

Ex 20.1

Definite Integrals Ex 20.1 Q1

We know that $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Now,

$$\begin{aligned} \int_4^9 \frac{1}{\sqrt{x}} dx &= \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_4^9 \\ &= \left[\frac{\sqrt{x}}{\frac{1}{2}} \right]_4^9 \\ &= 2[\sqrt{9} - \sqrt{4}] \\ &= 2[3 - 2] \\ &= 2 \end{aligned}$$

$$\therefore \int_4^9 \frac{1}{\sqrt{x}} dx = 2$$

Definite Integrals Ex 20.1 Q2

We know that $\int \frac{dx}{x} = \log x + C$

Now,

$$\begin{aligned} \int_{-2}^3 \frac{1}{x+7} dx &= [\log(x+7)]_{-2}^3 \\ &= [\log 10 - \log 5]_{-2}^3 \\ &= \log \frac{10}{5} \quad \left[\because \log a - \log b = \log \frac{a}{b} \right] \\ &= \log 2 \end{aligned}$$

$$\therefore \int_{-2}^3 \frac{1}{x+7} dx = \log 2$$

Definite Integrals Ex 20.1 Q3

$$\text{Let } x = \sin \theta$$

$$\Rightarrow dx = \cos \theta d\theta$$

Now,

$$x = 0 \Rightarrow \theta = 0$$

$$x = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1 - \sin^2 \theta}} \cos \theta d\theta \\ &= \int_0^{\frac{\pi}{6}} \frac{\cos \theta d\theta}{\cos \theta} \\ &= \int_0^{\frac{\pi}{6}} d\theta \\ &= [\theta]_0^{\frac{\pi}{6}} \\ &= \left[\frac{\pi}{6} - 0 \right] \\ &= \frac{\pi}{6} \end{aligned}$$

$$\therefore \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} = \frac{\pi}{6}$$

Definite Integrals Ex 20.1 Q4

We have,

$$I = \int_0^1 \frac{1}{1 + x^2} dx$$

$$\begin{aligned} &= [\tan^{-1} x]_0^1 \\ &= [\tan^{-1} 1 - \tan^{-1} 0] \\ &= \left[\frac{\pi}{4} - 0 \right] \quad \left[\because \tan^{-1} 1 = \frac{\pi}{4} \right] \\ &= \frac{\pi}{4} \end{aligned}$$

$$\therefore \int_0^1 \frac{1}{1 + x^2} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q5

$$\begin{aligned}\text{Let } x^2 + 1 &= t \\ \Rightarrow 2x \, dx &= dt \\ \Rightarrow x \, dx &= \frac{dt}{2}\end{aligned}$$

Now,

$$x = 2 \Rightarrow t = 5$$

$$x = 3 \Rightarrow t = 10$$

$$\begin{aligned}\therefore \int_2^3 \frac{x}{x^2 + 1} dx &= \frac{1}{2} \int_5^{10} \frac{dt}{t} = \frac{1}{2} [\log t]_5^{10} \\ &= \frac{1}{2} [\log 10 - \log 5] \\ &= \frac{1}{2} \left[\log \frac{10}{5} \right] \\ &= \frac{1}{2} [\log 2] \\ &= \log \sqrt{2}\end{aligned}$$

$$\therefore \int_2^3 \frac{x}{x^2 + 1} = \log \sqrt{2}$$

Definite Integrals Ex 20.1 Q6

We have,

$$\int_0^{\infty} \frac{1}{a^2 + b^2 x^2} dx = \frac{1}{b^2} \int_0^{\infty} \frac{1}{\left(\frac{a}{b}\right)^2 + x^2} dx$$

$$\text{We know that } \int \frac{1}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\begin{aligned}\therefore \frac{1}{b^2} \int_0^{\infty} \frac{1}{\left(\frac{a}{b}\right)^2 + x^2} dx &= \frac{1}{b^2} \left[\frac{b}{a} \tan^{-1} \left(\frac{bx}{a} \right) \right]_0^{\infty} \\ &= \frac{1}{ab} \left[\tan^{-1} \left(\frac{bx}{a} \right) \right]_0^{\infty} \\ &= \frac{1}{ab} [\tan^{-1} \infty - \tan^{-1} 0] \\ &= \frac{1}{ab} \left[\frac{\pi}{2} - 0 \right] \\ &= \frac{\pi}{2ab} \\ \Rightarrow \int_0^{\infty} \frac{1}{a^2 + b^2 x^2} dx &= \frac{\pi}{2ab}\end{aligned}$$

Definite Integrals Ex 20.1 Q7



We have,

$$\int_{-1}^1 \frac{1}{1+x^2} dx$$

We know that $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

Now,

$$\begin{aligned} & \int_{-1}^1 \frac{1}{1+x^2} dx \\ &= \left[\tan^{-1} x \right]_{-1}^1 \\ &= \left[\tan^{-1} 1 - \tan^{-1}(-1) \right] \\ &= \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] \quad \left[\because \tan^{-1}(-1) = -\frac{\pi}{4} \right] \\ &= \left[\frac{\pi}{4} + \frac{\pi}{4} \right] \\ &= \frac{2\pi}{4} \end{aligned}$$

$$\therefore \int_{-1}^1 \frac{1}{1+x^2} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.1 Q8

We have,

$$\int_0^{\infty} e^{-x} dx$$

We know that $\int e^{-x} dx = -e^{-x}$

Now,

$$\begin{aligned} & \int_0^{\infty} e^{-x} dx \\ &= \left[-e^{-x} \right]_0^{\infty} \\ &= \left[-e^{-\infty} + e^{-0} \right] \quad \left[\because e^{\infty} = 0, e^0 = 1 \right] \\ &= \left[-0 + 1 \right] \end{aligned}$$

$$\therefore \int_0^{\infty} e^{-x} dx = 1$$

Definite Integrals Ex 20.1 Q9

We have,

$$\int_0^1 \frac{x}{x+1} dx \quad \text{[Add and subtract 1 in numerator]}$$

$$\begin{aligned} &= \int_0^1 \frac{(x+1) - 1}{x+1} dx \\ &= \int_0^1 1 dx - \int_0^1 \frac{1}{x+1} dx \\ &= \left[x \right]_0^1 - \left[\log(x+1) \right]_0^1 \\ &= 1 - [\log 2 - \log 1] \\ &= 1 - \log \frac{2}{1} \\ &= 1 - \log 2 \\ &= \log e - \log 2 \quad \left[\because \log e = 1 \right] \\ &= \log \frac{e}{2} \end{aligned}$$

$$\therefore \int_0^1 \frac{x}{x+1} dx = \log \frac{e}{2}$$

Definite Integrals Ex 20.1 Q10

We have,

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + [\sin x]_0^{\frac{\pi}{2}}$$

$$= \left[\cos \frac{\pi}{2} + \cos 0 \right] + \left[\sin \frac{\pi}{2} - \sin 0 \right]$$

$$= [-0 + 1] + 1$$

$$= 1 + 1$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2$$

Definite Integrals Ex 20.1 Q11

We have,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

We know that $\int \cot x dx = \log(\sin x)$

Now,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

$$= [\log(\sin x)]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\log\left(\sin \frac{\pi}{2}\right) - \log\left(\sin \frac{\pi}{4}\right) \right]$$

$$= \left[\log 1 - \log \frac{1}{\sqrt{2}} \right]$$

$$= [0 - (\log 1 - \log \sqrt{2})]$$

$$= \log \sqrt{2}$$

$$[\because \log a^n = n \log a]$$

$$= \frac{1}{2} \log 2$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2} \log 2$$

Definite Integrals Ex 20.1 Q12

We have,

$$\int_0^{\frac{\pi}{4}} \sec x dx$$

We know that $\int \sec x dx = \log(\sec x + \tan x)$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx$$

$$= [\log(\sec x + \tan x)]_0^{\frac{\pi}{4}}$$

$$= [\log(\sqrt{2} + 1) - \log(1 + 0)]$$

$$= \log(\sqrt{2} + 1)$$

$$[\because \log 1 = 0]$$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$$

Definite Integrals Ex 20.1 Q13

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right) \\ &= \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right| \\ &= \log |\sqrt{2} - 1| - \log |2 - \sqrt{3}| \\ &= \log \left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}} \right) \end{aligned}$$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1-x}{1+x} \, dx$$

$$\text{Let } x = \cos 2\theta \Rightarrow dx = -2 \sin 2\theta \, d\theta$$

Now,

$$\begin{aligned} x = 0 &\Rightarrow \theta = \frac{\pi}{4} \\ x = 1 &\Rightarrow \theta = 0 \end{aligned}$$

Now,

$$\begin{aligned} &\int_0^1 \frac{1-x}{1+x} \, dx \\ &= \int_{\frac{\pi}{4}}^0 \frac{1 - \cos 2\theta}{1 + \cos 2\theta} \times (-2 \sin 2\theta) \, d\theta \\ &= \int_0^{\frac{\pi}{4}} \frac{2 \sin^2 \theta}{2 \cos^2 \theta} \times 2 \sin 2\theta \, d\theta \quad \left[\because -\int_a^b f(x) \, dx = \int_b^a f(x) \, dx \right] \\ &= \int_0^{\frac{\pi}{4}} \frac{4 \sin^3 \theta}{\cos \theta} \, d\theta \end{aligned}$$

$$\begin{aligned} \text{Let } \cos \theta &= t \\ \Rightarrow -\sin \theta \, d\theta &= dt \end{aligned}$$

Now,

$$\begin{aligned} \theta = 0 &\Rightarrow t = 1 \\ \theta = \frac{\pi}{4} &\Rightarrow t = \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} &\therefore \int_0^{\frac{\pi}{4}} \frac{4 \sin^3 \theta}{\cos \theta} \, d\theta \\ &= -4 \int_1^{\frac{1}{\sqrt{2}}} \frac{(1-t^2)}{t} \, dt \\ &= -4 \left[\log t - \frac{t^2}{2} \right]_1^{\frac{1}{\sqrt{2}}} \\ &= -4 \left[\log \left(\frac{1}{\sqrt{2}} \right) - \frac{1}{4} - 0 + \frac{1}{2} \right] \\ &= -4 \left[-\log \sqrt{2} + \frac{1}{4} \right] \end{aligned}$$

$$\therefore \int_0^1 \frac{1-x}{1+x} \, dx = 2 \log 2 - 1$$

Definite Integrals Ex 20.1 Q15



$$I = \int_0^{\pi} \frac{1}{1 + \sin x} dx$$

Multiplying Numerator and Denominator by $(1 - \sin x)$

$$\begin{aligned} I &= \int_0^{\pi} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \\ &= \int_0^{\pi} \frac{(1 - \sin x)}{(1^2 - \sin^2 x)} dx \\ &= \int_0^{\pi} \frac{1 - \sin x}{(\cos^2 x)} dx \\ &= \int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx \\ &= \int_0^{\pi} \sec^2 x dx - \int_0^{\pi} \tan x \cdot \sec x dx \\ &= [\tan x]_0^{\pi} - [\sec x]_0^{\pi} \\ &= [\tan \pi - \tan 0] - [\sec \pi - \sec 0] \\ &= [0 - 0] - [-1 - 1] \\ &= 2 \\ I &= 2 \end{aligned}$$

$$\therefore \int_0^{\pi} \frac{1}{1 + \sin x} dx = 2$$

Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx$$

We know,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\begin{aligned} \therefore \frac{1}{1 + \sin x} &= \frac{1}{1 + \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} = \frac{1 + \tan^2 \frac{x}{2}}{(1 + \tan \frac{x}{2})^2} = \frac{\sec^2 \frac{x}{2}}{(1 + \tan \frac{x}{2})^2} \\ \Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{(1 + \tan \frac{x}{2})^2} dx \end{aligned}$$

$$\text{If } f(x) \text{ is an even function } \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{(1 + \tan \frac{x}{2})^2} dx = 2 \int_0^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{(1 + \tan \frac{x}{2})^2} dx$$

$$\text{let } 1 + \tan \frac{x}{2} = t$$

$$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = -\frac{\pi}{4} \Rightarrow t = 1 - \tan \frac{\pi}{8}$$

$$x = \frac{\pi}{4} \Rightarrow t = 1 + \tan \frac{\pi}{8}$$

$$\therefore 2 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2}\right)^2} dx = 2 \int_{1 - \tan \frac{\pi}{8}}^{1 + \tan \frac{\pi}{8}} \frac{8 dt}{t^2}$$

$$= 2 \left[\frac{-1}{t} \right]_{1 - \tan \frac{\pi}{8}}^{1 + \tan \frac{\pi}{8}}$$

$$= 2 \left[\frac{1}{1 - \tan \frac{\pi}{8}} - \frac{1}{1 + \tan \frac{\pi}{8}} \right]$$

$$= 2 \left[\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} \right]$$

$$= 2 \tan \frac{\pi}{4} \quad \left[\because \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx = 2$$

