DIFFERENTIAL EQUATIONS WITH VARIABLE SEPARABLE (XII, R. S. AGGARWAL)

EXERCISE 19A (Pg. No.: 929)

Very-Short-Answer Questions

Find the general solution of each of the following differential equations

1. $\dfrac{dy}{dx} = (1+x^2)(1+y^2)$

Sol. $\dfrac{dy}{dx} = (1+x^2)(1+y^2)$

On separating variables we get $\dfrac{dy}{(1+y^2)} = (1+x^2)\,dx$

Integrate on the both sides we get $\displaystyle\int \dfrac{dy}{(1+y^2)} = \int (1+x^2)\,dx$

$\Rightarrow \dfrac{1}{1}\tan^{-1}\dfrac{y}{1} = x + \dfrac{x^3}{3} + C \qquad \Rightarrow \tan^{-1} y = x + \dfrac{x^3}{3} + C$

2. $x^4\dfrac{dy}{dx} = -y^4$

Sol. $x^4\dfrac{dy}{dx} = -y^4$

On separating variables we get $\dfrac{dy}{dx} = \dfrac{-y^4}{x^4}$

$\dfrac{dy}{y^4} = -\dfrac{1}{x^4}\,dx$

Integrate on the both sides we get $\displaystyle\int \dfrac{dy}{y^4} = \int -\dfrac{1}{x^4}\,dx$

$\displaystyle\int \dfrac{dy}{y^4} = -\int \dfrac{1}{x^4}\,dx$

$\displaystyle\int y^{-4}\,dy = -\int x^{-4}\,dx$

$y^{-4+1} = -x^{-4+1} + C$

$y^{-3} = -x^{-3} + C$

$y^{-3} + x^{-3} = C$

$\dfrac{1}{y^3} + \dfrac{1}{x^3} = C$

3. $\dfrac{dy}{dx} = 1 + x + u + xy$

Sol. $\dfrac{dy}{dx} = 1 + x + y + xy$

$$\frac{dy}{dx} = 1(1+x) + y(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y)$$

On separating variables we get $\dfrac{dy}{(1+y)} = (1+x)\,dx$

Integrating on the both sides we get $\displaystyle\int \frac{dy}{(1+y)} = \int (1+x)\,dx$

$$\log|1+y| = x + \frac{x^2}{2} + C$$

4. $\dfrac{dy}{dx} = 1 - x + y - xy$

Sol. $\dfrac{dy}{dx} = 1 - x + y - xy$

$$\frac{dy}{dx} = 1(1-x) + y(1-x)$$

$$\frac{dy}{dx} = (1-x)(1+y)$$

On separating variables we get $\dfrac{dy}{(1+y)} = (1-x)\,dx$

Integrating on the both sides we get $\displaystyle\int \frac{dy}{(1+y)} = \int (1-x)\,dx$

$$\log|1+y| = x - \frac{x^2}{2} + C$$

5. $(x+1)\dfrac{dy}{dx} = 2x^3 y$

Sol. $(x+1)\dfrac{dy}{dx} = 2x^3 y$

$$\frac{dy}{dx} = \frac{2x^3 y}{(x+1)}$$

On separating variables we get $\dfrac{dy}{y} = \dfrac{2x^3}{(x+1)}\,dx$

Integrating on the both sides we get $\displaystyle\int \frac{dy}{y} = \int \frac{2x^3}{(x+1)}\,dx$

$$\int \frac{dy}{y} = 2\int \frac{x^3}{(x+1)}\,dx$$

On dividing $x^3 dy(x-1)$

$$x-1 \overline{)\begin{array}{c} x^2+x+1 \\ x^3 \\ \underline{x^3-x^2} \\ x^2 \\ \underline{x^2-x} \\ x-1 \\ \underline{x-1} \\ 0 \end{array}}$$

$$\frac{x^3}{(x-1)} = x^2+x+1+\frac{1}{(x-1)}$$

$$\int \frac{dy}{y} = 2\int \frac{x^3}{(x-1)} dx$$

$$\int \frac{dy}{y} = 2\int \left[x^2+x+1+\frac{1}{(x-1)} \right] dx$$

$$\log|y| = 2\left[\frac{x^3}{3}+\frac{x^2}{2}+x+\log|x-1| \right]+C$$

$$\log|y| = \frac{2x^3}{3}+\frac{2x^2}{2}+2x+2\log|x-1|+C$$

$$\log|y| = \frac{2x^3}{3}+x^2+2x+2\log|x-1|+C$$

6. $\dfrac{dy}{dx} = e^{x+y}$

Sol. $\dfrac{dy}{dx} = e^{x+y}$

$$\frac{dy}{dx} = e^x \ge e^y$$

On separating variables we get $\dfrac{dy}{e^y} = e^x dx$

Integrating on the both sides we get $\int \dfrac{dy}{e^y} = \int e^x dx$

$$\int e^{-y} dy = \int e^x dx$$

$$-e^{-y} = e^x +C$$

$$-e^{-y}-e^x = C$$

$$-\left(e^x+e^{-y} \right) = C$$

$$e^x+e^{-y} = \frac{C}{-1}$$

$$e^x+e^{-y} = C$$

7. $\left(e^x+e^{-x} \right) dy - \left(e^x-e^{-x} \right) dx = 0$

Sol. $\left(e^x+e^{-x} \right) dy - \left(e^x-e^{-x} \right) dx = 0$

$$\left(e^x+e^{-x} \right) dy = \left(e^x-e^{-x} \right) dx$$

$$\frac{dy}{dx} = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

On separating variables we get $dy = \dfrac{(e^x - e^{-x})}{(e^x + e^{-x})} dx$

Integrating on the both sides we get $\displaystyle\int dy = \int \frac{(e^x - e^{-x})}{(e^x + e^{-x})} dx$

Let $(e^x + e^{-x}) = t$

Diff. on the both sides will respect to t

$$e^x - e^{-x} = \frac{dt}{dx}$$

$$dx = \frac{dt}{(e^x - e^{-x})}$$

$$\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$\int dy = \int \frac{(e^x - e^{-x})}{t} \times \frac{dt}{(e^x - e^{-x})}$$

$$\int dy = \int \frac{1}{t} dt$$

$$y = \log|t| + C$$

$$y = \log|e^x + e^{-x}| + C$$

8. $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Sol. $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

On separating variables we get $\dfrac{dy}{dx} = e^{-y}(e^x + x^2)$

$$\frac{dy}{e^{-y}} = (e^x + x^2) dx$$

$$e^y dy = (e^x + x^2) dx$$

Integrating on the both sides we get $\displaystyle\int e^y dy = \int (e^x + x^2) dx$

$$e^y = e^x + \frac{x^3}{3} + C$$

9. $e^{2x-3y} dx + e^{2y-3x} dy = 0$

Sol. $e^{2x-3y} dx + e^{2y-3x} dy = 0$

$$e^{2y-3x}dy = -e^{2x-3y}dx$$

$$\frac{dy}{dx} = \frac{-e^{2x-3y}}{e^{2y-3x}}$$

On separating variables we get $\dfrac{dy}{dx} = \dfrac{-e^{2x}e^{-3y}}{e^{2y}e^{-3x}}$

$$\frac{e^{2y}}{e^{-3y}}dy = \frac{-e^{2x}}{e^{-3x}}dx$$

$$e^{2y+3y}dy = -e^{2x+3x}dx$$

$$e^{5y}dy = -e^{5x}dx$$

Integrating on the both sides we get $\int e^{5y}dy = \int -e^{5x}dx$

$$\int e^{5y}dy = -\int e^{5x}dx$$

$$\frac{e^{5y}}{5} = -\frac{1}{5}e^{5x} + C$$

$$e^{5y} = -e^{5x} + C$$

$$e^{5y} + e^{5x} = C$$

$$e^{5x} + e^{5y} = C$$

10. $e^{x}\tan dx + \left(1-e^{x}\right)\sec^{2}ydy = 0$

Sol. $e^{x}\tan ydx + \left(1-e^{x}\right)\sec^{2}ydy = 0$

$$\left(1-e^{x}\right)\sec^{2}ydy = -e^{x}\tan ydx$$

$$\frac{dy}{dx} = \frac{-e^{x}\tan y}{\left(1-e^{x}\right)\sec^{2}y}$$

On separating variables we get $\dfrac{\sec^{2}y}{\tan y}dy = -\dfrac{e^{x}}{\left(1-e^{x}\right)}dx$

Integrating on the both sides we get $\displaystyle\int \dfrac{\sec^{2}y}{\tan y}dy = -\int \dfrac{e^{x}}{\left(1-e^{x}\right)}dx$

Let $\tan y = t$

Diff on the both sides w.r. to y

$$\sec^{2}y = \frac{dt}{dy}$$

$$dy = \frac{dt}{\sec^{2}y}$$

Let $\left(1-e^{x}\right) = m$

Diff on the both sides wr. To x

$$-e^{x} = \frac{dm}{dx}$$

$$dx = \frac{dm}{-e^{x}}$$

$$\int \frac{\sec^2 y}{t} \times \frac{dt}{\sec^2 y} = -\int \frac{e^x}{m} \times \frac{dm}{-e^x}$$

$$\int \frac{1}{t} dt = \int \frac{1}{m} dm$$

$$\log|t| = \log|m| + C$$

$$\log|\tan y| = \log|1 - e^x| + \log|C|$$

$$\log|\tan y| = \log|(1 - e^x)C|$$

$$\tan y = C(1 - e^x)$$

11. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$ s

Sol. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$$\sec^2 y \tan x\, dy = -\sec^2 x \tan y\, dx$$

$$\frac{dy}{dx} = \frac{-\sec^2 x \tan y}{\sec^2 y \tan x}$$

On separating on the both sides we get $\dfrac{ay}{dx} = \dfrac{-\sec^2 x \tan y}{\sec^2 y \tan x}$

$$\frac{\sec^2 y}{\tan y} dy = \frac{-\sec^2 x}{\tan x} dx$$

Integrating on the both sides $\int \dfrac{\sec^2 y}{\tan y} dy = \int \dfrac{-\sec^2 x}{\tan x} dx$

$$\int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x} dx$$

Let $\tan y = U$

Diff on the both sides w.r. to y $\sec^2 y = \dfrac{du}{dy}$

$$dy = \frac{du}{\sec^2 y}$$

Let $\tan x = V$

Diff on the both sides w.r.t x

$$\sec^2 x = \frac{dV}{dx}$$

$$dx = \frac{dV}{\sec^2 x}$$

$$\int \frac{\sec^2 y}{U} \times \frac{du}{\sec^2 y} = -\int \frac{\sec^2 x}{V} \times \frac{du}{\sec^2 x}$$

