Exercise – 12.1

1. A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 600. What is the height of the tower?

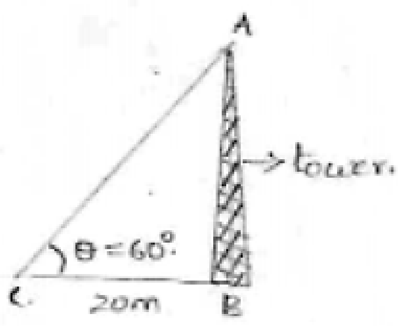
Sol:

Given

Distance between point of observation and foot of tower $= 20m = BC$

Angle of elevation of top of tower $= 60° = 0$

Height of tower $H = ? = AB$



Now from fig ABC

ΔABC is a right angle

$$\frac{1}{\tan} = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

$$\Rightarrow \boxed{\tan\theta = \frac{\text{Opposite side}(AB)}{\text{Adjacent side}(BC)}}$$

$$i.e., \tan 60° = \frac{AB}{20}$$

$$\Rightarrow AB = 20\tan 60°$$

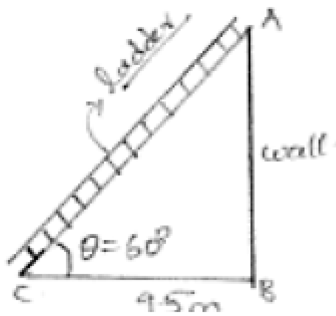
$$\Rightarrow H = 20\sqrt{3}$$

$$= 20\sqrt{3}$$

$\therefore$ Height of tower $H = 20\sqrt{3}m$

2. The angle of elevation of a ladder leaning against a wall is 600 and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

Sol:



Distance between foot ladder and wall $=9.5m=BC$

Angle of elevation $0=60°$

Length of ladder $=l=?=AC.$

Now fig. forms a right angle triangle ABC

We know

$$\boxed{\cos\theta=\frac{\text{Adjacent side}}{\text{hypotenuse}}}$$

$$\Rightarrow \cos 60°=\frac{BC}{AC}$$

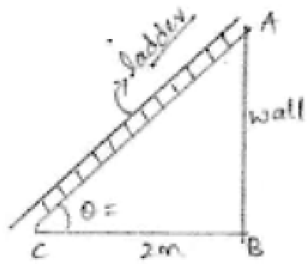
$$\Rightarrow \frac{1}{2}=\frac{9\cdot5}{AC}$$

$$\Rightarrow AC=2\times9.5=19m$$

$\therefore$ length of ladder $l=19m$

3. A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 600 with the level of the ground. Determine the height of the wall.

Sol:



Distance between foot and ladder and wall $=2m=BC$

Angle made by ladder with ground

$\theta=60°$

Height of wall $H=?=AB$

Now fig ABC forms a right angled triangle

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

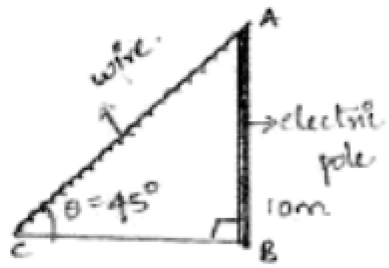
$\therefore$ height of wall $H = 2\sqrt{3}m.$

$\Rightarrow \tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{AB}{2} \Rightarrow AB = 2\sqrt{3}m.$

4. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Sol:



Height of the electric pole $H = 10m = AB$ angle made by steel wire with ground (horizontal) $\theta = 45°$

Let length of rope wire $= l = AC$

If we represent above data is

Form of figure thin it forms a right triangle ABC

Here $\boxed{\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}}$

$\Rightarrow \sin 45° = \frac{AB}{AC}$

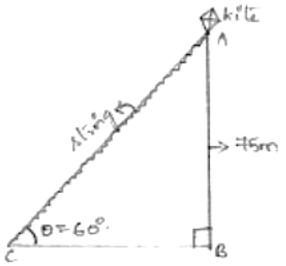
$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10m}{l}$

$\Rightarrow l = 10\sqrt{2}m$

$\therefore$ length of wire $l = 10\sqrt{2}m$

5. A kite is flying at a height of 75 meters from the ground level, attached to a string inclined at 600 to the horizontal. Find the length of the string to the nearest meter.

Sol:



Given

Height o kite from ground $= 75m = AB$

Inclination of string with ground

$\theta = 60°$

Length of string $l = ? = AC$

If we represent the above data is form of figure as shown then its form a right angled triangle ABC here

$$\sin\theta = \frac{\text{Opposite side}}{\text{hypotenuse}}$$

$$\sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{l}$$

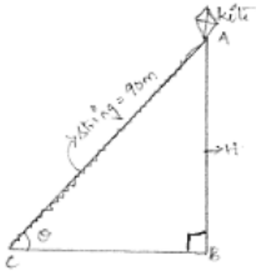
$$\Rightarrow l = \frac{75\times 2}{\sqrt{3}} = \frac{3\times 50}{\sqrt{3}}$$

$$\Rightarrow l = 50\sqrt{3}m$$

Length of string $l = 50\sqrt{3}m.$

6. The length of a string between a kite and a point on the ground is 90 meters. If the string makes an angle O with the ground level such that tan O = 15/8, how high is the kite? Assume that there is no slack in the string.

Sol:



Length of string between point on ground and kite $=90$.

Angle made by string with ground is θ and $\tan\theta=\dfrac{15}{8}$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{15}{8}\right)$$

Height of the kite be Hm

If we represent the above data in figure as shown then it forms right angled triangle ABC.

We have,

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\theta=\frac{AB}{BC}$$

$$\Rightarrow \frac{15}{8}=\frac{\text{H}}{BC}$$

$$\Rightarrow BC=\frac{84}{15} \qquad \ldots\ldots\ldots\ldots(1)$$

in ΔABC, by Pythagoras theorem we have

$$\boxed{AC^2=BC^2+AB^2}$$

$$\Rightarrow 90^2=\left(\frac{8H}{15}\right)^2+H^2$$

$$\Rightarrow 90^2=\frac{(8H)^2+(15H)^2}{15^2}$$

$$\Rightarrow H^2\left(8^2+15^2\right)=90^2\times15^2$$

$$\Rightarrow H^2(64+225)=(90\times15)^2$$

$$\Rightarrow H^2=\frac{(90\times15)^2}{289}$$

$$\Rightarrow H^2=\left(\frac{90\times15}{17}\right)^2$$

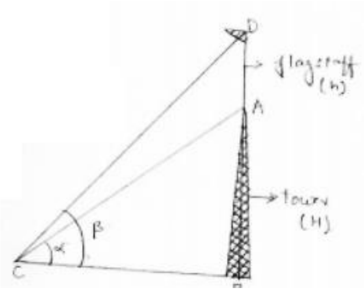
$$\Rightarrow H=\frac{90\times15}{17}=79\cdot41$$

$\therefore$ height of kite from ground $H=79\cdot41m$.

7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of

elevation of the top and the bottom of the flag-staff are respectively 600 and 45°. Find the height of the flag-staff and that of the tower.

Sol:



Given

Vertical tower is surmounted by flag staff distance between tower and observer $= 70m = BC$. Angle of elevation of top of tower $\alpha = 45°$

Angle of elevation of top of flag staff $\beta = 60°$

Height of flagstaff $= h = AD$

Height of tower $= H = AB$

If we represent the above data in the figure then it forms right angled triangles ΔABC and ΔCBD

When θ is angle in right angle triangle we know that

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{BC}$$

$$\Rightarrow \tan 45° = \frac{H}{70}$$

$$\Rightarrow H = 70 \times 1$$

$$= 70m.$$

$$\tan\beta = \frac{DB}{BC}$$

$$\Rightarrow \tan 60° = \frac{AD + AB}{70} = \frac{h + H}{70}$$

$$\Rightarrow h + 70 = 70\left(\sqrt{3}\right)$$

$$\Rightarrow h = 70\left(\sqrt{3} - 1\right)$$

$$= 70(1 \times 32 - 1) = 70 \times 0 \cdot 732$$

$$= 51 \cdot 24m. \qquad \therefore h = 51 \cdot 24m$$

Height of tower $= 70m$ height of flagstaff $= 51 \cdot 24m$

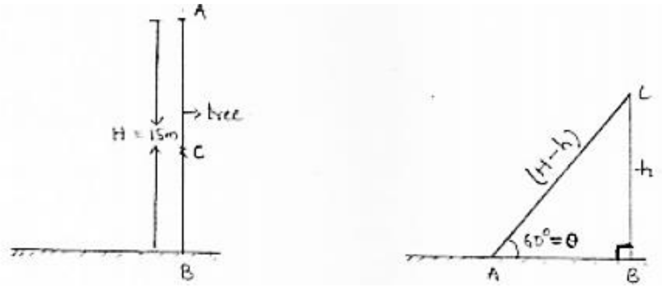
8. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

Sol:

Initial height of tree $H = 15m$

$= AB$

Let us assume that it is broken at pointe.



Then given that angle made by broken part with ground $\theta = 60°$

Height from ground to broken pointe $= h = BC$

$AB = AC + BC$

$\Rightarrow H = AC + h \Rightarrow AC = (H - h)m$

If we represent the above data in the figure as shown then it forms right angled triangle ABC from fig

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin 60° = \frac{BC}{CA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{H - h}$$

$$\Rightarrow \sqrt{3}(15 - h) = 2h$$

$$\Rightarrow 15\sqrt{3} - h\sqrt{3} = 2h$$

$$\Rightarrow (2 + \sqrt{3})h = 15\sqrt{3}$$

$$\Rightarrow h = \frac{15\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

Rationalizing denominator rationalizing factor of $a + \sqrt{b}$ is $a - \sqrt{b}$

$$\Rightarrow h = \frac{(15\sqrt{3})(2 - \sqrt{3})}{2^2 - (3)^2}$$

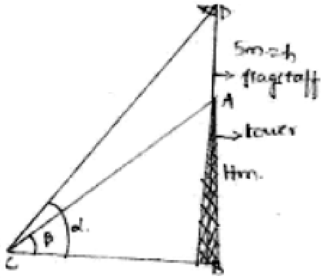
$$= 15(2\sqrt{3} - 3)$$

$\therefore h = 15\left(2\sqrt{3}-3\right)$

$\therefore$ height of broken point from ground $= 15\left(2\sqrt{3}-3\right)m$

9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 5 meters. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30^0 and 60^0. Find the height of the tower.

Sol:



Height of the flagstaff $h = 5m = AP$

Angle of elevation of the top of flagstaff $= 60° = \alpha$

Angle of elevation of the bottom of flagstaff $= 30° = \beta$

Let height of tower be $Hm = AB.$

If we represent the above data in forms of figure then it from triangle CBD in which ABC is included with $\angle B = 90°$

In right angle triangle, if

Angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{BD}{BC}$$

$$\Rightarrow \tan 60° = \frac{AB+AD}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{H+5}{BC} \qquad \ldots\ldots\ldots\ldots..(1)$$

$$\tan\beta = \frac{AB}{BC}$$

$$\Rightarrow \tan 30° = \frac{H}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{BC} \qquad \ldots\ldots\ldots\ldots..(2)$$

(1) and (2) $\Rightarrow \dfrac{\sqrt{3}}{\dfrac{1}{\sqrt{3}}} = \dfrac{H15/BC}{H/BC}$

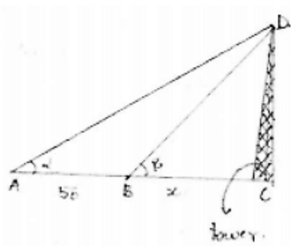
$\Rightarrow 3 = \dfrac{H+5}{H} \Rightarrow 3H = H+5$

$\Rightarrow 2H = 5 \Rightarrow H = \dfrac{5}{2} = 2 \times 5m.$

Height of tower $H = 2 \cdot 5m.$

10. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60^0. Find the height of the tower.

