Exercise 16.1

1. How many balls each of radius 1cm can be made from a solid sphere of lead of radius 8cm?

Sol:

Given that a solid sphere f radius $(r_1) = 8cm$

With this sphere we have to make spherical balls of radius $(r_2) = 1cm$

Since we don't know no of balls let us assume that no of balls formed be 'n'

We know that

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Volume of solid sphere should be equal to sum of volumes of n spherical balls

$$n \times \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi r^3$$

$$n = \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(1)^3}$$

$$n = 8^3$$

$$\boxed{n = 512}$$

$\therefore$ hence 512 no of balls can be made of radius 1cm from a solid sphere of radius $8cm$

2. How many spherical bullets each of 5cm in diameter can be cast from a rectangular block of metal $11dm \times 1m \times 5dm$?

Sol:

Given that a metallic block which is rectangular of diameter $11dm \times 1m \times 5dm$

Given that diameter of each bullet is 5cm

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Dimensions of rectangular block = $11dm \times 1m \times 5dm$

Since we know that $1dm = 10^{-1}m$

$$11 \times 10^{-1} \times 1 \times 5 \times 10^{-1} = 55 \times 10^{-2} m^3 \quad \ldots\ldots\ldots(1)$$

Diameter of each bullet $= 5cm$

Radius of bullet $(r) = \frac{d}{2} = \frac{5}{2} = 2 \cdot 5cm$

$= 25 \times 10^{-2} m$

So volume $=\frac{4}{3}\pi\left(25\times10^{-2}\right)^{3}$

Volume of rectangular block should be equal sum of volumes of n spherical bullets

Let no of bullets be 'n'

Equating (1) and (2)

$$55\times10^{-2}=n=\frac{4}{3}\pi\left(25\times10^{-2}\right)^{3}$$

$$\frac{55\times10^{-2}}{\frac{4}{3}\times\frac{22}{7}\left(25\times10^{-2}\right)^{3}}=n$$

$$n=8400$$

$$\boxed{\therefore \text{ No of bullets found were } 8400}$$

3. A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of the two of balls are $1\cdot9cm$ and $2cm$. Determine the diameter of the third ball?

Sol:

Given that a spherical ball of radius 3cm

We know that Volume of a sphere $=\frac{4}{3}\pi r^{2}$

So its volume $(v)=\frac{4}{3}\pi(3)^{2}$

Given that ball is melted and recast into three spherical balls

Radii of first ball $(v_1)=\frac{4}{3}\pi(1\cdot5)^{3}$

Radii of second ball $(v_2)=\frac{4}{3}\pi(2)^{3}$

Radii of third ball ____________?

Volume of third ball $=\frac{4}{3}\pi r^{3}=v_3$

Volume of spherical ball is equal to volume of 3 small spherical balls

$$\Rightarrow\frac{4}{3}\pi r^{2}+\frac{4}{3}\pi(1\cdot5)^{3}+\frac{4}{3}\pi(2)^{3}=\frac{4}{3}\pi(3)^{3}$$

$$\Rightarrow r^{2}+(1\cdot5)^{3}+(2)^{3}=(3)^{3}$$

$$\Rightarrow r^{3}=3^{3}-1\cdot5^{3}-2^{3}$$

$$\Rightarrow r=(15\cdot6)\frac{1}{3}$$

$$\Rightarrow r=2\cdot5cm$$

Diameter $(d) = 2r = 2\times 2\cdot 5 = 5cm$

$\boxed{\therefore\ Diameter\ of\ third\ ball\ = 5cm.}$

4. $2\cdot 2$ Cubic dm of grass is to be drawn into a cylinder wire $0\cdot 25cm$ in diameter. Find the length of wire?

Sol:

Given that $2\cdot 2dm^3$ of grass is to be drawn into a cylindrical wire $0.25cm$ in diameter

Given diameter of cylindrical wire $= 0.25cm$

Radius of wire $(r) = \frac{d}{2} = \frac{0\cdot 25}{2} = 0\cdot 125cm$

$= 0\cdot 125\times 10^{-2} m.$

We have to find length of wire?

Let length of wire be 'h' $\left(\because 1cm = 10^{-2} m\right)$

$\boxed{Volume\ of\ Cylinder = \pi r^2 h}$

Volume of brass of $2\cdot 2dm^3$ is equal to volume of cylindrical wire

$\frac{22}{7}\left(0\cdot 125\times 10^{-2}\right)h = 2\cdot 2\times 10^{-3}$

$\Rightarrow h = \frac{2\cdot 2\times 10^{-3}\times 7}{22\left(0\cdot 125\times 10^{-2}\right)^2}$

$\Rightarrow h = 448m$

$\boxed{\therefore\ Length\ of\ cylindrical\ wire = 448m}$

5. What length of a solid cylinder 2cm in diameter must be taken to recast into a hollow cylinder of length 16cm, external diameter 20cm and thickness $2\cdot 5mm$?

Sol:

Given that diameter of solid cylinder $= 2cm$

Given that solid cylinder is recast to hollow cylinder

Length of hollow cylinder $= 16cm$

External diameter = 20cm

Thickness $= 2\cdot 5mm = 0\cdot 25cm$

$\boxed{Volume\ of\ solid\ cylinder = \pi r^2 h}$

Radius of cylinder $= 1cm$

So volume of solid cylinder $= \pi(1)^2 h$ ……(i)

Let length of solid cylinder be h

$$\boxed{Volume\ of\ \ hollow\ cylinder = \pi h\left(R^2 - r^2\right)}$$

Thickness $= R - r$

$0 \cdot 25 = 10 - r$

$\Rightarrow$ Internal radius $= 9 \cdot 75cm$

So volume of hollow cylinder $= \pi \times 16\left(100 - 95 \cdot 0625\right)$ ....(2)

Volume of solid cylinder is equal to volume of hollow cylinder.

(1) = (2)

Equating equations (1) and (2)

$\pi(1)^2 h = \pi \times 16\left(100 - 95 \cdot 06\right)$

$\frac{22}{7}(1)^2 \times h = \frac{22}{7} \times 16\left(4 \cdot 94\right)$

$\boxed{h = 79 \cdot 04cm}$

$\therefore$ Length of solid cylinder $= 79cm$

6. A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42cm and height 21cm which are filled completely. Find the diameter of cylindrical vessel?

Sol:

Given that diameter is equal to height of a cylinder

So $h = 2r$

$\boxed{Volume\ of\ \ cylinder = \pi r^2 h}$

So volume $= \pi r^2 (2r)$

$= 2\pi r^3$

Volume of each vessel $= \pi r^2 h$

Diameter $= 42cm$

Height $= 21cm$

Diameter $(d) = 2r$

$2r = 42$

$r = 21$

$\therefore$ Radius $= 21cm$

Volume of vessel $= \pi (21)^2 \times 21$ ....(2)

Since volumes are equal

Equating (1) and (2)

$\Rightarrow 2\pi r^3 = \pi (21)^2 \times 21 \times 2$ ($\because$ 2 identical vessels)

$\Rightarrow r^3 = \dfrac{\pi(21)^2 \times 21 \times 2}{2 \times \pi}$

$\Rightarrow r^3 = (21)^3$

$\Rightarrow r = 21 \Rightarrow \boxed{d = 42cm}$

$\therefore$ Radius of cylindrical vessel $= 21cm$

Diameter of cylindrical vessel $= 42cm.$

7. 50 circular plates each of diameter 14cm and thickness $0\cdot 5cm$ are placed one above other to form a right circular cylinder. Find its total surface area?

Sol:

Given that 50 circular plates each with diameter $= 14cm$

Radius of circular plates $(r) = 7cm$

Thickness of plates $= 0\cdot 5$

Since these plates are placed one above other so total thickness of plates $= 0\cdot 5 \times 50$ $= 25cm.$

$\boxed{Total\ surface\ area\ of\ a\ cylinder = 2\pi rh + 2\pi r^2}$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \dfrac{22}{7} \times 7(25 + 7)$

$\boxed{T.S.A = 1408cm^2}$

$\therefore$ Total surface area of circular plates is $1408cm^2$

8. 25 circular plates each of radius $10\cdot 5cm$ and thickness $1\cdot 6cm$ are placed one above the other to form a solid circular cylinder. Find the curved surface area and volume of cylinder so formed?

Sol:

Given that 25 circular plates each with radius (r) = 10.5cm

Thickness $= 1\cdot 6cm$

Since plates are placed one above other so its height becomes $= 1\cdot 6 \times 25 = 40cm$

$\boxed{Volume\ of\ cylinder = \pi r^2 h}$

$= \pi(10\cdot 5)^2 \times 40$

$= 13860cm^3$

$\boxed{Curved\ surface\ area\ of\ a\ cylinder = 2\pi rh}$

$= 2 \times \pi \times 10\cdot 5 \times 40$

$=2\times\frac{22}{7}\times 10\cdot 5\times 40$

$=2640cm^2$

$\therefore$ Volume of cylinder $=13860cm^3$

Curved surface area of a cylinder $=2640cm^2$

9. A path 2m wide surrounds a circular pond of diameter 4cm. how many cubic meters of gravel are required to grave the path to a depth of 20cm

Sol:

Diameter of circular pond $=40m$

Radius of pond(r) $=20m.$

Thickness $=2m$

Depth $=20cm=0\cdot 2m$

Since it is viewed as a hollow cylinder

$\boxed{Thickness\ (t)=R-r}$

$2=R-r$

$2=R-20$

$R=22m$

$\boxed{\therefore Volume\ of\ hollow\ cylinder=\pi\left(R^2-r^2\right)h}$

$=\pi\left(22^2-20^2\right)h$

$=\pi\left(22^2-20^2\right)\times 0\cdot 2$

$=\pi(84)\times 0.2$

$\boxed{\therefore Volume\ of\ hollow\ cylinder=52\cdot\pi m^3}$

$\therefore 52\cdot 77m^3$ of gravel is required to have path to a depth of 20cm.

10. A 16m deep well with diameter $3\cdot 5m$ is dug up and the earth from it is spread evenly to form a platform $27\cdot 5m$ by 7m. Find height of platform?

Sol:

Let as assume well is a solid right circular cylinder

Radius of cylinder $(r)=\frac{3\cdot 5}{2}=1\cdot 75m$

Height (or) depth of well $=16m.$

$\boxed{Volume\ of\ right\ circular\ cylinder=\pi r^2h}$

$=\frac{22}{7}\times(1\cdot 75)^2\times 16$ ……..(1)

Given that length of platform $(l) = 27.5m$

Breath of platform $(b) = 7cm$

Let height of platform be xm

$\boxed{Volume\ of\ rec\tan gle = lbh}$

$= 27\cdot5 \times 7 \times x = 192\cdot5x$ ....(2)

Since well is spread evenly to form platform

So equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \frac{22}{7}(1\cdot75)^2 \times 16 = 192\cdot5x$

$\Rightarrow x = 0\cdot8m$

$\boxed{\therefore\ Height\ of\ \ platform\,(h) = 80cm.}$

11. A well of diameter 2m is dug14m deep. The earth taken out of it is spread evenly all around it to form an embankment of height 40cm. Find width of the embankment?

Sol:

Let us assume well as a solid circular cylinder

Radius of circular cylinder $= \frac{2}{2} = 1m$

Height (or) depth of well $= 14m$

$\boxed{Volume\ of\ \ solid\ \ circular\ \ cylindeer = \pi r^2 h}$

$= \pi(1)^2 14$ .....(1)

Given that height of embankment (h) = 40cm

Let width of embankment be ‘x’ m

Volume of embankment $= \pi r^2 h$

$= \pi\left(\left(1 + x^2\right) - 1\right)^2 \times 0\cdot4$ .....(2)

Since well is spread evenly to form embankment so their volumes will be same so equating (1) and (2)

$\Rightarrow \pi(1)^2 \times 14 = \pi\left((1+x)^2 - 1\right)^2 \times 0\cdot4$

$\Rightarrow x = 5m$

$\boxed{\therefore Width\ of\ \ embankment\ \ of\ (x) = 5m}$

12. Find the volume of the largest right circular cone that can be cut out of a cube where edge is 9cm_____?

Sol:

Given that side of cube =9cm

Given that largest cone is curved from cube

Diameter of base of cone = side of cube

$\Rightarrow 2x = 9$

$\Rightarrow r = \frac{9}{2} cm$

Height of cone = side of cube

$\Rightarrow$Height of cone (h) = 9cm

$$\boxed{Volume\ of\ l\arg est\ cone = \frac{1}{3}\pi r^2 h}$$

$= \frac{1}{3} \times \pi t \left(\frac{9}{2}\right)^2 \times 9$

$= \frac{\pi}{12} \times 9^3$

$= 190 \cdot 92 cm^3$

$\therefore$ Volume of largest cone $(v) = 190 \cdot 92 cm^3$

13. A cylindrical bucket, 32 cm high and 18cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Sol:

36cm, 43.27 cm

14. Rain water, which falls on a flat rectangular surface of length 6cm and breath 4m is transferred into a cylindrical vessel of internal radius 10cm. What will be the height of water in the cylindrical vessel if a rainfall of 1cm has fallen____?

