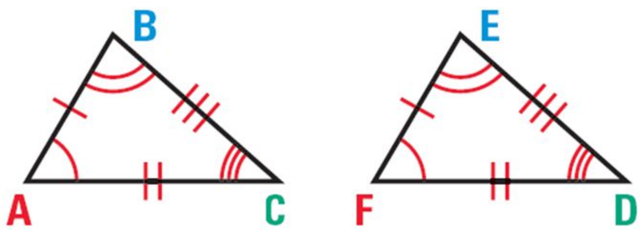
Congruence

Congruence Statement

When naming two congruent triangles, order is very impo

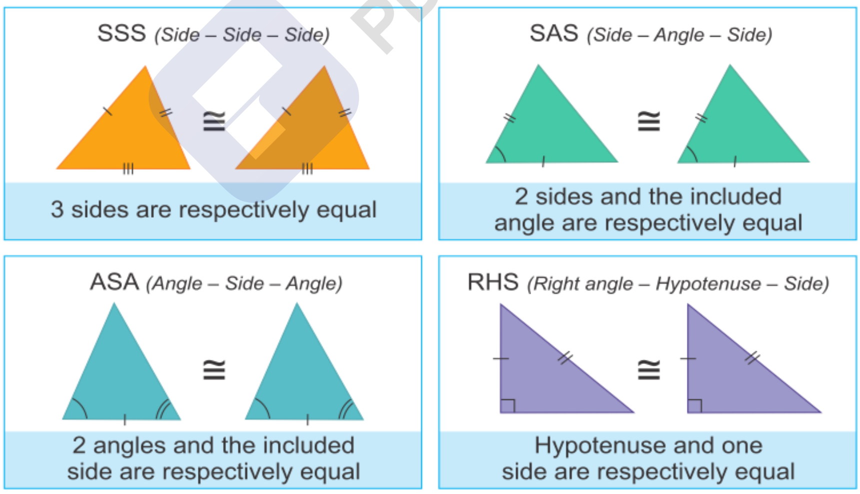


$\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.

Corresponding angles $\angle A \cong \angle F$ $\angle B \cong \angle E$ $\angle C \cong \angle D$

Corresponding sides $\overline{AB} \cong \overline{FE}$ $\overline{BC} \cong \overline{ED}$ $\overline{AC} \cong \overline{FD}$

Conditions for Congruence of Two Triangles



Congruent Triangles

Identical Triangles have all three Sides, and all three Angles exactly the same sizes.

If we gave several people three sticks: 5cm, 7cm and 9cm long, they would all only be able to make the exact sameTriangle.

Q1

Answer :

We have to state the correspondence between the vertices, sides and angles of the following pairs of congruent triangles.

(i) $\triangle ABC \cong \triangle EFD$

Correspondence between vertices :

$A \leftrightarrow E,\ B \leftrightarrow F,\ C \leftrightarrow D$

Correspondence between sides :

$AB = EF,\ BC = FD, CA = DE$

Correspondence between angles :

$\angle A = \angle E, \angle B = \angle F, \angle C = \angle D$

(ii) $\triangle CAB \cong \triangle QRP$

Correspondence between vertices :

$C \leftrightarrow Q,\ A \leftrightarrow R,\ B \leftrightarrow P$

Correspondence between sides :

$CA = QR,\ AB = RP,\ BC = PQ$

Correspondence between angles :

$\angle C = \angle Q,\ \angle A = \angle R,\ \angle B = \angle P$

(iii) $\triangle XZY \cong \triangle QPR$

Correspondence between vertices :

$X \leftrightarrow Q,\ Z \leftrightarrow P,\ Y \leftrightarrow R$

Correspondence between sides :

$XZ = QP,\ ZY = PR,\ YX = RQ$

Correspondence between angles :

$\angle X = \angle Q,\ \angle Z = \angle P,\ \angle Y = \angle R$

(iv) $\triangle MPN \cong \triangle SQR$

Correspondence between vertices :

$M \leftrightarrow S,\ P \leftrightarrow Q,\ N \leftrightarrow R$

Correspondence between sides :

$MP = SQ,\ PN = QR,\ NM = RS$

Correspondence between angles :

$\angle M = \angle S,\ \angle P = \angle Q,\ \angle N = \angle R$

Q2

Answer :

$(i)\ \triangle ACB \cong \triangle DEF$
(SAS congruence property)
$(ii)\ \triangle RPQ \cong \triangle LNM$
(RHS congruence property)
$(iii)\ \triangle YXZ \cong \triangle TRS$
(SSS congruence property)
$(iv)\ \triangle DEF \cong \triangle PNM$
(ASA congruence property)
$(v)\ \triangle ACB \cong \triangle ACD$
(ASA congruence property)