$$\int \frac{1}{U} du = -\int \frac{1}{V} dv$$

$$\log|u| = -\log|v| + C$$

$$\log|\tan y| = -\log|\tan x| + C$$

$$\log|\tan y| + \log|\tan x| = C$$

$$\log|\tan y \cdot \tan x| = \log|C|$$

$$\tan x \cdot \tan y = C$$

12. $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$

Sol. $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$

$$\cos x(1+\cos y)dx = \sin y(1+\sin x)dy$$

$$\sin y(1+\sin x)dy = \cos x(1+\cos y)dx$$

$$\frac{dy}{dx} = \frac{\cos x(1+\cos y)}{\sin y(1+\sin x)}$$

On separating variables we get

$$\frac{dy}{dx} = \frac{\cos x(1+\cos y)}{\sin y(1+\sin x)}$$

$$\frac{\sin y}{(1+\cos y)}dy = \frac{\cos x}{(1+\sin x)}dx$$

Integrating on the both sides we get

$$\int\frac{\sin y}{(1+\cos y)}dy = \int\frac{\cos x}{(1+\sin x)}dx$$

Let $(1+\cos y) = u$

Diff on the both sides w.r.t y $\quad 0 + (-\sin y) = \dfrac{du}{dy}$

$$dy = \frac{du}{-\sin y}$$

Let $(1+\sin x) = V$

Diff on the both sides w.r.t x

$$0 + \cos x = \frac{dv}{dx}$$

$$dx = \frac{dv}{\cos x}$$

$$\int\frac{\sin y}{u}\times\frac{du}{-\sin y} = \int\frac{\cos x}{v}\times\frac{dv}{\cos x}$$

$$-\int\frac{1}{u}du = \int\frac{1}{v}dv$$

$$-\log|u| = \log|v| + C$$

$$-\log|u| - \log|v| = C$$

$$-(\log|u| + \log v) = C$$

$$\log|u| + \log|v| = C$$

$$\log|1+\cos y| + \log|1+\sin x| = C$$

$$\log|(1+\cos y)(1+\sin x)| = \log|C|$$

$$(1+\cos y)(1+\cos x) = C$$

$$(1+\sin x)(1+\cos y) = C$$

For each of the following differential equations, find a particular solution satisfying the given condition

13. $\cos\left(\dfrac{dy}{dx}\right) = a$, c where $a \in R$ and $y = 2$ when $x = 0$

Sol. $\cos\left(\dfrac{dy}{dx}\right) = a$

On separating variables we get $\dfrac{dy}{dx} = \cos^{-1} a$

$dy = \cos^{-1} a \, dx$

Integrating on the both sides we get

$\int dy = \int \cos^{-1} a \, dx$

$y = \cos^{-1} a \int dx$

$y = \cos^{-1} a\, x + C$ ..... (i)

Putting when $x = 0$, then $y = 2$

$2 = \cos^{-1} a \times 0 + C$

$C = 2$

Now putting the value of $C = 2$ equation (i)

$y = \cos^{-1} a \cdot x + C$

$y = \cos^{-1} a \cdot x + 2$

$y^{-2} = \cos^{-1} a \cdot x$

$\dfrac{y-2}{x} = \cos^{-1} a$

$\cos\left(\dfrac{y-2}{x}\right) = a$

14. $\dfrac{dy}{dx} = -4xy^2$ it being given that $y = 1$ when $x = 0$

Sol. $\dfrac{dy}{dx} = -4xy^2$

On separating variables we get $\dfrac{dy}{dx} = -4xy^2$

$\dfrac{dy}{y^2} = -4x\, dx$

$y^{-2} dy = -4x\, dx$

Integrating on the both sides we get

$\int y^{-1} dy = \int -4x\, dx$

$\int y^{-2} dy = -4\int x\, dx$

$-y^{-1} = -4 \times \dfrac{x^2}{2} + C$

$$-\frac{1}{y} = -2x^2 + C \qquad \text{... (i)}$$

Putting when $x = 0$, they $y = 1$

$$-\frac{1}{1} = -2(0)^2 + C$$

$$-1 = C \implies C = -1$$

Now putting the value of $C = -1$ in equation (i)

$$-\frac{1}{y} = -2x^2 + (-1)$$

$$-\frac{1}{y} = -(2x^2 + 1)$$

$$\frac{1}{y} = (2x^2 + 1)$$

$$(2x^2 + 1)y = 1$$

$$y = \frac{1}{(2x^2 + 1)}$$

15. $x\,dy = (2x^2 + 1)dx\,(x \neq 0)$, given that $y = 1$ when $x = 1$

Sol. $x\,dy = (2x^2 + 1)dx$

On separating variables we get $x\,dy = (2x^2 + 1)dx$

$$\frac{dy}{dx} = \frac{(2x^2 + 1)}{x}$$

$$dy = \left(\frac{2x^2 + 1}{x}\right)dx$$

Integrating on the both sides we get $\int dy = \int\left(\frac{2x^2 + 1}{x}\right)dx$

$$\int dy = \int\left(\frac{2x}{x} + \frac{1}{x}\right)dx$$

$$\int dy = 2\int x\,dx + \int\frac{1}{x}dx$$

$$y = 2\frac{x}{2} + \log|x| + C$$

$$y = x^2 + \log|x| + C \qquad \text{... (i)}$$

Putting when $x = 1$ then $y = 1$

$$1 = 1^2 + \log|1| + C$$

$$1 = 1 + 0 + C$$

$$1 = 1 + 0 + C$$

$$1 - 1 = C$$

$$C = 0$$

Now putting the values of $C = 0$ in equation (i)

$$y = x^2 + \log|x| + 0$$

$$y = x^2 + \log|x|$$

16. $\dfrac{dy}{dx} = y \tan x$, it being given that $y = 1$ when $x = 0$

Sol. $\dfrac{dy}{dx} = y \tan x$

On separating variables we get $\dfrac{dy}{dx} = y \tan x$

$$\dfrac{dy}{y} = \tan x \, dx$$

Integrating on the both sides we get

$$\int \dfrac{dy}{y} = \int \tan x \, dx$$

$$\log|y| = \log|\sec x| + C$$

$$y = \sec x + C \qquad \dots \text{(i)}$$

Putting when $x = 0$, then $y = 1$

$$1 = \sec 0 + C$$

$$1 = 1 + C$$

$$1 - 1 = C$$

$$\therefore C = 0$$

EXERCISE 19B (Pg.No.: 931)

Very-Short-Answer Questions

Find the general solution of each of the following differential equations

1. $\dfrac{dy}{dx} = \dfrac{x-1}{y+2}$

Sol. Given differential equations $\dfrac{dy}{dx} = \dfrac{x-1}{y+2}$

$\Rightarrow (y+2)dy = (x-1)dx$ [separating the variables]

$\Rightarrow \int (y+2)dy = \int (x-1)dx$

$\Rightarrow \dfrac{y^2}{2} + 2y = \dfrac{x^2}{2} - x + c_1$

$\Rightarrow y^2 + 4y \Rightarrow y^2 + 4y = x^2 - 2x + 2c_1$

$\Rightarrow y^2 + 4y - x^2 + 2x = 2c_1$

$\Rightarrow y^2 + 4y - x^2 + 2x = c$ {Here $c = 24$ this is the required solution of given differential equation

2. $\dfrac{dy}{dx} = \dfrac{x}{(x^2+1)}$

Sol. Given differential equation is $\dfrac{dy}{dx} = \dfrac{x}{x^2+1}$

$\Rightarrow dy = \dfrac{xdx}{x^2+1}$ {separating the variables

$\Rightarrow \int dy = \int \dfrac{xdx}{x^2+1} \Rightarrow y = \dfrac{1}{2}\log(x^2+1) + c$

This is the required solution

3. $\dfrac{dy}{dx} = (1+x)(1+y^2)$

Sol. Given, $\dfrac{dy}{dx} = (1+x)(1+y^2)$

$\Rightarrow dy = (1+x)(1+y^2)dx$

$\Rightarrow \dfrac{dy}{1+y^2} = (1+x)dx$ [On separating the variables]

$\Rightarrow \int \dfrac{1}{1+y^2}dy = \int (1+x)dx$ [Integrating both sides] $\quad \therefore \tan^{-1}(y) = x + \dfrac{x^2}{2} + c$

4. $(1+x^2)\dfrac{dy}{dx} = xy$

Sol. Given, $(x^2+1)\dfrac{dy}{dx} = xy \Rightarrow (x^2+1)dy = xy\,dx \Rightarrow \dfrac{dy}{y} = \dfrac{x}{x^2+1}dx$ [On separating the variables]

$\Rightarrow \int \dfrac{1}{y}dy = \int \dfrac{x}{x^2+1}dx$ [Integrating both sides]

$\Rightarrow \log y = \dfrac{1}{2}\log|x^2+1| + \log c$

$\Rightarrow \log|y^2| = \log\{(x^2+1).c\}$

$$\therefore \quad y^2 = (x^2 + 1) + c$$

5. $\dfrac{dy}{dx} + y = 1 \, (y \neq 1)$

Sol. Given, $\dfrac{dy}{dx} + y = 1 \Rightarrow \dfrac{dy}{dx} = 1 - y \Rightarrow \dfrac{dy}{1-y} = dx$ [On separating the variables]

$\Rightarrow \displaystyle\int \dfrac{1}{1-y} \, dy = \int dx$ [Integrating both sides]

$\Rightarrow \dfrac{\log|1-y|}{-1} = x + c$

$\Rightarrow -\log|1-y| = x + c$

$\therefore \quad c = x + \log|1-y|$

6. $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

Sol. $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

$\Rightarrow \dfrac{1}{\sqrt{1-y^2}} \, dy + \dfrac{1}{\sqrt{1-x^2}} \, dx = 0$ [on separating the variables]

$\Rightarrow \displaystyle\int \dfrac{dy}{\sqrt{1-y^2}} + \int \dfrac{dx}{\sqrt{1-x^2}} = C$ [integrating both sides]

$\Rightarrow \sin^{-1} y + \sin^{-1} x = C$

Hence $\sin^{-1} y + \sin^{-1} x = C$ is the required solution

7. $x\dfrac{dy}{dx} + y = y^2$

Sol. $x\dfrac{dy}{dx} + y = y^2$

Sol. Given, $x\dfrac{dy}{dx} + y = y^2$

$\Rightarrow x\dfrac{dy}{dx} = y^2 - y$

$\Rightarrow x \, dy = (y^2 - y) \, dx$

$\Rightarrow \displaystyle\int \dfrac{dy}{y(y-1)} = \int \dfrac{dx}{x}$ [Integrating both sides]