Sol:



Given,

Angle of elevation of top of tour, from first point of elevation $(A)\alpha = 30°$

Let the walked $50m$ from first point (A) to B then $AB = 50m$

Angle of elevation from second point $B \Rightarrow Gb = 60°$

Now let is represent the given data in form of then it forms triangle ACD with triangle BCD in it $\angle c = 90°$

Let height of tower, be

$Hm = CD$

$BC = xm.$

If in a right angle triangle θ is the angle then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \dfrac{CD}{AC}$

$\Rightarrow \tan 30° = \dfrac{H}{AB+BC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{H}{50+x}$

$\Rightarrow 50 + x = H\sqrt{3}$ …………..(1)

$\tan\beta = \frac{CD}{BC}$

$\Rightarrow \tan 60° = \frac{H}{x}$

$\Rightarrow \sqrt{3} = \frac{H}{x}$

$\Rightarrow x = \frac{11}{\sqrt{3}}$ …………..(2)

(2) in (1)

$\Rightarrow 50 + \frac{H}{\sqrt{3}} = H\sqrt{3}$

$\Rightarrow H\sqrt{3} - \frac{H}{\sqrt{3}} = 50$

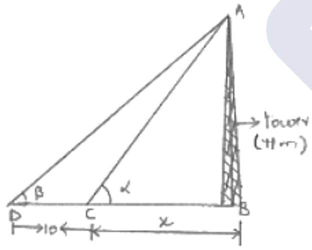
$\Rightarrow H\left(\frac{3-1}{\sqrt{3}}\right) = 50$

$\Rightarrow H = \frac{50\sqrt{3}}{2} = 25\sqrt{3}$

$\therefore$ Height of tower $H = 25\sqrt{3}m$

11. The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 m. longer than when it was 60^0. Find the height of the tower.

Sol:



Let the length of shadow of tower when angle of elevation is $(\alpha = 60°)$ be $x\,m = BC$ then according to problem

Length of the shadow with angle of elevation $(\beta = 45°)$ is $(10 + x)m = BD.$

If we represent the, above data in form of figure then it forms a triangle ABD is which triangle ABC is included with $\angle B = 90°$

Let height of tower be $Hm = AB$

If in right angle triangle one of the angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{BC}$

$\Rightarrow \tan 60° = \frac{H}{x}$

$\Rightarrow x = \frac{H}{\sqrt{3}}$ ……………(1)

$\tan\beta = \frac{AB}{BD}$

$\Rightarrow \tan 45° = \frac{H}{x+10}$

$\Rightarrow x+10 = H$

$\Rightarrow x = H-10$ ……………(2)

Substitute $x = H-10$ in (1)

$H-10 = \frac{H}{\sqrt{3}}$

$\Rightarrow \sqrt{3}H - 10\sqrt{3} = H$

$\Rightarrow (\sqrt{3}-1)H = 10\sqrt{3}$

$\Rightarrow H = \frac{10\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow H = \frac{10\sqrt{3}\times\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{10\sqrt{3}(\sqrt{3}+1)}{2}$

$= 5(3+\sqrt{3})$

$= 23\cdot 66m$

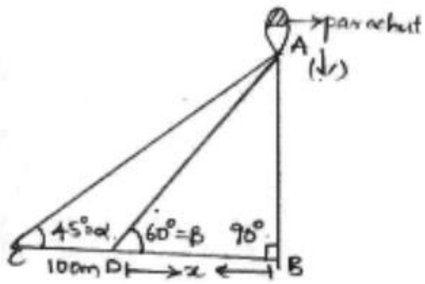
Rationalize denominator rationalizing factor of $a+\sqrt{b}$ is $a-\sqrt{b}$

$\therefore$ Height of tower

$= 23\cdot 66m$

12. A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

Sol:



Let is the parachutist at highest point A. Let C and D be points which are $100m$ a part on ground where from then $CD = 100m$

Angle of elevation from point $C = 45°[\alpha]$

Angle of elevation from point $B = 60°[\beta]$

Let B be the point just vertically down the parachute

Let us draw figure according to above data then it forms the figure as shown in which ABC is triangle and ABD included in it with

ABD triangle included

Maximum height of parachute

From ground $= AB = Hm$

Distance of point where parachute falls to just nearest observation point $= xm$

If in right angle triangle one of the included angle θ. Then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{CB}$$

$$\tan 45° = \frac{H}{100+x}$$

$$100 + x = H \qquad \text{..........(1)}$$

$$\tan\beta = \frac{AB}{DB}$$

$$\tan 60° = \frac{H}{x}$$

$$H = \sqrt{3}x \qquad \text{..........(2)}$$

From (1) and (2)

$$\sqrt{3}x = 100 + x$$

$\left(\sqrt{3}-1\right)x=100$

$x=\frac{100}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$=\frac{100\left(\sqrt{3}+1\right)}{2}$

$\Rightarrow x=50\left(\sqrt{3}+1\right)m.$

$\Rightarrow x=50\left(1\times 732+1\right)$

$\Rightarrow x=50\left(2\times 732\right)$

$\Rightarrow x=136.6m$ in (2)

$H=\sqrt{3}\times 136\times 6=1\cdot 732\times 136\cdot 6=236\cdot 6m$

Maximum height of parachute from ground

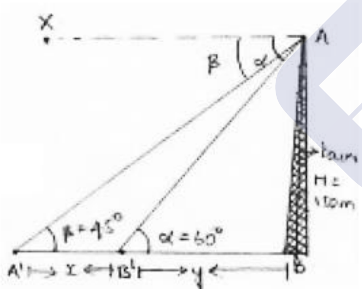
$H=236\cdot 6m$

Distance between point where parachute falls on ground and just observation is

$x=136\cdot 6m$

13. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

Sol:



Height of tower, $H=AB=150m.$

Let A and B be two objects m the ground

Angle of depression of objects $A^{'}\left[\angle A'Ax\right]=\beta=45°=\angle AA^{'}B\left[Ax\right]\left[A'B\right]$

Angle of depression of objects B'

$\angle xAB'=\alpha=60°=\angle AB'B\left[Ax\right]\left[A'B\right]$

Let $A'B'=x \quad B'B=y$

In we figure the above data in figure, then it is as shown with $\angle B=90°$

In any right angled triangle if one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{BB'}$$

$$\Rightarrow \tan 60° = \frac{150}{y}$$

$$\Rightarrow y = \frac{150}{\sqrt{3}} \qquad \text{..........(1)}$$

$$\tan\beta = \frac{AB}{A'B}$$

$$\Rightarrow \tan 45° = \frac{150}{x+y}$$

$$\Rightarrow x + y = 150 \qquad \text{..........(2)}$$

$$\text{(1) and (2)} \Rightarrow x + \frac{150}{\sqrt{3}} = 150$$

$$\Rightarrow x + \frac{50\times 3}{\sqrt{3}} = 150$$

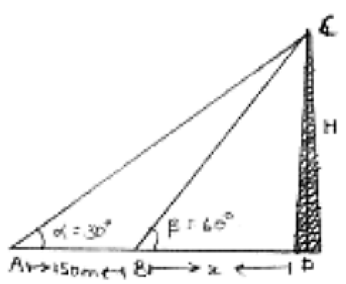
$$\Rightarrow x = 150 - 50\sqrt{3} = 150 - 50(1732)$$

$$= 150 - 86\cdot 6 = 63\cdot 4m$$

Distance between objects $A'B' = 63\cdot 4m$

14. The angle of elevation of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes 60°. Show that the height of the tower is 129.9 meters (Use $\sqrt{3}$ = 1.732).

Sol:



Angle of elevation of top of tower from first point $A, \alpha = 30°$

Let we advanced through A to b by 150m then $AB = 150m$

Angle of elevation of top of lower from second point $B, \beta = 60°$

Let height of tower $CD = H\ m$

If we represent the above data in from of figure then it forms figure as shown with $\angle D = 90°$

If in right angled triangle, one of included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \frac{CD}{AD}$

$\Rightarrow \tan 30° = \frac{H}{150+x}$

$150 + x = H\sqrt{3}$ ....(1)

$\tan\beta = \frac{CD}{BD}$

$\Rightarrow \tan 60° = \frac{H}{x}$

$\Rightarrow H = x\sqrt{3} \Rightarrow x = \frac{H}{\sqrt{3}}$ ....(2)

(2) in (1)

$150 + \frac{H}{\sqrt{3}} = H\sqrt{3} \Rightarrow H\left(\sqrt{3} - \frac{1}{3}\right) = 150$

$\Rightarrow H\left(\frac{3-1}{\sqrt{3}}\right) = 150 \Rightarrow H = \frac{150\times\sqrt{3}}{2} = 75\sqrt{3} = 75\times 1\cdot 732$

$= 129\cdot 9m$

$\therefore$ height of tower $= 129\cdot 9m$

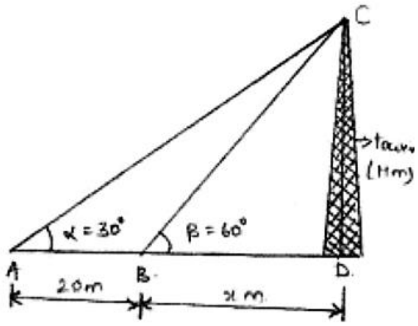
15. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626]

Sol:

91.65m, 146.7m

16. The angle of elevation of the top of a tower from a point A on the ground is 30°. Moving a distance of 20metres towards the foot of the tower to a point B the angle of elevation increases to 60°. Find the height of the tower & the distance of the tower from the point A.

Sol:



Angle of elevation of top of tower from points A $\alpha = 30°$

Angle of elevation of top of tower from points B $\beta = 60°$

Distance between A and B, $AB = 20m$

Let height of tower $CD = 'h'm$

Distance between second point B from foot of tower $bc\ 'x'm$

If we represent the above data in the figure, then it forms figure as shown with $\angle D = 90°$

In right angled triangle if one of the included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CD}{AD}$$

$$\tan 30° = \frac{h}{20+x}$$

$$20 + x = h\sqrt{3} \qquad \ldots\ldots\ldots\ldots(1)$$

$$\tan\beta = \frac{CD}{BD}$$

$$\tan 60° = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \qquad \ldots\ldots\ldots\ldots(2)$$

(2) in (1) $\Rightarrow 20 + \dfrac{h}{\sqrt{3}} = h\sqrt{3} \Rightarrow h\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right) = 20$

$$\Rightarrow h\left(\frac{3-1}{\sqrt{3}}\right) = 20 \Rightarrow h = \frac{20\sqrt{3}}{2} = 10\times\sqrt{3} = 17\cdot 32m$$

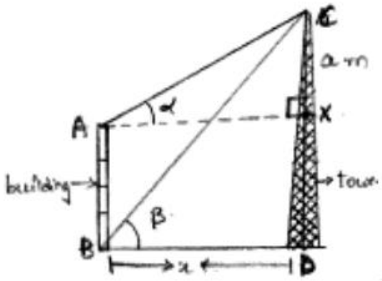
$$x = \frac{10\sqrt{3}}{\sqrt{3}} = 10m.$$

Height of tower $h = 17\times 32m$

Distance of tower from point $A = (20+10) = 30m$

17. From the top of a building 15 m high the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and building.

Sol:



Let AB be the building and CD be the tower height of the building is $15m = h = AB.$

Angle of elevation of top of tower from top of building $\alpha = 30°$

Angle of elevation of top of tower from bottom of building $\beta = 60°$

Distance between tower and building $BD = x$

Let height of tower above building be $'a'$ m

If we represent the above data is from of figure then it forms figure as shown with $\angle D = 90°$ also draw $AX \parallel BD, \angle AXC = 90°$