Definite Integrals Ex 20.1 Q17

$$\text{Let } I = \int_0^{\pi} \cos^2 x \, dx$$

$$\int \cos^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \left[F\left(\frac{\pi}{2}\right) - F(0) \right] \\ &= \frac{1}{2} \left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right] \\ &= \frac{1}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right] \\ &= \frac{\pi}{4} \end{aligned}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos 3x + 3 \cos x}{4} dx \quad \left[\because \cos 3x = 4 \cos^3 x - 3 \cos x \right]$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (\cos 3x + 3 \cos x) dx$$

$$= \frac{1}{4} \left[\frac{\sin 3x}{3} + 3 \sin x \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} \left[\left(\frac{\sin 3 \frac{\pi}{2}}{3} + 3 \sin \frac{\pi}{2} \right) - \left(\frac{\sin 0}{3} + 3 \sin 0 \right) \right]$$

$$= \frac{1}{4} \left[\left(\frac{-1}{3} + 3 \right) - (0 + 0) \right] = \frac{2}{3}$$

$$= \frac{1}{4} \left[\frac{8}{3} \right]$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^3 x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \quad \left[\because 2 \cos C \cos D = \cos(C+D) + \cos(C-D) \right]$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} 2 \cos x \cos 2x dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} (\cos 3x + \cos x) dx$$

$$= \frac{1}{2} \left[\frac{\sin 3x}{3} + \sin x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2} \left[\left(\frac{\sin 3 \frac{\pi}{6}}{3} + \sin \frac{\pi}{6} \right) - (\sin 0 + \sin 0) \right]$$

$$= \frac{1}{2} \left[\frac{\sin \frac{\pi}{2}}{3} + \sin \frac{\pi}{6} \right]$$

$$= \frac{1}{2} \left(\frac{1}{3} + \frac{1}{2} \right)$$

$$= \frac{1}{2} \left(\frac{5}{6} \right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} 2 \sin x \sin 2x dx \quad [\because 2 \sin C \times \sin D = \cos(D - C) - \cos(D + C)]$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin 3x}{3} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left[\left(\sin \frac{\pi}{2} - \sin 0 \right) - \left(\frac{\sin 3 \frac{\pi}{2}}{3} - \frac{\sin 0}{3} \right) \right]$$

$$= \frac{1}{2} \left[(1 - 0) - \left(\frac{-1}{3} - 0 \right) \right] \quad [\because \sin 3 \frac{\pi}{2} = -1]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\sin^2 x + \cot^2 x}{\sin x \cos x} \right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{1}{\sin x \cos x} \right)^2 dx$$

Multiplying numerator and denominator by 2

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{2 \sin x \cos x} \right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{\sin 2x} \right)^2 dx \quad [\because 2 \sin x \cos x = \sin 2x]$$

$$= 4 \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x dx$$

$$= 4 \left[-\frac{\cot 2x}{2} \right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= 2 \left[-\cot \frac{\pi}{2} + \cot 2 \frac{\pi}{3} \right]$$

$$= 2 \left[\frac{-1}{\sqrt{3}} - 0 \right]$$

$$= \frac{-2}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 + \cos 2x)^2 dx \quad \left[\because 2 \cos^2 x = 1 + \cos 2x \right]$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 + \cos^2 2x + 2 \cos 2x) dx$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} \left(1 + \frac{1 + \cos 4x}{2} + 2 \cos 2x \right) dx$$

$$= \frac{1}{4} \left[x + \frac{1}{2}x + \frac{\sin 4x}{8} + \sin 2x \right]_0^{\frac{\pi}{2}} \quad \left[\because \int \cos 4x dx = \frac{\sin 4x}{4} \right]$$

$$= \frac{1}{4} \left[\frac{\pi}{2} + \frac{\pi}{4} + 0 + 0 - 0 - 0 - 0 - 0 \right]$$

$$= \frac{1}{4} \times \frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

Definite Integrals Ex 20.1 Q23

We have,

$$\int_0^{\frac{\pi}{2}} \{a^2 \cos^2 x + b^2 (1 - \cos^2 x)\} dx$$

$$= \int_0^{\frac{\pi}{2}} \{(a^2 - b^2) \cos^2 x + b^2\} dx$$

$$= \frac{a^2 - b^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2x) dx + b^2 \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{a^2 - b^2}{2} \left[x + \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}} + b^2 \left[x \right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2 - b^2}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right] + b^2 \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{a^2 - b^2}{2} \left[\frac{\pi}{2} \right] + b^2 \left[\frac{\pi}{2} \right]$$

$$= a^2 \frac{\pi}{4} + b^2 \left[\frac{\pi}{2} - \frac{\pi}{4} \right]$$

$$= \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$$

$$= \frac{\pi}{4} (a^2 + b^2)$$

$$\therefore \int_0^{\frac{\pi}{2}} (a^2 \cos^2 x + b^2 \sin^2 x) dx = \frac{\pi}{4} (a^2 + b^2)$$

Definite Integrals Ex 20.1 Q24

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + \sin x} \, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} \, dx \quad \text{We use } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sqrt{1 + \sin x} \, dx &= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1 + \tan \frac{x}{2}\right)^2}{1 + \tan^2 \frac{x}{2}}} \, dx \\ &= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1 + \tan \frac{x}{2}\right)^2}{\sec^2 \frac{x}{2}}} \, dx \\ &= \int_0^{\frac{\pi}{2}} \left(\frac{1 + \tan \frac{x}{2}}{\sec \frac{x}{2}} \right) \, dx \\ &= \int_0^{\frac{\pi}{2}} \left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) \, dx \\ &= \left[2 \sin \frac{x}{2} - 2 \cos \frac{x}{2} \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 0 + 1 \right] \\ \therefore \int_0^{\frac{\pi}{2}} \sqrt{1 + \sin x} \, dx &= 2 \end{aligned}$$

Definite Integrals Ex 20.1 Q25

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + \cos x} \, dx$$

$$\text{We use } 1 + \cos x = 2 \cos^2 \frac{x}{2}$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} \sqrt{2 \cos^2 \frac{x}{2}} \, dx \\ &= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos \frac{x}{2} \, dx \\ &= \sqrt{2} \left[2 \sin \frac{x}{2} \right]_0^{\frac{\pi}{2}} \\ &= 2\sqrt{2} \left[\frac{1}{\sqrt{2}} \right] \\ &= 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1 + \cos x} \, dx = 2$$

Definite Integrals Ex 20.1 Q26

We have,

$$\begin{aligned} \int x \sin x \, dx &= x \int \sin x \, dx - \int \left(\int \sin x \, dx \right) \left(\frac{dx}{dx} \right) \, dx \\ &= -x \cos x + \int \cos x \, dx \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x \sin x \, dx = \left[-x \cos x + \sin x \right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2} \times 0 \right) + 1 + 0 - 0 = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x \sin x \, dx = 1$$

Definite Integrals Ex 20.1 Q27

We have,

$$\int x \cos x \, dx = x \int \cos x \, dx - \int \left(\int \cos x \, dx \right) \frac{dx}{dx} dx = x \sin x - \int \sin x \, dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x \, dx = \left[x \sin x + \cos x \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} + 0 - 0 - 1 \right] = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x \, dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\begin{aligned} \int x^2 \cos x \, dx &= x^2 \int \cos x \, dx - \int (2x) \left(\int \cos x \, dx \right) dx = x^2 \sin x - \int \sin x \cdot 2x \, dx \\ &= x^2 \sin x - 2 \left[x \int \sin x - \int \left(\int \sin x \, dx \right) dx \right] \\ &= x^2 \sin x - 2 \left[-x \cos x + \int \cos x \, dx \right] \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx &= \left[x^2 \sin x + 2x \cos x - 2 \sin x \right]_0^{\frac{\pi}{2}} \\ &= \left[\frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0 \right] \\ &= \frac{\pi^2}{4} - 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\begin{aligned} \int x^2 \sin x \, dx &= x^2 \int \sin x \, dx - \int 2x \left(\int \sin x \, dx \right) dx = x^2 \cos x + \int 2x \cos x \, dx \\ &= x^2 \cos x + 2 \left[x \int \cos x \, dx - \int \left(\int \cos x \, dx \right) dx \right] \\ &= -x^2 \cos x + 2 \left[x \sin x - \int \sin x \, dx \right] \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{4}} x^2 \sin x \, dx &= \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{4}} \\ &= \frac{-\pi^2}{16} \cdot \frac{1}{\sqrt{2}} + \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} + 0 - 0 - 2 \\ &= \frac{1}{\sqrt{2}} \left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2 \right] - 2 \\ &= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x \, dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30



We have,

$$\begin{aligned}
 \int x^2 \cos 2x \, dx &= x^2 \int \cos 2x \, dx - \int 2x \left(\int \cos 2x \, dx \right) dx \\
 &= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2} dx \\
 &= \frac{x^2 \sin 2x}{2} - \left[x \int \sin 2x \, dx - \int \left(\int \sin 2x \, dx \right) dx \right] \\
 &= \frac{x^2 \sin 2x}{2} + \left[\frac{x \cos 2x}{2} - \int \frac{x \cos 2x}{2} dx \right]
 \end{aligned}$$

$$\begin{aligned}
 \therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx &= \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}} \\
 &= \left[\frac{\pi^2}{8} \times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0 \right] \\
 &= \frac{-\pi}{4}
 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx = \frac{-\pi}{4}$$

Definite Integrals Ex 20.1 Q31

We have,

$$\int x^2 \cos^2 x \, dx = \int x^2 \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{1}{2} \int (x^2 + x^2 \cos 2x) dx = \frac{1}{2} \left[\int x^2 dx + \int x^2 \cos 2x dx \right] \quad \dots (A)$$

Now,

$$\int_0^{\frac{\pi}{2}} x^2 dx = \left[\frac{x^3}{3} \right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24} \quad \dots (B)$$

$$\begin{aligned}
 \int x^2 \cos 2x \, dx &= x^2 \int \cos 2x \, dx - \int 2x \left(\int \cos 2x \, dx \right) dx = \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{2} \cdot 2x \, dx \\
 &= \frac{x^2 \sin 2x}{2} - \left[x \int \sin 2x \, dx - \int \left(\int \sin 2x \, dx \right) dx \right] \\
 &= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \int \frac{\cos 2x}{2} dx
 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}} = \frac{-\pi}{4} \quad \dots (C)$$

Now, Put (B) & (C) in (A), we get,

$$\int_0^{\frac{\pi}{2}} x^2 \cos^2 x \, dx = \int_0^{\frac{\pi}{2}} x^2 dx + \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx = \frac{1}{2} \left[\frac{\pi^3}{24} - \frac{\pi}{4} \right] = \frac{\pi^3}{48} - \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q32

We have,

$$\int \log x \, dx = \int 1 \cdot \log x \, dx = \log x \int 1 \, dx - \int \left(\int dx \right) \cdot \frac{1}{x} dx = x \log x - \int x \cdot \frac{1}{x} dx = x \log x - \int 1 \, dx$$

$$\therefore \int_1^2 \log x \, dx = \left[x \log x - x \right]_1^2 = 2 \log 2 - 2 - 0 + 1 = 2 \log 2 - 1$$

Definite Integrals Ex 20.1 Q33

We have,

$$\begin{aligned}
 \int \frac{\log x}{(x+1)^2} dx &= \int \frac{1}{(x+1)^2} \log x \, dx = \log x \int \frac{1}{(x+1)^2} dx - \int \left(\int \frac{1}{(x+1)^2} dx \right) \frac{1}{x} dx \\
 &= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)} dx \\
 &= \frac{-\log x}{(x+1)} + \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx
 \end{aligned}$$

$$\therefore \int_1^3 \frac{\log x}{(x+1)^2} dx = \left[\frac{-\log x}{x+1} + \log x - \log(x+1) \right]_1^3 = \frac{3}{4} \log 3 - \log 2$$

Definite Integrals Ex 20.1 Q34

$$\text{Let } I = \int_1^e \frac{e^x}{x} (1 + x \log x) dx$$

$$I = \int_1^e \frac{e^x}{x} dx + \int_1^e e^x \log x dx$$

$$I = \left[e^x \log x \right]_1^e - \int_1^e e^x \cdot \log x + \int_1^e e^x \log x$$

$$I = \left[e^x \log x \right]_1^e$$

$$I = \left[e^x \log e - e^1 \cdot \log 1 \right]$$

$$I = \left[e^e \cdot 1 - 0 \right]$$

$$I = e^e$$

$$\therefore \int_1^e \frac{e^x}{x} (1 + x \log x) dx = e^e$$

Definite Integrals Ex 20.1 Q35

We have,

$$\int_1^e \frac{\log x}{x} dx$$

$$\text{Let } \log x = t$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Now,

$$x = 1 \Rightarrow t = 0$$

$$x = e \Rightarrow t = 1$$

$$\therefore \int_1^e \frac{\log x}{x} dx = \int_0^1 t dt$$

$$= \left[\frac{t^2}{2} \right]_0^1$$

$$= \frac{1}{2}$$

$$\therefore \int_1^e \frac{\log x}{x} dx = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q36