$\Rightarrow -\log y + \log|y-1| = \log x + \log c$

$\Rightarrow \log|y-1| = \log(xyc) \quad \therefore \quad y - 1 = xyc$

8. $x^2(y+1)dx + y^2(x-1)dy = 0$

Sol. Given differential equation is $x^2(y+1)dx + y^2(x-1)dy = 0$

$\Rightarrow y^2(x-1)dy = -x^2(y+1)dx$

$$\Rightarrow \frac{y^2 + dy}{y+1} = -\frac{x^2}{x-1}dx$$

$$\Rightarrow \int \frac{y^2 dy}{y+1} = -\int \frac{x^2 dx}{x-1}$$

$$\Rightarrow \int \frac{y^2 - 1 + 1}{y+1}dy = -\int \frac{x^2 - 1 + 1}{x-1}dx$$

$$\Rightarrow \int \frac{(y-1)(y+1)+1}{y+1}dy = -\int \frac{(x-1)(x+1)+1}{x-1}dx$$

$$\Rightarrow \int \left\{ y - 1 + \frac{1}{y+1} \right\}dy = -\int \left\{ x + 1 + \frac{1}{x-1} \right\}dx$$

$$\Rightarrow \frac{y^2}{2} - y + \log y + 1 = -\frac{x^2}{2} - x - \log|x-1| + c_1$$

$$\Rightarrow y^2 - 2y + 2\log|y+1| = -x^2 - 2x - 2\log|x-1| + 2c_1$$

$$\Rightarrow x^2 + y^2 + 2x + 2y + 2\log|(y+1)(x-1)| = c$$

Here $2c_1 = c$

This is the required solution of given differential equation

9. $y(1-x^2)\dfrac{dy}{dx} = x(1+y^2)$

Sol. Given, $y(1-x^2)dy = x(1+y^2)dx \Rightarrow \dfrac{y}{1+y^2}dy = \dfrac{x}{1-x^2}dx$ [On separating the variables]

$$\Rightarrow \int \frac{y}{1+y^2}dy = \int \frac{x}{1-x^2}dx \qquad \text{[Integrating both sides]}$$

Let $1 + y^2 = t$ & $1 - x^2 = z$, $2y = \dfrac{dt}{dy}$ & $-2x = \dfrac{dz}{dx} \Rightarrow y\,dy = \dfrac{dt}{2}$ & $x\,dx = -\dfrac{dz}{2}$

$$\Rightarrow \frac{1}{2}\int \frac{1}{t}dt = -\frac{1}{2}\int \frac{1}{z}dz \Rightarrow \log|t| + \log|z| = \log c \Rightarrow (tz) = c \quad \therefore \quad (1+y^2)(1-x^2) = c$$

10. $y \log y\, dx - x\, dy = 0$

Sol. Given differential equation is $y \cdot \log dx - x \cdot dy = 0$

$$\Rightarrow x\, dy = y \log y\, dx$$

$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

$$\Rightarrow \int \frac{dy}{y \log y} = \int \frac{dx}{x}$$

$$\Rightarrow \log(\log y) = \log|x| + \log c_1$$

$$\Rightarrow \log(\log y) = \log c_1 |x|$$

$$\Rightarrow (\log y) = c_1 |x| \Rightarrow \log y = \pm c_1 x$$

$$\Rightarrow \log y = cx$$

{here $c = \pm c_1$,

This is the required solution of given differential equation

11. $x\left(x^2 - x^2 y^2\right)dy + y\left(y^2 + x^2 y^2\right)dx = 0$

Sol. Given differential equation is $x\left(x^2 - x^2 y^2\right)dy + y\left(y^2 + x^2 y^2\right)dx = 0$

$\Rightarrow x^3\left(1 - y^2\right)dy + y^3\left(1 + x^2\right)dx = 0$

$\Rightarrow x^3\left(1 - y^2\right)dy = -y^3\left(1 + x^2\right)dx$

$\Rightarrow \dfrac{\left(1 - y^2\right)}{y^3}dy = -\dfrac{\left(1 + x^2\right)}{x^3}dx$

$\Rightarrow \left(\dfrac{1}{y^3} - \dfrac{1}{y}\right)dy = -\left(\dfrac{1}{x^3} + \dfrac{1}{x}\right)dx$

$\Rightarrow \int\left\{\dfrac{1}{y^3} - \dfrac{1}{y}\right\}dy = -\int\left(\dfrac{1}{x^3} + \dfrac{1}{x}\right)dx$

$\Rightarrow -\dfrac{1}{2y^2} - \log|y| = \dfrac{1}{2x^2} - \log|x| + c$

$\Rightarrow -\dfrac{1}{2x^2} - \dfrac{1}{2y^2} + \log|x| - \log|y| = c$

$\Rightarrow -\dfrac{1}{2x^2} - \dfrac{1}{2y^2} + \log\left|\dfrac{x}{y}\right| = c$

This is the required solution of given differential equation

12. $\left(1 - x^2\right)dy + xy(1 - y)dx = 0$

Sol. Given differential equation is $\left(1 - x^2\right)dy + xy(1 - x)dx = 0$

$\Rightarrow \left(1 - x^2\right)dy = +xy(1 - y)dx$

$\Rightarrow \dfrac{dy}{y(1 - y)} = \dfrac{-xdx}{1 - x^2}$

$\Rightarrow \int\dfrac{dy}{y(1 - y)} = \dfrac{1}{2}\int\dfrac{-2xdx}{1 - x^2}$

$\Rightarrow \int\left\{\dfrac{1}{y} + \dfrac{1}{y - 1}\right\}dy = \dfrac{1}{2}\int\dfrac{-2xdx}{1 - x^2}$

$\Rightarrow \log|t| = \log|1 - y| = \dfrac{1}{2}\log|1 - x^2| + \log c_1$

$\Rightarrow \log\left|\dfrac{y}{1 - y}\right| = \log\left(\sqrt{1 - x^2}\right)c_1$

$\Rightarrow \left|\dfrac{y}{1 - y}\right| = c_1\sqrt{1 - x^2} \quad \Rightarrow \dfrac{y}{1 - y} = \pm c_1\sqrt{1 - x^2} \quad \Rightarrow y = c(1 - y)\sqrt{1 - x^2}$

{ where $c = \pm c_1$

This is the required solution of given diff equation

13. $\left(1 - x^2\right)(1 - y)dx = xy(1 + y)dy$

Sol. Given, $\left(1 - x^2\right)(1 - y)dx = xy(1 + y)dy$

$$\Rightarrow \frac{1-x^2}{x}dx = \frac{y(1+y)}{1-y}dx \qquad \text{[On separating the variable]}$$

$$\Rightarrow \frac{1-x^2}{x}dx = \frac{y(1+y)}{1-y}dy \qquad \text{[On intergrading both sides]}$$

$$\Rightarrow \int\frac{1}{x}dx - \int x\,dx = \int\frac{y^2}{1-y}dy + \int\frac{y}{1-y}dy \quad \Rightarrow \log|x| - \frac{x^2}{2} = -\int\frac{1-y^2-1}{1-y}dy + \int\frac{y}{1-y}dy$$

$$\Rightarrow \log|x| - \frac{x^2}{2} = -\int\frac{(1-y)(1+y)}{1-y}dy + \int\frac{1}{1-y}dy + \int\frac{y}{1-y}dy$$

$$\Rightarrow \log|x| - \frac{x^2}{2} = -\int(1+y)dy - \log|1-y| - \int\frac{(1-y)-1}{1-y}dy$$

$$\Rightarrow \log|x| - \frac{x^2}{2} = -y - \frac{y^2}{2} - \log|1-y| - \int dy + \int\frac{1}{1-y}dy$$

$$\Rightarrow \log|x| - \frac{x^2}{2} = -y - \frac{y^2}{2} - \log|1-y| - y - \log|1-y|$$

$$\Rightarrow \log|x| - \frac{x^2}{2} = -2y - \frac{y^2}{2} - 2\log|1-y| \quad \Rightarrow \log|x| - \frac{x^2}{2} = -2y - \frac{y^2}{2} - \log|1-y|^2$$

$$\Rightarrow \log|x| + \log|1-y|^2 = -2y - \frac{y^2}{2} + \frac{x^2}{2} \quad \therefore \log\left(x(1-y)^2\right) = \frac{x^2}{2} - 2y - \frac{y^2}{2} + c$$

14. $(y+xy)dx + (x-xy^2)dy = 0$

Sol. Given, $(y+xy)dx + (x-xy^2)dy = 0 \quad \Rightarrow (y+xy)dx = -(x-xy^2)dy$

$$\Rightarrow y(1+x)dx = x(y^2-1)dy \quad \Rightarrow \frac{1+x}{x}dx = \frac{y^2-1}{y}dy \qquad \text{[On separating the variable]}$$

$$\Rightarrow \int\frac{1+x}{x}dx = \int\frac{y^2-1}{y}dy \quad \Rightarrow \int\frac{1}{x}dx + \int dx = \int y\,dy - \int\frac{1}{y}dy$$

$$\Rightarrow \log|x| + x = \frac{y^2}{2} - \log|y| + c \quad \therefore \log|xy| + x - \frac{y^2}{2} = c$$

15. $(x^2-yx^2)dy + (y^2+xy^2)dx = 0$

Sol. Given, $(x^2-yx^2)dy + (y^2+xy^2)dx = 0 \quad \Rightarrow x^2(1-y)dy = -y^2(1+x)dx$

$$\Rightarrow \frac{1-y}{y^2}dy = -\frac{1+x}{x^2}dx \qquad \text{[On separating the variable]}$$

$$\Rightarrow \int\frac{1-y}{y^2}dy = -\int\frac{1+x}{x^2}dx \qquad \text{[Integrating both sides]}$$

$$\Rightarrow \int\frac{1}{y^2}dy - \int\frac{y}{y^2}dx = -\int\frac{1}{x^2}dx - \int\frac{x}{x^2}dx \quad \Rightarrow -\frac{1}{y} - \log|y| = \frac{1}{x} - \log|x|$$

$$\Rightarrow \log|x| - \log|y| = \frac{1}{x} + \frac{1}{y} + c \quad \therefore \log\left|\frac{x}{y}\right| = \frac{1}{x} + \frac{1}{y} + c$$

16. $(x^2y-x^2)dx + (xy^2-y^2)dy = 0$

Sol. Given, $(x^2y-x^2)dx + (xy^2-y^2)dy = 0 \quad \Rightarrow (x^2y-x^2)dx = -(xy^2-y^2)dy$

$\Rightarrow x^2(y-1)dx = -y^2(x-1)dy$

$\Rightarrow \int \dfrac{x^2}{x-1}dx = \int \dfrac{y^2}{1-y}dy$

$\Rightarrow \int \dfrac{(x^2-1)+1}{x-1}dx = -\int \dfrac{(y^2-1)+1}{y-1}dy$

$\Rightarrow \int \dfrac{(x-1)(x+1)}{x-1}dx + \int \dfrac{1}{x-1}dx = -\left[\int \dfrac{(y-1)(y+1)}{y-1}dy + \int \dfrac{1}{y-1}dy\right]$

$\Rightarrow \int (x+1)dx + \log|x-1| = -\left[\int (y+1)dy + \log|y-1|\right] + c$

$\Rightarrow \dfrac{x^2}{2} + x + \log|x-1| = -\left[\dfrac{y^2}{2} + y + \log|y-1|\right] + c$

$\Rightarrow \dfrac{x^2}{2} + \dfrac{y^2}{2} + x + y + \log|x-1| + \log|y-1| = c$