Here $ABDX$ is a rectangle

$\therefore BD = DX = 'x'm \quad AB = XD = h = 15m$

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CX}{AX}$$

$$\Rightarrow \tan 30° = \frac{a}{x}$$

$$\Rightarrow x = a\sqrt{3} \qquad \ldots\ldots\ldots\ldots..(1)$$

$$\tan\beta = \frac{CD}{BD}$$

$$\Rightarrow \tan 60° = \frac{a+15}{x}$$

$$\Rightarrow x\sqrt{3} = a + 15 \qquad \ldots\ldots\ldots\ldots..(2)$$

(1) and (2)

$$\Rightarrow \frac{x}{x\sqrt{3}} = \frac{a\sqrt{3}}{a+15} \Rightarrow a + 15 = a\sqrt{3}\left(\sqrt{3}\right)$$

$$a + 15 = 3a$$

$$\Rightarrow 2a = 15$$

$\Rightarrow a=\frac{15}{2}=7\cdot 5m$

$x=a\sqrt{3}$

$=7\cdot 5\times 1\cdot 732$

$=12\cdot 99m$

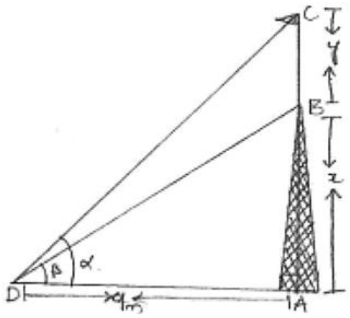
Height of tower above ground $=h+a$

$=15+7\cdot 5=22\cdot 5m$

Distance between tower and building $=12\cdot 99m$

18. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 meters away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Sol:



Let AB be the tower and BC be flagstaff on the tower

Distance of point of observation from foot of tower $BD=9m$

Angle of elevation of top of flagstaff $[c]\alpha=60°$

Angle of elevation of bottom of flag pole $[B]\beta=30°$

Let height of tower $='x'=AB$

Height of pole $='y'=BC$

The above data is represented in form of figure a shown with $\angle A=90°$

If in right triangle one of the included is $\theta,$ then

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha=\frac{AC}{AD}$

$\tan 60°=\frac{x+y}{9}$

$x+y=9\sqrt{3}$

$y=9\sqrt{3}-3\sqrt{3}$

$\tan\beta = \frac{AB}{AD}$

$\tan 30° = \frac{x}{9}$

$x = \frac{9}{\sqrt{3}} = 3\sqrt{3} = 5\cdot196m$

$= 6\sqrt{3} = 6\times1\cdot732$

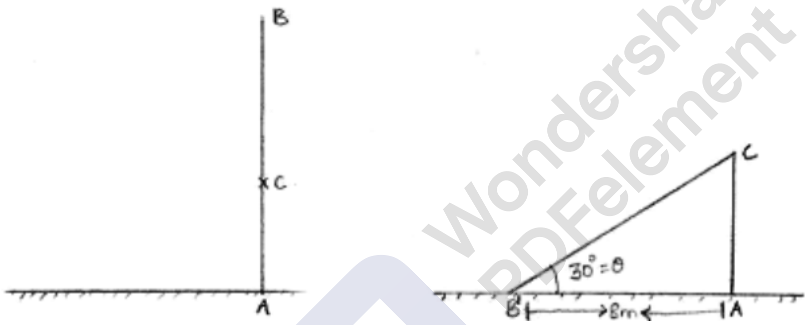
$= 10\cdot392m$

Height of tower $x = 5\cdot196m$

Height of pole $y = 10\cdot392m$

19. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol:



Let initially tree height be AB

Let us assumed that the tree is broken at point C

Angle made by broken part CB' with ground is $30° = \theta$

Distance between foot of tree of point where it touches ground $B'A = 8m$

Height of tree $= h = AC + CB' = AC + CB$

The above information is represent in the form of figure as shown

$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$\cos 30° = \frac{AB'}{CB'}$

$\frac{\sqrt{3}}{2} = \frac{B}{CB'}$

$CB' = \frac{16}{\sqrt{3}}$

$\tan 30° = \frac{CA}{AB'}$

$\frac{1}{\sqrt{3}} = \frac{CA}{8}$

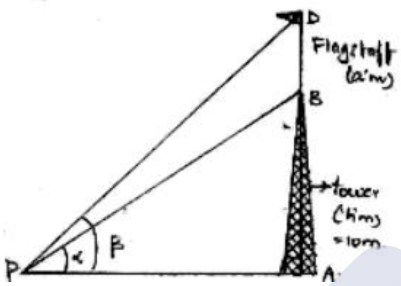
$CA = \frac{8}{\sqrt{3}}$

Height of tree $= CB' + CA = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{8\times 3}{\sqrt{3}}$

$= 8\sqrt{3}m$

20. From a point P on the ground the angle of elevation of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from P is 45°. Find the length of the flag-staff and the distance of the building from the point P.(Take $\sqrt{3}$= 1.732).

Sol:



Let AB be the tower and 80 be the flagstaff Angle of elevation of top of building from P $\alpha = 30°$

$AB =$ height of tower $= 10m$

Angle of elevation of top of flagstaff from P $\beta = 45°$

Let height of flagstaff $BD = 'a'm$

The above information is represented in form of figure as shown with $\angle A = 90°$

In a right angled triangle if one of the included

Angle is θ

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{AP'}$

$\tan 30° = \frac{10}{AP}$

$AP = 10\sqrt{3}$

$= 10\times 1\cdot 732$

$=17\cdot 32$

$\tan\beta=\frac{AD}{AP}$

$\tan 45°=\frac{10+a}{AP}$

$10+a=AP$

$a=17\cdot 32-10$

$=7\cdot 32m$

Height of flagstaff $'\theta'=7\cdot 32m$

Distance between P and foot of tower $=17\cdot 32m.$

21. A 1 .6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

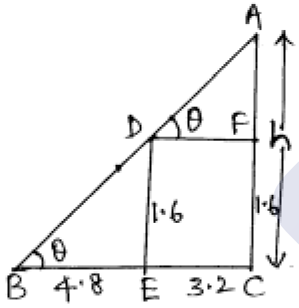
Sol:

Let AC be the lamp past of height $'h'$

We assume that $ED=1\cdot 6m, BE=4\cdot 8m$ and $EC=3\cdot 2m$

We have to find the height of the lamp post

Now we have to find height of lamp post using similar triangles



Since triangle BDE and triangle ABC are similar,

$\frac{AC}{BC}=\frac{ED}{BE}$

$\Rightarrow\frac{h}{4\cdot 8+3\cdot 2}=\frac{1\cdot 6}{4\cdot 8}$

$\Rightarrow h=\frac{8}{3}$

Again we have to find height of lamp post using trigonometry ratios

In ΔADE, $\tan\theta=\frac{1\cdot 6}{4\cdot 8}$

$\Rightarrow\tan\theta=\frac{1}{3}$

Again in ΔABC,

$$\tan\theta = \frac{h}{4\cdot 8 + 3\cdot 2}$$

$$\Rightarrow \frac{1}{3} = \frac{h}{8}$$

$$\Rightarrow h = \frac{8}{3}$$

Hence the height of lamp post is $\frac{8}{3}$.

22. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol:

$\mathbf{19\sqrt{3}}$

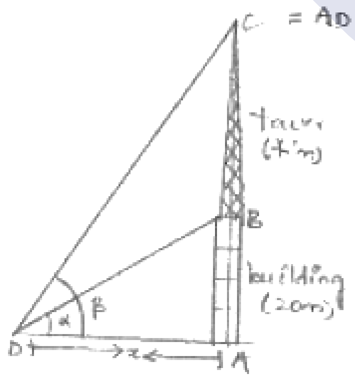
23. The shadow of a tower standing on a level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Find the height of the tower

Sol:

$\mathbf{20\sqrt{3}}$

24. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower.

Sol:



Given height of building $= 20m = AB$

Let height of tower above building $= 'h' = BC$

Height of tower + building $= (h+20)\,m$ [from ground] $= CA$

Angle of elevation of bottom of tour, $\alpha = 45°$

Angle of elevation of top of tour, $\beta = 60°$

Let distance between tower and observation point $='x'm$
The above data is represented in $= AD$
The form of figure as shown is one of the included angle is right angle triangle is a then

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\alpha = \frac{AB}{AD}$$

$$\Rightarrow \tan 45° = \frac{20}{x}$$

$$\Rightarrow x = 20m$$

$$\tan\beta = \frac{CA}{DA}$$

$$\Rightarrow \tan 60° = \frac{h+20}{x}$$

$$\Rightarrow h + 20 = 20\sqrt{3}$$

$$\Rightarrow h = 20\left(\sqrt{3}-1\right)$$

Height of tower $h = 20\left(\sqrt{3}-1\right)$

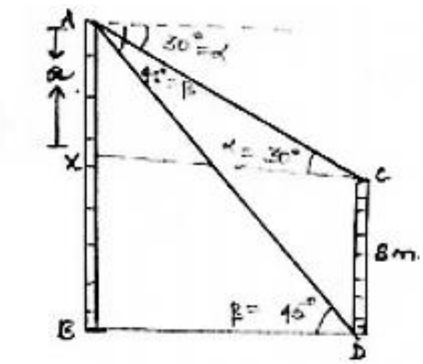
$$= 20(1\cdot 732 - 1)$$

$$= 20 \times 0\cdot 732$$

$$= 14\cdot 64m$$

25. The angles of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of the multistoried building and the distance between the two buildings.

Sol:



Let height of multistoried building $'h'm = AB$
Height of tall building $= 8m = CD$
Angle of depression of top of tall building $\alpha = 30°$

Angle of depression of bottom of tall building $\beta = 45°$

Distance between two building $= 'x'm = BD$

Let $Ax = x$

$AB = AX + XB$ but $XB = CD$ [$\because AXCD$ is rectangle]

$AB = 'a'm + 8m$

$AB = (a+8)m$

The above information is represented in the form of figure e as shown

If in right triangle are of included angle is θ

Then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In ΔAXB

$\tan 30° = \frac{AX}{CX}$

$\frac{1}{\sqrt{3}} = \frac{a}{BD} = \frac{a}{x}.$

$\Rightarrow x = a\sqrt{3}$ ....(1)

In ΔABD

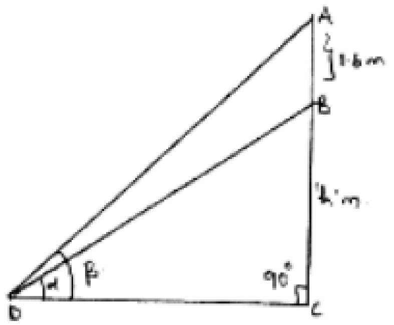
$\tan 45° = \frac{AB}{BD} = \frac{a+8}{x}$

$1 = \frac{a+8}{x}$

$\Rightarrow a+8 = x$ ....(2)

26. A statue I .6 m tall stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol:



Let height of pedestal be $'h'm$

Height of status $= 1\cdot 6m$

Angle of elevation of top of status $\alpha = 60°$

Angle of elevation of pedestal of status $\alpha = 60°$

The above data is represented in the form of figure as shown.