Sol:

Given length of rectangular surface $= 6cm$

Breath of rectangular surface $= 4cm$

Height (h) $1cm$

$$\boxed{Volume\ of\ a\ flat\ rec\tan gular\ surface = lbh}$$

$= 6000 \times 400 \times 1$

Volume $= 240000 cm^3$ __________(1)

Given radius of cylindrical vessel $= 20cm$

Let height off cylindrical vessel be h_1

Since rains are transferred to cylindrical vessel.
So equating (1) with (2)

$\boxed{Volume\ of\ \ cylindrical\ vessel = \pi r_1^2 h_1}$

$= \frac{22}{7}(20)^2 \times h_1$ ________(2)

$24000 = \frac{22}{7}(20)^2 \times h_1$

$\Rightarrow \boxed{h_1 = 190 \cdot 9cm}$

$\therefore$ height of water in cylindrical vessel $= 190 \cdot 9cms$

15. A conical flask is full of water. The flask has base radius r and height h. the water is proved into a cylindrical flask off base radius one. Find the height of water in the cylindrical flask?

Sol:

Given base radius of conical flask be r
Height of conical flask is h

$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h}$

So its volume $= \frac{1}{3}\pi r^2 h$ ________(1)

Given base radius of cylindrical flask is ms.
Let height of flask be h_1

$\boxed{Volume\ of\ cylinder = \pi r^2 h_1}$

So its volume $= \frac{22}{7}(mr)^2 h_1$ ________(2)

Since water in conical flask is poured in cylindrical flask their volumes are same
(1) = (2)

$\Rightarrow \frac{1}{3}\pi r^2 h = \pi (mr)^2 \times h_1$

$\Rightarrow \boxed{h_1 = \frac{h}{3m^2}}$

$\therefore$ Height of water in cylindrical flask $= \frac{h}{3m^2}$

16. A rectangular tank 15m long and 11m broad is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21m and length 5m. Find least height of tank that will serve purpose______?

Sol:

Given length of rectangular tank = 15m
Breath of rectangular tank = 11m
Let height of rectangular tank be h

$$\boxed{Volume\ of\ rec\tan gular\ \tan k = lbh}$$

Volume $= 15\times 11\times h$ ______(1)

Given radius of cylindrical tank $(r) = \frac{21}{2}m$

Length/height of tank = 5m

$$\boxed{Volume\ of\ cylindrical\ \tan k = \pi r^2 h}$$

$= \pi\left(\frac{21}{2}\right)^2 \times 5$ _______(2)

Since volumes are equal
Equating (1) and (2)

$$15\times 11\times h = \pi\left(\frac{21}{2}\right)^2\times 5$$

$$\Rightarrow h = \frac{\frac{22}{7}\times\left(\frac{21}{2}\right)^2\times 5}{15\times 11}$$

$$\Rightarrow \boxed{h = 10\cdot 5m}$$

$\therefore$ Height of tank $= 10\cdot 5m.$

17. A hemisphere tool of internal radius 9cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3cm and height 4cm. how many bottles are necessary to empty the bowl.

Sol:

Given that internal radius of hemisphere bowl $= 90m$

$$\boxed{Volume\ of\ hemisphere = \frac{4}{3}\pi r^3}$$

$= \frac{2}{3}\times\pi(9)^3$ ______(1)

Given diameter of cylindrical bottle $= 3cm$

Radius $= \frac{3}{2}cm$

Height $= 4cm$

$$\boxed{Volume\ of\ cylindrical = \pi r^2 h}$$

$$= \pi\left(\frac{3}{2}\right)^2 \times 4 \qquad \_\_\_\_\_\_(2)$$

Volume of hemisphere bowl is equal to volume sum of n cylindrical bottles

(1) = (2)

$$\frac{2}{3}\pi(9)^3 = \pi\left(\frac{3}{2}\right)^2 \times 4 \times n$$

$$\Rightarrow n = \frac{\frac{2}{3}\pi(9)^3}{\pi\left(\frac{3}{2}\right)^2 \times 4}$$

$$\Rightarrow \boxed{n = 54}$$

$\therefore$ No of bottles necessary to empty the bottle $= 54$.

18. The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.

Sol:

Internal diameter of hollow spherical shell $= 6cm$

Internal radius of hollow spherical shell $= \frac{6}{3} = 3cm$

External diameter of hollow spherical shell $= 10\,cm$

External radius of hollow spherical shell $= \frac{10}{2} = 5\,cm$

Diameter of cylinder $= 14\ cm$

Radius of cylinder $= \frac{14}{2} = 7cm$

Let height of cylinder $= xcm$

According to the question

Volume of cylinder = Volume of spherical shell

$$\Rightarrow \pi(7)^2 x \times = \frac{4}{3}\pi\left(5^3 - 3^3\right)$$

$$\Rightarrow 49x\times = \frac{4}{3}(125 - 27)$$

$$\Rightarrow 49x\times = \frac{4}{3} \times 98$$

$$x = \frac{4 \times 98}{3 \times 49} = \frac{8}{3}cm$$

$\therefore$ Height off cylinder $= \frac{8}{3} cm$

19. A hollow sphere of internal and external diameter 4cm and 8cm is melted into a cone of base diameter 8cm. Calculate height of cone?

Sol:

Given internal diameter of hollow sphere (r) $= 4cm$

External diameter $(R) = 8cm$

$$\boxed{Volume\ of\ hollow\ sphere = \frac{4}{3}\pi\left(R^2 - r^2\right)}$$

$= \frac{4}{3}\pi\left(B^2 - 4^2\right)$ ____(1)

Given diameter of cone $= 8cm$

Radius of cone $= 4cm$

Let height of cone be h

$$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h}$$

$= \frac{1}{3} \times \pi (4)^2 h$ ____(2)

Since hollow sphere is melted into a cone so there volumes are equal

(1) = (2)

$\Rightarrow \frac{4}{3}\pi(64-16) = \frac{1}{3}\pi(4)^2 h$

$\Rightarrow \dfrac{\frac{4}{3}\pi(48)}{\frac{1}{3}\pi(16)} = h$

$\Rightarrow \boxed{h = 12cm}$

$\therefore$ Height of cone $= 12cm$

20. A cylindrical tube of radius 12cm contains water to a depth of 20cm. A spherical ball is dropped into the tube and the level of the water is raised by $6 \cdot 75cm$. Find the radius of the ball____?

Sol:

Given that radius of a cylindrical tube (r) $= 12cm$

Level of water raised in tube $(h) = 6 \cdot 75cm$

$$\boxed{Volume\ of\ cylinder = \pi r^2 h}$$

$= \pi (12)^2 \times 6 \cdot 75 cm^3$

$= \frac{22}{7} (12)^2 6 \cdot 25 cm^3$ ....(1)

Let 'r' be radius of a spherical ball

$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^3}$ ....(2)

To find radius of spherical balls

Equating (1) and (2)

$\pi \times (12)^2 \times 6 \cdot 75 = \frac{4}{3}\pi r^3$

$r^3 = \frac{\pi \times (12)^2 \times 6 \cdot 75}{\frac{4}{3} \times \pi}$

$r^3 = 729$

$r^3 = 9^3$

$\boxed{r = 9cm}$

$\therefore$ Radius of spherical ball $(r) = 9cm$

21. 500 persons have to dip in a rectangular tank which is 80m long and 50m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is $0 \cdot 04 m^3$ ____?

Sol:

Given that length of a rectangular tank (r) = 80m

Breath of a rectangular tank (b) $= 50m$

Total displacement of water in rectangular tank

By 500 persons $= 500 \times 0 \cdot 04 m^3$

$= 20 m^3$ ______(1)

Let depth of rectangular tank be h

$\boxed{Volume\ of\ \ rectangular\ \ \tan k = lbh}$

$= 80 \times 50 \times hm^3$ ______(2)

Equating (1) and (2)

$\Rightarrow 20 = 80 \times 50 \times h$

$\Rightarrow 20 = 4000h$

$\Rightarrow \frac{20}{4000} = h$

$\Rightarrow h = 0 \cdot 005m$

$\boxed{h = 0\cdot 5cm}$

$\therefore$ Rise in level of water in tank $(h) = 0\cdot 05cm.$

22. A cylindrical jar of radius 6cm contains oil. Iron sphere each of radius $1\cdot 5cm$ are immersed in the oil. How many spheres are necessary to raise level of the oil by two centimetress?

Sol:

Given that radius of a cylindrical jar $(r) = 6cm$

Depth/height of cylindrical jar $(h) = 2cm$

Let no of balls be 'n'

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

$$V_1 = \frac{22}{7} \times (6)^2 \times 2cm^3 \qquad \ldots\ldots\ldots(1)$$

Radius of sphere $1\cdot 5cm$

$\boxed{So\ volume\ of\ sphere = \frac{4}{3}\pi r^3}$

$$V_2 = \frac{4}{3} \times \frac{22}{7}(1\cdot 5)^3\ cm^3 \qquad \ldots\ldots\ldots(2)$$

Volume of cylindrical jar is equal to sum of volume of n spheres

Equating (1) and (2)

$$\frac{22}{7} \times (6)^2 \times 2 = n \times \frac{4}{3} \times \frac{22}{4}(1\cdot 5)^3$$

$$n = \frac{v_1}{v_2} \Rightarrow n = \frac{\frac{22}{7} \times (6)^2 \times 2}{\frac{4}{3} \times \frac{22}{7}(1\cdot 5)^3}$$

$\boxed{n = 16}$

$\therefore$ No of spherical balls $(n) = 16$

23. A hollow sphere of internal and external radii 2cm and 4cm is melted into a cone of basse radius 4cm. find the height and slant height of the cone______?

Sol:

Given that internal radii of hollow sphere (r) = 2cm

External radii of hollow sphere $(R) = 4cm$

$\boxed{Volume\ of\ hollow\ sphere = \frac{4}{3}\pi\left(R^2 - r^2\right)}$

$v_1 = \frac{4}{3} \times \pi (4^2 - 2^2)$ ....(1)

Given that sphere is melted into a cone

Base radius of cone $= 4cm$

Let slant height of cone be l

Let height of cone be h

$l^2 = r^2 + h^2$

$l^2 = 16 + h^2$ ....(3)

$$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h}$$

$v_2 = \frac{1}{3}\pi (4)^2 h$ ....(2)

$v_1 = v_2$ Equating (1) and (2)

$\frac{4}{3}\pi (4^2 - 2^2) = \frac{1}{3}\pi (4)^2 h$

$$\frac{\frac{4}{3}\pi (16-4)}{\frac{1}{3}\pi (16)} = h$$

$h = 14cm$

Substituting 'h' value in (2)

$l^2 = 16 + h^2$

$l^2 = 16 + 14^2$

$l^2 = 16 + 196$

$\boxed{l = 14\cdot 56cm}$

$\therefore$ Slant height of cone $= 14\cdot 56cm$

24. The internal and external diameters of a hollow hemisphere vessel are $21cm$ and $25\cdot 2cm.$ The cost of painting $1cm^2$ of the surface is 10paise. Find total cost to paint the vessel all over______?

Sol:

Given that internal diameter of hollow hemisphere $(r) = \frac{21}{2} cm = 10\cdot 5cm$

External diameter $(R) = \frac{25\cdot 2}{2} = 12\cdot 6cm$

Total surface area of hollow hemisphere

$= 2\pi R^2 + 2\pi r^2 + \pi (R^2 - r^2)$

$= 2\pi(12\cdot6)^2 + 2\pi(10\cdot5)^2 + \pi(12\cdot6^2 - 10\cdot5^2)$

$= 997\cdot51 + 692\cdot72 + 152\cdot39$

$= 1843\cdot38cm^2$

Given that cost of painting $1cm^2$ of surface $= 10\,ps$

Total cost for painting $1843\cdot38cm^2$

$= 1843\cdot38 \times 10\,ps$

$= 184\cdot338\,Rs.$

$\therefore$ Total cot to paint vessel all over $= 184\cdot338\,Rs.$

25. A cylindrical tube of radius 12cm contains water to a depth of 20cm. A spherical ball of radius 9cm is dropped into the tube and thus level of water is raised by hcm. What is the value of h_____?