We have,

$$\int_e^{e^2} \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

$$I = \int \frac{1}{\log x} \cdot 1 dx = \frac{1}{\log x} \int dx - \int \left(\int dx \right) \cdot \frac{d}{dx} \left(\frac{1}{\log x} \right) dx = \frac{x}{\log x} + \int \frac{1}{(\log x)^2} \cdot x \cdot \frac{1}{x} dx$$

$$= \frac{x}{\log x} + \int \frac{dx}{(\log x)^2}$$

$$\int_e^{e^2} \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx = \left[\frac{x}{\log x} \right]_e^{e^2} + \int_e^{e^2} \frac{dx}{(\log x)^2} - \int_e^{e^2} \frac{dx}{(\log x)^2}$$

$$= \left[\frac{x}{\log x} \right]_e^{e^2}$$

$$= \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q37

We have,

$$\begin{aligned} & \int_1^2 \frac{x+3}{x(x+2)} dx \\ &= \int_1^2 \frac{x}{x(x+2)} dx + \int_1^2 \frac{3}{x(x+2)} dx \\ &= \int_1^2 \frac{dx}{x+2} + \int_1^2 \frac{3}{x(x+2)} dx \\ &= \left[\log(x+2) \right]_1^2 + \left[\frac{3}{2} \frac{1}{x} - \frac{1}{x+2} \right]_1^2 \quad [\text{using partial fraction}] \\ &= \left[\log(x+2) \right]_1^2 + \left[\frac{3}{2} \log x - \frac{3}{2} \log(x+2) \right]_1^2 \\ &= \left[\frac{3}{2} \log x - \frac{1}{2} \log(x+2) \right]_1^2 \\ &= \frac{1}{2} [3 \log 2 - \log 4 + \log 3] \\ &= \frac{1}{2} [3 \log 2 - 2 \log 2 + \log 3] \quad [\because \log 4 = 2 \log 2] \\ &= \frac{1}{2} [\log 2 + \log 3] \\ &= \frac{1}{2} [\log 6] \\ &= \frac{1}{2} \log 6 \end{aligned}$$

$$\therefore \int_1^2 \frac{x+3}{x(x+2)} dx = \frac{1}{2} \log 6$$

Definite Integrals Ex 20.1 Q38

$$\begin{aligned} \text{Let } I &= \int_0^1 \frac{2x+3}{5x^2+1} dx \\ \int \frac{2x+3}{5x^2+1} dx &= \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{1} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}} \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}x) \\ &= F(x) \end{aligned}$$

Definite Integrals Ex 20.1 Q39



$$\begin{aligned}
 \int_0^2 \frac{dx}{x+4-x^2} &= \int_0^2 \frac{dx}{-(x^2-x-4)} \\
 &= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)} \\
 &= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]} \\
 &= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}
 \end{aligned}$$

$$\text{Let } x - \frac{1}{2} = t \Rightarrow dx = dt$$

$$\text{When } x = 0, t = -\frac{1}{2} \text{ and when } x = 2, t = \frac{3}{2}$$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t} \right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2} - \frac{1}{2}}{\frac{\sqrt{17}}{2} + \frac{1}{2}} \right]$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1} \right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{(5+\sqrt{17})(5+\sqrt{17})}{25-17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4} \right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\int_0^1 \frac{1}{2x^2 + x + 1} dx$$

$$= \frac{1}{2} \int_0^1 \frac{1 dx}{\left(x^2 + \frac{1}{2}x + \frac{1}{2}\right)}$$

$$= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \frac{1}{2} - \frac{1}{16}} \quad \left[\text{Adding } \frac{1}{16} \text{ \& subtracting } \frac{1}{16} \text{ in numerator} \right]$$

$$= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \frac{7}{16}}$$

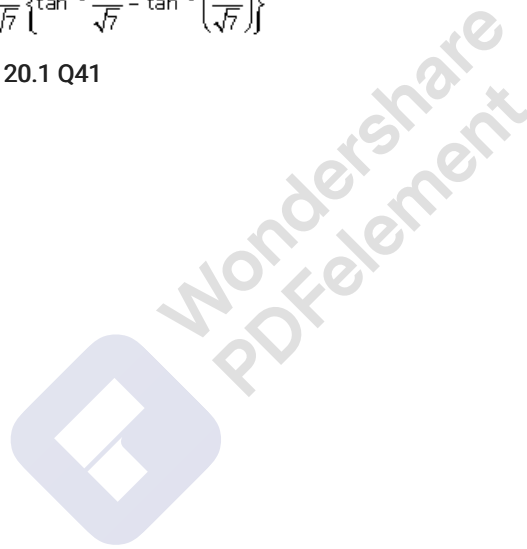
$$= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1}{2} \cdot \frac{4}{\sqrt{7}} \left[\tan^{-1} \left(\frac{x + \frac{1}{4}}{\frac{\sqrt{7}}{4}} \right) \right]_0^1$$

$$= \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\}$$

$$\therefore \int_0^1 \frac{1}{2x^2 + x + 1} dx = \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\}$$

Definite Integrals Ex 20.1 Q41





$$\text{Let } I = \int_0^1 \sqrt{x(1-x)} \, dx$$

$$\text{let } x = \sin^2 \theta$$

$$\Rightarrow dx = 2 \sin \theta \cdot \cos \theta \, d\theta$$

Now,

$$x = 0 \Rightarrow \theta = 0$$

$$x = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 \theta (1 - \sin^2 \theta)} \cdot 2 \sin \theta \cdot \cos \theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 2 \sin^2 \theta \cdot \cos^2 \theta \, d\theta$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} 4 \sin^2 \theta \cdot \cos^2 \theta \, d\theta$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin^2 2\theta) \, d\theta$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 4\theta}{2} \right) d\theta$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 - \cos 4\theta) \, d\theta$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4} \int_0^{\frac{\pi}{2}} \cos 4\theta \, d\theta$$

$$= \frac{1}{4} [\theta]_0^{\frac{\pi}{2}} - \frac{1}{4} \left[\frac{\sin 4\theta}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} \left[\frac{\pi}{2} - 0 \right] - \frac{1}{16} [\sin \pi - \sin 0]$$

$$= \frac{\pi}{8} - \frac{1}{16} [0 - 0]$$

$$= \frac{\pi}{8}$$

$$I = \frac{\pi}{8}$$

$$\therefore \int_0^1 \sqrt{x(1-x)} \, dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q42

We have,

$$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$$

$$\int_0^2 \frac{dx}{\sqrt{3+1-(x^2-2x+1)}}$$

[Add and subtract 1 in denominator]

$$= \int_0^2 \frac{dx}{\sqrt{(2)^2 - (x-1)^2}}$$

$$\left[\because \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} \right]$$

$$= \left[\sin^{-1} \left(\frac{x-1}{2} \right) \right]_0^2$$

$$= \sin^{-1} \frac{1}{2} - \sin^{-1} \left(\frac{-1}{2} \right)$$

$$= \sin^{-1} \left(\sin \frac{\pi}{6} \right) - \sin^{-1} \left[\sin \left(\frac{-\pi}{6} \right) \right]$$

$$= \frac{\pi}{6} + \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\therefore \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$$

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Definite Integrals Ex 20.1 Q43

We have,

$$\int_0^4 \frac{dx}{\sqrt{4x - x^2}}$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - 4 + 4x - x^2}} \quad [\text{Add and subtract 4 in denominator}]$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - (x^2 - 4x + 4)}}$$

$$= \int_0^4 \frac{dx}{\sqrt{(2)^2 - (x - 2)^2}}$$

$$= \left[\sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^4 \quad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} \right]$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2} \right)$$

$$= \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4x - x^2}} = \pi$$

Definite Integrals Ex 20.1 Q44

$$\int_{-1}^1 \frac{dx}{x^2 + 2x + 5} = \int_{-1}^1 \frac{dx}{(x^2 + 2x + 1) + 4} = \int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2}$$

$$\text{Let } x + 1 = t \Rightarrow dx = dt$$

$$\text{When } x = -1, t = 0 \text{ and when } x = 1, t = 2$$

$$\begin{aligned} \therefore \int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2} &= \int_0^2 \frac{dt}{t^2 + 2^2} \\ &= \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right]_0^2 \\ &= \frac{1}{2} \tan^{-1} 1 - \frac{1}{2} \tan^{-1} 0 \\ &= \frac{1}{2} \left(\frac{\pi}{4} \right) = \frac{\pi}{8} \end{aligned}$$

Definite Integrals Ex 20.1 Q45

We have,

$$\int_1^4 \frac{x^2 + x}{\sqrt{2x+1}} dx$$

$$\begin{aligned} \text{Let } 2x+1 &= t^2 \\ \Rightarrow 2dx &= 2t dt \end{aligned}$$

Now,

$$x=1 \Rightarrow t=\sqrt{3}$$

$$x=4 \Rightarrow t=3$$

$$\begin{aligned} \therefore \int_1^4 \frac{x^2 + x}{\sqrt{2x+1}} dx &= \int_{\sqrt{3}}^3 \frac{\left(\frac{t^2-1}{2}\right)^2 + \left(\frac{t^2-1}{2}\right)}{t} t dt \\ &= \frac{1}{4} \int_{\sqrt{3}}^3 (t^4 - 2t^2 + 1 + 2t^2 - 2) dt \\ &= \frac{1}{4} \int_{\sqrt{3}}^3 t^4 - 1 dt \\ &= \frac{1}{4} \left[\frac{t^5}{5} - t \right]_{\sqrt{3}}^3 \\ &= \frac{1}{4} \left[\frac{243 - 9\sqrt{3}}{5} - 3 + \sqrt{3} \right] \\ &= \frac{1}{4} \left[\frac{228}{5} - \sqrt{3}(4) \right] \\ &= \frac{57 - \sqrt{3}}{5} \end{aligned}$$

$$\therefore \int_1^4 \frac{x^2 + x}{\sqrt{2x+1}} dx = \frac{57 - \sqrt{3}}{5}$$

Definite Integrals Ex 20.1 Q46

We have,

$$\int_0^1 x(1-x)^5 dx$$

Expanding $(1-x)^5$ by Binomial theorem

$$\begin{aligned} \therefore (1-x)^5 &= 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5 \\ &= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5 \\ &= \int_0^1 x(1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5) dx \\ &= \left[\frac{x^2}{2} - \frac{5x^3}{3} + \frac{10x^4}{4} - \frac{10x^5}{5} + \frac{5x^6}{6} - \frac{x^7}{7} \right]_0^1 \\ &= \frac{1}{2} - \frac{5}{3} + \frac{10}{4} - \frac{10}{5} + \frac{5}{6} - \frac{1}{7} \\ &= \frac{1}{42} \end{aligned}$$

$$\therefore \int_0^1 x(1-x)^5 dx = \frac{1}{42}$$

Definite Integrals Ex 20.1 Q47



We have,

$$\int_1^2 \left(\frac{x-1}{x^2} \right) e^x dx = \int_1^2 \frac{x e^x}{x^2} - \int_1^2 \frac{e^x}{x^2} dx = \int_1^2 \frac{e^x}{x} - \int_1^2 \frac{e^x}{x^2} dx$$

Expanding 1st integral by by parts we get

$$\begin{aligned} &= \frac{1}{x} \int_1^2 e^x dx - \int_1^2 \left(\int e^x \cdot \frac{d\left(\frac{1}{x}\right)}{dx} dx \right) - \int_1^2 \frac{e^x}{x^2} dx \\ &= \left[\frac{e^x}{x} \right]_1^2 + \int_1^2 \frac{e^x}{x^2} dx - \int_1^2 \frac{e^x}{x^2} dx \\ &= \left[\frac{e^x}{x} \right]_1^2 \\ &= \frac{e^2}{2} - e \end{aligned}$$

$$\therefore \int_1^2 \left(\frac{x-1}{x^2} \right) e^x dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1 \left(x e^{2x} + \sin \frac{\pi x}{2} \right) dx = \int_0^1 x e^{2x} dx + \int_0^1 \sin \frac{\pi x}{2} dx$$

Applying by parts in first integral

$$\begin{aligned} &= x \int_0^1 e^{2x} dx - \int_0^1 \left(\int e^{2x} dx \right) \frac{dx}{dx} dx + \left[\frac{-\cos \frac{\pi x}{2}}{\frac{\pi}{2}} \right]_0^1 \\ &= \frac{x e^{2x}}{2} - \frac{1}{2} \int_0^1 e^{2x} dx + \frac{2}{\pi} [1-0] \\ &= \frac{x e^{2x}}{2} - \frac{1}{2} \int_0^1 e^{2x} dx + \frac{2}{\pi} [1-0] \\ &= \left[\frac{x e^{2x}}{2} - \frac{1}{4} e^{2x} \right]_0^1 + \frac{2}{\pi} [1-0] \\ &= \frac{e^2}{2} - \frac{1}{4} e^2 + \frac{1}{4} + \frac{2}{\pi} [1-0] \\ &= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4} \\ &= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi} \end{aligned}$$

$$\therefore \int_0^1 \left(x e^{2x} + \sin \frac{\pi x}{2} \right) dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\begin{aligned} & \int_0^1 \left(xe^x + \cos \frac{\pi x}{4} \right) dx \\ &= \int_0^1 x e^x dx + \int_0^1 \cos \frac{\pi x}{4} dx \end{aligned}$$