If in right angle triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{BC}{BD}$$

$$\tan 45° = \frac{h}{DC}$$

$$DC = h\sqrt{8}.1$$

$$DC = 'h'm \quad ....(1)$$

$$\tan\beta = \frac{AC}{DC}$$

$$\tan 60° = \frac{h+1\cdot 6}{DC}$$

$$DC = \frac{h+1\cdot 6}{BC} \quad ....(2)$$

From (1) and (2) $h = \dfrac{h+1\cdot 6}{\sqrt{3}}$

$$\Rightarrow h\sqrt{3} = h + 1\cdot 6$$

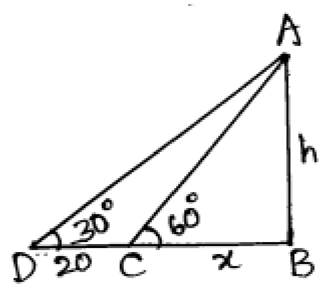
$$\Rightarrow h\left(\sqrt{3}-1\right) = 1\cdot 6$$

$$\Rightarrow h = \frac{1\cdot 6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 0.5\left(\sqrt{3}+1\right)$$

Height of pedestal $= 0\cdot 6\left(\sqrt{3}+1\right)m.$

27. A T.V. Tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the river.

Sol:



Let AB be the T.V tower of height $'h'm$ on a bank of river and $'D'$ be the point on the opposite of the river. An angle of elevation at top of tower is $60°$ and form the point $20m$ away them angle of elevation of tower at the same point is $30°$

Let $AB = h$ and $BC = x$

Here we have to find height and width of river the corresponding figure is here

In ΔCAB,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3}x = h$$

$$\Rightarrow \boxed{x = \frac{h}{\sqrt{3}}}$$

Again in ΔDBA,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow \sqrt{3}h = 20 + x$$

$$\Rightarrow \sqrt{3}h = 20 + \frac{h}{\sqrt{3}} \left[\because x = \frac{h}{\sqrt{3}}\right]$$

$$\Rightarrow \sqrt{3}h - \frac{h}{\sqrt{3}} = 20$$

$$\Rightarrow \frac{2h}{\sqrt{3}} = 20$$

$$\Rightarrow h = 10\sqrt{3}$$

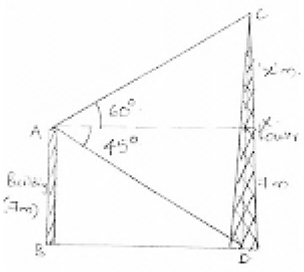
$$\Rightarrow x = \frac{h}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \boxed{x = 10}$$

Hence the height of the tower is $10\sqrt{3}m$ and width of the river is $10\,m.$

28. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol:



Given

Height of building $=7m=AB$

Height of cable tower $='H'm=CD$

Angle of elevation of top of tower, from top of building $\alpha=60°$

Angle of depression of bottom of tower, from top of building $\beta=45°$

The above data is represented in form of figure as shown

Let $CX='x'm$

$CD=DX+XC=7m+'x'm$

$=x+7m.$

In ΔADX

$$\tan 45°=\frac{\text{Opposite side}(\text{XD})}{\text{Adjacent side}(\text{AX})}$$

$$1=\frac{7}{AX}$$

$\Rightarrow AX=7m$

In ΔAXD

$$\tan 60°=\frac{XC}{AX}$$

$$\sqrt{3}=\frac{x}{H}$$

$\Rightarrow x=7\sqrt{3}$

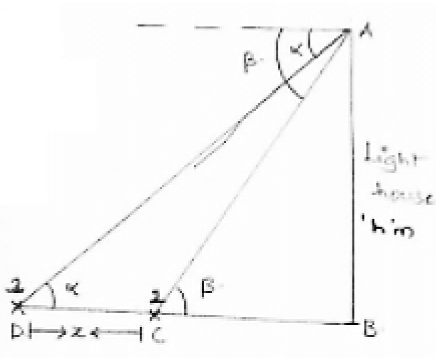
But $CD=x+7$

$=7\sqrt{3}+7=7\left(\sqrt{3}+1\right)m.$

Height of cable tower $=7\left(\sqrt{3}+1\right)m$

29. As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol:



Given

Height of light house $=75m='h'm=AB$

Angle of depression of ship 1 $\alpha=30°$

Angle of depression of ship 2 $\beta=45°$

The above data is represented in form of figure as shown.

Let distance between ships be $'x'm=CD$

In right triangle if one of included angle is θ then

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha=\frac{AB}{DB}$$

$$\tan 30°=\frac{75}{x+BC}$$

$$x+BC=75\sqrt{3} \qquad \ldots\ldots\ldots\ldots(1)$$

$$\tan\beta=\frac{AB}{CB}$$

$$\tan 45°=\frac{75}{BC}$$

$$BC=75 \qquad \ldots\ldots\ldots\ldots(2)$$

(2) in (1) $\Rightarrow x+75=75\sqrt{3}$

$\Rightarrow x=75\left(\sqrt{3}-1\right)$

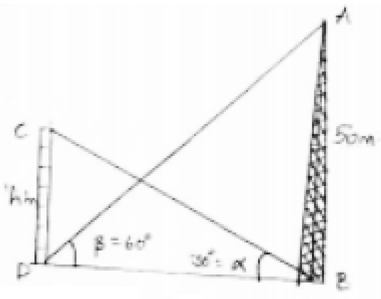
$\therefore$ Distance between ships

$='x'm$

$=75\left(\sqrt{3}-1\right)m.$

30. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol:



Angle of elevation of top of building from foot of tower $= 30° = \alpha$

Angle of elevation of top of tower, from foot of building $= 60° = \beta$

Height of tower $= 50m = AB$

Height of building $= 'h'm$

$= CD$

The above information is represented in form of figure as shown

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In ΔABC

$$\tan\beta = \frac{AB}{BD}$$

$$\tan 60° = \frac{50}{BD}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔCBD

$$\tan\alpha = \frac{CD}{BD}$$

$$\tan 30° = \frac{h}{\frac{50}{\sqrt{3}}}$$

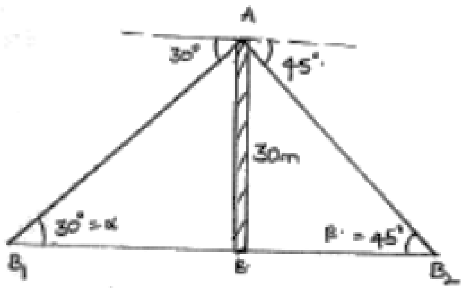
$$h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{50}{3}$$

$\therefore$ height of building $= \frac{50}{3} m$

31. From a point on a bridge across a river the angles of depression of the banks on opposite side of the river are 30° and 45° respectively. If bridge is at the height of 30 m from the banks, find the width of the river.

Sol:



Height of the bridge $= 30m[AB]$

Angle of depression of bank 1 i.e., $\alpha = 30°.[B_1]$

Angle of depression of bank 2 i.e., $\beta = 30°.[B_2]$

Given banks are on opposite sides

Distance between banks $B_1B_2 = B_1B + BB_2$

The above information is represented is the form of figure as shown in right angle triangle if one of the included angle is O then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

In ΔABB_1

$$\tan\alpha = \frac{AB}{B_1B}$$

$$\tan 30 = \frac{30}{B_1B}$$

$$B_1B = 30\sqrt{3}m$$

In ΔABB_2

$$\tan\beta = \frac{AB}{BB_2}$$

$$\tan 45° = \frac{30}{BB_2}$$

$$BB_2 = 30m$$

$$B_1B_2 = B_1B + BB_2 = 30\sqrt{3} + 30$$

$$= 30\left(\sqrt{3}+1\right)$$

Distance between banks $= 30\left(\sqrt{3}+1\right)m$

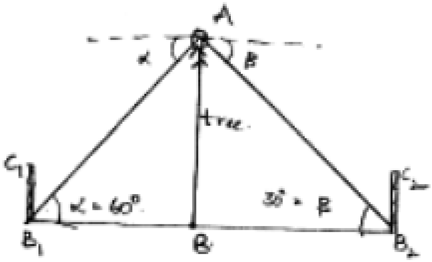
32. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol:

$20\sqrt{3}$m

33. A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

Sol:



Height of tree $AB = 20m$

Angle of depression of pole 1 feet $\alpha = 60°$

Angle of depression of pole 2 feet $\beta = 30°$

B_1C_1 be one pole and B_1C_2 be other sides width of river $= B_1B_2$

$= B_1B + BB_2$

The above information is G represent in from of figure as shown

In right triangle, if one of included angle is 0

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{B_1B}$$

$$\tan 60° = \frac{20}{B_1B}$$

$$B_1B = \frac{20}{\sqrt{3}}$$

$$\tan\beta = \frac{AB}{BB_2}$$

$\tan 30° = \dfrac{20}{BB_2}$

$BB_2 = 20\sqrt{3}$

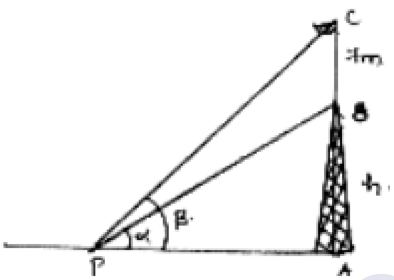
$B_1B_2 = B_2B + BB_2 = \dfrac{20}{\sqrt{3}} + 20\sqrt{3} = 20\left[\dfrac{1+3}{\sqrt{3}}\right] = \dfrac{20}{\sqrt{3}}$

Width of river $= \dfrac{80}{\sqrt{3}} m.$

$= \dfrac{80\sqrt{3}}{3} m.$

34. A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

Sol:



Given

Height of flagstaff $= 7m = BC$

Let height of tower $= 'h'm = AB$

Angle of elevation of bottom of flagstaff $\alpha = 30°$

Angle of elevation of top of flagstaff $\beta = 45°$

Points of desecration be $'p'$

The above data is represented in form of figure as shown

In right angle triangle if one of the induced angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \dfrac{AB}{AP}$

$\tan 30° = \dfrac{h}{AP}$

$AP = h\sqrt{3}$ ....(1)

$\tan\beta = \dfrac{AC}{AP}$

$\tan 45° = \frac{h+7}{AP}$

$AP = h+7$ ....(2)

From (1) and (2)

$h\sqrt{3} = h+7$

$h\sqrt{3} - h = 7$

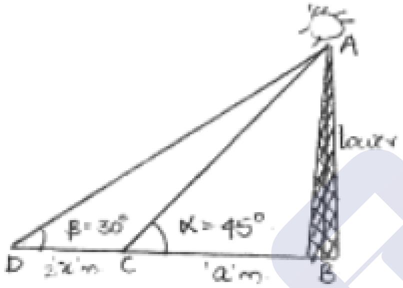
$h\left(\sqrt{3}-1\right) = 7 \Rightarrow h = \frac{7}{3-1} + \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{7\times\left(\sqrt{3}+1\right)}{2} = 3\cdot5\left(\sqrt{3}+1\right)$

Height of tower $= 3\cdot5\left(\sqrt{2}+1\right)m.$

35. The length of the shadow of a tower standing on level plane is found to be 2x metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of tower is x ($\sqrt{3}$ + 1) metres.