$\therefore \dfrac{1}{2}(x^2+y^2) + (x+y) + \log|(x-1)(y-1)| = c$

17. $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$

Sol. Given, $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$

$\Rightarrow x\sqrt{1+y^2}\,dx = -y\sqrt{1+x^2}\,dy$

$\Rightarrow \dfrac{x}{\sqrt{1+x^2}}dx = -\dfrac{y}{\sqrt{1+y^2}}dy \qquad$ [On separating both sides]

$\Rightarrow \int \dfrac{x}{\sqrt{1+x^2}}dx = -\int \dfrac{y}{\sqrt{1+y^2}}dy \quad$ [Integrating both sides]

Let $1+x^2 = t$ & $1+y^2 = z$

$\Rightarrow 2x = \dfrac{dt}{dx}$ & $2y = \dfrac{dz}{dy}$

$\Rightarrow x\,dx = \dfrac{dt}{2}$ & $y\,dy = -\dfrac{dz}{2}$

Now, $\dfrac{1}{2}\int \dfrac{1}{\sqrt{t}}dt = -\dfrac{1}{2}\int \dfrac{1}{\sqrt{z}}dz$

$\Rightarrow \int t^{-\frac{1}{2}}dt = -\int z^{-\frac{1}{2}}dz$

$\Rightarrow \dfrac{t^{1/2}}{1/2} = -\dfrac{z^{1/2}}{1/2} + c$

$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = c$

18. $\dfrac{dy}{dx} = e^{x+y} + x^2 e^y$

Sol. We have $\dfrac{dy}{dx} = e^{x+y} + x^2 \cdot e^y$

$= e^x \cdot e^y + x^2 \cdot e^y = (e^x + x^2) \cdot e^y$

$\Rightarrow e^{-y} dy = \left(e^x + x^2\right) dx$ [separating the variables]

$\Rightarrow \int e^{-y} dy = \int \left(e^x + x^2\right) dx$

$\Rightarrow -e^{-y} = e^x + \dfrac{x^3}{3} + C_1$

$\Rightarrow e^x + e^{-y} + \dfrac{x^3}{3} = C,$ where $C = -C_1$

Hence $e^x + e^{-y} + \dfrac{x^3}{3} = C$ is the required solution

19. $\dfrac{dy}{dx} = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

Sol. Given differential equation is $\dfrac{dy}{dx} = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

$\Rightarrow dy = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}} dx$

$\Rightarrow dy = \left\{ \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}} \right\} \cdot \dfrac{e^x}{e^x} dx$

$\Rightarrow dy = \dfrac{3e^{3x} + 3e^{5x}}{e^{2x} + 1} dx$

$\Rightarrow dy = \dfrac{3e^{3x}\left\{1 + e^{2x}\right\}}{e^{2x} + 1} dx$

$\Rightarrow dy = 3e^{3x} dx \Rightarrow \int dy = 3\int e^{3x} dx$

$\Rightarrow y = 2x \times \dfrac{1}{9} \cdot e^{3x} + c$

$\Rightarrow y = e^{3x} + c$

Is the required solution of given differential equation

20. $3e^x \tan y\, dx + \left(1 - e^x\right)\sec^2 y\, dy = 0$

Sol. Given equation is $3e^x \tan y\, dx + \left(1 - e^x\right)\sec^2 y\, dy = 0$

$\Rightarrow \left(1 - e^x\right)\sec^2 y\, dy = -3e^x \tan y\, dx$

$\Rightarrow \dfrac{\sec^2 y\, dy}{\tan y} = \dfrac{-3e^x dx}{1 - e^x}$

$\Rightarrow \int \dfrac{\sec^2 y\, dy}{\tan y} = 3\int \dfrac{-e^x dx}{1 - e^x}$

$\Rightarrow \log|\tan y| = 3\log\left|1 - e^x\right| + \log c_1$

$\Rightarrow \log|\tan y| = \log c_1 \left|1 - e^x\right|^3$

$\Rightarrow |\tan y| = c_1 \left|1 - e^x\right|^3$

$\Rightarrow \tan y = \pm c_1 \left(1 - e^x\right)^3$

$\Rightarrow \tan y = c\left(1 - e^x\right)$

Where $c = \pm c_1$

This is the required solution of given differention equation

21. $e^y (1+x^2) dy - \dfrac{x}{y} dx = 0$

Sol. Given differential equation is $e^y (1+x^2) dy - \dfrac{x}{y} dx = 0$

$\Rightarrow e^y (1+x^2) dy = \dfrac{x}{y} dx \quad \Rightarrow y e^y dy = \dfrac{x dx}{1+x^2}$

$\Rightarrow \displaystyle\int y \cdot e^y dy = \int \dfrac{x dx}{1+x^2}$

$\Rightarrow y \displaystyle\int e^y dy - \int \left\{ \dfrac{dy}{dx} \int e^y dy \right\} dy = \dfrac{1}{2} \int \dfrac{2x dx}{1+x^2}$

$\Rightarrow y \cdot e^y - \displaystyle\int e^y dy = \dfrac{1}{2} \log(1+x^2) + c$

$\Rightarrow y e^y - e^y = \dfrac{1}{2} \log(1+x^2) + c$

This is the required solution of given differential equation

22. $\dfrac{dy}{dx} = e^{x+y} + e^{x-y}$

Sol. Given differential equation is $\dfrac{dy}{dx} = e^{x+y} + e^{x-y}$

$\Rightarrow \dfrac{dy}{dx} = e^x \cdot e^y + \dfrac{e^x}{e^y} \quad \Rightarrow \dfrac{dy}{dx} = e^x \left\{ \dfrac{e^{2y}+1}{e^y} \right\}$

$\Rightarrow \dfrac{e^y}{e^{2y}+1} dy = e^x dx \quad \Rightarrow \displaystyle\int \dfrac{e^y dy}{(e^y)^2 + 1} = \int e^x dx \quad \Rightarrow \tan^{-1}(e^y) = e^y + c$

This is the required solution of given differential equation

23. $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$

Sol. Given differential equation is $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$

$\Rightarrow e^y \sin x\, dy = -(e^y + 1) \cos x\, dx$

$\Rightarrow \dfrac{e^y dy}{e^y + 1} = -\dfrac{\cos x\, dx}{\sin x} \quad \Rightarrow \displaystyle\int \dfrac{e^y dy}{e^y + 1} = -\int \cot x\, dx$

$\Rightarrow \log|e^y + 1| = -\log|\sin x| + \log c \quad \Rightarrow \log|e^y + 1| + \log|\sin x| = \log c_1$

$\Rightarrow \log|(e^y + 1)\sin x| = \log c_1 \quad \Rightarrow |(e^y + 1)\sin x| = c_1$

$\Rightarrow (e^y + 1)\sin x = \pm c_1 \quad \Rightarrow (e^y + 1)\sin x = c \quad \{\text{where } \pm c_1 - c\}$

This is the required solution of given differential equation

24. $\dfrac{dy}{dx} + \dfrac{xy + y}{xy + x} = 0$

Sol. Given, $\dfrac{dy}{dx} + \dfrac{xy+y}{xy+x} = 0 \quad \Rightarrow \dfrac{dy}{dx} = -\dfrac{y(x+1)}{x(y+1)} \quad \Rightarrow \dfrac{y+1}{y} dy = -\left(\dfrac{x+1}{x} \right) dx$ [On separating the variables]

$$\Rightarrow \int \frac{y+1}{y}dy = -\int \frac{x+1}{x}dx \quad \text{[Integrating both sides]}$$

$$\Rightarrow \int dy + \int \frac{1}{y}dy = -\int dx - \int \frac{1}{x}dx \quad \Rightarrow y + \log y = -x - \log x + c \quad \therefore \ x + y + \log(xy) = c$$

25. $\sqrt{1-x^4}\,dy = x\,dx$

Sol. Given differential equation is $\sqrt{1-x^4}\,dy = x\,dx$

$$\Rightarrow dy = \frac{x\,dx}{\sqrt{1-x^4}}$$

$$\Rightarrow \int dy = \frac{1}{2}\int \frac{2x\,dx}{\sqrt{1-\left(x^2\right)^2}} \quad \Rightarrow \int dy = \frac{1}{2}\int \frac{dz}{\sqrt{1-z^2}} \quad \{ \text{Let } z = x^2$$

$$\Rightarrow y = \frac{1}{2}\sin^{-1}z + c_1 \quad \Rightarrow y = \frac{1}{2}\sin^{-1}\left(2x\right) + c_1$$

26. $\operatorname{cosec} x \log y \dfrac{dy}{dx} + x^2 y = 0$

Sol. Given differential equation is $\operatorname{cosec} cx \cdot \log y \dfrac{dy}{dx} + x^2 y = 0$

$$\Rightarrow \operatorname{cosec} x \cdot \log y \frac{dy}{dx} = -x^2 y \quad \Rightarrow \int \frac{\log y}{y}dy = \int \left(-x^2 \sin x\right)dx$$

$$\Rightarrow \int z\,dx = -\left[x^2 \int \sin x\,dx - \int \left\{ \frac{dx^2}{dx}\int \sin x\,dx \right\}dx \right]$$

$$\left\{ \text{Let } z = \log y \ \therefore \ dz = \frac{dy}{y} \right\}$$

$$\Rightarrow \frac{z^2}{2} = -\left[-x^2 \cos x + 2\int x\cos x\,dx \right] + c$$

$$\Rightarrow \frac{z^2}{2} = x^2 \cos x - 2\left[x\int \cos dx - \int \left\{ \frac{dx}{dx}\int \cos x\,dx \right\}dx \right] + c$$

$$\Rightarrow \frac{z^2}{2} = x^2 \cos x - 2\left[x\sin x - \int \sin x\,dx \right] + c \quad \Rightarrow \frac{z^2}{2} = x^2 \cos x - 2x\sin x - 2\cos x + c$$

$$\Rightarrow \frac{1}{2}\left(\log y\right)^2 = \left(x^2 - 2\right)\cos x - 2x\sin x + c \quad \Rightarrow \frac{1}{2}\left(\log y\right)^2 + \left(2 - x^2\right)\cos x + 2x\sin x = c$$

This is the required solution of given differential equation

27. $y\,dx + \left(1+x^2\right)\tan^{-1}x\,dy = 0$

Sol. Given differential equation is $y\,dx + \left(1+x^2\right)\tan^{-1}x\,dy = 0$

$$\Rightarrow \left(1+x^2\right)\tan^{-1}x\,dy = -y\,dx \quad \Rightarrow \frac{dy}{y} = \frac{-dx}{\left(1+x^2\right)\tan^{-1}x}$$

$$\Rightarrow \int \frac{dy}{dx} = -\int \frac{dx}{\left(1+x^2\right)\tan^{-1}x} \quad \Rightarrow \log|y| = -\log\left|\tan^{-1}x\right| + \log c_1$$

$$\Rightarrow \log\left| y\tan^{-1}x \right| = \log c_1 \quad \Rightarrow \left| y\tan^{-1}x \right| = c_1 \quad \Rightarrow y\tan^{-1}x = \pm c_1 \quad \Rightarrow y\tan^{-1}x = c$$