Applying by parts in 1st integral we get,

$$\begin{aligned} &= x \int_0^1 e^x dx - \int_0^1 (e^x dx) \frac{dx}{dx} dx + \int_0^1 \cos \frac{\pi x}{4} dx \\ &= \left[x e^x \right]_0^1 - \int_0^1 e^x dx + \left[\frac{\sin \frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1 \\ &= \left[x e^x - e^x \right]_0^1 + \frac{4}{\pi} \left[\frac{1}{\sqrt{2}} - 0 \right] \\ &= \left[e^x (x-1) \right]_0^1 + \frac{4}{\pi} \left[\frac{1}{\sqrt{2}} \right] \\ &= 0 + 1 + \frac{4}{\pi \sqrt{2}} \\ &= 1 + \frac{2\sqrt{2}}{\pi} \end{aligned}$$

$$\therefore \int_0^1 \left(xe^x + \cos \frac{\pi x}{4} \right) dx = 1 + \frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\begin{aligned} \int_{\frac{\pi}{2}}^{\pi} \frac{e^x (1 - \sin x)}{1 - \cos x} dx &= \int_{\frac{\pi}{2}}^{\pi} \frac{e^x \frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} dx}{\left[1 - \cos x = 2 \sin^2 \frac{x}{2} \right]} \\ &= \int_{\frac{\pi}{2}}^{\pi} e^x \left(-\frac{1}{2} \csc^2 \frac{x}{2} + \cot \frac{x}{2} \right) dx \\ &= -e^x \cot \frac{x}{2} \Big|_{\frac{\pi}{2}}^{\pi} \quad \left[\int e^x (F(x) + F'(x)) dx = e^x F(x) \right] \\ &= e^{\frac{\pi}{2}} \end{aligned}$$

Definite Integrals Ex 20.1 Q51

We have,

$$\begin{aligned} \int_0^{2\pi} e^{x/2} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx &= \int_0^{2\pi} e^{x/2} \left(\sin \frac{x}{2} \cos \frac{\pi}{4} + \cos \frac{x}{2} \sin \frac{\pi}{4} \right) dx \\ &= \int_0^{2\pi} e^{x/2} \sin \frac{x}{2} \cdot \frac{1}{\sqrt{2}} dx + \int_0^{2\pi} e^{x/2} \cos \frac{x}{2} \cdot \frac{1}{\sqrt{2}} dx \end{aligned}$$

Expanding 1st part by parts, we get,

$$\begin{aligned} \int_0^{2\pi} e^{x/2} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \int_0^{2\pi} e^{x/2} dx - \int_0^{2\pi} \left(\int_0^{2\pi} e^{x/2} dx \right) \cdot \frac{d \left(\sin \frac{x}{2} \right)}{dx} dx \right\} + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cos \frac{x}{2} dx \\ &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \cdot 2e^{x/2} \right\}_0^{2\pi} - \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cdot \frac{1}{2} \cos \frac{x}{2} dx + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cos \frac{x}{2} dx \\ &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \cdot 2e^{x/2} \right\}_0^{2\pi} = \frac{1}{\sqrt{2}} \{0 - 0\} = 0 \end{aligned}$$

$$\therefore \int_0^{2\pi} e^{x/2} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx = 0$$

Definite Integrals Ex 20.1 Q52

$$\begin{aligned}
 \text{Let } I &= \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = \left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} + \frac{1}{2} \int_0^{2\pi} e^x \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\
 \Rightarrow I &= \left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} + \frac{1}{2} \left[\sin\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} - \frac{1}{2} \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\
 I &= \left[\cos\left(\pi + \frac{\pi}{4}\right) e^{2\pi} - \cos\frac{\pi}{4} \right] + \frac{1}{2} \left[\sin\left(\pi + \frac{\pi}{4}\right) e^{2\pi} - \sin\frac{\pi}{4} - \frac{1}{2} I \right] \\
 I &= \left[-\cos\frac{\pi}{4} e^{2\pi} - \cos\frac{\pi}{4} \right] + \frac{1}{2} \left[-\sin\frac{\pi}{4} e^{2\pi} - \sin\frac{\pi}{4} \right] - \frac{I}{4} \\
 \frac{5I}{4} &= -\frac{1}{\sqrt{2}} (e^{2\pi} + 1) - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} (e^{2\pi} + 1) = \frac{-3}{2\sqrt{2}} (e^{2\pi} + 1) \\
 I &= \frac{-3\sqrt{2}}{5} (e^{2\pi} + 1)
 \end{aligned}$$

$$\therefore \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = \frac{-3\sqrt{2}}{5} (e^{2\pi} + 1)$$

Definite Integrals Ex 20.1 Q53

$$\begin{aligned}
 \text{Let } I &= \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}} \\
 I &= \int_0^1 \frac{1}{(\sqrt{1+x} - \sqrt{x})} \times \frac{(\sqrt{1+x} + \sqrt{x})}{(\sqrt{1+x} + \sqrt{x})} dx \\
 &= \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1+x-x} dx \\
 &= \int_0^1 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx \\
 &= \left[\frac{2}{3} (1+x)^{\frac{3}{2}} \right]_0^1 + \left[\frac{2}{3} (x)^{\frac{3}{2}} \right]_0^1 \\
 &= \frac{2}{3} \left[(2)^{\frac{3}{2}} - 1 \right] + \frac{2}{3} [1] \\
 &= \frac{2}{3} (2)^{\frac{3}{2}} \\
 &= \frac{2 \cdot 2\sqrt{2}}{3} \\
 &= \frac{4\sqrt{2}}{3}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q54

$$\begin{aligned}
 \int_1^3 \frac{x}{(x+1)(x+2)} dx &= -\int_1^3 \frac{1}{x+1} dx + \int_1^3 \frac{2}{x+2} dx \quad [\text{Using Partial Fraction}] \\
 &= -\log(x+1) \Big|_1^3 + 2\log(x+2) \Big|_1^3 \\
 &= -(\log 3 - \log 2) + 2(\log 4 - \log 3) \\
 &= -3\log 3 + 5\log 2 \\
 &= \log \frac{32}{27}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q55



$$\begin{aligned}
 \text{Let } I &= \int_0^{\pi} \sin^3 x \, dx \\
 I &= \int_0^{\pi} \sin^2 x \cdot \sin x \, dx \\
 &= \int_0^{\pi} (1 - \cos^2 x) \sin x \, dx \\
 &= \int_0^{\pi} \sin x \, dx - \int_0^{\pi} \cos^2 x \cdot \sin x \, dx \\
 &= [-\cos x]_0^{\pi} + \left[\frac{\cos^3 x}{3} \right]_0^{\pi} \\
 &= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}
 \end{aligned}$$

Hence, the given result is proved.

Definite Integrals Ex 20.1 Q56

$$\begin{aligned}
 \text{Let } I &= \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx \\
 &= - \int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx \\
 &= - \int_0^{\pi} \cos x \, dx \\
 \int \cos x \, dx &= \sin x = F(x)
 \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F(\pi) - F(0) \\
 &= \sin \pi - \sin 0 \\
 &= 0
 \end{aligned}$$

Definite Integrals Ex 20.1 Q57

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$$

$$\text{Let } 2x = t \Rightarrow 2dx = dt$$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\begin{aligned}
 \therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx &= \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2} \right) e^t dt \\
 &= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt
 \end{aligned}$$

$$\text{Let } \frac{1}{t} = f(t)$$

$$\text{Then, } f'(t) = -\frac{1}{t^2}$$

$$\begin{aligned}
 \Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t dt &= \int_2^4 e^t [f(t) + f'(t)] dt \\
 &= [e^t f(t)]_2^4 \\
 &= \left[e^t \cdot \frac{1}{t} \right]_2^4 \\
 &= \left[\frac{e^t}{t} \right]_2^4 \\
 &= \frac{e^4}{4} - \frac{e^2}{2} \\
 &= \frac{e^4 - 2e^2}{4}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q58

$$\begin{aligned}
 & \int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}} dx \\
 &= \int_1^2 \frac{1}{\sqrt{-\left(x-\frac{3}{2}\right)^2 + \left(\frac{1}{4}\right)}} dx \\
 &= \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x-\frac{3}{2}\right)^2}} dx \\
 &= \left[\sin^{-1}(2x-3) \right]_1^2 \\
 &= \sin^{-1}(1) - \sin^{-1}(-1) \\
 &= \pi
 \end{aligned}$$

Definite Integrals Ex 20.1 Q59

We have,

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8} \int_0^k \frac{dx}{\left(\frac{1}{2}\right)^2 + x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8} \left[2 \tan^{-1} 2x \right]_0^k = \frac{\pi}{16} \quad \left[\because \int \frac{dx}{a^2 + x^2} = \tan^{-1} \frac{x}{a} \right]$$

$$\Rightarrow \frac{1}{4} \left[\tan^{-1} 2k - \tan^{-1} 0 \right] = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1} 2k - 0 = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} 2k = \frac{\pi}{4}$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q60

We have,

$$\int_0^a 3x^2 dx = 8$$

$$\Rightarrow \left[x^3 \right]_0^a = 8$$

$$\Rightarrow a^3 = 8$$

$$\Rightarrow a = 2$$

Definite Integrals Ex 20.1 Q61

$$\begin{aligned}
 & \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)} dx \\
 & \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sqrt{2\sin^2 x} dx \\
 & \sqrt{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x dx \\
 & \sqrt{2} (-\cos x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \\
 & = \sqrt{2}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q62

$$\begin{aligned}
 I &= \int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx \\
 \Rightarrow I &= \int_0^{2\pi} \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2 \sin \frac{x}{4} \cos \frac{x}{4}} dx \\
 \Rightarrow I &= \int_0^{2\pi} \sqrt{\left(\sin \frac{x}{4} + \cos \frac{x}{4}\right)^2} dx \\
 \Rightarrow I &= \int_0^{2\pi} \left(\sin \frac{x}{4} + \cos \frac{x}{4}\right) dx \\
 \Rightarrow I &= \left[\frac{-\cos \frac{x}{4}}{\frac{1}{4}} + \frac{\sin \frac{x}{4}}{\frac{1}{4}} \right]_0^{2\pi} \\
 \Rightarrow I &= 4(0 + 1 + 1 - 0) \\
 \Rightarrow I &= 8
 \end{aligned}$$

Definite Integrals Ex 20.1 Q63

$$\begin{aligned}
 I &= \int_0^{\pi/4} (\tan x + \cot x)^{-2} dx \\
 I &= \int_0^{\pi/4} \frac{1}{(\tan x + \cot x)^2} dx \\
 I &= \int_0^{\pi/4} \frac{1}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^2} dx \\
 I &= \int_0^{\pi/4} (\sin x \cos x)^2 dx \\
 I &= \int_0^{\pi/4} \sin^2 x (1 - \sin^2 x) dx \\
 I &= \int_0^{\pi/4} \sin^2 x dx - \int_0^{\pi/4} \sin^4 x dx
 \end{aligned}$$

We know that by reduction formula,

$$\int \sin^n x dx = \frac{n-1}{n} \int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

For $n = 2$

$$\begin{aligned}
 \int \sin^2 x dx &= \frac{2-1}{2} \int 1 dx - \frac{\cos x \sin x}{2} \\
 \int \sin^2 x dx &= \frac{1}{2} x - \frac{\cos x \sin x}{2}
 \end{aligned}$$

For $n = 4$

$$\begin{aligned}
 \int \sin^4 x dx &= \frac{4-1}{4} \int \sin^2 x dx - \frac{\cos x \sin^3 x}{4} \\
 \int \sin^4 x dx &= \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^3 x}{4}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 I &= \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\}_0^{\pi/4} - \left\{ \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^3 x}{4} \right\}_0^{\pi/4} \\
 &= \left\{ \frac{\pi}{8} - \frac{1}{4} \right\} - \left\{ \frac{3}{4} \left(\frac{\pi}{8} - \frac{1}{4} \right) - \frac{1}{16} \right\} \\
 &= \frac{\pi}{32}
 \end{aligned}$$



$$\int_0^{\frac{\pi}{2}} (\sin x \cos x)^2 dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x (1 - \sin^2 x) dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x - \sin^4 x dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx - \int_0^{\frac{\pi}{2}} \sin^4 x dx$$

We know, By reduction formula

$$\int \sin^n x dx = \frac{n-1}{n} \int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

For n=2

$$\int \sin^2 x dx = \frac{2-1}{2} \int 1 dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 x dx = \frac{1}{2} x - \frac{\cos x \sin x}{2}$$

For n=4

$$\int \sin^4 x dx = \frac{4-1}{4} \int \sin^2 x dx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 x dx = \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^3 x}{4}$$

Hence

$$\left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\}_0^{\frac{\pi}{2}} - \left\{ \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^3 x}{4} \right\}_0^{\frac{\pi}{2}}$$

$$\frac{\pi}{4} - \frac{3}{4} \left\{ \frac{\pi}{4} \right\}$$

$$\frac{\pi}{16}$$

Definite Integrals Ex 20.1 Q64

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\int_0^1 x \log(1+2x) dx$$

$$= \left[\frac{x^2 \log(1+2x)}{2} \right]_0^1 - \int_0^1 \frac{2x^2}{2(2x+1)} dx$$

$$= \frac{\log(3)}{2} - \int_0^1 \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)} dx$$

$$= \frac{\log(3)}{2} - \left[\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8} \log|2x+1| \right]_0^1$$

$$= \frac{\log(3)}{2} - \frac{1}{8} \log(3)$$

$$= \frac{3}{8} \log_e(3)$$

Definite Integrals Ex 20.1 Q65



$$\begin{aligned}
 & \int_0^{\frac{\pi}{6}} (\tan^2 x + 2 \tan x \cot x + \cot^2 x) dx \\
 & \int_0^{\frac{\pi}{6}} \left((\sec^2 x - 1) + 2 + (\cos^2 x - 1) \right) dx \\
 & \int_0^{\frac{\pi}{6}} (\sec^2 x + \cos^2 x) dx \\
 & \int_0^{\frac{\pi}{6}} \sec^2 x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2 x dx \\
 & \left\{ \tan x \right\}_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \left\{ -\cot x \right\}_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\
 & \left\{ \sqrt{3} - \frac{1}{\sqrt{3}} \right\} - \left\{ \frac{1}{\sqrt{3}} - \sqrt{3} \right\} \\
 & 2 \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right) \\
 & \frac{4}{\sqrt{3}}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q66