Q3

Answer :

Given :
$PL \perp OA$
$PM \perp OB$
$PL = PM$
To prove :
$\triangle PLO \cong \triangle PMO$
Proof :
In $\triangle PLO$ and $\triangle PMO$:
$\angle PLO = \angle PMO$ (90° each)
$PO = PO$ (common)
$PL = PM$ (given)
By RHS congruence property :
$\triangle PLO \cong \triangle PMO$

Q4

Answer :

Given :
$AD = BC$
$AD \parallel BC$
We have to show that $AB = DC$.
Proof :
$AD \parallel BC$
$\therefore \angle BCA = \angle DAC$ (alternate angles)
In $\triangle ABC$ and $\triangle CDA$:
$BC = DA$ (given)
$\angle BCA = \angle DAC$ (proved above)
$AC = AC$ (common)
By SAS congruence property :
$\triangle ABC \cong \triangle CDA$
$=> AB = CD$ (corresponding parts of the congruent triangles)

Q5

Answer :

Given :
$AB = AC$, $BD = DC$
To prove : $\triangle ADB \cong \triangle ADC$
Proof :
(i) In $\triangle ADB$ and $\triangle ADC$:
$AB = AC$ (given)
BD = DC (given)
DA = DA (common)
By SSS congruence property :
$\triangle ADB \cong \triangle ADC$
$\angle ADB = \angle ADC$ (corresponding parts of the congruent triangles) ...(1)
$\angle ADB$ and $\angle ADC$ are on the straight line.
$\therefore \angle ADB + \angle ADC = 180^\circ$
$\angle ADB + \angle ADB = 180^\circ$
$=> 2\angle ADB = 180^\circ$
$=> \angle ADB = 90^\circ$
From (1) :
$\angle ADB = \angle ADC = 90^\circ$
(ii) $\angle BAD = \angle CAD$ (corresponding parts of the congruent triangles)

Q6

Answer :

Given :
AD is a bisector of $\angle A$.
$=> \angle DAB = \angle DAC$ ...(1)
$AD \perp BC$
$=> \angle BDA = \angle CDA$ (90° each)
To prove :
$\triangle ABC$ is isosceles.
Proof :
In $\triangle DAB$ and $\triangle DAC$:
$\angle BDA = \angle CDA$ (90° each)
$DA = DA$ (common)
$\angle$DAB = $\angle$DAC (from 1)
By ASA congruence property :
$\triangle DAB \cong \triangle DAC$
$=> AB = AC$ (corresponding parts of the congruent triangles)
Therefore, $\triangle$ ABC is isosceles.

Q7

Answer :

Given :
$AB = AD$
$CB = CD$
To prove :
$\triangle ABC \cong \triangle ADC$
Proof :
In $\triangle ABC$ and $\triangle ADC$:
$AB = AD$ (given)
BC = DC (given)
AC = AC (common)
$\therefore \triangle ABC \cong \triangle ADC$ (*by* SSS congruence property)

Q8

Answer :

Given :
$PA \perp AB$
$QB \perp AB$
$PA = QB$
To prove : $\triangle OAP \cong \triangle OBQ$
Find whether $OA = OB$.
Proof :
In $\triangle OAP$ and $\triangle OBQ$:
$\angle POA = \angle QOB$ (vertically opposite angles)
$\angle OAP = \angle OBQ$ (90° each)
$PA = QB$ (given)
By AAS congruence property :
$\triangle OAP \cong \triangle OBQ$
$=> OA = OB$ (corresponding parts of the congruent triangles)

Q9

Answer :

Given :
Triangles ABC *and* DCB are right angled at A and D, respectively.
$AC = DB$
To prove : $\triangle ABC \cong \triangle DCB$
In $\triangle ABC$ and $\triangle DCB$:
$\angle CAB = \angle BDC$ (90° each)
$BC = BC$ (common)
$AC = DB$ (given)
By R.H.S. congruence property :
$\triangle ABC \cong \triangle DCB$

Q10

Answer :

Given :
$\triangle ABC$ is an isosceles triangle in which $AB = AC$.
E and F are midpoints of AC and AB, respectively.
To prove :
$BE = CF$
Proof :
E and F are midpoints of AC and AB, respectively.
$=> AF = FB,\ AE = EC$
$AB = AC$
$=> \frac{1}{2} AB = \frac{1}{2} AC$
$=> FB = EC$
$\angle ABC = \angle ACB$ (angle opposite to equal sides are equal)
$=> \angle FBC = \angle ECB$
Consider $\triangle BCF$ and $\triangle CBE$:
$BC = BC$ (common)