$\{\text{where } c = \pm c_1 \ \}$

This is the required solution of given differential equation

28. $\dfrac{1}{x} \cdot \dfrac{dy}{dx} = \tan^{-1} x$

Sol. Given differential equation $\dfrac{1}{x} \cdot \dfrac{dy}{dx} = \tan^{-1} x$

$\Rightarrow dy = x \tan^{-1} x\, dx \Rightarrow \int dy = \int \left(\tan^{-1} x\right) \cdot x\, dx$

$\Rightarrow \int dy = \tan^{-1} x \int x\, dx - \int\left\{\dfrac{d}{dx}\left(\tan^{-1} x\right) \int x\, dx\right\} dx$

$\Rightarrow \int dy = \dfrac{x^2}{2} \tan^{-1} x - \int \dfrac{1}{1+x^2} \cdot \dfrac{x^2}{2} dx$

$\Rightarrow \int dy = \dfrac{x^2}{2} \tan^{-1} x - \dfrac{1}{2} \int \dfrac{x^2}{1+x^2} dx$

$\Rightarrow \int dy = \dfrac{x^2}{2} \tan^{-1} x - \dfrac{1}{2} \int \dfrac{1+x^2-1}{1+x^2} dx$

$\Rightarrow \int dy = \dfrac{x^2}{2} \tan^{-1} x - \dfrac{1}{2} \int\left\{1 - \dfrac{1}{1+x^2}\right\} dx$

$\Rightarrow y = \dfrac{x^2}{2} \tan^{-1} x - \dfrac{1}{2} x + \dfrac{1}{2} \tan^{-1} x + c$

$\Rightarrow y = \dfrac{1}{2}\left(x^2+1\right) \tan^{-1} x - \dfrac{1}{2} x + c$

This is the required solution of given differential equation

29. $e^x \sqrt{1-y^2}\, dx + \dfrac{y}{x}\, dy = 0$

Sol. Given differential equation is

$e^x \sqrt{1-y^2}\, dx + \dfrac{y}{x}\, dy = 0$

$\Rightarrow x^2 dy = -e^x \sqrt{1-y^2}\, dx$

$\Rightarrow \dfrac{y\, dx}{\sqrt{1-y^2}} = -x \cdot e^x dx$

Integrating both sides we have $\int \dfrac{y\, dx}{\sqrt{1-y^2}} = -\int x \cdot e^x dx$

$\Rightarrow -\dfrac{1}{2} \int \dfrac{-2y\, dy}{\sqrt{1-y^2}} = -\int x \cdot e^x dx$

$\Rightarrow -\dfrac{1}{2} \int \dfrac{dz}{\sqrt{z}} = -\left[x \int e^x dx - \int\left\{\dfrac{dx}{dy} \int e^x dx\right\} dx\right] + C$

{Let $Z = 1-y^2$ }

$\Rightarrow -\dfrac{1}{2} \cdot 2\sqrt{Z} = -x \cdot e^x + e^x + C$

$\Rightarrow xe^x - e^x - \sqrt{1-y^2} = C$

$\Rightarrow e^x(x-1) - \sqrt{1-y^2} = C$

This is the required general solution of given differential equation

30. $\dfrac{dy}{dx} = \dfrac{1 - \cos x}{1 + \cos x}$

Sol. Given, $\dfrac{dy}{dx} = \dfrac{1 - \cos x}{1 + \cos x}$ $\Rightarrow dy = \dfrac{1 - \cos x}{1 + \cos x} dx$ $\Rightarrow dy = \dfrac{2 \sin^2 \dfrac{x}{2}}{2 \cos^2 \dfrac{x}{2}} dx$

$\Rightarrow dy = \tan^2 \dfrac{x}{2} dx$ [On integrating both sides]

$\Rightarrow \int dy = \int \tan^2 \dfrac{x}{2} dx$ [Integrating both sides]

$\Rightarrow y = \int \left(\sec^2 \dfrac{x}{2} - 1 \right) dx$ $\Rightarrow y = \dfrac{\tan \dfrac{x}{2}}{\dfrac{1}{2}} - x + c$ $\therefore y = 2\tan \dfrac{x}{2} - x + c$

31. $(\cos x)\dfrac{dy}{dx} + \cos 2x = \cos 3x$

Sol. Given differential equation is $\cos x \dfrac{dy}{dx} + \cos 2x = \cos 3x$

$\Rightarrow \cos x \dfrac{dy}{dx} = \cos 3x - \cos 2x$

$\Rightarrow dy = \dfrac{\cos 3x - \cos 2x}{\cos 2x} dx$

$\Rightarrow dy = \dfrac{4\cos^3 x - 3\cos x - 2\cos^2 x + 1}{\cos x} dx$

$\Rightarrow dy = \{4\cos^2 x - 3 - 2\cos x + \sec x\} dx$

$\Rightarrow dy = \left\{ \dfrac{4(1 + \cos 2x)}{2} - 3 - 2\cos x + \sec x \right\} dx$

$\Rightarrow dy = \{2 + 2\cos x - 3 - 2\cos x + \sec x\} dx$

Integrating both sides we have $\int dy = \int \{2\cos 2x - 2\cos x + \sec x - 1\} dx$

$\Rightarrow y = \sin 2x - 2\sin x - x + \log|\sec x + \tan x| + C$

This is the required grated solution of given differential equation

32. $\dfrac{dy}{dx} + \dfrac{(1 + \cos 2y)}{(1 - \cos 2y)} = 0$

Sol. Given, $\dfrac{dy}{dx} + \dfrac{1 + \cos 2y}{1 - \cos 2x} = 0$ $\Rightarrow \dfrac{dy}{dx} = -\dfrac{1 + \cos 2y}{1 - \cos 2x}$ $\Rightarrow \dfrac{1}{1 + \cos 2y} dy = -\dfrac{1}{1 - \cos 2x} dx$

$\Rightarrow \dfrac{1}{2\cos^2 y} dy = -\dfrac{1}{2\sin^2 x} dx$ [On separating the variables]

$\Rightarrow \dfrac{1}{2} \sec^2 y \, dy = -\dfrac{1}{2} \operatorname{cosec}^2 x \, dx$ [Integrating both sides]

$\Rightarrow \dfrac{1}{2} \int \sec^2 y \, dy = -\dfrac{1}{2} \int \operatorname{cosec}^2 x \, dx$ $\therefore \tan y = \cot x + c$

33. $\dfrac{dy}{dx} + \dfrac{\cos x \sin y}{\cos y} = 0$

Sol. Given differential equation is $\dfrac{dy}{dx} + \dfrac{\cos x \sin y}{\cos y} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{\cos x - \sin y}{\cos y}$

$\Rightarrow \dfrac{\cos y}{\sin y} dy = -\cos\, dx$

$\Rightarrow \cot y \cdot dy = -\cos x\, dx$

Integrating both sides we have $\int \cot y \cdot dy = -\int \cos x\, dx$

$\Rightarrow \log|\sin y| = -\sin x + C$

$\Rightarrow \log|\sin y| + \sin x = C$

This is the required solution of given differential equation

34. $\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$

Sol. We have $\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$ ... (i)

$\Rightarrow \dfrac{\cos x}{(1 + \sin x)}dx - \dfrac{\sin y}{(1 + \cos y)}dy = 0$

$\Rightarrow \int \dfrac{\cos x}{(1 + \sin x)}dx - \int \dfrac{\sin y}{(1 + \cos y)}dy = \log C$, where C is a constant

$\Rightarrow \log|1 + \sin x| + \log|1 + \cos y| = \log C$

$\Rightarrow \log|(1 + \sin x)(1 + \cos y)| = \log C$

$\Rightarrow (1 + \sin x)(1 + \cos y) = C$

Hence $(1 + \sin x)(1 + \cos y) = C$ is the required solution

35. $\sin^3 x\, dx - \sin y\, dy = 0$

Sol. Given differential equation is $\sin^3 x\, dx - \sin y\, dy = 0$

$\Rightarrow \sin y\, dy = \sin^3 x\, dx$

Integrating both sides we have $\int \sin y\, dy = \int \sin^3 x\, dx$

$\Rightarrow \int \sin y\, dy = \dfrac{1}{4}\int (3\sin x - \sin 3x)dx$

$\Rightarrow -\cos y = -\dfrac{3}{4}\cos x + \dfrac{1}{12}\cos 3x + C_1$

$\Rightarrow -12\cos y = -9\cos x + \cos 3x + 124$

$\Rightarrow \cos 3x - 9\cos 3x - 12\cos y + C = 0$

Where $12c = c$ this is the required solution of given differential equation

36. $\dfrac{dy}{dx} + \sin(x + y) = \sin(x - y)$

Sol. Given differential equation is $\dfrac{dy}{dx} + \sin(x + y) = \sin(x - y)$

$\Rightarrow \dfrac{dy}{dx} + \sin(x+y) - \sin(x-y) = 0$

$\Rightarrow \dfrac{dy}{dx} + 2\cos x \cdot \sin x = 0$

$\Rightarrow \dfrac{dy}{dx} = -2\cos x \cdot \sin y$

$\Rightarrow \sec y\, dy = -2\cos x\, dx$

Integrating both sides w.r.t x we have

$\displaystyle\int \sec y\, dy = -2\int \cos x\, dx$

$\Rightarrow \log|\sec y + \tan y| = -2\sin x + C$

$\Rightarrow \log|\sec y + \tan y| + 2\sin x = C$

This is the required general solution at given differential equation

37. $\dfrac{1}{x}\cos^2 y\, dy + \dfrac{1}{y}\cos^2 x\, dx = 0$

Sol. Given differential equation is $\dfrac{1}{x} \cdot \cos^2 y\, dy + \dfrac{1}{y}\cos^2 x\, dx = 0$

$\Rightarrow \dfrac{1}{x}\cdot \cos^2 y\, dy = -\dfrac{1}{y}\cos^2 x\, dx$

$\Rightarrow y\cos^2 y\, dy = -x\cos^2 x$

$\Rightarrow y\,\dfrac{1+\cos 2y}{2}\, dy = -x\,\dfrac{1+\cos 2x}{2}\, dx$

$\Rightarrow \left(\dfrac{y}{2} + \dfrac{y}{2}\cos 2y\right) dy = \left(\dfrac{-x}{2} - \dfrac{x\cos 2x}{2x}\right) dx$

$\Rightarrow y\, dy + y\cos 2y\, dy = -x\, dx - x\cdot \cos 2x\, dx$

Integrating both sides we have $\displaystyle\int y\, dy + \int y\cos 2y\, dy = -\int x\, dx - \int x\cos 2x\, dx$