Sol:



Let

Length of shadow be $'a'm\left[BC\right]$ when sun attitude be $= 45°$

Length of shadow will be $\left(2x+a\right)m = 80$ when sun attitude is $\beta = 30°$

Let height of tower be $'h'm = AB$ the above information is represented in form of figure as shown

In right triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

In ABC

$\tan\alpha = \frac{AB}{BC}$

$\tan 45° = \frac{h}{a}$

$h = a$ ……….(1)

In ADB

$$\tan\beta = \frac{AB}{(2x+a)BC}$$

$$\tan 30° = \frac{h}{2x+a}$$

$2x + a = h\sqrt{3}$ ……….(2)

(1) in (2) $\Rightarrow 2x + h = h\sqrt{3}$

$\Rightarrow h\left(\sqrt{3}-1\right) = 2x$

$$\Rightarrow h = \frac{2x}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

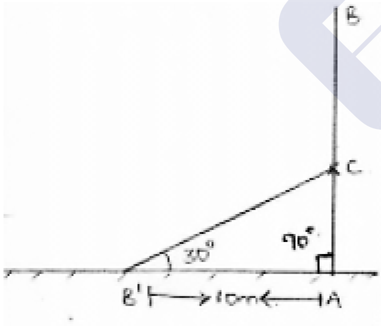
$$\Rightarrow \frac{2x\left(\sqrt{3}+1\right)}{2}$$

$\Rightarrow x\left(\sqrt{3}+1\right)$

Height of tower $= x\left(\sqrt{3}+1\right)m$

36. A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 meters. Find the height of the tree.

Sol:



Let AB be height of tree it is broken at pointe and top touches ground at B'

Angle made by top $\alpha = 30°$

Distance from foot of tree from point where A touches ground = O meter

The above information is represented in form of figure as shown

Height of tree $= AB = AC + CB$

$= AC + CB'$

In right triangle If one of angle is θ then

$\tan\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$	$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$$\tan 30° = \frac{AC}{B'A}$$

$$AC = \frac{10}{\sqrt{3}} m$$

$$\cos 30 = \frac{AB'}{B'C}$$

$$\frac{\sqrt{3}}{2} = \frac{10}{B'C}$$

$$B'C = \frac{20}{\sqrt{3}} m.$$

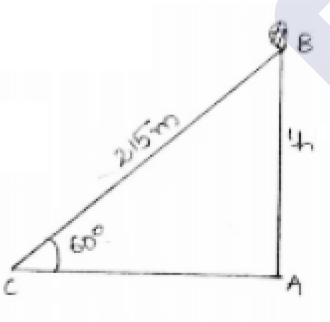
$$AB = CA + CB' = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

Height of tree $= 10\sqrt{3}m$

37. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 600 to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Sol:



Length of cable connected to balloon $= 215m[CB]$

Angle of inclination of cable with ground $\alpha = 60°$

Height of balloon from ground $= 'h'm = AB$

The above data is represented in form of figure as shown

In right triangle one of the included angle is θ then

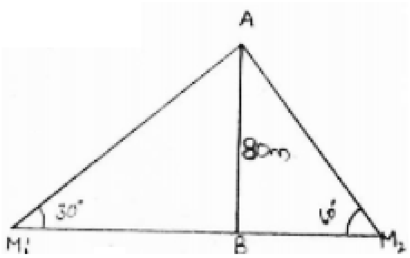
$\sin\theta = \frac{\text{Opposite side}}{\text{hypotenuse}}$

$\sin 60° = \frac{AB}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{215} \Rightarrow h = \frac{215\sqrt{3}}{2} = 107 \cdot 5\sqrt{3}m$

$\therefore$ Height of balloon from ground $= 107 \cdot 5\sqrt{3}m.$

38. Two men on either side of the cliff 80 m high observes the angles of elevation of the top of the cliff to be 300 and 600 respectively. Find the distance between the two men.

Sol:



Height of cliff $= 80m = AB.$

Angle of elevation from Man 1, $\alpha = 30° [M_1]$

Angle of elevation from Man 2, $\beta = 60° [M_2]$

Distance between two men $= M_1M_2 = BM_1 + BM_2.$

The above information is represented in form of figure as shown

In right angle triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{M_1B}$

$\tan 30° = \frac{80}{M_1B}$

$M_1B = 80\sqrt{3}$

$\tan\beta = \frac{AB}{BM_2}$

$\tan 60° = \frac{80}{BM_2}$

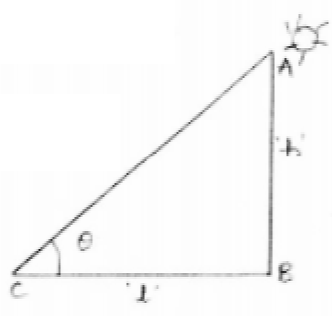
$BM_2 = \frac{80}{\sqrt{3}}$

$M_1M_2 = M_1B + BM_1 = 80\sqrt{3} + \frac{80}{\sqrt{3}} = \frac{80 \times 4}{\sqrt{3}} = \frac{320}{\sqrt{3}}$

Distance between men $= \frac{320\sqrt{3}}{3}$ meters

39. Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height.

Sol:



Let

Height of pole $='h'm=$ sun's altitude from ground length of shadow be $'l'$

Given that $l=h.$

Angle of elevation of sun's altitude be θ the above data is represented in form of figure as shown

In right triangle if one of the included angle is 0 then.

$$\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\theta=\frac{AB}{BC}\Rightarrow\tan\theta=\frac{h}{l}$$

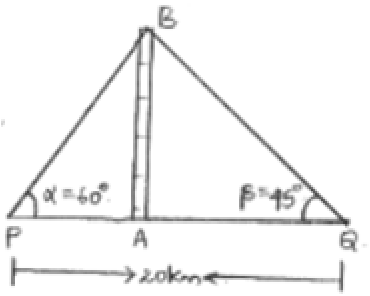
$$\Rightarrow\tan\theta=\frac{l}{l}[\because h=1]$$

$$\Rightarrow\theta=\tan^{-1}(1)=45°$$

Angle of sun's altitude is $45°$

40. A fire in a building B is reported on telephone to two fire stations P and 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road and Q observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel?

Sol:



Let AB be the building

Angle of elevation from point P [Fire station 1] $\alpha = 60°$

Angle of elevation from point Q [Fire station 1] $\beta = 45°$

Distance between fire stations $PQ = 20km$

The above information is represented in form of figure as shown

In right triangle if one of the angle is θ then.

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\alpha = \frac{AB}{AP}$$

$$\tan 60°- = \frac{AB}{AP}$$

$$AP = \frac{AB}{\sqrt{3}} \qquad \text{..........(1)}$$

$$\tan\beta = \frac{AB}{AQ}$$

$$\tan 45°- = \frac{AB}{AQ}$$

$$AQ = AB \qquad \text{..........(2)}$$

$$(1) + (2) \Rightarrow AP + AQ = \frac{AB}{\sqrt{3}} + AB = AB\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$$

$$\Rightarrow 20 = AB\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) \Rightarrow AB = \frac{20\sqrt{3}}{\sqrt{3}+1}$$

$$AB = \frac{20\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 10\sqrt{3}\left(\sqrt{3}-1\right) = 10\left(3-\sqrt{3}\right)$$

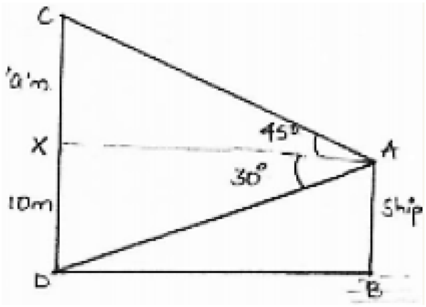
$$AQ = AB = 10\left(3-\sqrt{3}\right) = 10(3-1\cdot 732) = 12\cdot 64km$$

$$Ap = \frac{AB}{\sqrt{3}} = 10\left(\sqrt{3}-1\right) = 10\times 0\cdot 732 = 7\cdot 32km$$

Station 1 should send its team and they have to travel $7\cdot 32km$

41. A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 300. Calculate the distance of the cliff from the ship and the height of the cliff.

Sol:



Height of ship from water level $=10cm=AB$

Angle of elevation of top of cliff $\alpha=45°$

Angle of depression of bottom of cliff $\alpha=30°$

Height of cliff $CD='h'm.$

Distance of ship from foot of tower cliff

Height of cliff above ship be $'a'm$

Then height of cliff $=DX+XC$

$=(10+0)m$

The above data is represented in form of figure as shown

In right triangle, if one of the included angle is θ, then $\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan 45°=\frac{CX}{AX}$$

$$1=\frac{a}{AX}$$

$$AX='a'm$$

$$\tan 30°=\frac{XD}{AX}$$

$$\frac{1}{\sqrt{3}}=\frac{10}{AX}$$

$$AX=10\sqrt{3}$$

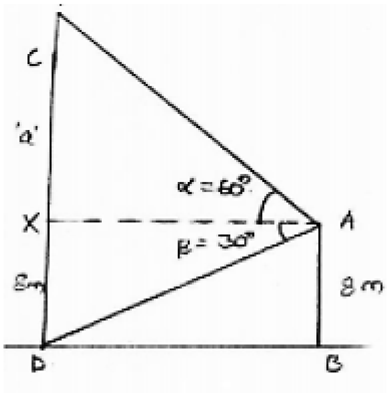
$$\therefore a=10\sqrt{3}m.$$

Height of cliff $=10+10\sqrt{3}=10+\left(\sqrt{3}+1\right)m.$

Distance between ship and cliff $=10\sqrt{3}m.$

42. A man standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Sol:



Height of ship above water level $= 8m = AB$

Angle of elevation of top of cliff (hill) $\alpha = 60°$

Angle of depression of bottom of hill $\beta = 30°$

Height of hill $= CD$

Distance between ship and hill $= AX$.

Height of hill above ship $= CX = 'a'm$

Height of hill $= (a+8)m.$

The above data is represented in form of figure as shown

In right triangle if one of included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CX}{AX}$$

$$\tan 60° = \frac{a}{AX}$$

$$AX = \frac{a}{\sqrt{3}}$$

$$\tan\beta = \frac{XD}{AX}$$

$$\tan 30° = \frac{8}{AX}$$

$$AX = 8\sqrt{3}$$

$$\therefore \frac{a}{\sqrt{3}} = 8\sqrt{3} \Rightarrow a = 24m.$$

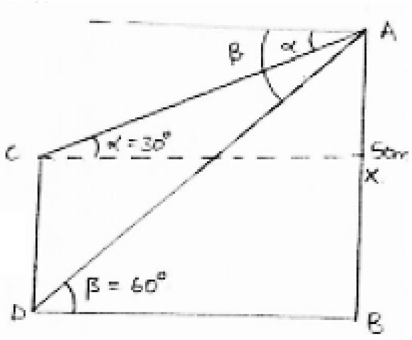
$$AX = 8\sqrt{3}m$$

$\therefore$ Height of cliff hill $= (24+8)m = 32m$

Distance between hill and ship $8\sqrt{3}m.$

43. There are two temples, one on each bank of a river, just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Sol:



Height of temple $1(AB) = 50m$

Angle of depression of top of temple 2, $\alpha = 30°$

Angle of depression of bottom of temple 2, $\beta = 60°$

Height of temple $2(CD) = 'h'm$

Width of river $= BD = 'x'm.$ the above data is represents in form of figure as shown

In right triangle if one of $'h'm$ included angle is θ, then

$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$ here $BD = CX, CD = BX,$

$$\tan\alpha = \frac{AX}{CX}$$

$$\tan 30° = \frac{AX}{CX}$$

$$CX = A \times \sqrt{3}$$

$$\tan\beta = \frac{AB}{BD}$$

$$\tan 60° = \frac{50}{CX}$$

$$CX = \frac{50}{\sqrt{3}}$$

$$AX\left(\sqrt{3}\right) = \frac{50}{\sqrt{3}} \Rightarrow AX = \frac{50}{3}m.$$

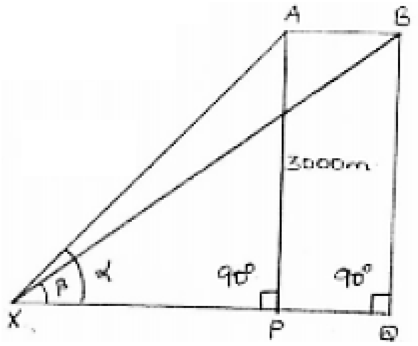
$$CD = XB = AB - AX = 50 - \frac{50}{3} = \frac{100}{3}m$$

Width of river $= \frac{50}{\sqrt{2}} m$

Height of temple 2 $= \frac{100}{3} m$

44. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 meters, find the speed of the aeroplane.