$=> \angle FBC = \angle ECB$
Consider $\triangle BCF$ and $\triangle CBE$:
$BC = BC$ (common)
$\angle FBC = \angle ECB$ (proved above)
$FB = EC$ (proved above)
By SAS congruence property :
$\triangle BCF \cong \triangle CBE$
$BE = CF$ (corresponding parts of the congruent triangles)

Q11

Answer :

Given :
$AB = AC$
$\triangle ABC$ is an isosceles triangle.
$AP = AQ$
To prove :
$BQ = CP$
Proof :
$AB = AC$ (given)
$AP = AQ$ (given)
$AB - AP = AC - AQ$
$=> BP = CQ$
$\angle ABC = \angle ACB$ (angle opposite to the equal sides are equal)
$=> \angle PBC = \angle QCB$
In $\triangle PBC$ and $\triangle QCB$:
$PB = QC$ (proved above)
$\angle PBC = \angle QCB$ (proved above)
$BC = BC$ (common)
By SAS congruence property :
$\triangle PBC \cong \triangle QCB$
$BQ = CP$ (corresponding parts of the congruent triangles)

Q12

Answer :

Given :
ABC is an isosceles triangle.
$AB = AC$
$BD = CE$
To prove :
$BE = CD$
Proof :
$AB + BD = AC + CE \quad (As,\ AB = AC,\ BD = CE)$
$=> AD = AE$
Consider $\triangle ACD$ and $\triangle ABE$:
$AC = AB \quad$ (given)
$\angle CAD = \angle BAE \quad$ (common)
$AD = AE \quad$ (proved above)
By SAS congruence property :
$\triangle ACD \cong \triangle ABE$
$=> CD = BE \quad$ (corresponding parts of the congruent triangles)

Q13

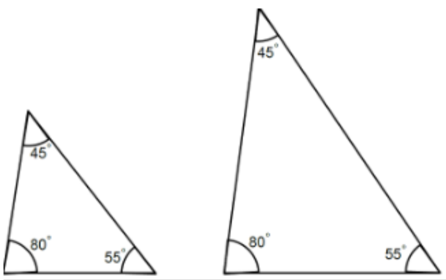
Answer :

Given :
$\triangle ABC$ is an isosceles triangle.
$AB = AC$
$BD = CD$
To prove :
AD bisects $\angle A$ and $\angle D$.
Proof :
Consider $\triangle ABD$ and $\triangle ACD$:
$AB = AC \quad$ (given)
BD = CD (given)
AD = AD (common)
By SSS congruence property :
$\triangle ABD \cong \triangle ACD$
$=> \angle BAD = \angle CAD \quad$ (by cpct)
$=> \angle BDA = \angle CDA \quad$ (by cpct)

Q14

Answer :

No, its not necessary. If the corresponding angles of two triangles are equal, then they may or may not be congruent.
They may have proportional sides as shown in the following figure:



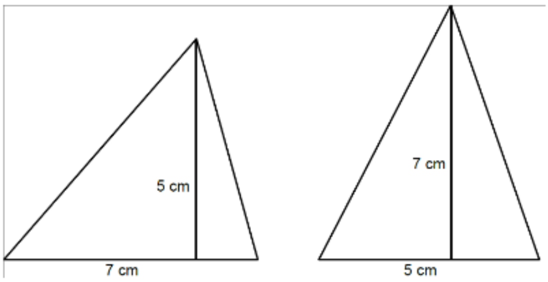
Q15

Answer :

No, two triangles are not congruent if their two corresponding sides and one angle are equal. They will be congruent only if the said angle is the included angle between the sides.

Q16

Answer :



Both triangles have equal area due to the the same product of height and base. But they are not congruent.

Q17

Answer :

(i) the same length

(ii) the same measure

(iii)the same side length

(iv) the same radius

(v) the same length and the same breadth

(vi) equal parts

Q18

Answer :

(i) False
This is because they can be equal only if they have equal sides.

(ii) True
This is because if squares have equal areas, then their sides must be of equal length.

(iii) False
For example, if a triangle and a square have equal area, they cannot be congruent.

(iv) False
For example, an isosceles triangle and an equilateral triangle having equal area cannot be congruent.

(v) False
They can be congruent if two sides and the included angle of a triangle are equal to the corresponding two sides and the included corresponding angle of another triangle.

(vi) True
This is because of the AAS criterion of congruency.

(vii) False
Their sides are not necessarily equal.

(viii) True
This is because of the AAS criterion of congruency.

(ix) False
This is because two right triangles are congruent if the hypotenuse and one side of the first triangle are respectively equal to the hypotenuse and the corresponding side of the second triangle.

(x) True