$\Rightarrow \dfrac{y^2}{2} + y\displaystyle\int \cos 2y\, dy - \int\left\{\dfrac{dy}{dx}\int \cos 2y\, dy\right\}dy = -\dfrac{x^2}{2} - \left[x\int \cos 2x - \int\left\{\dfrac{dx}{dx}\int \cos 2x\right\}dx\right] + C_1$

$\Rightarrow \dfrac{y^2}{2} + \dfrac{1}{2}y\sin 2y - \dfrac{1}{2}\displaystyle\int \sin 2y\, dy = -\dfrac{x^2}{2} - \dfrac{1}{2}x\sin 2x + \dfrac{1}{2}\int \sin 2x\, dx$

$\Rightarrow \dfrac{y^2}{2} + \dfrac{1}{2}\cdot y\cdot \sin 2y + \dfrac{1}{4}\cos 2y = -\dfrac{x^2}{2} - \dfrac{1}{2}x\sin 2x - \dfrac{1}{4}\cos 2x + C_1$

$\Rightarrow \dfrac{x^2}{2} + \dfrac{y^2}{2} + \dfrac{1}{2}x\sin 2x + \dfrac{1}{2}y\sin 2y + \dfrac{1}{4}\cos 2y + \dfrac{1}{4}\cos 2x = C_1$

$\Rightarrow 2(x^2 + y^2) + 2\{x\sin 2x + y\sin 2y\} + \{\cos 2x + \cos 2y\} = C$

{where $C = 4C_1$

This is the required general solution of given differential equation

38. $\dfrac{dy}{dx} = \sin^3 x\cos^2 x + xe^x$

Sol. Given differential equation is $\dfrac{dy}{dx} = \sin^3 x \cdot \cos^2 x + x\cdot e^x$

$\Rightarrow dy = \sin^3 x \cos^2 x dx + x e^x dx$

$\Rightarrow \int dy = \int \sin^3 x \cdot \cos^2 x dx + \int x e^x dx$

$\Rightarrow \int dy = \int \sin^2 x \cdot \cos^2 x \cdot \sin dx + \int x \cdot e^x dx$

$\Rightarrow \int dy = \int (1 - \cos^2 x) \cos^2 x \sin x dx + \int x e^x dx$

Let $\cos x = Z \Rightarrow \sin x dx = -dz$

$\Rightarrow \int dy = -\int (1 - z^2) \cdot z^2 dz + x \int e^x dx - \int \left\{ \dfrac{dx}{dx} \int e^x dx \right\} dx$

$\Rightarrow y = -\int z^2 dz + \int z^4 dz + x \cdot e^x - e^x + c$

$\Rightarrow Y = -\dfrac{z^3}{3} + \dfrac{z^5}{5} + e^x (x - 1) + c$

$\Rightarrow y = -\dfrac{1}{3} \cos^3 + \dfrac{1}{5} \cos^5 x + e^x (x - 1) + c_1$

This is the Required general solution of given differential equation

39. Find the particular solution of the differential equation $\dfrac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$

Sol. Given differential equation is $\dfrac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \dfrac{dy}{dx} = (1 + x) + y(1 + x)$

$\Rightarrow \dfrac{dy}{dx} = (1 + x)(1 + y)$

$\Rightarrow \dfrac{dy}{1 + y} = (1 + x) dx$

$\Rightarrow \int \dfrac{dy}{1 + y} = \int (1 + x) dx \cdot \{\text{integrating both side}\}$

$\Rightarrow \log|1 + y| = x + \dfrac{x^2}{2} + c \quad \ldots\ldots (i)$

Putting $x = 1$ and $y = 0$ in (i) we have $\log|1 + 0| = 1 + \dfrac{1}{2} + c$

$\Rightarrow 0 = \dfrac{3}{2} + c \Rightarrow c = -\dfrac{3}{2}$

Putting $c = -\dfrac{3}{2}$ in (i) we have $\log|1 + y| = x + \dfrac{x^2}{2} - \dfrac{3}{2}$

This is the required particular solution of given differential equation

40. Find the particular solution of the differential equation $x(1 + y^2) dx - y(1 + x^2) dy = 0$, given that $y = 1$ when $x = 0$

Sol. Given differential equation is $x(1 + y^2) dx - y(1 + x^2) dy = 0$

$\Rightarrow x(1 + y^2) dx = y(1 + x^2) dy$

$$\Rightarrow \frac{y\,dy}{1+y^2} = \frac{x\,dx}{1+x^2}$$

Integrating both sides we have $\int \frac{y\,dy}{1+y^2} = \int \frac{x\,dx}{1+x^2}$

$$\Rightarrow \int \frac{2y\,dy}{1+y^2} = \int \frac{2x\,dx}{1+x^2}$$

$$\Rightarrow \log\left|1+y^2\right| = \log\left|1+x^2\right| + \log c$$

$$\Rightarrow \log\left|1+y^2\right| = \log c\left|1+x^2\right|$$

$$\Rightarrow \left|1+y^2\right| = c_1\left|1+x^2\right|$$

$$\Rightarrow \left(1+y^2\right) = \pm\left(1+x^2\right)$$

$$\Rightarrow \left(1+y^2\right) = c\left(1+x^2\right) \quad \{ \text{Let } c = \pm c_1 \quad \ldots\ldots\ldots \text{ (i)}$$

Putting $x = 0$ and $y = 1$ in (i) we have $2 = c$

Putting $c = 2$ (i) we have $\left(1+y^2\right) = 2\left(1+x^2\right)$

$$\Rightarrow y^2 = 2 + 2x^2 - 1 \Rightarrow y^2 = 2x^2 + 1$$

This is the particular solution of given differential equation

41. Find the particular solution of the differential equation $\log\left(\dfrac{dy}{dx}\right) = 3x+4y$, given that $y = 0$ where

$x = 0$

Sol. Given differential equation is $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$

$$\Rightarrow \frac{dy}{dx} = e^{3x+4y} \Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{4y}$$

$$\Rightarrow e^{-4y}\,dy = e^{3x} \cdot dx$$

Integrating both sides we have $\int e^{-4y} \cdot dy = \int e^{3x} \cdot dx$

$$\Rightarrow -\frac{1}{4} \cdot e^{-4y} = \frac{1}{3}e^{3x} + c \quad \ldots\ldots \text{ (i)}$$

Putting $x = 0$ and $y = 0$ we have $-\dfrac{1}{4} \cdot e^0 = \dfrac{1}{3}e^0 + c$

$$c = -\frac{1}{4} - \frac{1}{3} \Rightarrow c = \frac{-7}{12}$$

Putting $c = -\dfrac{7}{12}$ in (i) we have

$$-\frac{1}{4} \cdot e^{-4y} = \frac{1}{3} \cdot e^{3x} - \frac{7}{12}$$

$$\Rightarrow -3.e^{-4y} = 4 \cdot e^{3x} - 7$$

$$\Rightarrow 4e^{3x} + 3e^{-4y} = 7$$

This is the required particular solution of given differential equation

42. Solve the differential equation $\left(x^2 - yx^2\right)dy + \left(y^2 + x^2y^2\right)dx = 0$, given that $y = 1$ when $x = 1$

Sol. Given differential equation is $x^2(1-y)dy + y^2\left(1+x^2\right)dx = 0$

$\Rightarrow x^2(1-y)dy = -y^2(1+x^2)dx$

$\Rightarrow \dfrac{(1-y)dy}{y^2} = \dfrac{-(1+x^2)}{x^2}dx$

Integrating both sides we have $\displaystyle\int \dfrac{(1-y)dy}{y^2} = -\int \dfrac{1+x^2}{x^2}dx$

$\Rightarrow \displaystyle\int \left\{\dfrac{1}{y^2} - \dfrac{1}{y}\right\}dy = -\int \left\{\dfrac{1}{x^2} + 1\right\}dx$

$\Rightarrow -\dfrac{1}{y} - \log|y| = \dfrac{1}{x} - x + c$ .... (i)

Putting $x = 1$ and $y = 1$ we get $-1 - \log 1 = 1 - 1 + c$

$c = -1$

Putting $c = -1$ in (i) we have $-\dfrac{1}{y} - \log|y| = \dfrac{1}{x} - x - 1$

$\Rightarrow x - \dfrac{1}{x} - \dfrac{1}{y} - \log|y| + 1 = 0$ is the particular solution of given differential equation

43. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \dfrac{y}{x}dy = 0$ given that $y = 1$ when $x = 0$

Sol. Given differential equation is $e^x\sqrt{1-y^2}\,dx + \dfrac{y}{x}dy = 0$

$\Rightarrow e^x\sqrt{1-y^2}\,dx = -\dfrac{y}{x}dy$

$\Rightarrow x.e^x\,dx = -\dfrac{ydy}{\sqrt{1-y^2}}$

$\Rightarrow \displaystyle\int x\cdot e^x\,dx = \int \dfrac{-ydy}{\sqrt{1-y^2}}$

$\Rightarrow x\displaystyle\int e^x\,dx - \int\left\{\dfrac{dx}{dx}\int e^x\,dx\right\}dx = \dfrac{1}{2}\int\dfrac{-2ydy}{\sqrt{1-y^2}}$

$\Rightarrow x\cdot e^x - e^x = \dfrac{1}{2}2\sqrt{1-y^2} + c$

$\Rightarrow e^x(x-1) = \sqrt{1-y^2} + c$

Putting $x = 0$ and $y = 1$ we have $= c$

Putting $c = 1$, in (i) we have $e^x(x-1) = \sqrt{1-y^2} = 1$

This is the required particular solution of given differential equation

44. Find the particular solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{(\sin y + y\cos y)}$, given that $y = \dfrac{\pi}{2}$ when $x = 1$

Sol. Given differential equation is $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$

$\Rightarrow (\sin y + y\cos y)dy = x(2\log x + 1)dx$

Integrating both sides we have

$$\int (\sin y + y \cos y) \, dy = \int x (2 \log x + 1) \, dx$$

$$\Rightarrow \int \sin y \, dy + \int y \cos y \, dy = 2 \int x \log x \, dx + \int x \, dx$$

$$\Rightarrow -\cos y + y \int \cos y \, dy - \int \left\{ \frac{dy}{dy} \int \cos y \, dy \right\} dy$$

$$= 2 \left[\log x \int x \, dx - \int \left\{ \frac{d \log x}{dx} \int x \, dx \right\} dx \right] + \frac{x^2}{2} + c$$

$$\Rightarrow -\cos y + y \sin y - \int \sin y \, dy = 2 \left[\frac{x^2}{2} \log x - \int \frac{1}{x} \frac{x^2}{2} \, dx \right] + \frac{x^2}{2} + c$$

$$\Rightarrow -\cos y + y \sin y + \cos y = x^2 \cdot \log x - \int x \, dx + \frac{x^2}{2} + c$$

$$\Rightarrow y \sin y = x^2 \log x - \frac{x^2}{2} + \frac{x^2}{2} + c \qquad \dots (i)$$