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{4}} (a^2 \cos^2 x + b^2 \sin^2 x) dx \\
 I &= \int_0^{\frac{\pi}{4}} (a^2 (1 - \sin^2 x) + b^2 \sin^2 x) dx \\
 I &= \int_0^{\frac{\pi}{4}} (a^2 - a^2 \sin^2 x + b^2 \sin^2 x) dx \\
 I &= \int_0^{\frac{\pi}{4}} a^2 + (b^2 - a^2) \sin^2 x dx \\
 I &= \int_0^{\frac{\pi}{4}} a^2 + (b^2 - a^2) \frac{(1 + \cos 2x)}{2} dx \\
 I &= \left[a^2 x + \frac{(b^2 - a^2)}{2} \left(x + \frac{\sin 2x}{2} \right) \right]_0^{\frac{\pi}{4}} \\
 I &= \left[\frac{a^2 \pi}{4} + \frac{(b^2 - a^2)}{2} \left(\frac{\pi}{4} + \frac{1}{2} \right) \right] \\
 I &= \frac{(b^2 + a^2) \pi}{8} + \frac{(b^2 - a^2)}{4}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q67



$$\begin{aligned}& \int_0^1 \frac{1}{x^4 + 2x^3 + 2x^2 + 2x + 1} dx \\& \int_0^1 \frac{1}{(x+1)^2(x^2+1)} dx \\& \int_0^1 \left\{ -\frac{x}{2(x^2+1)} + \frac{1}{2(x+1)} + \frac{1}{2(x+1)^2} \right\} dx \\& - \int_0^1 \frac{x}{2(x^2+1)} dx + \int_0^1 \frac{1}{2(x+1)} dx + \int_0^1 \frac{1}{2(x+1)^2} dx \\& - \left\{ \frac{\log(x^2+1)}{4} \right\}_0^1 + \left\{ \frac{\log(x+1)}{2} \right\}_0^1 - \left\{ \frac{1}{2(x+1)} \right\}_0^1 \\& - \frac{\log 2}{4} + \frac{\log 2}{2} - \frac{1}{4} + \frac{1}{2} \\& \frac{\log 2}{4} + \frac{1}{4} \\& = (1/4)\log(2e)\end{aligned}$$





Ex 20.2

Definite Integrals Ex 20.2 Q1

$$\text{Let } I = \int_2^4 \frac{x}{x^2+1} dx$$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \log(1+x^2) = F(x)$$

By the second fundamental theorem of calculus, we obtain

$$I = F(4) - F(2)$$

$$= \frac{1}{2} [\log(1+4^2) - \log(1+2^2)]$$

$$= \frac{1}{2} [\log 17 - \log 5]$$

$$= \frac{1}{2} \log\left(\frac{17}{5}\right)$$

Definite Integrals Ex 20.2 Q2

$$\text{Let } 1 + \log x = t$$

Differentiating w.r.t. x , we get

$$\frac{1}{x} dx = dt$$

$$\text{Now, } x = 1 \Rightarrow t = 1$$

$$x = 2 \Rightarrow t = 1 + \log 2$$

$$\therefore \int_1^2 \frac{1}{x(1+\log x)^2} dx = \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[\frac{-1}{t} \right]_1^{1+\log 2}$$

$$= \left[\frac{-1}{1+\log 2} + 1 \right]$$

$$= \left[\frac{-1+1+\log 2}{1+\log 2} \right]$$

$$= \left[\frac{\log 2}{1+\log 2} \right]$$

$$[\because \log e = 1]$$

$$= \frac{\log 2}{\log e + \log 2}$$

$$[\log a + \log b = \log ab]$$

$$= \frac{\log 2}{\log 2e}$$

$$\therefore \int_1^2 \frac{1}{x(1+\log x)^2} dx = \frac{\log 2}{\log 2e}$$

Definite Integrals Ex 20.2 Q3

$$\text{Let } 9x^2 - 1 = t$$

Differentiating w.r.t. x , we get

$$18x dx = dt$$

$$3x dx = \frac{dt}{6}$$

$$\text{Now, } x = 1 \Rightarrow t = 8$$

$$x = 2 \Rightarrow t = 35$$

$$\therefore \int_1^2 \frac{3x}{9x^2-1} dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6} [\log t]_8^{35}$$

$$= \frac{1}{6} (\log 35 - \log 8)$$

$$\therefore \int_1^2 \frac{3x}{9x^2-1} dx = \frac{1}{6} (\log 35 - \log 8)$$

Definite Integrals Ex 20.2 Q4

$$\begin{aligned}\text{Put } \sin x &= \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \\ \cos x &= \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \\ \int_0^{\frac{\pi}{2}} \frac{dx}{5 \cos x + 3 \sin x} &= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{5 \left(1 - \tan^2 \frac{x}{2} \right) + 6 \tan \frac{x}{2}} \\ &= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{5 - 5 \tan^2 \frac{x}{2} + 6 \tan \frac{x}{2}}\end{aligned}$$

$$\text{Let } \tan \frac{x}{2} = t$$

Differentiating w.r.t. x , we get

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\text{Now, } x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{5 - 5 \tan^2 \frac{x}{2} + 6 \tan \frac{x}{2}} = \int_0^1 \frac{2dt}{5 - 5t^2 + 6t} = \frac{2}{5} \int_0^1 \frac{dt}{1 - t^2 + \frac{6}{5}t}$$

Forming perfect square by adding and subtracting $\frac{9}{25}$

$$\begin{aligned}& \frac{2}{5} \int_0^1 \frac{dt}{1 - t^2 + \frac{6}{5}t} \\ &= \frac{2}{5} \int_0^1 \frac{dt}{\frac{34}{25} - \left(t - \frac{3}{5}\right)^2} \\ &= \frac{2}{5} \cdot \frac{1}{2} \sqrt{\frac{34}{25}} \log \left(\frac{\sqrt{\frac{34}{25}} + t - \frac{3}{5}}{\sqrt{\frac{34}{25}} - t + \frac{3}{5}} \right) \quad \left[\because \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left(\frac{x+a}{x-a} \right) \right] \\ &= \frac{1}{\sqrt{34}} \left\{ \log \left(\frac{\sqrt{34} + 2}{\sqrt{34} - 2} \right) - \log \left(\frac{\sqrt{34} - 3}{\sqrt{34} + 3} \right) \right\} \\ &= \frac{1}{\sqrt{34}} \log \left(\frac{(\sqrt{34} + 2)(\sqrt{34} - 3)}{(\sqrt{34} - 2)(\sqrt{34} + 3)} \right) \\ &= \frac{1}{\sqrt{34}} \log \left(\frac{40 + 5\sqrt{34}}{40 - 5\sqrt{34}} \right) \\ &= \frac{1}{\sqrt{34}} \log \left(\frac{8 + \sqrt{34}}{8 - \sqrt{34}} \right)\end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{5 \cos x + 3 \sin x} = \frac{1}{\sqrt{34}} \log \left(\frac{8 + \sqrt{34}}{8 - \sqrt{34}} \right)$$

Definite Integrals Ex 20.2 Q5

Let $a^2 + x^2 = t^2$

Differentiating w.r.t. x , we get

$$2x dx = 2t dt$$

$$x dx = t dt$$

Now, $x = 0 \Rightarrow t = 0$

$$x = a \Rightarrow t = \sqrt{2}a$$

$$\begin{aligned} \therefore \int_0^a \frac{x dx}{\sqrt{a^2 + x^2}} &= \int_a^{\sqrt{2}a} \frac{t dt}{t} \\ &= \int_a^{\sqrt{2}a} dt \\ &= [t]_a^{\sqrt{2}a} \\ &= [\sqrt{2}a - a] \\ &= a(\sqrt{2} - 1) \end{aligned}$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}} dx = a(\sqrt{2} - 1)$$

Definite Integrals Ex 20.2 Q6

Let $e^x = t$

Differentiating w.r.t. x , we get

$$e^x dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$$x = 1 \Rightarrow t = e$$

$$\begin{aligned} \therefore \int_0^1 \frac{e^x}{1 + e^{2x}} dx &= \int_1^e \frac{dt}{1 + t^2} \\ &= [\tan^{-1} t]_1^e \\ &= [\tan^{-1} e - \tan^{-1} 1] \\ &= \tan^{-1} e - \frac{\pi}{4} \end{aligned} \quad \begin{aligned} \left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t \right] \\ \left[\because \tan \frac{\pi}{4} = 1 \right] \end{aligned}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}} dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$

Differentiating w.r.t. x , we get

$$2x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = 1 \Rightarrow t = 1$$

$$\begin{aligned} \therefore \int_0^1 x e^{x^2} dx &= \int_0^1 \frac{e^t dt}{2} \\ &= \frac{1}{2} \int_0^1 e^t dt \\ &= \frac{1}{2} [e^t]_0^1 \\ &= \frac{1}{2} [e^1 - e^0] \quad [\because e^0 = 1] \\ &= \frac{1}{2} (e - 1) \end{aligned}$$

$$\therefore \int_0^1 x e^{x^2} dx = \frac{1}{2} (e - 1)$$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$

Differentiating w.r.t. x , we get

$$\frac{1}{x} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = 3 \Rightarrow t = \log 3$$

$$\int_1^3 \frac{\cos(\log x)}{x} dx$$

$$= \int_0^{\log 3} \cos t dt \quad [\because \int \cos t = \sin t]$$

$$= [\sin t]_0^{\log 3}$$

$$= \sin(\log 3) - \sin 0$$

$$= \sin(\log 3)$$

$$\int_1^3 \frac{\cos(\log x)}{x} dx = \sin(\log 3)$$

Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x , we get

$$2x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = 1 \Rightarrow t = 1$$

$$\therefore \int_0^1 \frac{2x}{1+x^4} dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= [\tan^{-1} t]_0^1$$

$$= [\tan^{-1} 1 - \tan^{-1} 0] \quad \left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{2x}{1+x^4} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q10

Let $x = a \sin \theta$

Differentiating w.r.t. x , we get

$$dx = a \cos \theta d\theta$$

Now,

$$x = 0 \Rightarrow \theta = 0$$

$$x = a \Rightarrow \theta = \frac{\pi}{2}$$

$$\therefore \int_0^a \sqrt{a^2 - x^2} dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{a^2 (1 - \sin^2 \theta)} a \cos \theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta \quad \left[\because (1 - \sin^2 \theta) = \cos^2 \theta \text{ and } \frac{1 + \cos 2\theta}{2} = \cos^2 \theta \right]$$

$$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$$

$$= \frac{a^2}{2} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right]$$

$$= \frac{\pi a^2}{4}$$

$$\therefore \int_0^a \sqrt{a^2 - x^2} dx = \frac{\pi a^2}{4}$$

Definite Integrals Ex 20.2 Q11

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^4 \phi \cos \phi d\phi$$

$$\text{Also, let } \sin \phi = t \Rightarrow \cos \phi d\phi = dt$$

$$\text{When } \phi = 0, t = 0 \text{ and when } \phi = \frac{\pi}{2}, t = 1$$

$$\therefore I = \int_0^1 \sqrt{t} (1 - t^2)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}} (1 + t^4 - 2t^2) dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right] dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231}$$

$$= \frac{64}{231}$$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x , we get

$$\cos x \, dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} \, dx$$

$$= \int_0^1 \frac{dt}{1 + t^2}$$

$$= \left[\tan^{-1} t \right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0 \right]$$

$$\left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} \, dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1 + \cos \theta = t^2$

Differentiating w.r.t. θ , we get

$$-\sin \theta \, d\theta = 2t \, dt$$

$$\sin \theta \, d\theta = -2t \, dt$$

Now,

$$x = 0 \Rightarrow t = \sqrt{2}$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin \theta \, d\theta}{\sqrt{1 + \cos \theta}}$$

$$= \int_{\sqrt{2}}^1 \frac{-2t \, dt}{t}$$

$$= -2 \int_{\sqrt{2}}^1 dt$$

$$= -2 \left[t \right]_{\sqrt{2}}^1$$

$$= -2 \left[1 - \sqrt{2} \right]$$

$$= 2 \left[\sqrt{2} - 1 \right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin \theta \, d\theta}{\sqrt{1 + \cos \theta}} = 2 \left[\sqrt{2} - 1 \right]$$