Sol:

Given that radius of cylindrical tube $(r_1) = 12cm$

Let height of cylindrical tube (h)

$\boxed{Volume\ of\ \ a\ cylinder = \pi r_1^2 h}$

$v_1 = \pi(12)^2 \times h \qquad \ldots\ldots(1)$

Given spherical ball radius $(r_2) = 9cm$

$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r_2^3}$

$v_2 = \frac{4}{3} \times \pi \times 9^3 \qquad \ldots\ldots(2)$

Equating (1) and (2)

$v_1 = v_2$

$\pi(12)^2 \times h = \frac{4}{3} \times \pi \times 9^3$

$h = \frac{\frac{4}{3} \times \pi \times 9^3}{\pi(12)^2}$

$h = 6\cdot75cm$

Level of water raised in tube (h) $= 6\cdot75cm$

26. The difference between outer and inner curved surface areas of a hollow right circular cylinder 14cm long is $88cm^2$. If the volume of metal used in making cylinder is $176cm^3$. find the outer and inner diameters of the cylinder____?

Sol:

Given height of a hollow cylinder = 14cm
Let internal and external radii of hollow
Cylinder be 'r' and R
Given that difference between inner and outer
Curved surface $= 88 cm^2$
Curved surface area of cylinder (hollow)

$= 2\pi (R - r) h\ cm^2$

$\Rightarrow 88 = 2\pi (R - r) h$

$\Rightarrow 88 = 2\pi (R - r) 14$

$\Rightarrow R - r = 1$ ……(1)

Volume of cylinder (hollow) $= \pi (R^2 - r^2) h\ cm^3$

Given volume of a cylinder $= 176 cm^3$

$\Rightarrow \pi (R^2 - r^2) h = 176$

$\Rightarrow \pi (R^2 - r^2) \times 14 = 176$

$\Rightarrow R^2 - r^2 = 4$

$\Rightarrow (R + r)(R - r) = 4$

$\Rightarrow R + r = 4$ ……(2)

$R - r = 1$

$R + r = 4$

$2R \quad = 5$

$2R = 5 \Rightarrow \boxed{R = \frac{5}{2} = 2 \cdot 5 cm}$

Substituting 'R' value in (1)

$\Rightarrow R - r = 1$

$\Rightarrow 2 \cdot 5 - r = 1$

$\Rightarrow 2 \cdot 5 - 1 = r$

$\Rightarrow \boxed{r = 1 \cdot 5 cm}$

$\therefore$ Internal radii of hollow cylinder $= 1 \cdot 5 cm$

External radii of hollow cylinder $= 2 \cdot 5 cm$

27. Prove that the surface area of a sphere is equal to the curved surface area of the circumference cylinder__?

Sol:

Let radius of a sphere be r

$\boxed{Curved\ surface\ area\ of\ sphere = 4\pi r^2}$

$S_1 = 4\pi r^2$

Let radius of cylinder be $'r'cm$

Height of cylinder be $'2r'cm$

$\boxed{Curved\ surface\ area\ of\ cylinder = 2\pi rh}$

$S_2 = 2\pi r(2r) = 4\pi r^2$

S_1 and S_2 are equal. Hence proved

So curved surface area of sphere = surface area of cylinder

28. The diameter of a metallic sphere is equal to 9cm. it is melted and drawn into a long wire of diameter 2mm having uniform cross-section. Find the length of the wire?

Sol:

Given diameter of a sphere (d) $= 9cm$

Radius (r) $= \frac{9}{2} = 4 \cdot 5cm$

$\boxed{Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3}$

$V_1 = \frac{4}{3} \times \pi \times 4 \cdot 5^3 = 381 \cdot 70cm^3$ ......(1)

Since metallic sphere is melted and made into a cylindrical wire

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

Given radius of cylindrical wire $(r) = \frac{2mm}{2}$

$= 1mm = 0 \cdot 1cm$

$V_2 = \pi(0 \cdot 1)^2 h$ ......(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow 381 \cdot 703 = \pi(0 \cdot 1)^2 h$

$\Rightarrow \boxed{h = 12150cm}$

$\therefore$ Length of wire (h) $= 12150cm$

29. An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is $\frac{1}{4}$ of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

Sol:

Given that radius of each of smaller ball $= \frac{1}{4}$ Radius of original ball.

Let radius of smaller ball be r.
Radius of bigger ball be $4r$

Volume of big spherical ball $= \frac{4}{3}\pi r^3 \quad (\because r = 4r)$

$$V_1 = \frac{4}{3}\pi(4r)^3 \qquad \ldots\ldots(1)$$

$$\boxed{Volume\ of\ each\ small\ ball = \frac{4}{3}\pi r^3}$$

$$V_2 = \frac{4}{3}\pi r^3 \qquad \ldots\ldots(2)$$

Let no of balls be $'n'$

$$n = \frac{V_1}{V_2}$$

$$\Rightarrow n = \frac{\frac{4}{3}\pi(4r)^3}{\frac{4}{3}\pi(r)^3}$$

$$\Rightarrow n = 4^3 = 64$$

$$\boxed{\therefore\ No\ of\ small\ balls = 64}$$

Curved surface area of sphere $= 4\pi r^2$

Surface area of big ball $(S_1) = 4\pi(4r)^2 \qquad \ldots\ldots.(3)$

Surface area of each small ball $(S_1) = 4\pi r^2$

Total surface area of 64 small balls

$(S_2) = 64 \times 4\pi r^2 \qquad \ldots\ldots.(4)$

By combining (3) and (4)

$$\Rightarrow \frac{S_2}{3} = 4$$

$$\Rightarrow \boxed{S_2 = 4s}$$

$\therefore$ Total surface area of small balls is equal to 4 times surface area of big ball.

30. A tent of height 77dm is in the form a right circular cylinder of diameter 36m and height 44dm surmounted by a right circular cone. Find the cost of canvas at Rs.3.50 per m^2 ?

Sol:

Given that height of a tent = 77dm

Height of cone $= 44dm$

Height of a tent without cone $=77-44=33dm$
$=3\cdot 3m$
Given diameter of cylinder (d) = 36m
Radius $(r)=\frac{36}{2}=18m$
Let 'l' be slant height of cone
$l^2=r^2+h^2$
$l^2=18^2+3\cdot 3^2$
$l^2=324+10\cdot 89$
$l^2=334\cdot 89$
$l=18\cdot 3$
Slant height of cone $l=18\cdot 3$
Curved surface area of cylinder $(S_1)=2\pi rh$
$=2\times\pi\times 18\times 4\cdot 4m^2$ ....(1)
Curved surface area of cone $(S_2)=\pi rl$
$=\pi\times 18\times 18\cdot 3m^2$ ....(2)
Total curved surface of tent $=S_1+S_2$
T.C.S.A $=S_1+S_2$
$=1532\cdot 46m^2$
Given cost canvas per m^2 = Rs $3\cdot 50$
Total cost of canvas per $1532\cdot 46\times 3\cdot 50$
$=1532\cdot 46\times 3\cdot 50$
$=5363\cdot 61$
$\therefore$ Total cost of canvas $=Rs$ 5363.61

31. Metal spheres each of radius 2cm are packed into a rectangular box of internal dimension $16cm\times 8cm\times 8cm$ when 16 spheres are packed the box is filled with preservative liquid. Find volume of this liquid?

Sol:

Given radius of metal spheres $=2cm$

$$Volume\ of\ sphere(v)=\frac{4}{3}\pi r^3$$

So volume of each metallic sphere $=\frac{4}{3}\pi(2)^3\ cm^3$

Total volume of 16 spheres $(v_1)=16\times\frac{4}{3}\pi(2)^3\ cm^3$ ...(1)

Volume of rectangular box $=lbh$

$V_2 = 16\times 8\times 8 cm^3$ …(2)

Subtracting (2) – (1) we get volume of liquid

$\Rightarrow V_2 - V_1 =$ Volume off liquid

$\Rightarrow 16\times 8\times 8 - \frac{4}{3}\pi(2)^3\times 16$

$\Rightarrow 1024 - 536\cdot 16 = 488 cm^3$

$\therefore$ Hence volume of liquid $= 488 cm^3$

32. The largest sphere is to be curved out of a right circular of radius 7cm and height 14cm. find volume of sphere?

Sol:

Given radius of cylinder $(r) = 7cm$

Height of cylinder $(h) = 14cm$

Largest sphere is curved out from cylinder

Thus diameter of sphere = diameter of cylinder

Diameter of sphere $(d) = 2\times 7 = 14cm$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi(7)^3$

$= \frac{1372\pi}{3}$

$= 1436\cdot 75 cm^3$

$\therefore$ Volume of sphere $= 1436\cdot 75 cm^3$

33. A copper sphere of radius 3cm is melted and recast into a right circular cone of height 3cm. find radius of base of cone?

Sol:

Given radius of sphere $= 3cm$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi\times 3^3 cm^3$ …..(1)

Given sphere is melted and recast into a right circular cone

Given height of circular cone $= 3cm$.

Volume of right circular cone $= \pi r^2 h \times \frac{1}{3}$

$= \frac{\pi}{3}(r)^2 \times 3cm^2$ .....(1)

Equating 1 and 2 we get

$\frac{4}{3}\pi \times 3^3 = \frac{1}{3}\pi(r)^2 \times 3$

$r^2 = \frac{\frac{4}{3}\pi \times 3^3}{\pi}$

$r^2 = 36cm$

$\boxed{r = 6cm}$

$\therefore$ Radius of base of cone $(r) = 6cm$

34. A vessel in the shape of cuboid contains some water. If these identical spheres are immersed in the water, the level of water is increased by 2cm. if the area of base of cuboid is $160cm^2$ and its height 12cm, determine radius of any of spheres?

Sol:

Given that area of cuboid $= 160cm^2$

Level of water increased in vessel $= 2cm$

Volume of a vessel $= 160 \times 2cm^3$ ......(1)

Volume of each sphere $= \frac{4}{3}\pi r^3 cm^3$

Total volume of 3 spheres $= 3 \times \frac{4}{3}\pi r^3 cm^3$ ......(2)

Equating (1) and (2) ($\because$ Volumes are equal $V_1 = V_2$)

$160 \times 2 = 3 \times \frac{4}{3}\pi r^3$

$r^3 = \frac{160 \times 2}{3 \times \frac{4}{3}\pi}$

$r^3 = \frac{320}{4\pi}$

$\boxed{r = 2 \cdot 94cm}$

$\therefore$ Radius of sphere $= 2 \cdot 94cm$

35. A copper rod of diameter 1cm and length 8cm is drawn into a wire of length 18m of uniform thickness. Find thickness of wire?

Sol:

Given diameter of copper rod $(d_1) = 1cm$

Radius $(r_1) = \frac{1}{2} = 0\cdot5cm$

Length of copper rod $(h_1) = 8cm$

$\boxed{Volume\ of\ \ cylinder = \pi r_1^2 h_1}$

$V_1 = \pi(0\cdot5)^2 \times 8cm^3$ ……(1)

$V_2 = \pi r_2^2 h_2$

Length of wire $(h_2) = 18m = 1800cm$

$V_2 = \pi r_2^2 (1800) cm^3$ …….(2)

Equating (1) and (2)

$V_1 = V_2$

$\pi(0\cdot5)^2 \times 8 = \pi r_2^2 (1800)$

$\frac{\pi(0\cdot5)^2 \times 8}{\pi(1800)} = r_2^2$

$\boxed{r_2 = 0\cdot033cm}$

$\therefore$ Radius thickness of wire $= 0\cdot033cm.$

36. The diameters of internal and external surfaces of hollow spherical shell are 10cm and 6cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

Sol:

Given diameter of internal surfaces of a hollow spherical shell $= 10cm$

Radius $(r) = \frac{10}{2} = 5cm.$

External radii $(R) = \frac{6}{2} = 3cm$

$\boxed{Volume\ of\ \ a\ \ spherica\ shell\ (hollow) = \frac{4}{3}\pi(R^2 - r^2)}$

$V_1 = \frac{4}{3}\pi(5^2 - 3^2)cm^3$ ………(1)

Given length of solid cylinder $(h) = \frac{8}{3}$

Let radius of solid cylinder be $'r'$

$$\boxed{Volume\ of\ \ a\ cylinder = \pi r^2 h}$$

$$V_2 = \pi r^2 \left(\frac{8}{3}\right) cm^3 \qquad \ldots\ldots\ldots(2)$$

$$V_1 = V_2$$

Equating (1) and (2)

$$\Rightarrow \frac{4}{3}\pi(25-9) = \pi r^2\left(\frac{8}{3}\right)$$

$$\Rightarrow \frac{\frac{4}{3}\pi(16)}{\pi\left(\frac{8}{3}\right)} = r^2$$

$$\Rightarrow r^2 = 49cm$$

$$\Rightarrow r = 7cm$$

$$d = 2r = 14cm$$

$$\boxed{\therefore Diameter\ of\ \ cylinder\ = 14cm}$$

37. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two days. Find the difference in volumes of the two cones so formed. Also, find their curved surfaces.