Sol:



Let aeroplane travelled from A to B in 15 sec

Angle of elevation of point A $\alpha = 45°$

Angle of elevation of point B $\beta = 30°$

Height of aeroplane from ground $= 3000$ meters

$= AP = BQ$

Distance travelled in 15 sees $= AB = PQ$

Velocity (or) speed = distance travelled time the above data is represents is form of figure as shown

In right triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AP}{XP}$$

$$\tan 45° = \frac{3000}{XP}$$

$$XP = 3000m$$

$$\tan\beta = \frac{BQ}{XQ}$$

$$\tan 30° = \frac{3000}{XQ}$$

$$XQ = 3000\sqrt{3}$$

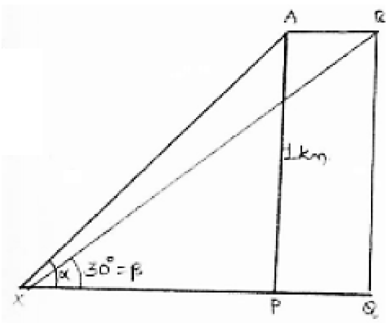
$$PQ = XQ - XP = 3000\left(\sqrt{3}-1\right)m$$

$$\text{Speed } = \frac{PQ}{time} = \frac{3000\left(\sqrt{3}-1\right)}{15} = 200\left(\sqrt{3}-1\right)$$
$$= 2000 \times 0\cdot 732$$
$$= 146\cdot 4\ m/\sec$$

Speed of aeroplane $= 146.4\ m/\sec$

45. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.

Sol:



Let aeroplane travelled from A to B in 10 secs

Angle of elevation of point $A = \alpha = 60°$

Angle of elevation of point $B = \beta = 30°$

Height of aeroplane from ground $= 1km = AP = BQ$

Distance travelled in 10 sec $= AB = PQ$

The above data is represent in form of figure as shown

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{AP}{PX}$$

$$\tan 60° = \frac{1}{PX}$$

$$PX = \frac{1}{\sqrt{3}} km$$

$$\tan\beta = \frac{BQ}{XQ}$$

$$\tan 30° = \frac{1}{XQ}$$

$XQ=\sqrt{3}km$

$PQ=XQ-PX=\sqrt{3}-\frac{1}{\sqrt{3}}=\frac{2}{\sqrt{3}}km.=\frac{2\sqrt{3}}{2}km.$

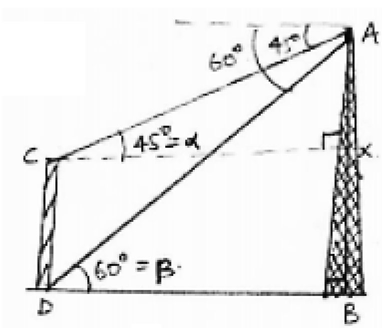
Speed $=\frac{PQ}{time}=\frac{2\sqrt{3}/3km}{\frac{10}{60\times 60}hr}=\frac{2\sqrt{3}}{\cancel{3}}\times 60\times \cancel{6}^{2}$

$=240\sqrt{3}\ km/hr$

Speed of aeroplane $=240\sqrt{3}\ km/hr$

46. From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

Sol:



$AB=$ height of tower $=50m.$
$CD=$ height of (Pole)
Angle of depression of top of building $\alpha=45°$
Angle of depression of bottom of building $\beta=60°$
The above data is represent in the form of figure as shown

In right triangle one of included angle is θ then $\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha=\frac{AX}{CX}$

$\tan 45°=\frac{AX}{CX}$

$AX=CX$

$\tan\beta=\frac{AB}{BD}$

$\tan 60°=\frac{50}{BD}$

$CX=\frac{50}{\sqrt{3}}$

$AX = \frac{50}{3} m = BD$

$CD + AB - AX = 50 - \frac{50}{\sqrt{3}} = \frac{50\left(\sqrt{3}-1\right)}{\sqrt{3}}$

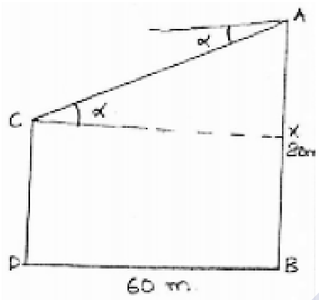
$= \frac{50}{3}\left(3-\sqrt{3}\right)$

Height of building (pole) $= \frac{50}{3}\left(3-\sqrt{3}\right) m.$

Distance between pole and tower $= \frac{50}{\sqrt{3}} m.$

47. The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree.

Sol:



Distance between trees $= 60m.[80]$

Height of second tree $= 80m[CD]$

Let height of first tree $= 'h'm[AB]$

Angle of depression from second tree top from first tree top $\alpha = 45°$

The above information is represent in form of figure as shown

In right triangle if one of the included angle is 0 their

$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

Draw $CX \perp AB, CX = BD = 60n.$

$XB = CD = AB - AX$

$\tan\alpha = \frac{AX}{CX}$

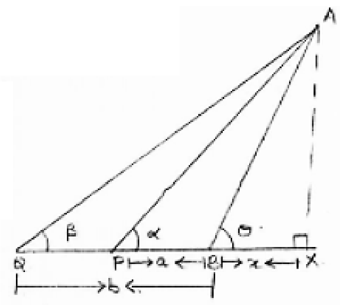
$\tan 45° = \frac{AX}{60} \Rightarrow AX = 60m.$

$XB = CD = AB - AX$

$= 80 - 60$
$= 20m$
Height of second tree $= 80m$
Height of first tree $= 20m$

48. A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β Prove that the height of the op from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$

Sol:



AB be the tree leaning east
From distance $'a'm$ from tree, Angle of elevation be α at point P.
From distance $'b'm$ from tree, Angle of elevation be β at point Q.
The above data is represented in the form of figure as shown in right triangle if one of the included angle is θ then

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

Draw $AX \perp QB$ let $BX = 'a'm$

$$\tan\alpha = \frac{AX}{PX}$$

$$\tan\alpha = \frac{AX}{x+a}$$

$$\cot\alpha = \frac{x+a}{Ax}$$

$x + B = AX \cot$ ....(1)

$$\tan\beta = \frac{AX}{QX}$$

$$\tan\beta = \frac{AX}{x+b}$$

$$\cot\beta = \frac{x+b}{AX}$$

$x + B = AX \cot\beta$ ....(2)

(2) and (1) $\Rightarrow (x+b)-(x+a) = AX\cot\beta - AX\cot\alpha$

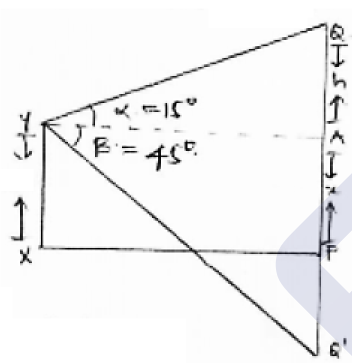
$$\Rightarrow b-a = AX\left[\frac{\tan\alpha - \tan\beta}{\tan\alpha\cdot\tan\beta}\right]$$

$$\Rightarrow AX = \frac{(b-a)\tan\alpha\cdot\tan\beta}{\tan\alpha-\tan\beta}$$

$\therefore$ Height of top from ground $= \dfrac{(b-0)\tan\alpha\cdot\tan\beta}{\tan\alpha-\tan\beta}$

49. The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower.
Sol:

50. The angle of elevation of a stationery cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level? (Use tan 15° = 0.268)
Sol:



Let cloud be at height PQ as represented from lake level

From point x, 2500 meters above the lake angle of elevation of top of cloud $\alpha = 15°$

Angle of depression of shadow reflection in water $\beta = 45°$

Here $PQ = PQ'$ draw $AY \perp PQ$

Let $AQ = 'h' m AP = 'x' m.$

$PQ = (h+x)m \quad PQ' = (h+x)m$

The above data is represented in from of figure as shown

In right triangle if one of included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan 15° = \frac{AQ}{AY}$$

$\Rightarrow 0\cdot268 = \dfrac{h}{AY}$

$\Rightarrow AY = \dfrac{h}{0\cdot268}$ ....(1)

$\tan 45° = \dfrac{AB'}{AY} = \dfrac{AP + PQ'}{AY}$

$\Rightarrow AY = x + (h + x)$

$= h + 2x$

$\Rightarrow AY = h + 2x$ ....(2)

From (1) and (2) $\dfrac{h}{0\cdot268} = h + 2x \Rightarrow 3\cdot131h - h = 2\times2500$

$\Rightarrow h = \dfrac{5000}{0\cdot731} = 1830\cdot8312$

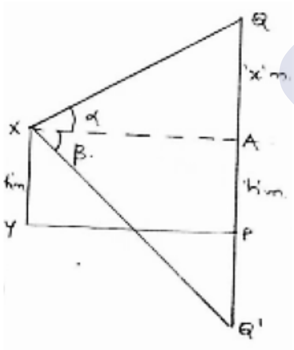
Height of cloud above lake $= h + x$

$= 1830\cdot8312 + 2500$

$= 4300\cdot8312\ m$

51. If the angle of elevation of a cloud from a point h meters above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is $\frac{2\,h\sec\alpha}{\tan\beta - \tan\alpha}$

Sol:



Let x be point $'b'$ meters above lake

Angle of elevation of cloud from $X = \alpha$

Angle of depression of cloud refection in lake $= \beta$

Height of cloud from lake $= PQ$

PQ' be the reflection then $PQ' = PQ$

Draw $XA \perp PQ, AQ = 'x'm \quad AP = XY = 'h'm.$

Distance of cloud from point of observation is XQ

The above data is represented in form of figure as shown

In ΔAQX

$$\tan\alpha = \frac{AQ}{AX}$$

$$\tan\alpha = \frac{x}{AX} \qquad \ldots\ldots\ldots.(1)$$

In $\Delta AXQ'$

$$\tan\beta = \frac{AQ'}{AX}$$

$$\tan\beta = \frac{h+x+h}{AX} \qquad \ldots\ldots\ldots.(2)$$

(2) and (1) $\Rightarrow \tan\beta - \tan\alpha = \frac{2h}{AX} \Rightarrow AX = \frac{2h}{\tan\beta - \tan\alpha}$

In ΔAXQ

$$\cos\alpha = \frac{AX}{XQ} \Rightarrow QX = AX \sec$$

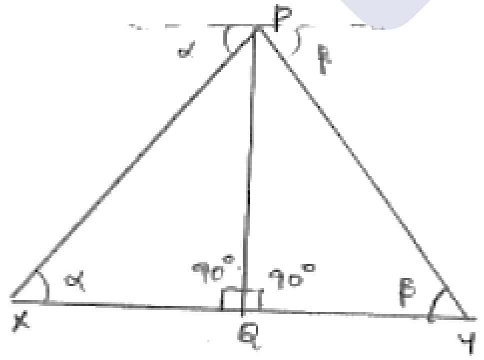
$$\Rightarrow XQ = \frac{2h\sec}{\tan\beta - \tan\alpha}$$

$\therefore$ Distance of cloud from point of observation

$= 2h\sec\alpha / \tan\beta - \tan\alpha$

52. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β Show that the height in miles of aeroplane above the road is given by $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

Sol:



Let PQ be height of aeroplane from ground x and y be two mile stones on opposite sides of the aeroplane $xy = 1$ mile

Angle of depression of x from $p = \alpha$

Angle of depression of y from $p = \beta$

The above data is represented in form of figure as shown

In right triangle, if one of included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In $\Delta P \times Q$

$$\tan\alpha = \frac{PQ}{XQ}$$

$$XQ = \frac{PQ}{\tan\alpha}$$

In PQY

$$\tan\beta = \frac{PQ}{QY}$$

$$QY = \frac{PQ}{QY}$$

$$XQ + QY = \frac{PQ}{\tan\alpha} + \frac{PQ}{\tan\beta} \Rightarrow XY = PQ\left[\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right]$$

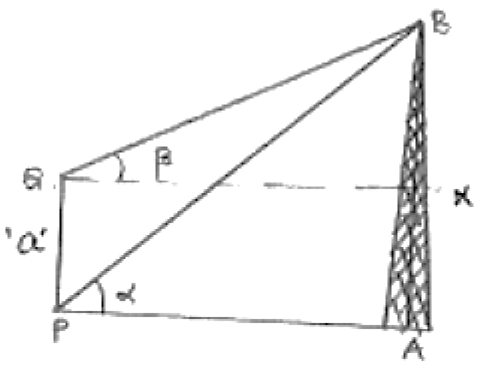
$$\Rightarrow 1 = PQ\left[\frac{\tan\alpha + \tan\beta}{\tan\alpha \cdot \tan\beta}\right]$$

$$\Rightarrow PQ = \frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}$$

Height of aeroplane $= \frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}$ miles