Definite Integrals Ex 20.2 Q14

Let $3 + 4\sin x = t$

Differentiating w.r.t. x , we get

$$4 \cos x dx = dt$$

$$\cos x dx = \frac{dt}{4}$$

Now,

$$x = 0 \Rightarrow t = 3$$

$$x = \frac{\pi}{3} \Rightarrow t = 3 + 2\sqrt{3}$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3 + 4\sin x} dx$$

$$= \int_3^{3+2\sqrt{3}} \frac{dt}{4t}$$

$$= \frac{1}{4} [\log t]_3^{3+2\sqrt{3}}$$

$$= \frac{1}{4} [\log(3 + 2\sqrt{3}) - \log 3]$$

$$= \frac{1}{4} \log \left(\frac{3 + 2\sqrt{3}}{3} \right)$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3 + 4\sin x} dx = \frac{1}{4} \log \left(\frac{3 + 2\sqrt{3}}{3} \right)$$

Definite Integrals Ex 20.2 Q15

Let $\tan^{-1} x = t$

Differentiating w.r.t. x , we get

$$\frac{1}{1+x^2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$

$$= \int_0^{\frac{\pi}{4}} t^{1/2} dt$$

$$= \left[\frac{t^{3/2}}{3/2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3} \left[t^{3/2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3} \left[\left(\frac{\pi}{4} \right)^{3/2} - 0 \right]$$

$$= \frac{1}{12} \pi^{3/2}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \frac{1}{12} \pi^{3/2}$$

Definite Integrals Ex 20.2 Q16

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$$\begin{aligned}\therefore \int_0^2 x\sqrt{x+2}dx &= \int_{\sqrt{2}}^2 (t^2 - 2)\sqrt{t^2} 2tdt \\ &= 2 \int_{\sqrt{2}}^2 (t^2 - 2)t^2 dt \\ &= 2 \int_{\sqrt{2}}^2 (t^4 - 2t^2) dt \\ &= 2 \left[\frac{t^5}{5} - \frac{2t^3}{3} \right]_{\sqrt{2}}^2 \\ &= 2 \left[\frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3} \right] \\ &= 2 \left[\frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15} \right] \\ &= 2 \left[\frac{16 + 8\sqrt{2}}{15} \right] \\ &= \frac{16(2 + \sqrt{2})}{15} \\ &= \frac{16\sqrt{2}(\sqrt{2} + 1)}{15}\end{aligned}$$

Definite Integrals Ex 20.2 Q17

Let $x = \tan \theta$

Differentiating w.r.t. x , we get

$$dx = \sec^2 \theta d\theta$$

Now,

$$x = 0 \Rightarrow \theta = 0$$

$$x = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\begin{aligned}&\int_0^1 \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx \\ &= \int_0^{\frac{\pi}{4}} \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) \sec^2 \theta d\theta \quad \left[\because \tan^2 \theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \right] \\ &= \int_0^{\frac{\pi}{4}} \tan^{-1} (\tan 2\theta) \sec^2 \theta d\theta \\ &= \int_0^{\frac{\pi}{4}} 2\theta \sec^2 \theta d\theta\end{aligned}$$

Applying by parts, we get

$$\begin{aligned}&= 2 \left[\theta \int_0^{\frac{\pi}{4}} \sec^2 \theta d\theta - \int_0^{\frac{\pi}{4}} (\sec^2 \theta) \frac{d\theta}{d\theta} d\theta \right] \\ &= 2 \left[\theta \tan \theta \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan \theta d\theta \\ &= 2 \left[\theta \tan \theta + \log(\cos \theta) \right]_0^{\frac{\pi}{4}} \\ &= 2 \left[\frac{\pi}{4} + \log \left(\frac{1}{\sqrt{2}} \right) - 0 - 0 \right] \\ &= 2 \left[\frac{\pi}{4} + \frac{1}{2} \log 2 \right] \\ &= \frac{\pi}{2} - \log 2\end{aligned}$$

$$\therefore \int_0^1 \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx = \frac{\pi}{2} - \log 2$$

Definite Integrals Ex 20.2 Q18



Let $\sin^2 x = t$

Differentiating w.r.t. x , we get

$$2 \sin x \cos x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\begin{aligned} & \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 + \sin^4 x} dx \\ &= \frac{1}{2} \int_0^1 \frac{dt}{1 + t^2} \\ &= \frac{1}{2} \left[\tan^{-1} t \right]_0^1 \\ &= \frac{1}{2} \left[\tan^{-1}(1) - \tan^{-1}(0) \right] \\ &= \frac{1}{2} \left[\tan^{-1} \left(\tan \frac{\pi}{4} \right) - \tan^{-1}(\tan 0) \right] \\ &= \frac{1}{2} \times \frac{\pi}{4} \\ &= \frac{\pi}{8} \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 + \sin^4 x} dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.2 Q19

Putting $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1 - \tan^2 \frac{x}{2}}{\sec^2 \frac{x}{2}}$

$$\sin x = \frac{2 \tan \frac{x}{2}}{\sec^2 \frac{x}{2}}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{1}{a \cos x + b \sin x} dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{a \left(1 - \tan^2 \frac{x}{2} \right) + 2b \tan^2 \frac{x}{2}} dx$$

Put $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

If $x = 0$, $t = 0$ and if $x = \frac{\pi}{2}$, $t = 1$

$$\begin{aligned} \Rightarrow I &= 2 \int_0^1 \frac{dt}{a(1 - t^2) + 2bt} \\ &= 2 \int_0^1 \frac{dt}{-at^2 + 2bt + a} \\ &= 2 \int_0^1 \frac{dt}{-a \left[t^2 - \frac{2b}{a}t - 1 \right]} \\ &= \frac{2}{a} \int_0^1 \frac{dt}{-\left[\left(t - \frac{b}{a} \right)^2 - 1 - \frac{b^2}{a^2} \right]} \\ &= \frac{2}{a} \int_0^1 \frac{dt}{\left(\frac{b^2}{a^2} + 1 \right) - \left(t - \frac{b}{a} \right)^2} \\ &= \frac{2}{a} \left[\frac{1}{2\sqrt{\frac{b^2}{a^2} + 1}} \left(\log \left| \frac{\sqrt{\frac{b^2}{a^2} + 1} + \left(t - \frac{b}{a} \right)}{\sqrt{\frac{b^2}{a^2} + 1} - \left(t - \frac{b}{a} \right)} \right| \right) \right]_0^1 \\ &= \frac{1}{\sqrt{b^2 + a^2}} \log \left(\frac{a + b + \sqrt{a^2 + b^2}}{a + b - \sqrt{a^2 + b^2}} \right) \end{aligned}$$

Definite Integrals Ex 20.2 Q20

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We know that $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \sin x} dx = \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \sin \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{5 \left(1 + \tan^2 \frac{x}{2} \right) + 4 \left(2 \tan \frac{x}{2} \right)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1 + \tan^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\text{Differentiating w.r.t. } x, \text{ we get}$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

$$\begin{aligned}
 &= \int_0^1 \frac{2dt}{5 + 5t^2 + 8t} \\
 &= \frac{2}{5} \int_0^1 \frac{dt}{1 + t^2 + \frac{8}{5}t} \\
 &= \frac{2}{5} \int_0^1 \frac{dt}{1 - \frac{16}{25} + \frac{16}{25} + t^2 + \frac{8}{5}t} \quad \left[\text{Adding and subtracting } \frac{16}{25} \right] \\
 &= \frac{2}{5} \int_0^1 \frac{dt}{\left(\frac{3}{2}\right)^2 + \left(t + \frac{4}{5}\right)^2} \\
 &= \frac{2}{5} \left[\frac{5}{3} \tan^{-1} \left(t + \frac{4}{5} \right) \times \frac{5}{3} \right]_0^1 \\
 &= \frac{2}{3} \left[\tan^{-1} \left(1 + \frac{4}{5} \right) \times \frac{5}{3} - \tan^{-1} \frac{4}{5} \times \frac{5}{3} \right]_0^1 \\
 &= \frac{2}{3} \left[\tan^{-1} 3 - \tan^{-1} \frac{4}{3} \right]_0^1 \\
 &= \frac{2}{3} \left[\tan^{-1} \left(\frac{3 - \frac{4}{3}}{1 + 3 \times \frac{4}{3}} \right) \right]_0^1 \quad \left[\because \tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A - B}{1 + AB} \right) \right] \\
 &= \frac{2}{3} \left[\tan^{-1} \frac{5}{3} \right] \\
 &= \frac{2}{3} \tan^{-1} \frac{1}{3}
 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \sin x} dx = \frac{2}{3} \tan^{-1} \frac{1}{3}$$

Definite Integrals Ex 20.2 Q21

We have,

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

$$\begin{aligned}
 \text{Let } \sin x &= K (\sin x + \cos x) + L \frac{d}{dx} (\sin x + \cos x) \\
 &= K (\sin x + \cos x) + L (\cos x - \sin x) \\
 &= \sin x (K - L) + \cos x (K + L)
 \end{aligned}$$

Equating similar terms

$$\begin{aligned}
 K - L &= 1 \\
 K + L &= 0
 \end{aligned}$$

$$\Rightarrow K = \frac{1}{2} \text{ and } L = -\frac{1}{2}$$

$$\begin{aligned}
 \therefore \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx &= \frac{1}{2} \int_0^{\frac{\pi}{2}} dx + \left(\frac{-1}{2} \right) \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\sin x + \cos x} dx \\
 &= \frac{1}{2} [x]_0^{\frac{\pi}{2}} - \frac{1}{2} (\log |\sin x + \cos x|)_0^{\frac{\pi}{2}} = \frac{\pi}{2} - \frac{1}{2} (0) = \frac{\pi}{2}
 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q22

We know,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\begin{aligned} \therefore \frac{1}{3 + 2 \sin x + \cos x} &= \frac{1}{3 + 2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} \\ &= \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{3 \left(1 + \tan^2 \frac{x}{2} \right) + 4 \tan \frac{x}{2} + \left(1 - \tan^2 \frac{x}{2} \right)} \\ &= \frac{\sec^2 \frac{x}{2} dx}{3 + 3 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} \end{aligned}$$

$$\therefore \int_0^{\pi} \frac{1}{3 + 2 \sin x + \cos x} dx = \int_0^{\pi} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4}$$

Let $\tan \frac{x}{2} = t$

Differentiating w.r.t. x , we get

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \pi \Rightarrow t = \infty$$

$$\begin{aligned} \therefore \int_0^{\pi} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4} &= \int_0^{\infty} \frac{dt}{t^2 + 2t + 2} \\ &= \int_0^{\infty} \frac{dt}{(t+1)^2 + 1} \\ &= \left[\tan^{-1}(t+1) \right]_0^{\infty} \\ &= \tan^{-1}(\infty) - \tan^{-1}(0+1) \\ &= \tan^{-1}(\infty) - \tan^{-1}(1) \\ &= \tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}\left(\tan \frac{\pi}{4}\right) \\ &= \frac{\pi}{2} - \frac{\pi}{4} \\ &= \frac{2\pi - \pi}{4} \\ &= \frac{\pi}{4} \end{aligned}$$

$$\therefore \int_0^{\pi} \frac{1}{3 + 2 \sin x + \cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q23

We have,

$$\begin{aligned}\int_0^1 1 \cdot \tan^{-1} x \, dx &= \tan^{-1} x \int_0^1 dx - \int_0^1 (1 \, dx) \frac{d}{dx} (\tan^{-1} x) \, dx \\ &= \left[x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} \, dx \\ &= \left[x \tan^{-1} x - \frac{1}{2} \log(1+x^2) \right]_0^1 \\ &= \frac{\pi}{4} - \frac{1}{2} (\log 2 - 0) \\ &= \frac{\pi}{4} - \frac{1}{2} \log 2\end{aligned}$$

$$\therefore \int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$

Definite Integrals Ex 20.2 Q24

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = \frac{x}{\sqrt{1-x^2}}, g = \sin^{-1} x$$

$$f = -\sqrt{1-x^2}, g' = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2} \sin^{-1} x - \int (-1) \, dx$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2} \sin^{-1} x + x$$

Hence

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx = \left[x - \sqrt{1-x^2} \sin^{-1} x \right]_0^{\frac{1}{2}}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx = \left\{ \frac{1}{2} - \sqrt{1-\left(\frac{1}{2}\right)^2} \sin^{-1} \frac{1}{2} \right\}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx = \left\{ \frac{1}{2} - \frac{\sqrt{3}}{2} \frac{\pi}{6} \right\}$$

Definite Integrals Ex 20.2 Q25

$$I = \int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} \right) dx$$

$$I = \sqrt{2} \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} \right) dx$$

$$I = \sqrt{2} \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} \right) dx$$

$$\text{Let } \sin x - \cos x = t$$

$$(\cos x + \sin x) dx = dt$$

$$x = 0 \Rightarrow t = -1 \text{ and } x = \frac{\pi}{4} \Rightarrow t = 0$$

$$I = \sqrt{2} \int_{-1}^0 \left(\frac{1}{\sqrt{1-t^2}} \right) dt$$

$$I = \sqrt{2} [\sin^{-1} t]_{-1}^0$$

$$I = \sqrt{2} [\sin^{-1}(0) - \sin^{-1}(-1)]$$

$$I = \frac{\pi}{\sqrt{2}}$$

Definite Integrals Ex 20.2 Q26

We have,

$$\int_0^{\pi/4} \frac{\tan^3 x}{1 + \cos 2x} dx = \int_0^{\pi/4} \frac{\tan^3 x}{2 \cos^2 x} dx = \frac{1}{2} \int_0^{\pi/4} \tan^3 x \sec^2 x dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{4} \Rightarrow t = 1$$

$$\therefore \frac{1}{2} \int_0^{\pi/4} \sec^2 x \tan^3 x dx = \frac{1}{2} \int_0^1 t^3 dt = \frac{1}{2} \left[\frac{t^4}{4} \right]_0^1 = \frac{1}{8}$$

$$\therefore \int_0^{\pi/4} \frac{\tan^3 x}{1 + \cos 2x} dx = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q27