Sol:

(i) Given that radius of cone $(r_1) = 4cm$

Height of cone $(h_1) = 3cm$

Slant height of cone $(l_1) = 5cm$

Volume of cone $(V_1) = \frac{1}{3}\pi r_1^2 h_1$

$$= \frac{1}{3}\pi(4)^2(3) = 16\pi cm^3$$

(ii) Given radius of second cone $(r_2) = 3cm$

Height of cone $(h_2) = 4cm$

Slant height of cone $(l_2) = 5cm$

Volume of cone $(V_2) = \frac{1}{3} r_2^2 h_2$

$$= \frac{1}{3}\pi(3)^2(4) = 12\pi cm^3$$

Difference in volumes of two cones $(V) = V_1 - V_2$

$V = 16\pi - 12\pi$

$V = 4\pi cm^3$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of second cone $(S_2) = \pi r_2 l_2$

$S_1 = \pi(3)(5) = 15\pi cm^2$

$S_1 = 20\pi cm^2 S_2 = 15\pi cm^2$

38. How many coins $1\cdot75cm$ in diameter and 2mm thick must be melted to form a cuboid $11cm \times 10cm \times 75cm$ ___?

Sol:

Given that dimensions of a cuboid $11cm \times 10cm \times 75cm$

So its volume $(V_1) = 11cm \times 10cm \times 7cm$

$= 11 \times 10 \times 7cm^3$ ....(1)

Given diameter (d) $= 1\cdot75cm$

Radius $(r) = \frac{d}{2} = \frac{1\cdot75}{2} = 0\cdot875cm$

Thickness $(h) = 2mm = 0\cdot2cm$

$\boxed{Volume\ \ of\ \ a\, cylinder = \pi r^2 h}$

$V_2 = \pi(0\cdot875)^2(0\cdot2)cm^3$ ....(2)

$V_1 = V_2 \times n$

Since volume of a cuboid is equal to sum of n volume of 'n' coins

$n = \frac{V_1}{V_2}$

$n = no$ of coins

$n = \frac{11 \times 10 \times 7}{\pi(0\cdot875)^2(0\cdot2)}$

$\boxed{n = 1600}$

$\therefore$ No of coins $(n) = 1600,$

39. A well with inner radius 4m is dug 14m deep earth taken out of it has been spread evenly all around a width of 3m it to form an embankment. Find the height of the embankment?

Sol:

Given that inner radius of a well $(a) = 4m$

Depth of a well $(h) = 14m$

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

$V_1 = \pi(4)^2 \times 14cm^3$ ……….(1)

Given well is spread evenly to form an embankment

Width of an embankment $= 3m$

Outer radii of a well (R) $= 4 + 3 = 7m.$

$\boxed{Volume\ of\ a\ hollow\ cylinder = \pi\left(R^2 - r^2\right) \times hm^3}$

$V_2 = \pi\left(7^2 - 4^2\right) \times hm^3$ ………..(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \pi(4)^2 \times 14 = \pi(49 - 16) \times h$

$\Rightarrow h = \dfrac{\pi(4)^2 \times 14}{\pi(33)}$

$\boxed{h = 6 \cdot 78m}$

40. Water in a canal $1.5m$ wide and 6m deep is flowering with a speed of $10km/hr$. how much area will it irrigate in 30 minutes if 8cm of standing water is desired?

Sol:

Given that water is flowering with a speed $= 10km/hr$

In 30 minutes length of flowering standing water $= 10 \times \dfrac{30}{60} km$

$= 5km = 5000m.$

Volume of flowering water in 30 minutes

$V = 5000 \times width \times depth\ m^3$

Given width of canal $= 1 \cdot 5m$

Depth of canal $= 6m$

$V = 5000 \times 1 \cdot 5 \times 6m^3$

$\boxed{V = 45000m^3}$

Irrigating area in 30 minutes if 8cm of standing water is desired $= \dfrac{45000}{0 \cdot 08}$

$$= \frac{45000}{0\cdot 08} = 562500m^2$$

$$\boxed{\therefore\ Irrigated\ area\ in\ 30\min utes = 562500m^2}$$

41. A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?
Sol:
$\frac{9}{8}\ m$

42. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.
Sol:
Given diameter of well = 3m

Radius of well $= \frac{3}{2}m = 4$

Depth of well $(b) = 14m$

With of embankment $= 4m$

$\therefore$ Radius of outer surface of embankment $= 4 + \frac{3}{2} = \frac{11}{2}m$

Let height of embankment $= hm$

$$\boxed{Volume\ of\ embankment\left(V_1\right) = \pi\left(r_2^2 - r_1^2\right)h}$$

($\because$ it is viewed as a hollow cylinder)

$$V_1 = \pi\left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)\right)^2 \times h - m^3 \quad \text{.....(1)}$$

$$\boxed{Volume\ of\ earth\ dugout\ \left(V_2\right) = \pi r_1^2 h}$$

$$V_2 = \pi\left(\frac{3}{2}\right)^2 \times 14\ m^3 \quad \text{.....(2)}$$

Given that volumes (1) and (2) are equal

So $V_1 = V_2$

$$\Rightarrow \left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) \times h = \pi\left(\frac{3}{2}\right)^2 \times 14$$

$$\Rightarrow \left(\frac{121}{4} - \frac{9}{4}\right)h = \frac{9}{4} \times 14$$

$\Rightarrow \boxed{h = \frac{9}{8}m}$

$\therefore$ Height of embankment $(h) = \frac{9}{8}m.$

43. The surface area of a solid metallic sphere is 616 cm2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed (Use it = $\frac{22}{7}$)

Sol:

Given height of cone $(h) = 28cm$

Given surface area of Sphere $= 616cm^2$

We know surface area of sphere $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$

$\Rightarrow \boxed{r = 7cm}$

$\therefore$ Radius of sphere $(r) = 7cm$

Let r_1 be radius of cone

Given volume of cone $=$ Volume of sphere

$\boxed{Volume\ of\ cone = \frac{1}{3}\pi (r^2) h}$

$V_1 = \frac{1}{3}\pi (r_1)^2 \times 28cm^3$ ....(1)

$\boxed{Volume\ of\ sphere = (V_2) = \frac{4}{3}\pi r^3}$

$V_2 = \frac{4}{3}\pi (7)^3 cm^3$ ....(1)

(1) = (2) $\Rightarrow V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi (r_1)^2 \times 28 = \frac{4}{3}\pi (7)^3$

$\Rightarrow r_1^2 = 49$

$r_1 = 7cm$

Radius of cone $(r_1) = 7cm$

$\boxed{Diameter\ of\ base\ of\ cone (d_1) = 2 \times 7 = 14cm}$

44. The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14cm long is $88cm^2$. If the volume of metal used in making cylinder is $176cm^3$ find outer and inner diameters of the cylinder?

Sol:

Given height of a hollow cylinder $=14cm$

Let internal and external radii of hollow

Cylinder be 'r' an 'R'

Given that difference between inner and outer curved surface $=88cm^2$

$$\boxed{Curved\ surface\ area\ of\ hollow\ cylinder = 2\pi(R-r)h}$$

$$\Rightarrow 88 = 2\pi(R-0)h$$

$$\Rightarrow 88 = 2\pi(R-r)14$$

$$\Rightarrow R-r=1 \quad \text{........(1)}$$

$$\boxed{Volume\ of\ hollow\ cylinder = \pi\left(R^2-r^2\right)h\ cm^3}$$

Given volume of cylinder $=176cm^3$

$$\Rightarrow \pi\left(R^2-r^2\right)h=176$$

$$\Rightarrow \pi\left(R^2-r^2\right)\times 14=176$$

$$\Rightarrow R^2-r^2=4$$

$$\Rightarrow (R+r)(R-r)=4$$

$$\Rightarrow R+4=4 \quad \text{........(2)}$$

By using (1) and (2) equations and solving we get

$$R-r=1 \quad ..(1)$$

$$R+r=4 \quad ..(2)$$

$$2R=5$$

$$\Rightarrow \boxed{R=\frac{5}{2}=2\cdot 5cm}$$

Substituting 'R' value in (1)

$$\Rightarrow R-r=1$$

$$\Rightarrow 2\cdot 5-r=1$$

$$\Rightarrow 2\cdot 5-1=r$$

$$\Rightarrow \boxed{r=1\cdot 5cm}$$

External radii of hollow cylinder $(R)=2\cdot 5cm$

Internal radii of hollow cylinder $(r)=1\cdot 5cm$

45. The volume of a hemisphere is $2425\frac{1}{2}cm^3$. Find its curved surface area?

Sol:

Given that volume of a hemisphere $=2424\frac{1}{2}cm^3$

Volume of a hemisphere $=\frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = 2425\frac{1}{2}$

$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4841}{2}$

$\Rightarrow r^3 = \frac{4851\times 3}{2\times 2\times \pi}$

$\Rightarrow r^3 = \frac{4851\times 3}{4\pi}$

r^3

$r = 10\cdot 50cm$

$\therefore$ Radius of hemisphere $=10\cdot 5cm$

Curved surface area of hemisphere $=2\pi r^2$

$=2\pi(10\cdot 5)^2$

$=692\cdot 72$

$\Rightarrow 693xm^2$

$\therefore$ curved surface area off hemisphere $=693cm^2$

46. A cylindrical bucket 32cm high and with radius of base 18cm is filled with sand. This bucket is emptied out on the ground and a conical heap of sand is formed. If the height of the conical heap of sand is formed. If the height of the conical heap is 24cm. find the radius and slant height of the heap?

Sol:

Given that height of cylindrical bucket $(h)=32cm$

Radius $(r)=18cm$

Volume of cylinder $=\pi r^2 h$

$=\frac{22}{7}(18)^2\times 32cm^3$ ....(1)

Given height of conical heap $=24cm$

Let radius of conical heap be r_1

Slant height of conical heap be l_1

$\Rightarrow l_1^2 = r_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 - (24)^2$ ....(2)

Volume of cone $= \frac{1}{3}\pi r^2 h$

So its volume $= \frac{1}{3}\pi \Rightarrow r_1^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times r_1^2 \times 24$

$= \frac{22}{7} \times r_1^2 \times 8cm^3$ ....(3)

So equating (1) and (3)

(1) = (3)

$\Rightarrow \frac{22}{7}(18)^2 \times 32 = \frac{22}{7} \times r_1^2 \times 8$

$\Rightarrow \frac{(18)^2 \times 32}{8} = r_1^2$

$\Rightarrow r_1^2 = 1296$

$\Rightarrow \boxed{r_1 = 36cm}$

Radius of conical heap is 36cm

Substituting r_1 in (2)

$\Rightarrow r_1^2 = l_1^2 - (24)^2$

$\Rightarrow 1296 = l_1^2 - 576$

$\Rightarrow 1296 + 576 = l_1^2$

$\Rightarrow 1872 = l_1^2$

$\Rightarrow \boxed{l_1 = 43 \cdot 26cm}$

$\therefore$ Slant height of conical heap $= 43 \cdot 26cm$

Exercise 16.2

47. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Sol:

Given diameter of cylinder $24m$

Radius $(r)=\frac{24}{2}=12m$

Given height of cylindrical part $(h_1)=11m$

$\therefore$ Height of cone part $(h_2)=5m$

Vertex of cone above ground $=11+5=16m$

Curved surface area of cone $(S_1)=\pi rl$

$=\frac{22}{7}\times 12\times l$

Let l be slant height of cone

$\Rightarrow l=\sqrt{r^2+h_2^2}$

$\Rightarrow l=\sqrt{12^2+5^2}=13m$

$l=13m$

$\therefore$ Curved surface area of cone $(5)=\frac{22}{7}\times 12\times 13m^2$ ……….(1)

Curved surface area of cylinder $(S_2)=2\pi rh$

$S_2=2\pi(12)(11)m^2$ ………..(2)

To find area of canvas required for tent

$S=S_1+S_2=(1)+(2)$

$S=\frac{22}{7}\times 12\times 13+2\pi(12)(11)$

$S=490+829\cdot 38$

$S=1320m^2$

$\therefore$ Total canvas required for tent $(S)=1320m^2$

48. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius $2\cdot 5m$ and height 21m and the cone has a slant height 8m. Calculate total surface area and volume of the rocket?