53. PQ is a post of given height a, and AB is a tower at some distance. If α and β are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

Sol:



PQ is part height $= 'a'm$ AB is tower height

Angle of elevation of B from $P = \alpha$

Angle of elevation of B from $Q = \beta$

The above information is represented in form of figure as shown

In right triangle if one of the included angle is θ, then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

Draw $QX \perp AB, PQ = AK$

In ΔBQX

$$\tan\beta = \frac{BX}{QX}$$

$$\Rightarrow \tan\beta = \frac{AB - AX}{QX}$$

$$\Rightarrow \tan\beta = \frac{AB - a}{QX} \qquad \ldots\ldots\ldots.(1)$$

In ΔBPA

$$\tan\alpha = \frac{AB}{AP}$$

$$\Rightarrow \tan\beta = \frac{AB}{QX} \qquad \ldots\ldots\ldots.(2)$$

(1) divided by (2)

$$\Rightarrow \frac{\tan\beta}{\tan\alpha} = \frac{AB - a}{AB} = 1 - \frac{a}{AB}$$

$$\Rightarrow \frac{a}{AB} = 1 - \frac{\tan\beta}{\tan\alpha} = \frac{\tan\alpha - \tan\beta}{\tan\alpha}$$

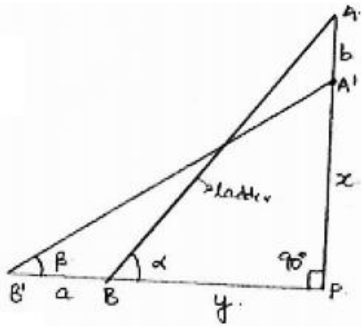
$$\Rightarrow \boxed{AB = \frac{a\tan\alpha}{\tan\alpha - \tan\beta}} Q \times \frac{AB}{\tan\alpha} = \frac{a}{\tan\alpha - \tan\beta}$$

Height of power $= a\tan\alpha\left(\tan\alpha - \tan\beta\right)$

Distance between past and tower $= a\left(\tan\alpha - \tan\beta\right)$

54. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Sol:



Let AB be ladder initially at an inclination α to ground
When its foot is pulled through distance $'a'$ let $BB' = 'a'm$ and $AA' = 'b'm$
New angle of elevation from $B' = B$ the above information is represented in form of figure as shown
Let $AP \perp$ ground $B'P \quad AB = A'B'$
$A'P = x \qquad BP = y$
In ΔABP

$$\sin\alpha = \frac{AP}{AB} \Rightarrow \sin\alpha = \frac{x+b}{AB} \qquad \ldots\ldots\ldots(1)$$

$$\cos\alpha = \frac{BP}{AB} \Rightarrow \cos\alpha = \frac{y}{AB} \qquad \ldots\ldots\ldots(2)$$

In $\Delta A'B'P.$

$$\sin\beta = \frac{A'P}{A'B'} \Rightarrow \sin\beta = \frac{x}{AB} \qquad \ldots\ldots\ldots(3)$$

$$\cos\beta = \frac{B'P}{A'B'} \Rightarrow \cos\beta = \frac{y+a}{AB} \qquad \ldots\ldots\ldots(4)$$

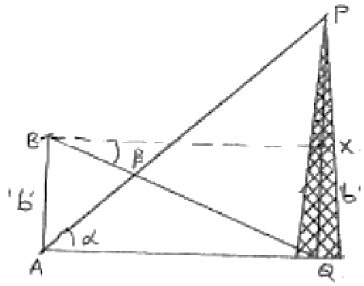
(1) and (3) $\Rightarrow \sin\alpha - \sin\beta = \frac{b}{AB}$

(4) and (2) $\Rightarrow \cos\beta - \cos\alpha = \frac{a}{AB}$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}}$$

55. A tower subtends an angle α at a point A in the plane of its base and the angle if depression of the foot of the tower at a point b metres just above A is β. Prove that the height of the tower is b tan α cot β

Sol:



Let height of tower be $'h'm = PQ$
Angle of elevation at point A on ground $= \alpha$
Let B be point $'b'm$ above the A.

Angle of depression of foot of tower from $B=\beta$ the above data is represented in ffrom of figure as shown draw $BX \perp PQ$ from figure $QX = 'b'm$

In ΔPBX

$$\tan\alpha = \frac{PQ}{BX\,(AD)} \qquad \ldots\ldots..(1)$$

In ΔQBX

$$\tan\beta = \frac{QX}{BX} \qquad \ldots\ldots..(2)$$

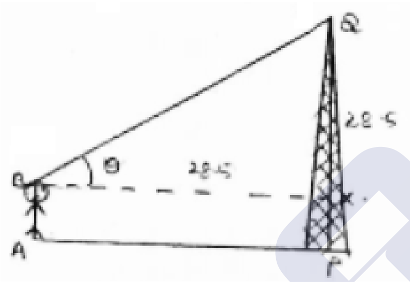
(1) and (2) $\Rightarrow \dfrac{\tan\alpha}{\tan\beta} = \dfrac{PQ}{QX}$

$$\Rightarrow PQ = QX \cdot \frac{\tan\alpha}{\tan\beta} = b \cdot \tan\alpha \cdot \cot\beta$$

$\therefore$ Height of tour $= b \cdot \tan\alpha \cdot \cot\beta$

56. An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye.

Sol:



Height of observer $= AB = 1\cdot 5m$

Height of tower $= PQ = 30m$

Height of tower above the observe eye $= 30 - 1\cdot 5$

$QX = 28\cdot 5m.$

Distance between tower and observe $XB = 28\cdot 5m.$

θ be angle of elevation of tower top from eye

The above data is represented in form of figure as shown from figure

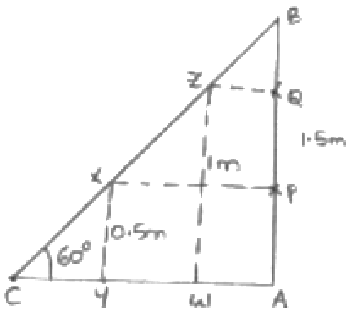
$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\theta = \frac{QX}{BX} = \frac{28\cdot 5}{28\cdot 5} = 1 \Rightarrow \theta = \tan^{-1}(1) = 45°$$

Angle of elevation $= 45°$

57. A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also, each leg is inclined at an angle of 60° to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distances.

Sol:



Let AB be height of stool $= 1\cdot 5m.$

Let P *and* Q be equal distance then $AP = 0\cdot 5m, AQ = 1m$ the above information is represented in form of figure as shown

$BC =$ length of leg

$$\sin 60° = \frac{AB}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1\cdot 5}{BC}$$

$$\Rightarrow BC = \frac{1\cdot 5 \times 2}{\sqrt{3}} = \sqrt{3}m.$$

Draw $PX \perp AB, QZ \perp AB, XY \perp CA, ZW \perp CA$

$$\sin 60° = \frac{XY}{XC}$$

$$\Rightarrow XC = \frac{0\cdot 5}{\sqrt{3}} \times \sqrt{4}$$

$$= \left(\frac{\sqrt{3}}{4}\right) \times \frac{8}{3}$$

$$= \frac{2}{\sqrt{3}}$$

$$\Rightarrow XC = 1\cdot 1077m.$$

$$\sin 60° = \frac{ZW}{CZ}$$

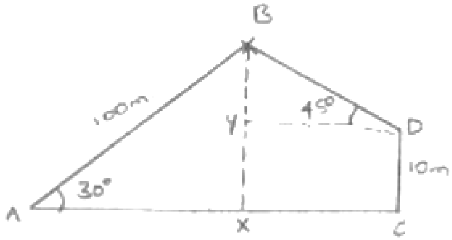
$$CZ = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

$$CZ = 1\cdot 654m.$$

58. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Sol:



For boy

Length of string $AB = 100m.$

Angle Made by string with ground $= \alpha = 30°$

For boy 2

Height of building $CD = 10m.$

Angle made by string with building top $\beta = 45°$ length of kite thread of boy 2 if both the kites meet must be $'DB'$

The above information is represented in form of figure as shown

Drawn $BX \perp AC, YD \perp BC$

In ΔABX

$\tan 30° = \frac{BC}{AX}$

$\sin 30° = \frac{BX}{AB} \Rightarrow \frac{1}{2} = \frac{BX}{100} \Rightarrow BX = 20m.$

$BY = BX - XY = 50 - 10m = 50m.$

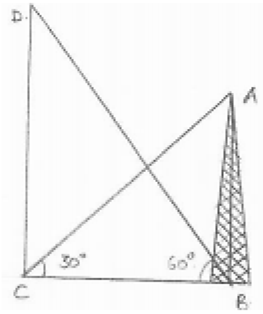
In $\Delta BYD \sin 45° = \frac{BY}{BD}$

$\frac{1}{\sqrt{2}} = \frac{40}{BD} \Rightarrow BD = 40\sqrt{2}m.$

Length of thread or string of boy $2 = 40\sqrt{2}m.$

59. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill?

Sol:



Height of towers $AB = 50m$
Height of hill $CD = 'h'm.$
Angle of elevation of top of hill from of tower $\alpha = 60°.$
Angle of elevation of top of tower from foot of hill $\beta = 30°.$
The above information is represented I form of figure as shown
From figure
In ΔABC

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{BC} \Rightarrow BC = 50\sqrt{3}.$$

In ΔBCD

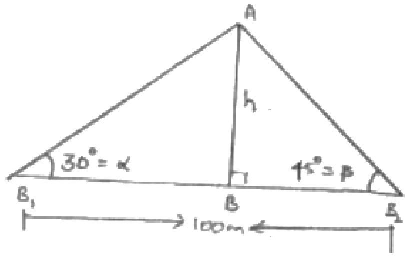
$$\tan 60° = \frac{Opposite\ side}{Adjacent\ side} = \frac{CD}{BC} = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}} \Rightarrow CD = 50 \times 3 = 150m$$

Height of hill $= 150m.$

60. Two boats approach a light house in mid-sea from opposite directions. The angles of elevation of the top of the light house from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the light house.

Sol:



Let B_1 be boat 1 and B_2 be boat 2.
Height of light house $= 'h'm = AB$
Distance between $B_1B_2 = 100m$

Angle of elevation of A from B_1 $\alpha = 30°$

Angle of elevation of B from B_2 $\beta = 45°$

The above information is represented in the form of figure as shown here

In ΔABB_1

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{B_1B}$$

$$B_1B = AB\sqrt{3} = h\sqrt{3} \qquad \ldots\ldots\ldots\ldots(1)$$

In ΔABB_2

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{B_1B} \qquad \ldots\ldots\ldots\ldots(2)$$

(1) + (2) $\Rightarrow B_1B + BB_2 = h\sqrt{3} + h$

$$\Rightarrow B_1B_2 = h\left(\sqrt{3}+1\right)$$

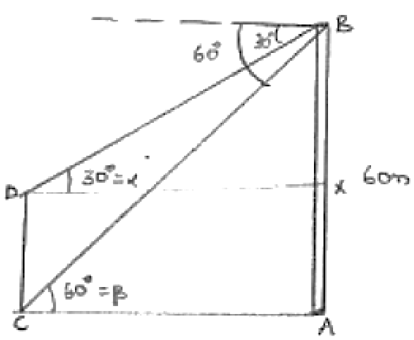
$$\Rightarrow h = \frac{B_1B_2}{\sqrt{3}+1} = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{100\left(\sqrt{3}-1\right)}{2} = 50\left(\sqrt{3}-1\right)$$

Height of light house $= 50\left(\sqrt{3}-1\right)$

61. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find
   (i) The horizontal distance between AB and CD.
   (ii) The height of the lamp post.
   (iii) The difference between the heights of the building and the lamp post.

