} = 16\text{ m}$$

Also $BO = OD$ [Diagonal of rhombus bisect each other at 90°]

$$\therefore BD = 2OB = 2 \times 16 = 32\text{ m}$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} \times 32 \times 24 = 384\text{m}^2 \quad [\because \text{Area of rhombus} = \frac{1}{2} \times BD \times AC]$$

7. A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs 5 per m^2 . Find the cost of painting.

Sol:

Given that,

Perimeter of a rhombus = 32 m

We know that,

Perimeter of rhombus = $4 \times \text{side}$

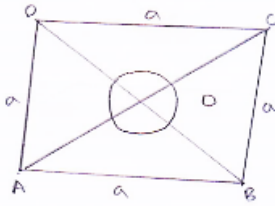
$$\Rightarrow 4a = 32\text{m}$$

$$\Rightarrow a = 8\text{ m}$$

$$\text{Let } AC = 10 = OA = \frac{1}{2}AC$$

$$= \frac{1}{2} \times 10$$

$$= 5\text{m}$$



By using Pythagoras theorem:

$$\therefore OB^2 = AB^2 - OA^2$$

$$\Rightarrow OB = \sqrt{AB^2 - OA^2}$$

$$\Rightarrow OB = \sqrt{8^2 - 5^2}$$

$$\Rightarrow OB = \sqrt{64 - 25}$$

$$\Rightarrow OB = \sqrt{39} \text{ m}$$

$$\text{Now, } BD = 2OB = 2\sqrt{39} \text{ m}$$

$$\therefore \text{Area of sheet} = \frac{1}{2} \times BD \times AC = \frac{1}{2} \times 2\sqrt{39} \times 10 = 10\sqrt{39} \text{ m}^2$$

$$\therefore \text{Cost of printing on both sides at the rate of Rs 5 per m}^2 = \text{Rs } 2 \times 10\sqrt{39} \times 5 \\ = \text{Rs. 625.00}$$

8. Find the area of a quadrilateral ABCD in which $AD = 24 \text{ cm}$, $\angle BAD = 90^\circ$ and BCD forms an equilateral triangle whose each side is equal to 26 cm. (Take $\sqrt{3} = 1.73$)

Sol:

Given that, a quadrilateral ABCD in which $AD = 24 \text{ cm}$, $\angle BAD = 90^\circ$

BCD is equilateral triangle and sides $BC = CD = BD = 26 \text{ cm}$

In $\triangle BAD$ By using Pythagoras theorem

$$BA^2 = BD^2 - AD^2$$

$$\Rightarrow BA = \sqrt{BD^2 - AD^2}$$

$$= \sqrt{676 - 576}$$

$$= \sqrt{100} = 10 \text{ cm}$$

$$\text{Area of } \triangle BAD = \frac{1}{2} \times BA \times AD$$

$$= \frac{1}{2} \times 10 \times 24$$

$$= 120 \text{ cm}^2$$

$$\text{Area of } \triangle BCD = \frac{\sqrt{3}}{4} \times (26)^2 = 292.37 \text{ cm}^2$$

\therefore Area of quadrilateral

ABCD = Area of $\triangle BAD$ + area of $\triangle BCD$

$$= 120 + 292.37$$

$$= 412.37 \text{ cm}^2$$



9. Find the area of a quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.

Sol:

Given that

Sides of a quadrilateral are AB = 42 cm, BC = 21 cm, CD = 29 cm

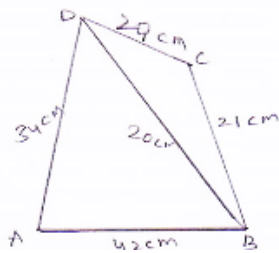
DA = 34 cm and diagonal BD = 20 cm

Area of quadrilateral = area of $\triangle ADB$ + area of $\triangle BCD$.