Sol:

Given radius of cylinder $(a)=2\cdot 5m$

Height of cylinder $(h)=21m$

Slant height of cylinder $(l)=8m$

Curved surface area of cone $(S_1)=\pi rl$

$S_1=\pi(2\cdot 5)(8)cm^2$ ………(1)

Curbed surface area of a cone $=2\pi rh+\pi r^2$

$S_2 = 2\pi(2\cdot5)(21) + \pi(2\cdot5)^2\ cm^2$ ……….(2)

$\therefore$ Total curved surface area = (1) + (2)

$S = S_1 + S_2$

$S = \pi(2\cdot5)(8) + 2\pi(2\cdot5)(21) + \pi(2\cdot5)^2$

$S = 62\cdot831 + 329\cdot86 + 19\cdot63$

$S = 412\cdot3m^2$

$\therefore$ Total curved surface area $= 412\cdot3m^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$V_1 = \frac{1}{3}\times\pi(2\cdot5)^2 h\ \ cm^3$ ………...(3)

Let 'h' be height of cone

$l^2 = r^2 + h^2$

$\Rightarrow l^2 - r^2 = h^2$

$\Rightarrow h = \sqrt{l^2 - r^2}$

$\Rightarrow h = \sqrt{8^2 - 25^2}$

$\Rightarrow h = 23\cdot685m$

Subtracting 'h' value in (3)

Volume of a cone $(V_1) = \frac{1}{3}\times\pi(2\cdot5)^2(23\cdot685)cm^2$ ………(4)

Volume of a cylinder $(V_2) = \pi r^2 h$

$= \pi(2\cdot5)^2 21m^3$ ………..(5)

Total volume = (4) + (5)

$V = V_1 + V_2$

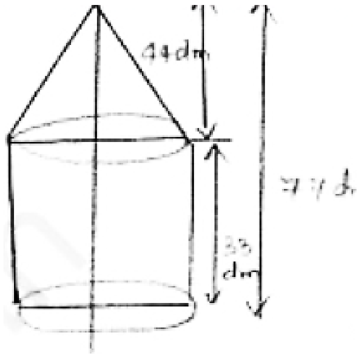
$\Rightarrow V = \frac{1}{3}\times\pi(2\cdot5)^2(23\cdot685) + \pi(2\cdot5)^2 = 1$

$\Rightarrow V = 461\cdot84m^2$

Total volume $(V) = 461\cdot84m^2$

49. A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 350 per m^2 (Use it = $\frac{22}{7}$).

Sol:



Given that height of a tent $=77dm$
Height of a surmounted cone $=44dm$
Height of cylinder part $=77-44$
$=33dm=3.3m$
Given diameter of cylinder (d) $=26m$

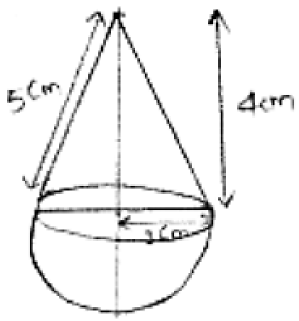
Radius $(r)=\frac{36}{2}=18m.$

Let 'l' be slant height of cone
$\Rightarrow l^2=r^2+h^2$
$\Rightarrow l^2=18^2+3\cdot3^2$
$\Rightarrow l^2=824+10\cdot89$
$\Rightarrow l=18\cdot3$
$\therefore$ Slant height of cone $(1)=18\cdot3$

Curved surface area of cylinder $(S_1)=2\pi rh$
$=2\times\pi\times18\times4\cdot4m^2$ ....(1)
Curved surface area of cone $(S_2)=\pi rh$
$=\pi\times18\times18\cdot3m^2$ ....(2)
Total curved surface of tent $=S_1+S_2$
$S=S_1+S_2$
$S=1532\cdot46m^2$
$\therefore$ Total curved surface area $(S)=12=1532\cdot46m^2$

50. A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of cone are 6cm and 4cm. determine surface area of toy?
Sol:
Given height of cone $(h)=4cm$
Diameter of cone $(d)=6cm$

$\therefore$ Radius (r) $= \frac{6}{2} = 3cm$



Let 'l' be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2} = 5cm$

$l = 5cm$

$\therefore$ Slant height of cone $(l) = 5cm.$

Curved surface area of cone $(S_1) = \pi rl$

$S_1 = \pi(3)(5) = 47 \cdot 1cm^2$

Curved surface area of hemisphere $(S_2) = 2\pi r^2$

$S_2 = 2\pi(3)^2 = 56 \cdot 52cm^2$

$\therefore$ Total surface area $(s) = 6_1 + S_2$

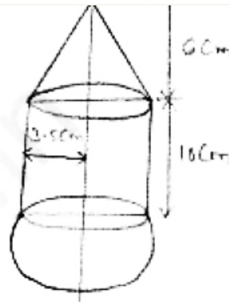
$= 47 \cdot 1 + 56 \cdot 52$

$= 103 \cdot 62cm^2$

$\therefore$ Curved surface area of toy $= 103 \cdot 62cm^2$

51. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of the cylindrical and conical portions are 10 cm. and 6 cm, respectively. Find the total surface area of the solid. (Use n = $\frac{22}{7}$)

Sol:



Given radius of common base $= 3\cdot5cm$

Height of cylindrical part $(h) = 10cm$

Height of conical part $(h) = 6cm$

Let $'l'$ be slant height of cone

$l = \sqrt{r^2 + h^2}$

$l = \sqrt{(3\cdot5)^2 + 6^2}$

$l = 48\cdot25cm$

Curved surface area of cone $(S_1) = \pi rl$

$= \pi(3\cdot5)(48\cdot25)$

$= 76\cdot408cm^2$

Curved surface area of cylinder $(S_2) = 2\pi rh$

$= 2\pi(3\cdot5)(10)$

$= 220cm^2$

Curved surface area of hemisphere $(S) = S_1 + S_2 + S_3$

$= 76\cdot408 + 220 + 77$

$= 373\cdot408cm^2$

$\therefore$ Total surface area of solid $(S) = 373\cdot408cm^2$

Cost of canvas per $m^2 = Rs\ 3\cdot50$

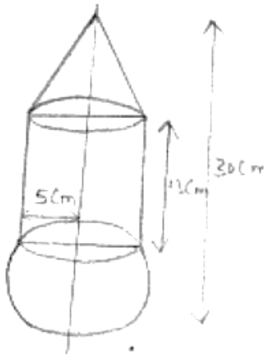
Cost of canvas for $1532\cdot46m^2 = 1532\cdot46 \times 3\cdot50$

$= 5363\cdot61Rs$

$\therefore$ Cost of canvas required for tent $= Rs\ 5363\cdot61pr$

52. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

Sol:



$S_1 = 2\pi(2)(13)$

$S_1 = 408\cdot 2cm^2$

Curved surface area of cone $(S_2) = \pi rl$

Let l be slant height of cone

$l = \sqrt{r^2 + h^2}$

$h = 30 - 13 - 5 = 12cn$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13cm$

$l = 13cm$

$\therefore$ Curved surface area of cone $(S_2) = \pi(5)(13)$

$= 204\cdot 1cm^2$

Curved surface area of hemisphere $(S_3) = 2\pi r^2$

$= 2\pi(5)^2$

$= 2\pi(25) = 50\pi = 157cm^2$

$S_3 = 157cm^2$

Total curved surface area $(S) = S_1 + S_2 + S_3$

$S = 408\cdot 2 + 204\cdot 1 + 157$

$S = 769\cdot 3cm^2$

$\therefore$ Surface area of toy $(S) = 769.3cm^2$

53. A cylindrical tube of radius 5cm and length $9\cdot 8cm$ is full of water. A solid in form of a right circular cone mounted on a hemisphere is immersed in tube. If radius of hemisphere is immersed in tube if the radius of hemisphere is $3\cdot 5cm$ and height of the cone outside hemisphere is 5cm. find volume of water left in the tube?

Sol:

Given radius of cylindrical tube $(r) = 5cm.$

Height of cylindrical tube $(h) = 9\cdot 8cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi(5)^2(9\cdot 8) = 770cm^3$

Given radius of hemisphere $(r) = 3\cdot 5cm$

Height of cone $(h) = 5cm$

Volume of hemisphere $= \dfrac{2}{3}\pi r^3$

$=\frac{2}{3}\times\pi(3\cdot5)^3=89\cdot79cm^3$

Volume of cone $=\frac{1}{3}\pi r^2h$

$=\frac{\pi}{3}(3\cdot5)^2 5=64\cdot14cm^3$

Volume of cone + volume of hemisphere $(V_2)=39\cdot79+64\cdot14=154cm^3$

54. A circular tent has cylindrical shape surmounted by a conical roof. The radius of cylindrical base is $20m.$ The height of cylindrical and conical portions are $4\cdot2m$ and $2\cdot1m.$ Find the volume of the tent?

Sol:

Given radius of cylindrical base $=20m$

Height of cylindrical part $(h)=4\cdot2m.$

Volume of cylindrical $=\pi r^2h_1$

$V_1=\pi(20)^2 4\cdot2=5280m^3$

Volume of cone $=\frac{1}{3}\pi r^2h_2$

Height of conical part $(h_2)=2\cdot1m$

$V_2=\frac{\pi}{3}(20)^2(2\cdot1)=880m^3$

Volume of tent $(v)=V_1+V_2$

$V=5280+880$

$V=6160m^3$

$\therefore$ Volume of tent $(v)=V_1+V_2$

$V=5280+880$

$V=6160m^3$

$\therefore$ Volume of tent $(v)=6160m^3$

55. A petrol tank is a cylinder of base diameter 21cm and length 18cm fitted with conical ends each of axis 9cm. determine capacity of the tank?

Sol:

Given base diameter of cylinder $=21cm$

Radius $(r)=\frac{21}{2}=11\cdot5cm$

Height of cylindrical part $(h)=18cm$

Height of conical part $(h_2) = 9cn$

Volume of cylinder $= \pi r^2 h_1$

$V_1 = \pi (11 \cdot 5)^2 18 = 7474 \cdot 77 cm^3$

Volume of cone $= \frac{1}{3} \pi r^2 h_2$ ($\because$ 2 conical end)

$V_2 = \frac{1}{3} \pi (11 \cdot 5)^2 (9) \times 2$

$V_2 = \frac{1}{3} \pi (1190 \cdot 25) = 2492 \cdot 25 cm^3$

Volume of tank = volume of cylinder + volume of cone

$V = V_1 + V_2$

$V = 7474 \cdot 77 + 2492 \cdot 85$

$V = 9966 \cdot 36 cm^3$

Volume of water left in tube = Volume of cylinder – Volume of hemisphere and cone

$V = V_1 - V_2$

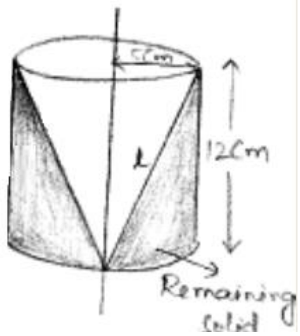
$= 770 - 154$

$= 616 cm^3$

$\therefore$ Volume of water left in tube $= 616 cm^3$

56. A conical hole is drilled in a circular cylinder of height 12cm and base radius 5cm. The height and base radius of the cone are also the same. Find the whole surface and volume of the remaining cylinder?

Sol:



Given base radius of cylinder $(r) = 5cm$

Height of cylinder $(h) = 12cm$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$

$l = 13cm$

$\therefore$ Height and base radius of cone and cylinder are same

Total surface area of remaining part $(s) = 2\pi rh + \pi r^2 + \pi rl$

$= 2\pi(5)(12) + \pi(5)^2 + \pi(5)(13)$

T.S.A $= 210\pi cm^2$

Volume of remaining part = Volume of cylinder – Volume of cone

$\Rightarrow V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

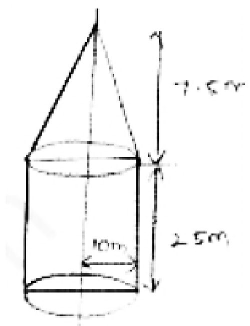
$\Rightarrow V = \pi(5)^2(12) - \frac{1}{3}\pi(5)^2(12)$

$\Rightarrow V = 200\pi cm^3$

$\therefore$ Volume of remaining part $(v) = 200\pi cm^3$

57. A tent is in form of a cylinder of diameter 20m and height $2\cdot5m$ surmounted by a cone of equal base and height $7\cdot5m$. Find capacity of tent and cost of canvas at Rs 100per square meter?

Sol:



Given radius of cylinder $(r) = \frac{20}{2} = 10m$

Height of a cylinder $(h_1) = 2\cdot5m$

Height of cone $(h_2) = 7\cdot5m$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h_2^2}$

$l = \sqrt{10^2 + 7\cdot5^2}$

$\Rightarrow l = 12\cdot5m$

Volume of cylinder $(V_1) = \pi r^2 h$

$V_1 = \pi(10)^2(2\cdot5)$ ……..(1)

Volume of cone $(V_2)=\frac{1}{3}\pi r^2 h_2$

$=\frac{1}{3}\pi(10)^2(7\cdot 5)m^3$ ....(2)

Total capacity of tent = (1) + (2)

$V=V_1+V_2$

$V=\pi(10)^2 2\cdot 5+\frac{1}{3}\pi(10)^2 7\cdot 5$

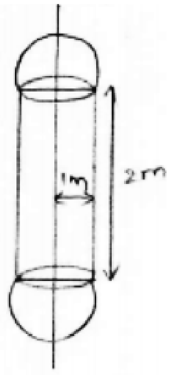
$V=250\pi+250\pi$

$V=500\pi cm^3$

$\therefore$ Total capacity of tent $=500\pi cm^2$

58. A boiler is in the form of a cylinder 2m long with hemispherical ends each of 2m diameter. Find the volume of the boiler?