Putting $x = 1$ and $y = \frac{\pi}{2}$ we have $\frac{\pi}{2} \cdot \sin \frac{\pi}{2} = x^2 \log y + c$

$$\Rightarrow \frac{\pi}{2} = +c$$

$$\Rightarrow c = \frac{\pi}{2}$$

Putting $c = \frac{\pi}{2} - \frac{1}{2}$ in (i) we have $y \sin y = x^2 \cdot \log x + \frac{\pi}{2}$

This is the required solution of given differential equation

45. Solve the differential equation $\frac{dy}{dx} = y \sin 2x$, given that $y(0) = 1$

Sol. Given differential equation is $\frac{dy}{dx} = y \cdot \sin 2x$

$$\Rightarrow \frac{dy}{y} = \sin x \cdot dx$$

Integrating both sides we have $\int \frac{dy}{y} = \int \sin 2x \, dx$

$$\Rightarrow \log |y| = -\frac{1}{2} \cos 2x + c \qquad \dots (i)$$

Putting $x = 0$ and $y - 1$ we have $0 = -\frac{1}{2} + c$

$$\Rightarrow c = \frac{1}{2}$$

Putting $= -\frac{1}{2}$ in (i) we have $\log |y| = -\frac{1}{2} \cos 2x + \frac{1}{2}$

$$\Rightarrow 2 \log |y| = 1 - \cos 2x \Rightarrow 2 \log |y| = 2 \sin^2 x$$

$$\Rightarrow \log |y| = \sin^2 c \Rightarrow |y| = e^{\sin^2 x}$$

This is the required solution of given differential equation

46. Solve the differential equation $(x+1)\dfrac{dy}{dx} = 2xy,$ given that $y(2) = 3$

Sol. Given differential equation is $(1+x)\cdot\dfrac{dy}{dx} = 2xy$

$\Rightarrow \dfrac{dy}{y} = \dfrac{2x}{x+1}dx$

Integrating both sides we have $\int\dfrac{dy}{y} = 2\int\dfrac{xdx}{x+1}$

$\Rightarrow \int\dfrac{dy}{y} = 2\int\dfrac{x+1-1}{x+1}dx$

$\Rightarrow \int\dfrac{dy}{y} = 2\int\left\{1 - \dfrac{1}{1+x}\right\}dx$

$\Rightarrow \log|y| = 2x - 2\log|x+1| + c_1$

$\Rightarrow \log|y| + \log(x+1)^2 = c_1 + 2x$

$\Rightarrow \log\left|y(x+1)^2\right| = C_1 + 2x$

$\Rightarrow e^{C_1+2x} = \left|y(x+1)^2\right|$

$\Rightarrow \pm e^{C_1}\cdot e^{2x} = y(x+1)^2$

$\Rightarrow C\cdot e^{2x} = y(x+1)^2$ ..... (i)

Putting $x = 2$ and $y = 3$ we get $C\cdot e^4 = 3(2+1)^2$

$\Rightarrow C = \dfrac{27}{e^4}$

Putting $C = \dfrac{27}{e^4}$ in (i) we have $\dfrac{27}{e^4}\cdot e^{2x} = y(x+1)^2$

$\Rightarrow y(x+1)^2 = 27e^{(2x-4)}$

This is the required solution of given differential equation

47. Solve $\dfrac{dy}{dx} = x(2\log x + 1),$ given that $y = 0$ when $x = 2$

Sol. Given differential equation is $\dfrac{dy}{dx} = x(2\log x + 1)$

$\Rightarrow dy = x(2\log x + 1)dx$

Integrating both sides we have $\int dy = 2\int x\log x\,dx + \int xdx$

$y = 2\left[\log x\int xdx - \int\left\{\dfrac{d}{dx}(\log x)\int xdx\right\}dx\right] + \int xdx + C$

$\Rightarrow 2\cdot\dfrac{x^2}{2}\log x - 2\int\dfrac{x^2}{2}\cdot\dfrac{1}{x}\cdot dx + \int xdx + C$

$\Rightarrow y = x^2\cdot\log x - \int xdx + \int xdx + C$

$\Rightarrow y = x^2\log x + C$ .... (i)

Putting $x = 2$ and $y = 0$ we have $0 = 4\log 2 + C$

$\Rightarrow C = -4\log 2$

Putting $C = -4\log 2$ in (i) we have $y = x^2 \cdot \log x - 4\log 2$

$\Rightarrow y = x^2 \cdot \log x - 4\log 2$

This is the required solution of given differential equation

48. Solve $\left(x^3 + x^2 + x + 1\right)\dfrac{dy}{dx} = 2x^2 + x$, given that $y = 1$ when $x = 0$

Sol. Given differential equation is $\left(x^3 + x^2 + x + 1\right)\dfrac{dy}{dx} = 2x^2 + x$

$\Rightarrow \dfrac{2x^2 + x}{x^3 + x^2 + x + 1}\,dx = dy$

Integrating both sides we get

$\displaystyle\int \dfrac{\left(2x^2 + x\right)dx}{x^2(x+1) + (x+1)} = \int dy$

$\Rightarrow \displaystyle\int \dfrac{2x^2 + x}{(x+1)(x^2+1)} = \int dy \quad \dots \text{(i)}$

Let $\dfrac{2x^2 + x}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx}{x^2+1} + \dfrac{C}{x^2+1}$

$\Rightarrow 2x^2 + x = A\left(x^2+1\right) + Bx(x+1) + C(x+1)$

Putting $x = -1$ we have $2 - 1 = 2A + 0 + 0$

$\Rightarrow C = -A = -\dfrac{1}{2}$

Putting $x = -2$, we have $8 - 2 = 5A + 2B - C$

$\Rightarrow 6 = +\dfrac{5}{2} + 2B + \dfrac{1}{2}$

$\Rightarrow 6 - \dfrac{5}{2} - \dfrac{1}{2} = 2B$

$\Rightarrow B = \dfrac{3}{2}$

Now $\dfrac{2x^2 + x}{(x+1)(x^2+1)} = \dfrac{1}{2(x+1)} + \dfrac{3x}{2(x^2+1)} - \dfrac{1}{2(x^2+1)}$

Now from (i) $\displaystyle\int dy = \int \dfrac{\left(2x^2 + x\right)}{(x+1)(x^2+1)}\,dx$

$\displaystyle\int dy = \dfrac{1}{2}\int \dfrac{dx}{x+1} + \dfrac{3}{2}\int \dfrac{x\,dx}{x^2+1} - \dfrac{1}{2}\int \dfrac{dx}{x^2+1}$

$\Rightarrow y = \dfrac{1}{2}\log|x+1| + \dfrac{3}{4}\log\left(x^2+1\right) - \dfrac{1}{2}\tan^{-1}x + C \quad \dots \text{(ii)}$

Putting $x = 0$ and $y = 1$ we have $1 = \dfrac{1}{2}\log 1 + \dfrac{3}{2}\log 2 - \dfrac{1}{2}\tan^{-1}0 + C$

$\Rightarrow 1 = 0 + 0 - 0 + C$

$\Rightarrow C = 1$

Putting $C = 1$ in (ii) we have

$$y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + 1$$

This is the required solution of given differential equation

49. Solve $\frac{dy}{dx} = y\tan x$, given that $y = 1$ and $x = 0$

Sol. Given differential equation is $\frac{dy}{dx} = y\tan x$

$$\Rightarrow \frac{dy}{y} = \tan x\, dx$$

Integrating both sides we get $\int \frac{dy}{y} = \int \tan x\, dx$

$$\Rightarrow \log|y| = \log|\sec x| + \log C_1$$

$$\Rightarrow \log|y| = \log C_1|\sec x|$$

$$\Rightarrow |y| = C_1|\sec x|$$

$$\Rightarrow y = \pm C_1 \sec x$$

$$\Rightarrow y = \pm C \cdot \sec x \quad \pm C_1 = e \text{ let}$$

$$\Rightarrow y\cos x = C$$

Putting $x = 0$ and $y = 1$ we have

$$1 \times \cos 0 = e$$

$$\Rightarrow c = 1$$

Putting $C = 1$ in equation (ii) we have $y\cos x = 1$

This is the required solution of given differential equation

50. Solve $\frac{dy}{dx} = y^2\tan 2x$, given that $y = 2$ when $x = 0$

Sol. Given differential equation is $\frac{dy}{dx} = y^2\tan 2x$

$$\Rightarrow y^2 dy = \tan 2x \cdot dx$$

Integrating both sides we have $\int y^{-2} dy = \int \tan 2x\, dx$

$$\Rightarrow -\frac{1}{y} = \frac{1}{2}\log|\sec 2x| + C$$

$$\Rightarrow -\frac{1}{y} = -\frac{1}{2}|\cos 2x|$$

$$\Rightarrow -\frac{1}{y} = -\frac{1}{2}\log|\cos 2x| + C \qquad \dots (i)$$

Putting $x = 0$ and $y = 2$ we have $-\frac{1}{2} = -\frac{1}{2}\log|\cos| + C$

$$\Rightarrow -\frac{1}{2} = C$$

Putting $C = -\dfrac{1}{2}$ in (i) we get $-\dfrac{1}{y} = -\dfrac{1}{2}\log|\cos 2x| - \dfrac{1}{2}$

$\Rightarrow 2 = y\{\log|\cos 2x| + 1\}$

This is the required solution of given differential equation

51. Solve $\dfrac{dy}{dx} = y\cot 2x$, given that $y = 2$ when $x = \dfrac{\pi}{4}$

Sol. Given differential equation is $\dfrac{dy}{dx} \cdot \cot 2x$

$\Rightarrow \dfrac{dy}{y} = \cot 2x \cdot dx$

Integrating both sides we have

$\Rightarrow \displaystyle\int \dfrac{dy}{y} = \int \cot 2x \cdot dx$

$\Rightarrow \log|y| = \dfrac{1}{2}\log|\sin x| + \log C_1$

$\Rightarrow \log|y| = \log\left(C_1 \sin^2 2x\right)$

$\Rightarrow |y| = C_1 \sin^2 2x$

$\Rightarrow y = \pm C_1 \sin^2 2x$

$\Rightarrow y = C\sin^2 2x \quad \{\text{Let } \pm C_1 = C \qquad \dots\text{(i)}$

Putting $x = \dfrac{\pi}{4}$ and $y = 2$ in (i) we have $2 = C\sin^2 \dfrac{\pi}{2}$

$\Rightarrow C = 2$ putting $C = 2$ in (i) we have $y = 2\sin^2 2x$

This is the required particular solution of given differential equation

52. Solve $\left(1 + x^2\right)\sec^2 y\, dy + 2x\tan y\, dx = 0$ given that $y = \dfrac{\pi}{4}$ when $x = 1$

Sol. Given, $\left(1 + x^2\right)\sec^2 y\, dy + 2x\tan y\, dx = 0 \Rightarrow \left(1 + x^2\right)\sec^2 y\, dy = -2x\tan y\, dx$

$\Rightarrow \dfrac{\sec^2 y}{\tan y}dy = -\dfrac{2x}{1 + x^2}dx \Rightarrow \displaystyle\int \dfrac{\sec^2 y}{\tan y}dy = -\int \dfrac{2x}{1 + x^2}dx$

Let $\tan y = t$ & $1 + x^2 = z \Rightarrow \sec^2 y\, dy = dt$ & $2x\, dx = dz$

$\Rightarrow \displaystyle\int \dfrac{1}{t}dt = -\int \dfrac{1}{z}dz \Rightarrow \log|t| + \log|z| = \log c \Rightarrow t.z = c$

$\Rightarrow \tan y.\left(1 + x^2\right) = c \qquad \dots(1)$

Given that $y = \dfrac{\pi}{4}$ when $x = 1$, $\tan\left(\dfrac{\pi}{4}\right)(1 + 1) = c \Rightarrow 1.2 = c \quad \therefore c = 2$

Putting the value of c in equation (1), we get, $\left(1 + x^2\right)\tan y = 2$.