Sol:



Height of building $AB = 60m.$

Height of lamp post $CD = h\,m$

Angle of depression of top of lamp post from top of building $\alpha = 30°$

Angle of depression of bottom of lamp post from top of building $\beta = 60°$

The above information is represented in the form of figure as shown

Draw $DX \perp AB, DX = AC, CD = AX$

In ΔBDX

$$\tan\alpha = \frac{Opposite\ side}{Adjacent\ side} = \frac{BX}{DX}$$

$$\tan 30° = \frac{60 - CD}{DX}$$

$$\frac{1}{\sqrt{3}} = \frac{60-h}{AC}$$

$$AC = (60-h)\sqrt{3}m \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

In ΔBCA

$$\tan\beta = \frac{AB}{AC} \Rightarrow \tan 60° = \frac{60}{AC}$$

$$\Rightarrow AC = \frac{60}{\sqrt{3}} = 20\sqrt{3}m \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$(60-h)\sqrt{3} = 20\sqrt{3}$$

$$60 - h = 20$$

$$\Rightarrow h = 40m$$

Height of lamp post $= 40m$

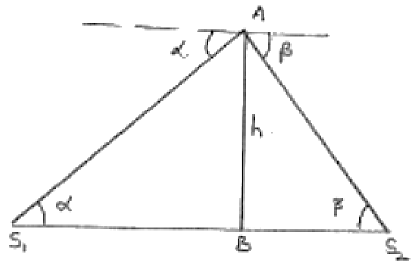
Distance between lamp posts building $AC = 20\sqrt{3}m.$

Difference between heights of building and lamp post

$= BX = 60 - h = 60 - 40 = 20m$

62. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be a and 3. If the height of the light house be h meters and the line joining the ships passes through the foot of the light house, show that the distance $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$ metres

Sol:



Height of light house $= 'h'$ meters $= AB$

S_1 *and* S_2 be two ships on opposite sides of light house $= \alpha$

Angle of depression of S_1 from top of light house $= \alpha$

Angle of depression of S_2 from top of light house

Required to prove that

Distance between ships $= \dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$ meters

The above information is represented in the form of figure as shown

In ΔABS_1

$$\tan\alpha = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{S_1B}$$

$$S_1B = \frac{h}{\tan\alpha} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

In ΔABS_2

$$\tan\beta = \frac{AB}{BS_2} \Rightarrow BS_2 = \frac{h}{\tan\beta} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

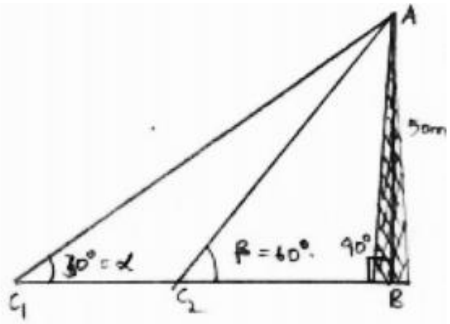
(1) and (2) $\Rightarrow BS_1 + BS_2 = \dfrac{h}{\tan\alpha} + \dfrac{h}{\tan\beta}$

$$\Rightarrow S_1S_2 = h\left\{\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right\} = \frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$$

$\therefore$ Distance between ships $= \dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$.meters

63. A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° respectively. What is the distance between the two cars and how far is each car from the tower?

Sol:



Height of towers $AB = 50mts$

C_1 and C_2 be two cars

Angle of depression of C_1 from top of towers $\alpha = 30°$

Angle of depression of C_2 from top of towers $\beta = 60°$

Distance between cars C_1C_2

The above information is represented in form of figure as shown

In ΔABC_2

$$\tan\beta = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{BC_2}$$

$$\tan 60° = \frac{50}{BC_1}$$

$$BC_2 = \frac{50}{\sqrt{3}}$$

In ΔABC_1

$$\tan\alpha = \frac{AB}{BC_1}$$

$$\tan 30° = \frac{50}{BC_1} \Rightarrow BC_1 = 50\sqrt{3},$$

$$C_1C_2 = BC_1 - BC_2 = 50\sqrt{3} - \frac{50}{\sqrt{3}} = 50\left(\frac{3-1}{\sqrt{3}}\right) = \frac{100}{\sqrt{3}} = \frac{100}{\sqrt{3}}\sqrt{3}mts.$$

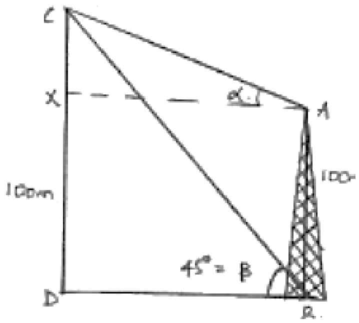
Distance between cars $C_1C_2 = \frac{100}{3}\sqrt{3}\,mts$

Distance of car1 from tower $= 50\sqrt{3}\,mts.$

Distance of car 2 from tower $= \frac{50}{\sqrt{3}}\,mts$

64. The angles of elevation of the top of a rock from the top and foot of a loo m high tower are respectively 30° and 45°. Find the height of the rock.

Sol:



Height of tour $AB = 100m$

Height of rock $CD = 'h'm$

Angle of elevation of top of root from top of tower $\alpha = 30°$

Angle of elevation of top of root from bottom of tower $\beta = 45°$

The above data is represented in form of figure as shown

Draw $AX \perp CD$

$XD = AB = 100m$

$XA = DB.$

In $\Delta CXA,\quad \tan\alpha = \frac{CX}{AX}$

$\Rightarrow \tan 30° = \frac{CX}{DB}$

$\Rightarrow DB = C \times \sqrt{3}$ …………..(1)

In $\Delta CBD,\quad \tan\beta = \frac{CD}{DB} = \frac{100+CX}{DB}$

$\tan 45° = \frac{100+CX}{DB} \Rightarrow DB = 100 + CX$ ……………(2)

From (1) and (2)

$100 + CX = C \times \sqrt{3} \Rightarrow C \times \left(\sqrt{3}-1\right) = 100$

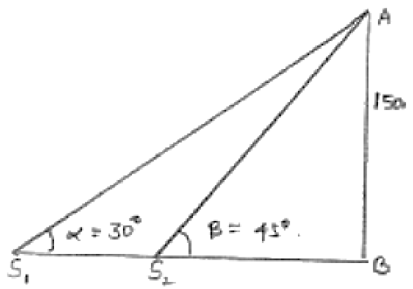
$\Rightarrow CX = \frac{100}{\sqrt{2}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$CX = 50\left(\sqrt{3}+1\right)$

Height of hill $= 100 + 50\left(\sqrt{3}+1\right) = 150\left(3+\sqrt{3}\right) mts.$

65. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships

Sol:



Height of light house $AB = 150 mts.$

Let S_1 *and* S_2 be two ships approaching each other.

Angle of depression of $S_1,\ \alpha = 50°$

Angle of depression of $S_2,\ \beta = 50°$

Distance between ships $= S_1S_2$.

The above data is represented in the form of figure as shown

In ΔABS_2

$$\tan\beta = \frac{AB}{BS_2}$$

$$\tan 45° = \frac{150}{BS_2}$$

$$BS_2 = 150m.$$

In ΔABS_1

$$\tan\alpha = \frac{AB}{BS_1}$$

$$\tan 30° = \frac{150}{BS_1}$$

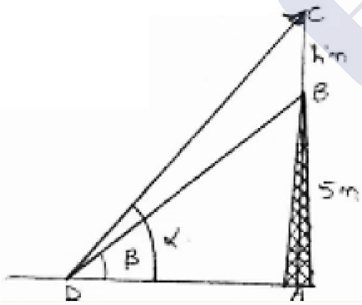
$$BS_1 = 150\sqrt{3}m.$$

$$S_1S_2 = BS_1 - BS_2 = 150\left(\sqrt{3}-1\right)mts$$

Distance between ships $= 150\left(\sqrt{3}-1\right)mts.$

66. A flag-staff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45°. Find the height of the flag-staff.

Sol:



Height of tower $= AB = 5m.$

Height of flagstaff $BC = 'h'm$

Angle of elevation of top of flagstaff $a = 60°$

Angle of elevation of bottom of flagstaff $\beta = 45°$

The above data is represented in form of figure as shown

In ΔADB $\tan\beta = \frac{AB}{DA} \Rightarrow \tan 45° = \frac{5}{DA}$

$\Rightarrow DA = 5m.$

In $\Delta ADC, \tan\alpha = \frac{AC}{AD}$,

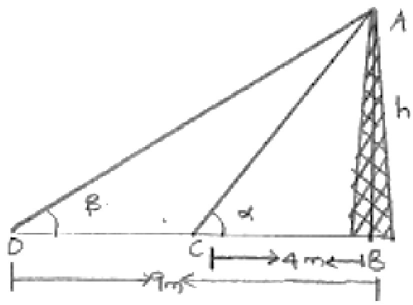
$$\tan 60° = \frac{AB+BC}{AD} = \frac{h+5}{5}$$

$$\sqrt{3} = \frac{h+5}{5}$$

$h+5 = 5\sqrt{3} \Rightarrow h = 5\left(\sqrt{3}-1\right) = 5\times 0\cdot 732 = 3\cdot 65$ meters height of flagstaff $= 3\cdot 65$ meters

67. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Sol:



Height of tower $AB = 'h'$ meters

Let point C be 4 meters from B, Angle of elevation be α given point D be 9 meters from B.

Angle of elevation be β. given α, β are complementary, $\alpha + \beta = 90° \Rightarrow \beta = 90° - \alpha$

required to prove that $h = 6$ meters

The above data is represented in the form of figure as shown

In $\Delta ABC, \tan\alpha = \frac{AB}{BC}$

$$\tan\alpha = \frac{h}{4}$$

$h = 4\tan\alpha$ ………………(1)

In $\Delta ABD, \tan\beta = \frac{AB}{BD} = \frac{h}{9}$

$$\tan(90-\alpha) = \frac{h}{9}$$

$h = 4\tan\alpha$ ………………(2)

Multiply (1) and (2) $h\times h = 4\tan\alpha \times 9\cot\alpha$

$= 36(\tan\alpha \cdot \cot\alpha)$

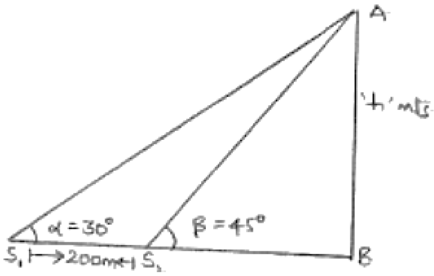
$h^2 = 36$

$h = \sqrt{36} = 6$ meters.

$\therefore$ height of tower $= 6$ meters.

68. The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30° respectively. If the ships are 200 m apart, find the height of the light house.

Sol:



Height of light house $AB = 'h'$ meters

Let S_1 *and* S_2 be ships distance between ships S_1S_2

Angle of depression of S_1 $[\alpha = 30°]$

Angle of depression of S_2 $[\beta = 45°]$

The above data is represented in form of figure as shown

In ΔABS_2

$$\tan\beta = \frac{AB}{BC_2}$$

$$\tan 45° = \frac{h}{BS_2}$$

$$BS_2 = h \qquad \text{..........(1)}$$

In ΔABS_1

$$\tan\alpha = \frac{AB}{BS_1}$$

$$\tan 30° = \frac{h}{BS_2}$$

$$BS_1 = h\sqrt{3} \qquad \text{..........(2)}$$

(2) and (1) $\Rightarrow BS_1 - BS_2 = h\left(\sqrt{3}-1\right)$

$$\Rightarrow 200 = h\left(\sqrt{3}-1\right)$$

$$\Rightarrow h = \frac{200}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{200}{2}\left(\sqrt{3}+1\right) = 100\left(\sqrt{3}+1\right) \text{ meters}$$

$$h = 100(1\cdot 732 + 1) = 273\cdot 2 \text{ meters}$$

Height of light house $= 273\cdot 2$ meters