Sol:



Given height of cylinder $(h)=2m$

Diameter of hemisphere $(d)=2m$

Radius $(r)=1m$

Volume of a cylinder $=\pi r^2 h$

$V_1=\pi(1)^2(2)cm^3$ ....(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

Since at ends of cylinder hemisphere are attached

Volumes of 2 hemispheres

$=2\times\frac{2}{3}\pi(1)^2 cm^2$ ....(2)

Volumes of boiler = (1) + (2)

$V=V_1+V_2$

$V = 2\times\frac{2}{3}\pi(1)^2 + \pi(1)^2(2)$

$V = \frac{220}{21}m^3$

$\therefore$ Volumes of boiler $= \frac{220}{21}m^3$

59. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of cylinder is $\frac{14}{3}m$ and internal surface area of the solid?

Sol:

Given radius of hemisphere $(r) = \frac{3\cdot5}{2} = 1\cdot75m$

Height of cylinder $(h) = \frac{14}{3}m$

Volume of cylinder $= \pi r^2 h$

$= \pi(1\cdot75)^2\left(\frac{14}{3}\right)cm^3$ ……..(1)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\pi(1\cdot75)^3 cm^3$ ……..(2)

Volume of vessel = (1) + (2)

$V = V_1 + V_2$

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$V = \pi(1\cdot75)^2\left(\frac{14}{3}\right) + \frac{2}{3}\pi(1\cdot75)^2$

$V = 56m^3$

$\therefore$ Volumes of vessel $(v) = 56m^3$

Internal surface area of solid $(s) = 2\pi rh + 2\pi r^2$

S = Surface area of cylinder + surface are of hemisphere

S = $2\pi(1\cdot75)\left(\frac{14}{3}\right) + 2\pi(1\cdot75)^2$

$S = 70\cdot51m^2$

$\therefore$ Internal surface area of solid $(s) = 70\cdot51m^2$

60. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104cm and radius of each of hemispherical ends is 7cm. find the cost of polishing its surface at the rate of $Rs\ 10\ per\ dm^2$?

Sol:

Given radius of hemispherical ends $= 7cm$

Height of body $(h+2r) = 104cm.$

Curved surface area of cylinder $= 2\pi rh$

$= 2\pi(7)h$ ....(1)

$\Rightarrow h+2x = 104$

$\Rightarrow h = 104 - 2(r)$

$\Rightarrow h = 90cm$

Substitute 'h' value in (1)

Curved surface area of cylinder $= 2\pi(7)(90)$

$= 3948\cdot 40cm^2$ ....(2)

Curved surface area of 2 hemisphere $= 2(2\pi r^2)$

$= 2(2\times\pi\times 7^2)$

$= 615\cdot 75cm^3$ ....(3)

Total curved surface area = (2) + (3)

$= 3958\cdot 40 + 615\cdot 75 = 4574\cdot 15cm^2 = 45\cdot 74dm^2$

Cost of polishing for $1dm^2 = Rs10$

Cost of polishing for $45\cdot 74dm^2 = 45\cdot 74\times 10$

$= Rs\ 457\cdot 4$

61. A cylindrical vessel of diameter 14cm and height 42cm is fixed symmetrically inside a similar vessel of diameter 16cm and height $42cm.$ The total space between two vessels is filled with cork dust for heat insulation purpose. How many cubic cms of cork dust will be required?

Sol:

Given height of cylindrical vessel $(h) = 42cm$

Inner radius of a vessel $(r_1) = \frac{14}{2}cm = 7cm$

Outer radius of a vessel $(r_2) = \frac{16}{2} = 8cm$

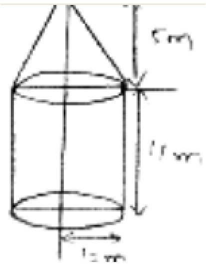
Volume of a cylinder $= \pi(r_2^2 - r_1^2)h$

$= \pi(8^2 - 7^2)42$

$= \pi (64 - 49) 42$
$= 15 \times 42 \times \pi$
$= 630\pi$
$= 1980 cm^3$
Volume of a vessel $= 1980 cm^2$

62. A cylindrical road solar made of iron is 1m long its internal diameter is 54cm and thickness of the iron sheet used in making roller is 9cm. Find the mass of roller if $1cm^3$ of iron has $7 \cdot 8 gm$ mas?

Sol:



Given internal radius of cylindrical road

Roller $(r_1) = \frac{54}{2} = 27cm$

Given thickness of road roller $\left(\frac{1}{b}\right) = 9cm$

Let order radii of cylindrical road roller be R
$\Rightarrow t = R - r$
$\Rightarrow 9 = R - 27$
$\Rightarrow R = 9 + 27 = 36cm$
$R = 36cm$
Given height of cylindrical road roller $(h) = 1m$
$h = 100cm.$
Volume of iron $= \pi h (R^2 - r^2)$
$= \pi (36^2 - 27^2) \times 100$
$= 1780 \cdot 38 cm^3$
Volume of iron $= 1780 \cdot 38 cm^3$
Mass of $1cm^3$ of iron $= 7 \cdot 8 gm$
Mass of $1780 \cdot 38 cm^3$ of iron $= 1780 \cdot 38 \times 7 \cdot 8$
$= 1388696 \cdot 4 gm$
$= 1388 \cdot 7 kg$

$\therefore$ Mass of roller $(m) = 1388 \cdot 7kg$

63. A vessel in from of a hollow hemisphere mounted by a hollow cylinder. The diameter of hemisphere is 14cm and total height of vessel is 13cm. find the inner surface area of vessel?

Sol:

Given radius of hemisphere and cylinder (r)

$= \frac{14}{2} = 7cm$

Given total height of vessel $= 13cm$

$(h + r) = 13cm$

Inner surface area of vessel $= 2\pi r(h + r)$

$= 2 \times \pi \times 7(13)$

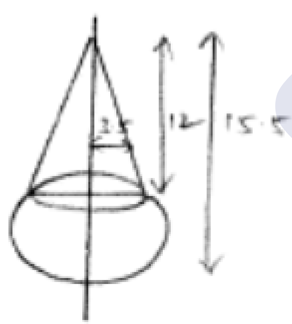
$= 182\pi$

$= 572cm^2$

$\therefore$ Inner surface area of vessel $= 572cm^2$

64. A toy is in the form of a cone of radius $3 \cdot 5cm$ mounted on a hemisphere of same radius. The total height of toy is $15 \cdot 5cm$. Find the total surface area of toy?

Sol:



Given radius of cone $(r) = 3 \cdot 5cm$

Total height of toy $(h) = 15 \cdot 5cm$

Length of cone $(l) = 15 \cdot 5 - 3 \cdot 5$

$= 12cm$

$\therefore$ Length of cone $(l) = 12cm$

Curved surface area of cone $= \pi rl$

$S_1 = \pi(3 \cdot 5)(12)$

$S_1 = 131 \cdot 94cm^2$ ......(1)

Curved surface area of hemisphere $=2\pi r^2$

$S_2=2\pi(3\cdot5)^2$

$S_2=76\cdot96cm^2$ ......(2)

$\therefore$ Total surface of toy = (1) + (2)

$S=S_1+S_2$

$S=181\cdot94+76\cdot96$

$S=208\cdot90$

$S=209cm^2$

$\therefore$ Total surface area of toy $=209cm^2$

65. The difference between outside and inside surface areas of cylindrical metallic pipe 14cm long is $44m^2$. If pipe is made of $99cm^3$ of metal. Find outer and inner radii of pipe?

Sol:

Let inner radius of pipe be r_1

Radius of outer cylinder be r_2

Length of cylinder $(h)=14cm.$

Surface area of hollow cylinder $=2\pi h(r_2-r_1)$

Given surface area of cylinder $=44m^2$

66. A radius circular cylinder bring having diameter 12cm and height 15cm is full ice-cream. The ice-cream is to be filled in cones of height 12cm and diameter 6cm having a hemisphere shape on top find the number of such cones which can be filled with ice-cream?

Sol:

Given radius of cylinder $(r_1)=\frac{12}{2}=6cm$

Given radius of hemisphere $(r_2)=\frac{6}{2}=3cm.$

Given height of cylinder $(h)=15cm..$

Height of cones $(l)=12cm.$

Volume of cylinder $=\pi r_1^2h$

$=\pi(6)^2(15)cm^3$ ......(1)

Volume of each cone = volume of cone + volume of hemisphere

$=\frac{1}{3}\pi r_2^2l+\frac{2}{3}\pi r_2^3$

$$= \frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3 cm^3 \quad \text{........(2)}$$

Let number of cones be 'n'

n(Volume of each cone) = volume of cylinder

$$n\left(\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3\right) = \pi(6)^2 15$$

$$\Rightarrow n = \frac{\pi(6)^2 15}{\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3}$$

$$\Rightarrow n = \frac{540}{5} = 10$$

$$\Rightarrow 2\pi h(r_2 - r_1) = 44$$

$$\Rightarrow 2\pi(14)(r_2 - r_1) = 44$$

$$\Rightarrow 28\pi(r_2 - r_1) = 44$$

$$\Rightarrow (r_2 - r_1) = \frac{44}{28\pi}$$

$$\Rightarrow (r_2 - r_1) = \frac{1}{2} \quad \text{..........(1)}$$

Given volume of a hollow cylinder $= 99cm^3$

Volume of a hollow cylinder $= \pi h\left(r_2^2 - r_1^2\right)$

$$\Rightarrow \pi h\left(r_2^2 - r_1^2\right) = 99$$

$$\Rightarrow 14\pi\left(r_2^2 - r_1^2\right) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2)(r_2 - r_1) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2)(1) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2) = 99$$

$$\Rightarrow (r_1 + r_2) = \frac{9}{2} \quad \text{..........(2)}$$

Equating (1) and (2) equations we get

$$r_1 + r_2 = \frac{9}{2}$$

$$-r_1 + r_2 = \frac{1}{2}$$

$$2r_2 = 5$$

$$r_2 = \frac{5}{2} cm.$$

Substituting r_2 value in (1)

$\Rightarrow r_1 = 2cm$

$\therefore$ Inner radius of pipe $(a) = 2cm$

Radius of outer cylinder $(r_2) = \frac{5}{2}cm.$

67. A solid iron pole having cylindrical portion 110cm high and of base diameter 12cm is surmounted by a cone 9cm high. Find the mass of the pole given that the mass of $1cm^3$ of iron is $8gm$?

Sol:

Given radius of cylindrical part $(r) = \frac{12}{2} = 6cm$

Height of cylinder $(h) = 110cm$

Length of cone $(l) = 9cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi(0)^2 110cm^3$ ....(1)

Volume of cone $= \frac{1}{3}\pi r^2 l$

$V_2 = \frac{1}{3}\pi(6)^2 9 = 108\pi cm^3$ ....(2)

Volume of pole = (1) + (2)

$V = V_1 + V_2$

$\Rightarrow V = \pi(6)^2 110 + 108\pi$

$\Rightarrow V = 12785 \cdot 14cm^3$

Given mass of $1cm^3$ of iron $= 8gm$

Mass of $12785 \cdot 14cm^3$ of iron $= 12785 \cdot 14 \times 8$

$= 102281 \cdot 12$

$= 102 \cdot 2kg$

$\therefore$ Mass of pole for $12785 \cdot 14cm^3$ of iron is $102 \cdot 2kg$

68. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover.

Sol:

Given radius of cone, cylinder and hemisphere $(r) = \frac{4}{2} = 2cm$

Height of cone $(l)=2cm$

Height of cylinder $(h)=4cm$

Volume of cylinder $=\pi r^2 h=\pi(2)^2(4)cm^3$ ....(1)

Volume of cone $=\frac{1}{3}\pi r^2 l$

$=\frac{1}{3}\pi(2)^2\times 2$

$=\frac{\pi}{3}(4)\times 2cm^3$ ....(2)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

$=\frac{2}{3}\times\pi(2)^3$

$=\frac{2}{3}\times\pi(8)cm^3$ ....(3)

So remaining volume of cylinder when toy is inserted to it $=\pi r^2 h-\left(\frac{1}{3}\pi r^2 l+\frac{2}{3}\pi r^3\right)$

= (1) – ((2) + (3))

$=\pi(2)^2(4)-\left(\frac{\pi}{3}\times 8+\frac{2}{3}\times\pi\times 8\right)$

$=16\pi-\frac{2}{3}\pi(4+8)=16\pi-8\pi=8\pi cm^3$

$\therefore$ So remaining volume of cylinder when toy is inserted to it $=8\pi cm^3$

69. A solid consisting of a right circular cone of height 120cm and radius 60cm is placed upright in right circular cylinder full of water such that it touches bottoms. Find the volume of water left in the cylinder. If radius of cylinder is 60cm and its height is $180cm$?