We know that,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\frac{1}{5 + 3 \cos x} = \frac{1}{5 + 3 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} = \frac{1 + \tan^2 \frac{x}{2}}{5 \left(1 + \tan^2 \frac{x}{2} \right) + 3 \left(1 - \tan^2 \frac{x}{2} \right)} = \frac{\sec^2 \frac{x}{2} dx}{8 + 2 \tan^2 \frac{x}{2}}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3 \cos x} = \frac{1}{2} \int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{2^2 + \tan^2 \frac{x}{2}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

Differentiating w.r.t. x , we get

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \pi \Rightarrow t = \infty$$

$$\therefore \frac{1}{2} \int_0^{\pi} \left(\frac{\sec^2 \frac{x}{2} dx}{2^2 + \tan^2 \frac{x}{2}} \right) dx$$

$$= \int_0^{\infty} \frac{dt}{2^2 + t^2}$$

$$= \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) \right]_0^{\infty}$$

$$= \frac{1}{2} \left[\tan^{-1}(\infty) - \tan^{-1}(0) \right]$$

$$= \frac{1}{2} \left[\tan^{-1} \left(\tan \frac{\pi}{2} \right) - \tan^{-1}(\tan 0) \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3 \cos x} = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\begin{aligned} & \int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx \\ &= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx \\ &= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx \end{aligned}$$

Let $\tan x = t$

Differentiating w.r.t. x , we get

$$\sec^2 x dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x \sec^2 \frac{x}{2}}{2} + \tan \frac{x}{2} \right) dx$$

$$= \left[x \tan \left(\frac{x}{2} \right) - \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\text{Let } t = \tan^{-1} x$$

$$dt = \frac{1}{1+x^2} dx$$

$$x=0, t=0$$

$$x=1, t = \frac{\pi}{4}$$

$$I = \int_0^{\frac{\pi}{4}} t dt$$

$$= \left[\frac{t^2}{2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \frac{\pi^2}{16}$$

$$= \frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$$

$$I = \int_0^{\frac{\pi}{4}} \left(\frac{\sin x + \cos x}{3 + 1 - (\cos x - \sin x)^2} \right) dx$$

$$I = \int_0^{\frac{\pi}{4}} \left(\frac{\sin x + \cos x}{4 - (\cos x - \sin x)^2} \right) dx$$

$$I = \frac{1}{4} \left[\log \left| \frac{2 + \sin x - \cos x}{2 - \sin x + \cos x} \right| \right]_0^{\frac{\pi}{4}}$$

$$I = -\frac{1}{4} \log \left(\frac{1}{3} \right)$$

$$I = \frac{1}{4} \log_e 3$$

Definite Integrals Ex 20.2 Q32

We have,

$$\int_0^1 x \tan^{-1} x dx = \tan^{-1} x \int_0^1 x dx - \int_0^1 \left(\int x dx \right) \frac{d}{dx} (\tan^{-1} x) dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1+x^2} dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{1+x^2-1}{1+x^2} dx$$

$$= \frac{1}{2} \left(\frac{\pi}{4} \right) - \frac{1}{2} \left[\int_0^1 dx - \int_0^1 \frac{dx}{1+x^2} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[x - \tan^{-1} x \right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[1 - \frac{\pi}{4} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_0^1 x \tan^{-1} x dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

$$\text{Let } I = \int \frac{1-x^2}{x^4+x^2+1} dx = -\int \frac{x^2-1}{x^4+x^2+1} dx.$$

Then,

$$I = -\int \frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx \quad \left[\begin{array}{l} \text{Dividing the numerator and} \\ \text{denominator by } x^2 \end{array} \right]$$

$$\Rightarrow I = -\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1^2} dx$$

$$\text{Let, } x + \frac{1}{x} = u. \text{ Then, } d\left(x + \frac{1}{x}\right) = du \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = du$$

$$\therefore I = -\int \frac{du}{u^2-1^2}$$

$$\Rightarrow I = -\frac{1}{2(1)} \log \left| \frac{u-1}{u+1} \right| + C$$

$$\Rightarrow I = -\frac{1}{2} \log \left| \frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1} \right| + C = -\frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| + C$$

$$\therefore \int_0^1 \frac{1-x^2}{x^4+x^2+1} dx = \left[-\frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| \right]_0^1 = \left(-\frac{1}{2} \log \left| \frac{1}{3} \right| \right) - \left(-\frac{1}{2} \log |1| \right) = \log \sqrt{3}$$

$$= \log 3^{\frac{1}{2}}$$

$$= \frac{1}{2} \log 3$$

Definite Integrals Ex 20.2 Q34

$$\text{Let } 1+x^2 = t$$

Differentiating w.r.t. x , we get

$$2x dx = dt$$

$$\text{Now, } x = 0 \Rightarrow t = 1$$

$$x = 1 \Rightarrow t = 2$$

$$\int_0^1 \frac{24x^3}{(1+x^2)^4} dx = \int_1^2 \frac{12(t-1)}{t^4} dt$$

$$= 12 \int_1^2 \left(\frac{1}{t^3} - \frac{1}{t^4} \right) dt$$

$$= 12 \left[-\frac{1}{2t^2} - \frac{1}{3t^3} \right]_1^2$$

$$= 12 \left[-\frac{1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3} \right]$$

$$= 12 \left[\frac{-3+1+12-8}{24} \right]$$

$$= \frac{12 \times 2}{24} = 1$$

$$\therefore \int_0^1 \frac{24x^3}{(1+x^2)^4} dx = 1$$

Definite Integrals Ex 20.2 Q35

Let $x - 4 = t^3$

Differentiating w.r.t. x , we get

$$dx = 3t^2 dt$$

Now, $x = 4 \Rightarrow t = 0$

$$x = 12 \Rightarrow t = 2$$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \int_0^2 (t^3 + 1)t \cdot 3t^2 dt$$

$$= 3 \int_0^2 (t^6 + 4t^3) dt$$

$$= 3 \left[\frac{t^7}{7} + t^4 \right]_0^2$$

$$= 3 \left[\frac{128}{7} + 16 \right]$$

$$= \frac{720}{7}$$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \frac{720}{7}$$

Definite Integrals Ex 20.2 Q36

We have,

$$\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$$

Using by parts, we get

$$x^2 \int \sin x \, dx - \int \left(\int \sin x \, dx \right) \frac{dx^2}{dx} \cdot dx$$

$$= x^2 \cos x + \int \cos x \cdot 2x \, dx$$

Again applying by parts

$$= x^2 \cos x + 2 \left[x \int \cos x \, dx - \int \left(\int \cos x \, dx \right) \cdot \frac{dx}{dx} \cdot dx \right]$$

$$= x^2 \cos x + 2 [x \sin x - \int \sin x \, dx]$$

$$= \left[x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{2}}$$

$$= \pi + 0 - 0 - 0 - 2$$

$$= \pi - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x \, dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37



Let $x = \cos 2\theta$

Differentiating w.r.t. x , we get

$$dx = -2 \sin 2\theta d\theta$$

$$\text{Now, } x = 0 \Rightarrow \theta = \frac{\pi}{4}$$

$$x = 1 \Rightarrow \theta = 0$$

$$\therefore \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \int_{\frac{\pi}{4}}^0 \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} (-2 \sin 2\theta) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} (2 \sin 2\theta) d\theta \quad \left[\because \sin 2\theta = 2 \sin \theta \cos \theta; \text{ and } \sin^2 \theta = \frac{1-\cos 2\theta}{2} \right]$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\cos \theta} \cdot \sin 2\theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{4}} \sin^2 \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} (1 - \cos 2\theta) d\theta$$

$$= 2 \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{4}}$$

$$= 2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{(1+x^2)^2} dx = \int_0^1 \frac{-x^2 \left(1 - \frac{1}{x^2}\right) dx}{x^2 \left(x + \frac{1}{x}\right)^2} = - \int_0^1 \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2}$$

$$\text{Let } x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2} dx = dt$$

$$\text{When } x = 0 \Rightarrow t = \infty$$

$$x = 1 \Rightarrow t = 2$$

$$\therefore \int_0^1 \frac{1-x^2}{(1+x^2)^2} dx = - \int_{\infty}^2 \frac{dt}{t^2} = \int_2^{\infty} \frac{dt}{t^2} = \left[-\frac{1}{t} \right]_2^{\infty} = \left(\frac{1}{2} - 0 \right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4 dx$.

$$\text{Therefore, } \int 5x^4 \sqrt{x^5 + 1} dx = \int \sqrt{t} dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} (x^5 + 1)^{\frac{3}{2}}$$

$$\begin{aligned} \text{Hence, } \int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx &= \frac{2}{3} \left[(x^5 + 1)^{\frac{3}{2}} \right]_{-1}^1 \\ &= \frac{2}{3} \left[(1^5 + 1)^{\frac{3}{2}} - ((-1)^5 + 1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} \left[2^{\frac{3}{2}} - 0^{\frac{3}{2}} \right] = \frac{2}{3} (2\sqrt{2}) = \frac{4\sqrt{2}}{3} \end{aligned}$$

Alternatively, first we transform the integral and then evaluate the transformed integral with new limits.

Let $t = x^5 + 1$. Then $dt = 5x^4 dx$.

Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

Thus, as x varies from -1 to 1 , t varies from 0 to 2

$$\begin{aligned} \text{Therefore } \int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx &= \int_0^2 \sqrt{t} dt \\ &= \frac{2}{3} \left[t^{\frac{3}{2}} \right]_0^2 = \frac{2}{3} \left[2^{\frac{3}{2}} - 0^{\frac{3}{2}} \right] = \frac{2}{3} (2\sqrt{2}) = \frac{4\sqrt{2}}{3} \end{aligned}$$



Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1 + 3\sin^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x (\sec^2 x + 3\tan^2 x)} dx$$

Put $\tan x = t$

$$\sec^2 x dx = dt$$

$$x = 0 \Rightarrow t = 0 \text{ and } x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$I = \int_0^{\infty} \frac{1}{(1+t^2)(1+4t^2)} dt$$

$$I = -\frac{1}{3} \int_0^{\infty} \left[\frac{1}{(1+t^2)} - \frac{1}{(1+4t^2)} \right] dt$$

$$I = -\frac{1}{3} \left[\tan^{-1} t - 2 \tan^{-1} 2t \right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

$$\text{Let } I = \int_0^{\pi/4} \sin^3 2t \cos 2t dt. \text{ consider } \int \sin^3 2t \cos 2t dt$$

$$\text{Put } \sin 2t = u \text{ so that } 2 \cos 2t dt = du \text{ or } \cos 2t dt = \frac{1}{2} du$$

$$\begin{aligned} \text{So } \int \sin^3 2t \cos 2t dt &= \frac{1}{2} \int u^3 du \\ &= \frac{1}{8} [u^4] = \frac{1}{8} \sin^4 2t = F(t) \text{ say} \end{aligned}$$

Therefore, by the second fundamental theorem of integrals calculus

$$I = F\left(\frac{\pi}{4}\right) - F(0) = \frac{1}{8} \left[\sin^4 \frac{\pi}{2} - \sin^4 0 \right] = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q42

$$\text{Let } 5 - 4 \cos \theta = t$$

Differentiating w.r.t. θ , we get

$$4 \sin \theta d\theta = dt$$

$$\text{Now, } \theta = 0 \Rightarrow t = 1$$

$$\theta = \pi \Rightarrow t = 9$$

$$\therefore \int_0^{\pi} (5 - 4 \cos \theta)^{\frac{1}{4}} \sin \theta d\theta$$

$$= \frac{5}{4} \int_1^9 t^{\frac{1}{4}} dt$$

$$= \frac{5}{4} \left[\frac{4}{5} t^{\frac{5}{4}} \right]_1^9$$

$$= 3^{\frac{5}{2}} - 1$$

$$= 9\sqrt{3} - 1$$

$$\therefore \int_0^{\pi} (5 - 4 \cos \theta)^{\frac{1}{4}} \sin \theta d\theta = 9\sqrt{3} - 1$$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta \cdot \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x , we get

$$2 \sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{6}} \tan 2\theta \cdot \sec^2 2\theta d\theta &= \frac{1}{2} \int_0^{\sqrt{3}} t dt = \frac{1}{2} \left[\frac{t^2}{2} \right]_0^{\sqrt{3}} \\ &= \frac{3}{4} \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

Definite Integrals Ex 20.2 Q44

Let $x^{\frac{2}{3}} = t$

Differentiating w.r.t. x , we get

$$\frac{3}{2} \sqrt{x} dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$$x = \pi^{\frac{2}{3}} \Rightarrow t = \pi$$

$$\begin{aligned} \therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx &= \frac{2}{3} \int_0^{\pi} \cos^2 t dt \\ &= \frac{1}{3} \int_0^{\pi} 1 + \cos 2t dt \quad \left[\because 2 \cos^2 t = 1 + \cos 2t \right] \\ &= \frac{1}{3} \left[t + \frac{\sin 2t}{2} \right]_0^{\pi} \end{aligned}$$

$$= \frac{1}{3} [\pi + 0 - 0 - 0] = \frac{\pi}{3}$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx = \frac{\pi}{3}$$

Definite Integrals Ex 20.2 Q45

Let $1 + \log x = t$

Differentiating w.r.t. x , we get

$$\frac{1}{x} dx = dt$$

When $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1 + \log 2$

$$\therefore \int_1^2 \frac{dx}{x(1 + \log x)^2}$$

$$= \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[-\frac{1}{t} \right]_1^{1+\log 2}$$

$$= 1 - \frac{1}{1 + \log 2}$$

$$= \frac{\log 2}{1 + \log 2}$$

$$\therefore \int_1^2 \frac{dx}{x(1 + \log x)^2} = \frac{\log 2}{1 + \log 2}$$

Definite Integrals Ex 20.2 Q46

We have,

$$\int_0^{\frac{\pi}{2}} \cos^5 x \, dx = \int_0^{\frac{\pi}{2}} (1 - \sin^2 x)^2 \cos x \, dx$$

Let $\sin x = t$

Differentiating w.r.t. x , we get

$$\cos x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} (1 - \sin^2 x)^2 \cos x \, dx$$

$$= \int_0^1 (1 - t^2)^2 dt$$

$$= \int_0^1 (1 - 2t^2 + t^4) dt$$

$$= \left[t - \frac{2}{3}t^3 + \frac{t^5}{5} \right]_0^1$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{8}{15}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^5 x \, dx = \frac{8}{15}$$

Definite Integrals Ex 20.2 Q47

Let $I = \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx$. We first find the anti derivative of the integrand.