53. Find the equation of the curve passing through the point $\left(0, \dfrac{\pi}{4}\right)$ whose differential equation is $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$

Sol. Given differential equation is $\sin x \cdot \cos y\, dx + \cos x \cdot \sin y\, dy = 0$

$\Rightarrow \cos x \cdot \sin y \cdot dy = -\sin x \cdot \cos y\, dx$

$\Rightarrow \dfrac{\sin y}{\cos y}\,dy = -\dfrac{\sin x}{\cos x}\,dx \quad \Rightarrow \tan y\,dy = -\tan x\,dx$

Integrating both sides we get $\displaystyle \int \tan y\,dy = -\int \tan x\,dx$

$\log y|\sec y| = -\log|\sec x| + \log C_1$

$\Rightarrow \log|\sec y| + \log|\sec x| = \log C_1$

$\Rightarrow \log|\sec x \cdot \sec y| = \log C_1$

$\Rightarrow |\sec x \cdot \sec y| = C_1$

$\Rightarrow \sec x \cdot \sec y = \pm C_1$

$\Rightarrow \sec x \geq \sec y = C \quad \{\text{Let } \pm C_1 = e$

$\because$ the curve passing through $\left(0, \dfrac{\pi}{4}\right)$

$\therefore \sec 0 \cdot \sec \dfrac{\pi}{4} = C$

$\Rightarrow \sqrt{2} = C$

Hence required solution is $\sec x \cdot \sec y = \sqrt{2}$

54. Find the equation of a curve which passes through the origin and whose differential equation is $\dfrac{dy}{dx} = e^x \sin x$

Sol. Given differential equation is $\dfrac{dy}{dx} = e^x \cdot \sin x$

$\Rightarrow dy = e^x \cdot \sin x \cdot dx$

Integrating both sides w.r.t x we have

$\Rightarrow \displaystyle \int dy = \int e^x \cdot \sin x\,dx$

$\Rightarrow y = I_1 + C \quad \{\text{Let } I_1 = \displaystyle \int e^x \sin x\,dx$

Now $I_1 = \displaystyle \int e^x \sin x\,dx$

$\Rightarrow I_1 = \sin x \displaystyle \int e^x dx \int \left\{ \dfrac{d}{dx}(\sin x)\int e^x dx \right\} dx$

$\Rightarrow I_1 = e^x \cdot \sin x - \displaystyle \int \cos x \cdot e^x dx$

$\Rightarrow I_1 = e^x \sin x - \left[\cos x \displaystyle \int e^x dx - \int \left\{ \dfrac{d}{dx}(\cos x)\int e^x dx \right\} dx \right]$

$\Rightarrow I_1 = e^x \cdot \sin x - \left[e^x \cos x + \displaystyle \int \sin x \cdot e^x dx \right]$

$\Rightarrow e^x \cdot \sin x - e^x \cdot \cos x - I_1$

$\Rightarrow 2I_1 = e^x (\sin x - \cos x)$

$\Rightarrow I_1 = \dfrac{1}{2} \cdot e^x (\sin x - \cos x) \quad \text{........ (ii)}$

From (i) and (ii) we have

$y = \dfrac{1}{2} e^x (\sin x - \cos x) + C \quad \text{...... (iii)}$

Since it passes through origin

$\therefore 0 = \frac{1}{2} e^{0} \left(\sin 0 - \cos 0 \right) + C$

$\Rightarrow 0 = \frac{1}{2} (0 - 1) + C \Rightarrow C = \frac{1}{2}$

Putting $C = \frac{1}{2}$ in equation (iii) we have

$y = \frac{1}{2} \cdot e^{x} \left(\sin x - \cos x \right) + \frac{1}{2}$

$\Rightarrow 2y = e^{x} \left(\sin x - \cos x \right) + 1$

This is the required solution of given differential equation

55. A curve passes through the point $(0, -2)$ and at any point (x, y) of the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point. Find the equation of the curve

Sol. We know that slope of tangent at (x, y) is $m = \frac{dy}{dx}$

According to question $y \cdot \frac{dy}{dx} = x$

$\Rightarrow y \, dy = x \, dx$

Integrating both sides we have $\int y \, dy = \int x \, dx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C_1$

$\Rightarrow y^2 = x^2 + 2C_1$

$\Rightarrow y^2 = x^2 + C$ {let $2C_1 = C$ ..... (i)

Since the curve passes through $(0, -2)$

$\therefore (-2)^2 = 0 + C$

$\Rightarrow C = 4$

Putting $C = 4$ in (i) we have $y^2 = x^2 + 4$

This is the required equation of curve

56. A curve passes through the point $(-2, 1)$ and at any point (x, y) of the curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve

Sol. We know that slope of tangent of (x, y) is $m = \frac{dy}{dx}$

Equation $\frac{dy}{dx} = 2 \left(\frac{y + 3}{x + 4} \right)$

$\Rightarrow \frac{dy}{y + 3} = \frac{2 \, dx}{x + 4}$

Integrating both sides we have $\int \frac{dy}{y + 3} = 2 \int \frac{dx}{x + 4}$

$\Rightarrow \log |y + 3| = 2 \log |x + 4| + \log C_1$

$\Rightarrow \log|y+3| = \log C_1 (x+4)^2$

$\Rightarrow |y+3| = C_1 (x+4)^2$

$\Rightarrow y+3 = \pm C_1 (x+4)^2$

$\Rightarrow y+3 = C(x+4)^2$ { let $C = \pm C_1$

Since the curve through $(-2, 1)$

$\therefore 1+3 = C(-2+4)^2$

$\Rightarrow 4 = 4C$

$\Rightarrow C = 1$

Putting $C = 1$ in equation (i) we have

$y+3 = (x+4)^2$

$\Rightarrow y = x^2 + 8x + 16 - 3$

$\Rightarrow y = x^2 + 8x + 15$

This is the required equation of curve

57. In a bank principal increases at the rate of r% per annum. Find the value of r Rs. 100 double itself in 10 years. (Given $\log_e 2 = 0.9631$)

Sol. Let P be the principal at any time t_1

$\therefore \dfrac{dp}{dt} = \dfrac{pr}{100}$

$\Rightarrow \dfrac{dp}{p} = \dfrac{r}{100} dt$

Integrating both sides we have $\int \dfrac{dp}{p} = \int \dfrac{rdt}{100}$

$\Rightarrow \log(p) = \dfrac{rt}{100} + C$ ...... (i)

At $t = 0$, we have $p = p_0$

$\therefore \log P_0 = C$

$\therefore \log P = \dfrac{rt}{100} + \log p_0$

$\Rightarrow \log p - \log p_0 = \dfrac{rt}{100}$

$\Rightarrow \log\left(\dfrac{p}{p_0}\right) = \dfrac{rt}{100}$ ..... (ii)

Putting $p_0 = 100, p = 2 p_0 = 200$

And $t = 10$, we have $\log 2 = \dfrac{r}{10}$

$\Rightarrow r = 10 \log 2 = 6.931$

58. In a bank principal increases at the rate of 5% per annum An amount of Rs. 1000 is deposited in the bank. How much will it worth after 10 years? (Given $e^{0.5} = 1.648$)

Sol. Let P be the principal at any instant t

$$\therefore \frac{dp}{dt} = \frac{5p}{50}$$

$$\Rightarrow \frac{dp}{p} = \frac{5dt}{100}$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides we have $\int \frac{dp}{p} = \frac{1}{20} \int dt$

$$\Rightarrow \log|p| = \frac{t}{20} + \log C$$

At $t = 0$ we have $p = 1000$

$$\therefore \log 1000 = \log C$$

$$\therefore \log p = \frac{t}{20} + \log 1000$$

Putting $t = 10$, we have $\log p = \frac{10}{20} + \log 1000$

$$\Rightarrow \log \frac{p}{1000} = \frac{1}{2}$$

$$\Rightarrow \frac{p}{1000} = e^{0.5}$$

$$\Rightarrow p = 1648$$

Hence $p = 1648$ after 10 years

59. The volume of a spherical balloon being inflated changes at a constant rate if initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds

Sol. The volume of a spherical balloon of radius r is given by $V = \frac{4}{3}\pi r^3$

Now, $\frac{dV}{dt} = -k$, where $k > 0$ [note that V is decreasing]

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k \Rightarrow \left(4\pi r^2\right)\frac{dr}{dt} = -k$$

$$\Rightarrow \int \left(4\pi r^2\right) dr = \int (-k) dt$$

$$\Rightarrow \frac{4}{3}\pi r^3 = -kt + C \quad \ldots\ldots \text{(i)}, \text{ where C is an arbitary constant}$$

Putting $t = 0$ and $r = 3$ in (i), we get $C = 36\pi$

$$\therefore \frac{4}{3}\pi r^3 = -\alpha t + 36\pi \quad \ldots\ldots \text{(iii)}$$

It is being given that when $t = 3$, then $r = 6$

Putting $t = 3$ and $r = 6$ in (ii), we get $k = -84\pi$

Putting $k = -84\pi$ in (ii), we get $r^3 = (63t + 27) \Rightarrow r = (63t + 27)^{1/3}$

60. In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours in how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present ?

Sol. Let at any time t, the bacteria count be N. then

$$\frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = kN \Rightarrow \int \frac{1}{N} dN = \int k dt \Rightarrow \log N = kt + \log(C)$$

At $t = 0$, we have $N = 100000$

$\therefore \log C = \log 100000$

$\Rightarrow \lg oN = kt + \log 100000$

At $t = 2$, we have $N = 110000$

Putting these values in (i), we get $k = \dfrac{1}{2}\log\dfrac{11}{10}$ .... (i)

$\therefore \log N = \dfrac{1}{2}t\log\left(\dfrac{11}{10}\right) + \log 100000$ ..... (ii)

When $N = 200000$, let $t = T$, then $\log 20000 = \dfrac{T}{2}\log\left(\dfrac{11}{16}\right) + \log 100000 \Rightarrow T = \dfrac{2\log 2}{\log\left(\dfrac{11}{10}\right)}$