Sol:

Given radius of circular cone $(a)=60cm$

Height of circular cone $(b)=120cm.$

Volume of a cone $=\frac{1}{3}\pi r^2 l$

$=\frac{1}{3}\pi(60)^2(120)cm^3$ ....(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

Given radius of hemisphere $=60cm$

$=\frac{2}{3}\pi(60)^2 cm^3$ ....(2)

Given radius of cylinder $=60cm$

Height of cylinder $(h)=180cm.$

Volume of cylinder $=\pi r^2 h$

$=\pi(60)^2\times 180cm^3$ ....(3)

Volume of water left in cylinder = (3) – ((1) + (2))

$\Rightarrow \frac{1}{3}\pi(60)^3(120)-\left(\frac{2}{3}\pi(60)^3+\pi(60)^2\times 180\right)$

$\Rightarrow 113\cdot 1cm^3=1\cdot 131m^3$

$\therefore$ Volume of water left in cylinder $=1\cdot 131m^3$

70. A cylindrical vessel with internal diameter 10cm and height $10\cdot 5cm$ is full of water. A solid cone of base diameter 7cm and height 6cm is completely immersed in water. Find value of water (i) displaced out of the cylinder (ii) left in the cylinder?

Sol:

Given internal radius $(r_1)=\frac{10}{2}=5cm$

Height of cylindrical vessel $(h)=10\cdot 5cm$

Outer radius of cylindrical vessel $(l_2)=\frac{7}{2}=3\cdot 5cm$

Length of cone $(l)=6cm.$

(i) Volume of water displaced = volume of cone

Volume of cone $=\frac{1}{3}\pi r_2^2 l$

$=\frac{1}{3}\pi\times 3\cdot 5^2\times 6=76\cdot 9cm^3$

$=77cm^3$

$\therefore$ Volume of water displaced $=77cm^3$

Volume of cylinder $=\pi r_1^2 h=\pi(5)^2 10\cdot 5$

$=824\cdot 6$

$=825cm^2$

(ii) Volume of water left in cylinder = volume of Cylinder – volume of cone

$=825-77=748cm^3$

$\therefore$ Volume of water left in cylinder $=748cm^3$

71. A hemispherical depression is cut from one face of a cubical wooden block of edge 21cm such that the diameter of hemisphere is equal to the edge of cube determine the volume and total surface area of the remaining block?

Sol:

Given edge of wooden block $(a) = 21cm$

Given diameter of hemisphere = edge of cube

Radius $= \frac{21}{2} = 10 \cdot 5cm$

Volume of remaining block = volume of box – volume of hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$= (2)^3 - \frac{2}{3}\pi(10 \cdot 5)^3$

$= 6835 \cdot 5cm^3$

Surface area of box $= 6a^2$ ……..(1)

Curved surface area of hemisphere $= 2\pi r^2$ ……..(2)

Area of base of hemisphere $= \pi r^2$ ……..(3)

So remaining surface area of box = (1) – (2) + (3)

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= 6(21)^2 - \pi(10 \cdot 5) + 2\pi(10 \cdot 5)^2$

$= 2992 \cdot 5cm^2$

$\therefore$ Remaining surface area of box $= 2992 \cdot 5cm^2$

Volume of remaining block $= 6835 \cdot 5cm^3$

72. A tag is in the form of a hemisphere surmounted by a right circular cone of same base radius as that of the hemisphere. If the radius of the base of cone is 21cm and its volume is $\frac{2}{3}$ of volume of hemisphere calculate height of cone and surface area of toy?

Sol:

Given radius of cone = radius of hemisphere

Radius $(r) = 21cm$

Given that volume of cone $= \frac{2}{3}$ Volume of hemisphere

$\Rightarrow$ Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

So $\frac{1}{3}\pi r^2 h = \frac{2}{3}\left(\frac{2}{3}\pi r^3\right)$

$\Rightarrow \frac{1}{3}\pi (21)^2 h = \frac{2}{3}\left(\frac{2}{3}\pi (21)^3\right)$

$\Rightarrow h = \frac{4(21)\pi \times 3}{4\pi (21)}$

$\Rightarrow h = \frac{4}{3} \times 21 = 28cm$

$\therefore$ Height of cone $(h) = 28cm$

Curved surface area of cone $= \pi rl$

$S_1 = \pi (21)(28) cm^2$ …………..(1)

Curved surface area off hemisphere $= 2\pi r^2$

$S_2 = 2 \times \pi (21)^2 cm^2$ …………..(2)

Total surface area $(s) = S_1 + S_2 = (1) + (2)$

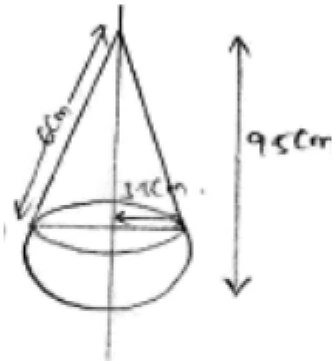
$S = \pi rl + 2\pi r^2$

$S = 5082cm^2$

$\therefore$ Curved surface area of toy $= 5082cm^2$

73. A solid is in the shape of a cone surmounted on hemisphere the radius of each of them is being $3\cdot5cm$ and total height of solid is $9\cdot5cm$. Find volume of the solid?

Sol:



Given radius of hemisphere and cone $=3\cdot5cm$

Given total height of solid $(h)=9\cdot5cm$

Length of cone $(l)=9\cdot5-3\cdot5=6cm$

Volume of a cone $=\frac{1}{3}\pi r^2 l$

$V_1=\frac{1}{3}\pi(3\cdot5)^2\times6\ cm^3$ ....(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

$V_2=\frac{2}{3}\pi(3\cdot5)^3\ cm^3$ ....(2)

Volume of solid = (1) + (2)

$V=V_1+V_2$

$V=\frac{1}{3}\pi(3\cdot5)^2\times6+\frac{2}{3}\pi(3\cdot5)^3$

$V=76\cdot96+89\cdot79=166\cdot75cm^3$

$\therefore$ Volume of solid $(v)=166\cdot75cm^3$

Exercise 16.3

1. A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs 1.20 per dm^2. (Use π = 3.14)

Sol:

Given diameter to top of bucket = 40cm

Radius $(r_1) = \frac{40}{2} = 20cm$

Depth of a bucket $(h) = 12cm$

Volume of a bucket $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{3}{1}\pi\left(20^2 + 10^2 + 20(10)\right)^{12}$

$= 8800cm^3$.

Let ‘l’ be slant height of bucket

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h_2}$

$\Rightarrow l = \sqrt{(20-10)^2 + 12^2}$

$\Rightarrow l = 2\sqrt{61} = 15\cdot620cm$

Total surface area of bucket $= \pi(r_1 + r_2)\times l + \pi r_2^2$

$= \pi(20+10)\times15\cdot620 + \pi(10)^2$

$= \frac{1320\sqrt{61} + 2200}{7}cm^2$

$= \frac{1320\sqrt{61} + 2200}{7\times100}dm^2 = 17\cdot87dm^2$

Given that cost of tin sheet used for making bucket per $dm^2 = Rs1.20$

So total cost for $17\cdot87dm^2 = 1\cdot20\times17\cdot87$

$= 21\cdot40\ Rs.$

$\therefore$ Cost of tin sheet for $17\cdot87dm^2 = Rs2140\,ps$

2. A frustum of a right circular cone has a diameter of base 20cm, of top 12cm and height 3cm. find the area of its whole surface and volume?

Sol:

Given base diameter of cone $(d_1) = 20cm$

Radius $(r_1) = \frac{20}{2} = 10cm$

Top diameter of cone $(d_2) = 12cm$

Radius $(r_2) = \frac{12}{2} = 6cm$

Height of cone $(h) = 3cm$

Volume of frustum right circular cone

$= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1r_2\right)h$

$= \frac{1}{3}\pi\left(10^2 + 6^2 + (10)(6)\right)3$

$= 616cm^3$

Let 'l' be slant height of cone

$\Rightarrow l = \sqrt{\left(r_1 - r_2\right)^2 + h_2}$

$\Rightarrow l = \sqrt{\left(10-6\right)^2 + 3^2}$

$\Rightarrow l = \sqrt{16+9} = \sqrt{25}cm = 5cm$

$\therefore$ Slant height of cone $(l) = 5cm$

Total surface area of cone $= \pi\left(r_1 + r_2\right)l + \pi r_1^2 + \pi r_2^2$

$= \pi(10+6)5 + \pi(10)^2 + \pi(6)^2$

$= \pi(80+100+36)$

$= \pi(216) = 678 \cdot 85cm^2$

$\therefore$ Total surface area of cone $= 678 \cdot 85cm^2$

3. The slant height of the frustum of a cone is 4cm and perimeters of it circular ends are 18cm and 6cm. find curved surface of the frustum?

Sol:

Given slant height of cone $(r) = 4cm$

Let radii of top and bottom circles be r_1 and r_2

Given perimeters of its ends as 18cm and 6cm

$\Rightarrow 2\pi r_1 = 18cm$

$\Rightarrow \pi r_1 = 9cm$ ......(1)

$\Rightarrow 2\pi r_2 = 6cm$

$\Rightarrow \pi r_2 = 3cm$ ......(2)

Curved surface area of frustum cone $= \pi\left(r_1 + r_2\right)l$

$= \pi\left(r_1 + r_2\right)l$

$= \left(\pi r_1 + \pi r_2\right)l$

$= (9+3)4$

$= (12)4 = 48cm^2$

$\therefore$ Curved surface area of frustum cone $= 48cm^2$

4. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Sol:

Given perimeters of ends of frustum right circular cone are 44cm an 33cm

Height of frustum cone $=16cm$

Perimeter $=2\pi r$

$2\pi r_1 = 44$

$r_1 = 7cm$

$2\pi r_2 = 33$

$r_2 = \frac{21}{4} = 5\cdot 25cm$

Let slant height of frustum right circular cone be l

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(7 - 5\cdot 25)^2 + 16^2 cm}$

$l = 16\cdot 1cm$

$\therefore$ Slant height of frustum cone $=16\cdot 1cm$

Curved surface area of frustum cone $=\pi(r_1 + r_2)l$

$=\pi(7 + 5\cdot 25)16\cdot 1$

C.S.A of cone $=619\cdot 65cm^2$

Volume of a cone $=\frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)\times h$

$=\frac{1}{3}(7^2 + (5\cdot 25)^2 + 7(5\cdot 25)\times 16$

$=1898\cdot 56cm^3$

$\therefore$ Volume of a cone $=1898\cdot 56\ cm^3$

Total surface area of frustum cone $=\pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$=\pi(7 + 5\cdot 25)16\cdot 1 + \pi(7^2 + 5\cdot 25^2)$

$=860\cdot 27cm^2$

$\therefore$ Total surface area of frustum cone $=860\cdot 27cm^2$

5. If the radii of circular ends of a conical bucket which is 45cm high be 28cm and 7cm, find the capacity of the bucket?