Now, area of $\triangle ABD$

Perimeter of $\triangle ABD$

We know that



$$2s = AB + BD + DA$$

$$\Rightarrow S = \frac{1}{2}(AB + BD + DA)$$

$$= \frac{1}{2}(34 + 42 + 20) = 96$$

$$= \frac{96}{2}$$

$$= 48 \text{ cm}$$

$$\text{Area of } \triangle ABD = \sqrt{S(S - AB)(S - BD)(S - DA)}$$

$$= \sqrt{48(48 - 42)(48 - 20)(48 - 34)}$$

$$= \sqrt{48(14)(6)(28)}$$

$$= 336 \text{ cm}^2$$

Also for area of $\triangle BCD$,

Perimeter of $\triangle BCD$

$$2s = BC + CD + BD$$

$$\Rightarrow S = \frac{1}{2}(29 + 21 + 20) = 35 \text{ cm}$$

By using heron's formulae

$$\text{Area of } \triangle BCD = \sqrt{s(s - bc)(s - cd)(s - db)}$$

$$= \sqrt{35(35 - 21)(35 - 29)(35 - 20)}$$

$$= \sqrt{210 \times 210} \text{ cm}^2$$

$$= 210 \text{ cm}^2$$

$$\therefore \text{Area of quadrilateral ABCD} = 336 + 210 = 546 \text{ cm}^2$$



10. Find the perimeter and area of the quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, $\angle ACB = 90^\circ$ and AC = 15 cm.

Sol:

The sides of a quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, $\angle ACB = 90^\circ$ and AC = 15 cm

Here, By using Pythagoras theorem

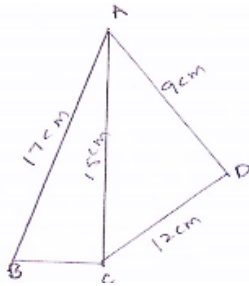
$$BC = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8 \text{ cm}$$

$$\text{Now, area of } \triangle ABC = \frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$$

For area of $\triangle ACD$,

Let a = 15 cm, b = 12 cm and c = 9 cm

$$\text{Therefore, } S = \frac{15+12+9}{2} = \frac{36}{2} = 18 \text{ cm}$$



$$\begin{aligned} \text{Area of } \triangle ACD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{18(18-15)(18-12)(18-9)} \\ &= \sqrt{18 \times 18 \times 3 \times 3} \\ &= \sqrt{(18 \times 3)^2} \end{aligned}$$

$$= 54 \text{ cm}^2$$

$$\therefore \text{ Thus, the area of quadrilateral ABCD} = 60 + 54 = 114 \text{ cm}^2$$

11. The adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. Find the area of the parallelogram.

Sol:

Given that adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm.

Area of parallelogram = Area of $\triangle ADC$ + area of $\triangle ABC$

[\because Diagonal of a parallelogram divides into two congruent triangles]

$$= 2 \times [\text{Area of } \triangle ABC]$$

Now for Area of $\triangle ABC$

Let $2s = AB + BC + CA$ [\because Perimeter of $\triangle ABC$]

$$\Rightarrow S = \frac{1}{2}(AB + BC + CA)$$

$$\Rightarrow S = \frac{1}{2}(34 + 20 + 42)$$



$$= \frac{1}{2}(96) = 48 \text{ cm}$$

$$\therefore \text{Area of } \triangle ABC = \sqrt{s(s-ab)(s-bc)(s-ca)} \quad [\text{heron's formula}]$$

$$= \sqrt{48(48-34)(48-20)(48-42)}$$

$$= \sqrt{48(14)(28)(6)} = 336 \text{ cm}^2$$

$$\therefore \text{Area of parallelogram } ABCD = 2[\text{Area of } \triangle ABC] = 2 \times 336 = 672 \text{ cm}^2$$

12. Find the area of the blades of the magnetic compass shown in Fig. 12.27. (Take $\sqrt{11} = 3.32$).

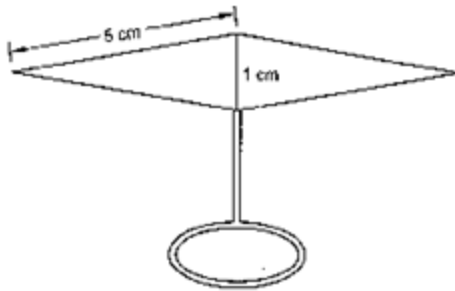


Fig.12.27

Sol:

Area of the blades of magnetic compass = Area of $\triangle ADB$ + Area of $\triangle CDB$

Now, for area of $\triangle ADB$

Let, $2s = AD + DB + BA$ (Perimeter of $\triangle ADB$)

$$\text{Semi perimeter } (S) = \frac{1}{2}(5 + 1 + 5) = \frac{11}{2} \text{ cm}$$

By using heron's formulae

$$\text{Now, area of } \triangle ADB = \sqrt{s(s-ad)(s-bd)(s-ba)}$$

$$= \sqrt{\frac{11}{2} \left(\frac{11}{2} - 5 \right) \left(\frac{11}{2} - 1 \right) \left(\frac{11}{2} - 5 \right)}$$

$$= 2.49 \text{ cm}^2$$

$$= \text{Also, area of triangle } ADB = \text{Area of } \triangle CDB$$

\therefore Area of the blades of magnetic compass

$$= 2 \times (\text{area of } \triangle ADB)$$

$$= 2 \times 2.49$$

$$= 4.98 \text{ m}^2$$

13. A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown in Fig. 12.28. The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.

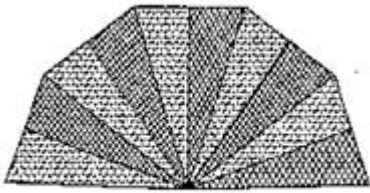


Fig. 12.28

Sol:Given that the sides of $\triangle AOB$ are

$$AO = 24 \text{ cm}$$

$$OB = 25 \text{ cm}$$

$$BA = 14 \text{ cm}$$

Area of each equal strips = Area of $\triangle AOB$ Now, for area of $\triangle AOB$ Perimeter of $\triangle AOB$

$$\text{Let } 2s = AO + OB + BA$$

$$\Rightarrow s = \frac{1}{2}(AO + OB + BA)$$

$$= \frac{1}{2}(25 + 25 + 14) = 32 \text{ cm}$$

 \therefore By using Heron's formulae

$$\text{Area of } (\triangle AOB) = \sqrt{s(s - ao)(s - ob)(s - ba)}$$

$$= \sqrt{32(32 - 25)(32 - 25)(32 - 14)}$$

$$= \sqrt{32(7)(4)(18)}$$

$$= 168 \text{ cm}^2$$

$$\therefore \text{Area of each type of paper needed to make the hand fan} = 5 \times (\text{area of } \triangle AOB)$$

$$= 5 \times 168$$

$$= 840 \text{ cm}^2$$

14. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13 cm, 14 cm and 15 cm and the parallelogram stands on the base 14 cm, find the height of the parallelogram.

Sol:

The sides of a triangle DCE are

$$DC = 15 \text{ cm}, CE = 13 \text{ cm}, ED = 14 \text{ cm}$$

Let h be the height of parallelogram ABCD

Given,

Perimeter of $\triangle DCE$



$$2s = DC + CE + ED$$

$$\Rightarrow S = \frac{1}{2}(15 + 13 + 4)$$

$$\Rightarrow S = \frac{1}{2}(42)$$

$$\Rightarrow S = 21 \text{ cm}$$

$$\text{Area of } \triangle DCE = \sqrt{s(s - dc)(s - ce)(s - ed)} \quad [\text{By heron's formula}]$$

$$= \sqrt{21(21 - 15)(21 - 13)(21 - 14)}$$

$$= \sqrt{21 \times 7 \times 8 \times 6}$$

$$= \sqrt{84 \times 84}$$

$$= 84 \text{ cm}^2$$

Given that

$$\text{Area of } \triangle DCE = \text{area of } ABCD$$

$$= \text{Area of parallelogram } ABCD = 84 \text{ cm}^2$$

$$\Rightarrow 24 \times h = 84 \quad [\because \text{Area of parallelogram} = \text{base} \times \text{height}]$$

$$\Rightarrow h = 6 \text{ cm}$$