Sol:

Given height of conical bucket $=45cm$

Give radii of 2 circular ends of a conical bucket is 28cm and 7cm

$r_1 = 28cm$

$r_2 = 7cm$

Volume of a conical bucket $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(28^2 + 7^2 + 28(7)\right)45$

$= \frac{1}{3}\pi(1029)45$

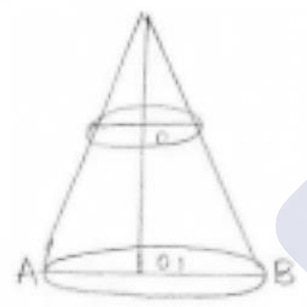
$= 15435$

$V = 48510cm^3$

Volume of a conical bucket $= 48510cm^3$

6. The height of a cone is 20cm. A small cone is cut off from the top by a plane parallel to the base. If its volumes be $\frac{1}{25}$ of the volume of the original cone, determine at what height above base the section is made

Sol:



V AB be a cone of height $h_1 = VO_1 = 20cm$

Fronts triangles Vo_1A and VoA_1

$\frac{VO_1}{VO} = \frac{O_1A}{OA_1} \Rightarrow \frac{20}{VO} = \frac{O_1A}{OA_1}$

Volumes of cone $VA_1O = \frac{1}{125}$ times volumes of cone VAB

We have $\frac{1}{3}\pi \times OA_1^2 \times VO = \frac{1}{125} \times \frac{1}{3}\pi \times O_1A_1^2 \times 20$

$\Rightarrow \left(\frac{OA_1}{O_1A}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow \left(\frac{VO}{20}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow (VO)^3 = \frac{4 \times 400}{25}$

$\Rightarrow VO^3 = 64$

$\Rightarrow VO = 4$

Height at which section is made $= 20 - 4 = 16cm.$

7. If the radii of circular ends of a bucket 24cm high are 5cm and 15cm. find surface area of bucket?

Sol:

Given height of a bucket $(R) = 24cm$

Radius of circular ends of bucket $5cm$ and $15cm$

$r_1 = 5cm$; $r_2 = 15cm$

Let $'l'$ be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(15-5)^2 + 24^2}$

$\Rightarrow l = \sqrt{100 + 576} = \sqrt{676}$

$l = 26cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(5+15)26 + \pi(15)^2$

$= \pi(20)26 + \pi(15)^2$

$= \pi(520 + 225)$

$= 745\pi cm^2$

$\therefore$ Curved surface area of bucket $= 745\pi cm^2$

8. The radii of circular bases of a frustum of a right circular cone are 12cm and 3cm and height is 12cm. find the total surface area volume of frustum?

Sol:

Let slant height of frustum cone be ‘l’

Given height of frustum cone $12cm$

Radii of a frustum cone are 12cm and 23cm

$r_1 = 12cm \quad r_2 = 3cm$

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(12-3)^2 + 12^2}$

$l = \sqrt{81 + 144} = 15cm$

$l = 15cm$

Total surface area of cone $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= \pi(12+3)15 + \pi(12)^2 + \pi(3)^2$

T.S.A $= 378\pi cm^2$

Volume of cone $= \frac{1}{3}\pi(r_1^2 + r_1 r_2 + r_2^2) \times h$

$= \frac{1}{3}\pi(12^2 + 3^2 + (12)(3))12$

$= 756\pi cm^3$

Volume of frustum cone $= 756\pi cm^3$

9. A tent consists of a frustum of a cone copped by a cone. If radii of ends of frustum be 13m and 7m the height of frustum be 8m and slant height of thee conical cap bee 12m. find canvas required for tent?

Sol:

Given height of frustum $(h) = 8m$

Radii of frustum cone are $13m$ and $7m$

$r_1 = 13m \quad r_2 = 7cm$

Let 'l' be slant height of frustum cone

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(13-7)^2 + 8^2} = \sqrt{36+64}$

$\Rightarrow l = 10m$

Curved surface area of friction $(S_1) = \pi(r_1 + r_2) \times l$

$= \pi(13+7) \times 10$

$= 200\pi m^2$

C.S.A of frustum $(S_1) = 200\pi m^2$

Given slant height of conical cap = 12m

Base radius of upper cap cone $= 7m$

Curved surface area of upper cap cone $(S_2) = \pi rl$

$= \pi \times 7 \times 12 = 264m^2$

Total canvas required for tent $(S) = S_1 + S_2$

$S = 200\pi + 264 = 892 \cdot 57m^2$

$\therefore$ Total canvas $= 892 \cdot 57m^2$

10. A reservoir in form of frustum of a right circular contains 44×10^7 liters off water which fills it completely. The radii of bottom and top of reservoir are 50m and 100m. find depth of water and lateral surface area of reservoir?

Sol:

Let depth of frustum cone be h

Volume of first cone $(V)=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$r_1=50m \quad r_2=100m$

$V=\frac{1}{3}\times\frac{22}{7}\times\left(50^2+100^2+50(100)\right)h$

$V=\frac{1}{3}\times\frac{22}{7}\times(2500+1000+5000)h \quad \ldots(1)$

Volumes of reservoir $=44\times10^7$ liters $\quad \ldots.(2)$

Equating (1) and (2)

$\frac{1}{3}\pi(2500)h=44\times10^2$

$h=24$

Let 'l' be slant height of cone

$l=\sqrt{(r_1-r_2)^2+h_2}$

$l=\sqrt{(50-100)^2+24^2}$

$l=55\cdot461m$

Lateral surface area of reservoir

$(S)=\pi(r_1+r_2)\times l$

$=\pi(50+100)55\cdot461$

$=1500(55\cdot461)\pi=26145\cdot225m^2$

Lateral surface area of reservoir $=26145\cdot225m^2$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(30^2+18^2+30(18)\right)9$

$=5292\pi cm^3$

Volume $=5292\pi cm^3$

Total surface area of frustum cone =

$=\pi(r_1+r_2)\times l+\pi r_1^2+\pi r_2^2$

$=(30+18)15+\pi(30)^2+(18)^2$

$=\pi\left(48(15)+(30)^2+(18)^2\right)$

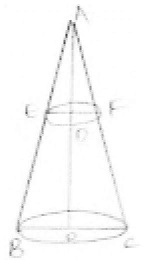
$=\pi(720+900+324)$

$=1944\pi cm^2$

$\therefore$ Total surface area $=1944\pi cm^2$

11. A metallic right circular cone 20cm high and whose vertical angle is $90°$ is cut into two parts at the middle point of its axis by a plane parallel to base. If frustum so obtained bee drawn into a wire of diameter $\left(\frac{1}{16}\right)cm.$ find length of the wire?

Sol:



Let ABC be cone. Height of metallic cone $AO=20cm$
Cone is cut into two parts at the middle point of its axis
Hence height of frustum cone $AD=10cm$
Since angle A is right angled. So each angles B and $C=45°$
Angles E and $F=45°$
Let radii of top and bottom circles of frustum cone bee r_1 and $r_2 cm$

From $\Delta^{le} ADE \Rightarrow \frac{DE}{AD}=\cot 45°$

$\Rightarrow \frac{r_1}{10}=1$

$\Rightarrow r_1=10cm.$

From $\Delta^{le} AOB$

$\Rightarrow \frac{OB}{OA}=\cot 45°$

$\Rightarrow \frac{r_2}{20}=1$

$\Rightarrow r_2=20cm$

12. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. (Use $\pi=3.14$).

Sol:

Given radii of top circular ends $(r_1) = 20cm$

Radii of bottom circular end of bucket $(r_2) = 12cm$

Let height of bucket be 'h'

Volume of frustum cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(20^2 + 12^2 + 20(12)\right)h$

$= \frac{784}{3}\pi h cm^3$ ……….(1)

Given capacity/volume of bucket $= 123308\cdot 8cm^3$ ……….(2)

Equating (1) and (2)

$\Rightarrow \frac{784}{3}\pi h = 12308\cdot 8$

$\Rightarrow h = \frac{12308\cdot 8\times 3}{784\times\pi}$

$\Rightarrow h = 15cm$

$\therefore$ Height of bucket $(h) = 15cm$

Let 'l' be slant height of bucket

$\Rightarrow l^2 = (r_1 - r_2)^2 + h^2$

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(20+2)^2 + 15^2} = \sqrt{64+225}$

$\Rightarrow l = 17cm$

Length of bucket/ slant height of

Bucket $(l) = 17cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(20+12)17 + \pi(12)^2$

$= \pi(32)17 + \pi(12)^2$

$= \pi(9248+144) = 2160\cdot 32cm^2$

$\therefore$ Curved surface area $= 2160\cdot 32cm^2$

13. A bucket made of aluminum sheet is of height 20cm and its upper and lower ends are of radius 25cm an 10cm, find cost of making bucket if the aluminum sheet costs Rs 70 per $100cm^2$

Sol:

Given height of bucket $(h) = 20cm$

Upper radius of bucket $(r_1) = 25cm$

Lower radius of bucket $(r_2) = 10cm$

Let 'l' be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(25-10)^2 + 20^2} = \sqrt{225+400}$

$l = 25m$

$\therefore$ Slant height of bucket (l) $= 25cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(25+10)25 + \pi(10)^2$

$= \pi(35)25 + \pi(100) = 975\pi$

C.S.A $= 3061 \cdot 5cm^2$

Curved surface area $= 3061 \cdot 5cm^2$

Cost of making bucket per $100cm^2 = Rs70$

Cost of making bucket per $3061 \cdot 5cm^2 = \frac{3061 \cdot 5}{100} \times 70$

$= Rs\ 2143 \cdot 05$

$\therefore$ Total cost for $3061 \cdot 5cm^2 = Rs\ 2143 \cdot 05\ per$

14. Radii of circular ends of a solid frustum off a cone re 33cm and 27cm and its slant height are 10cm. find its total surface area?

Sol:

Given slant height of frustum cone $= 10cm$

Radii of circular ends of frustum cone are 33 and 27cm

$r_1 = 33cm$; $r_2 = 27cm.$

Total surface area of a solid frustum of cone

$= \pi(r_1 + r_2) \times l + \pi r_1^2 + \pi r_2^2$

$= \pi(33+27) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(60) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(600 + 1089 + 729)$

$= 2418\pi cm^2$

$= 7599 \cdot 42cm^2$

$\therefore$ Total surface area of frustum cone $= 7599 \cdot 42cm^2$

15. A bucket made up of a metal sheet is in form of a frustum of cone of height 16cm with diameters of its lower and upper ends as 16cm and 40cm. find thee volume of bucket. Also find cost of bucket if the cost of metal sheet used is Rs 20 per 100 cm^2

Sol:

Given height off frustum cone $= 16cm$

Diameter of lower end of bucket $(d_1) = 16cm$

Lower and radius $(r_1) = \frac{16}{2} = 8cm$

Upper and radius $(r_2) = \frac{40}{2} = 20cm$

Let 'l' be slant height of frustum of cone

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(20-8)^2 + 16^2}$

$l = \sqrt{144 + 256}$

$l = 20cm$

$\therefore$ Slant height of frustum cone $(l) = 20cm.$

Volume of frustum cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)16$

$= \frac{1}{3}\pi(9984)$

Volume $= 10449 \cdot 92cm^3$

Curved surface area of frustum cone

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(20+8)20 + \pi(8)^2$

$= \pi(560 + 64) = 624\pi cm^2$

Cost of metal sheet per $100cm^2 = Rs20$

Cost of metal sheet for $624\pi cm^2 = \frac{624\pi}{100} \times 20$

$= Rs\ 391 \cdot 9$

$\therefore$ Total cost of bucket $= Rs\ 391 \cdot 9$

16. A solid is in the shape of a frustum of a cone. The diameter of two circular ends are $60cm$ and 36cm and height is 9cm. find area of its whole surface and volume?

Sol:

Given height of a frustum cone = 9cm

Lower end radius $(r_1)=\frac{60}{2}cm=30cm$

Upper end radius $(r_2)=\frac{36}{2}cm=18cm$

Let slant height of frustum cone be l

$l=\sqrt{(r_1-r_2)^2+h^2}$

$l=\sqrt{(8-30)^2+9^2}$

$l=\sqrt{144+81}$

$l=15cm$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(30^2+18^2+30(18)\right)9$

$=5292\pi cm^3$

Volume $=5292\pi cm^3$

Total surface area of frustum cone =

$=\pi(r_1+r_2)\times l+\pi r_1^2+\pi r_2^2$

$=\pi(30+18)15+\pi(30)^2+\pi(18)^2$

$=\pi\left(48(15)+(30)^2+(18)^2\right)$

$=\pi(720+900+324)$

$=1944\pi cm^2$

$\therefore$ Total surface area $=1944\pi cm^2$

17. A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7}cm^3$. The radii of its lower and upper circular ends are 8cm and 20cm. find the cost of metal sheet used in making container at rate of $Rs\ 1\cdot 4\ per\ cm^2$?

Sol:

Given lower end radius of bucket $(r_1)=8cm$

Upper end radius of bucket

Let height of bucket be 'h'

$V_1 = \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h\,cm^3$ ....(1)

Volume of milk container $= 10459\frac{3}{4}cm^3$

$V_2 = \frac{73216}{7}cm^3$ ....(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h = \frac{73216}{7}$

$\Rightarrow h = \frac{10459\cdot 42}{653\cdot 45}$

$\Rightarrow h = 16cm$

$\therefore$ Height of frustum cone $(h) = 16cm$

Let slant height of frustum cone be 'l'

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + 16^2} = \sqrt{144 + 256}$

$l = 20cm$

$\therefore$ Slant height of frustum cone $(l) = 20cm$

Total surface area of frustum cone

$= \pi(r_1 + r_2)l + \pi r_2^2 + \pi r_1^2$

$\Rightarrow \pi(20+8)20\pi(20)^2 + \pi(8)^2$

$= \pi(560 + 400 + 64)$

$= \pi(960 + 64) = 1024\pi = 3216\cdot 99cm^2$

Total surface area $= 3216\cdot 99cm^2$