Pu $30 - x^{\frac{3}{2}} = t$. Then $-\frac{3}{2}\sqrt{x} dx = dt$ or $\sqrt{x} dx = -\frac{2}{3} dt$

$$\text{Thus, } \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = -\frac{2}{3} \int \frac{dt}{t^2} = \frac{2}{3} \left[\frac{1}{t} \right] = \frac{2}{3} \left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)} \right] = f(x)$$

Therefore, by the second fundamental theorem of calculus, we have

$$\begin{aligned} I &= F(9) - F(4) = \frac{2}{3} \left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)} \right]_4^9 \\ &= \frac{2}{3} \left[\frac{1}{(30 - 27)} - \frac{1}{30 - 8} \right] = \frac{2}{3} \left[\frac{1}{3} - \frac{1}{22} \right] = \frac{19}{99} \end{aligned}$$

Definite Integrals Ex 20.2 Q48

Let $\cos x = t$

Differentiating w.r.t. x , we get

$$-\sin x dx = dt$$

When $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

Now,

$$\begin{aligned} &\int_0^{\pi} \sin^3 x (1 + 2 \cos x) (1 + \cos x)^2 dx \\ &= \int_0^{\pi} \sin^2 x (1 + 2 \cos x) (1 + \cos x)^2 \cdot \sin x dx \\ &= -\int_{-1}^1 (1 - t^2) (1 + 2t) (1 + t)^2 dt \quad [\sin^2 x = 1 - \cos^2 x] \\ &= -\int_{-1}^1 (1 + 2t - t^2 - 2t^3) (1 + t^2 + 2t) dt \\ &= -\int_{-1}^1 (1 - t^2 + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4) dt \\ &= -\int_{-1}^1 (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5) dt \\ &= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t^6}{3} \right]_{-1}^1 \\ &= \left[2 + 0 + \frac{8}{3} - 0 - 2 - 0 \right] = \frac{8}{3} \end{aligned}$$

$$\therefore \int_0^{\pi} \sin^3 x (1 + 2 \cos x) (1 + \cos x)^2 dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49



$$I = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx$$

Let $t = \sin x$

$$dt = \cos x dx$$

$$x = 0, t = 0$$

$$x = \frac{\pi}{2}, t = 1$$

$$I = \int_0^1 2t \tan^{-1}(t) dt$$

$$= 2 \left[\frac{1}{2} t^2 \tan^{-1} t - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1$$

$$= 2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let $\sin x = t$

Differentiating w.r.t. x , we get

$$\cos x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = 2 \int_0^1 t \tan^{-1} t dt \quad [\because \sin 2x = 2 \sin x \cos x]$$

Using by parts

$$= 2 \left\{ \tan^{-1} t \int t dt - \int (t dt) \frac{d \tan^{-1} t}{dt} dt \right\}$$

$$= 2 \left\{ \frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \int \frac{t^2}{1+t^2} dt \right\}$$

$$= 2 \left\{ \frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \left(\int dt - \int \frac{dt}{1+t^2} \right) \right\}$$

$$= 2 \left[\frac{t^2}{2} \tan^{-1} t - \frac{1}{2} (t - \tan^{-1} t) \right]_0^1$$

$$= 2 \left\{ \frac{1}{2} \frac{\pi}{4} - \frac{1}{2} \left(1 - \frac{\pi}{4} \right) \right\}$$

$$= 2 \left\{ \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} \right\}$$

$$= 2 \left(\frac{\pi}{4} - \frac{1}{2} \right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\begin{aligned}\int_0^1 (\cos^{-1} x)^2 dx &= (\cos^{-1} x)^2 \int_0^1 dx - \int_0^1 (1 dx) \frac{d(\cos^{-1} x)^2}{dx} dx \\ &= \left[x (\cos^{-1} x)^2 \right]_0^1 + \int_0^1 \frac{x \cdot 2 \cos^{-1} x}{\sqrt{1-x^2}} dx\end{aligned}$$

Now,

$$\text{Let } \cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\begin{aligned}\text{When } x = 0 &\Rightarrow t = \frac{\pi}{2} \\ x = 1 &\Rightarrow t = 0\end{aligned}$$

$$\begin{aligned}\therefore \int_0^1 \frac{2x \cos^{-1} x}{\sqrt{1-x^2}} dx &= -2 \int_{\frac{\pi}{2}}^0 t \cos t dt = 2 \int_0^{\frac{\pi}{2}} t \cos t dt \\ &= 2 \left[t \int \cos t dt - \int (\cos t dt) \frac{dt}{dt} dt \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[t \sin t - \int \sin t dt \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[t \sin t + \cos t \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[\frac{\pi}{2} - 1 \right]\end{aligned}$$

$$\begin{aligned}\int_0^1 (\cos^{-1} x)^2 dx &= \left[x (\cos^{-1} x)^2 \right]_0^1 + \int_0^1 \frac{x \cdot 2 \cos^{-1} x}{\sqrt{1-x^2}} dx = \left[x (\cos^{-1} x)^2 \right]_0^1 + 2 \left(\frac{\pi}{2} - 1 \right) \\ &= 0 - 0 + 2 \left(\frac{\pi}{2} - 1 \right) \\ &= (\pi - 2)\end{aligned}$$

$$\therefore \int_0^1 (\cos^{-1} x)^2 dx = (\pi - 2)$$

Definite Integrals Ex 20.2 Q53

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{\frac{3}{2}}} dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2 \cos^2 \frac{x}{2}}}{\left(2 \sin^2 \frac{x}{2}\right)^{\frac{3}{2}}} dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2} \cos \frac{x}{2}}{2\sqrt{2} \sin^3 \frac{x}{2}} dx$$

$$= \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx$$

$$\left[\begin{array}{l} \because 1 + \cos x = 2 \cos^2 \frac{x}{2} \\ 1 - \cos x = 2 \sin^2 \frac{x}{2} \end{array} \right]$$

$$\left[\begin{array}{l} \because \operatorname{cosec}^2 \frac{x}{2} = \frac{1}{\sin^2 \frac{x}{2}} \\ \cot \frac{x}{2} = \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \end{array} \right]$$

Let $\cot \frac{x}{2} = t$

Differentiating w.r.t. x , we get

$$\frac{-1}{2} \operatorname{cosec}^2 \frac{x}{2} = dt$$

Now, $x = \frac{\pi}{3} \Rightarrow t = \sqrt{3}$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx = - \int_{\sqrt{3}}^1 t dt = - \left[\frac{t^2}{2} \right]_{\sqrt{3}}^1 = \frac{-1}{2} [1 - 3] = 1$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{\frac{3}{2}}} dx = 1$$

Definite Integrals Ex 20.2 Q54

Substitute $x^2 = a^2 \cos 2\theta$

Differentiating w.r.t. x , we get

$$2x dx = -2a^2 \sin 2\theta d\theta$$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$$x = a \Rightarrow \theta = 0$$

$$\therefore \int_0^a x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} dx = \int_{\frac{\pi}{4}}^0 \frac{\sqrt{a^2 (1 - \cos 2\theta)}}{\sqrt{a^2 - (1 - \cos 2\theta)}} (-a^2 \sin 2\theta) d\theta$$

$$= -a^2 \int_{\frac{\pi}{4}}^0 \frac{\sin \theta}{\cos \theta} \sin 2\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} 2 \sin^2 \theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} (1 - \cos 2\theta) d\theta$$

$$= a^2 \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{4}}$$

$$= a^2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$\therefore \int_0^a x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} dx = a^2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$



Definite Integrals Ex 20.2 Q55

Let $x = a \cos 2\theta$

Differentiating w.r.t. x , we get

$$dx = -2a \sin 2\theta$$

$$\begin{aligned} \text{Now, } x = -a &\Rightarrow \theta = \frac{\pi}{2} \\ x = a &\Rightarrow \theta = 0 \end{aligned}$$

$$\therefore \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx = \int_{\frac{\pi}{2}}^0 \sqrt{\frac{a(1-\cos 2\theta)}{a(1+\cos 2\theta)}} (-2 \sin 2\theta) d\theta$$

$$= 2a \int_0^{\frac{\pi}{2}} \frac{\sin \theta}{\cos \theta} \cdot \sin 2\theta d\theta$$

$$\left[\begin{aligned} \because 1 - \cos 2\theta &= 2 \sin^2 \theta \\ 1 + \cos 2\theta &= 2 \cos^2 \theta \\ -\int_a^b f(x) dx &= \int_b^a f(x) dx \end{aligned} \right]$$

$$= 2a \int_0^{\frac{\pi}{2}} \frac{\sin \theta \cdot 2 \sin \theta \cos \theta}{\cos \theta} d\theta$$

$$= 4a \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta$$

$$= 2a \int_0^{\frac{\pi}{2}} (1 - \cos 2\theta) d\theta$$

$$= 2a \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 2a \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 2a \left[\frac{\pi}{2} - 0 - 0 + 0 \right] = \pi a$$

$$\therefore \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx = \pi a$$

Definite Integrals Ex 20.2 Q56

Let $\cos x = t$

Differentiating w.r.t. x , we get

$$-\sin x dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$$x = \frac{\pi}{2} \Rightarrow t = 0$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3 \cos x + 2}$$

$$= -\int_1^0 \frac{tdt}{t^2 + 3t + 2}$$

$$= \int_0^1 \frac{tdt}{(t+2)(t+1)}$$

$$\left[\because -\int_a^b f(x) = \int_b^a f(x) \right]$$

$$= \int_0^1 \left(-\frac{1}{t+1} + \frac{2}{t+2} \right) dt$$

[Applying partial fraction]

$$= [-\log|1+t| + 2 \log|t+2|]_0^1$$

$$= -\log 2 + 2 \log 3 + 0 - 2 \log 2$$

$$= 2 \log 3 - 3 \log 2$$

$$= \log \frac{9}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3 \cos x + 2} = \log \frac{9}{8}$$

Definite Integrals Ex 20.2 Q57

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$$I = \int_0^{\pi/2} \frac{\tan x}{1 + m^2 \tan^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x} dx$$

Put $\sin^2 x = t$ then $2 \sin x \cos x dx = dt$

$$x = 0 \Rightarrow t = 0 \text{ and } x = \frac{\pi}{2} \Rightarrow t = 1$$

$$I = \frac{1}{2} \int_0^1 \frac{1}{(1-t) + m^2 t} dt$$

$$I = \frac{1}{2} \int_0^1 \frac{1}{(m^2 - 1)t + 1} dt$$

$$I = \frac{1}{2} \left[\frac{1}{m^2 - 1} \log |(m^2 - 1)t + 1| \right]_0^1$$

$$I = \frac{1}{2} \left[\frac{1}{m^2 - 1} \log |m^2| - \frac{1}{m^2 - 1} \ln |1| \right]$$

$$I = \frac{1}{2} \left[\frac{\log |m^2|}{m^2 - 1} \right]$$

$$I = \frac{1}{2} \left[\frac{2 \log |m|}{m^2 - 1} \right]$$

$$I = \frac{\log |m|}{m^2 - 1}$$

Definite Integrals Ex 20.2 Q58

$$I = \int_0^{\pi/2} \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$$

Let $x = \sin u$

$$dx = \cos u du$$

$$I = \int_0^{\pi/2} \frac{1}{(1+\sin^2 u)} du$$

$$I = \int_0^{\pi/2} \frac{\sec^2 u}{(1+2 \tan^2 u)} du$$

Let $\tan u = v$

$$dv = \sec^2 u du$$

$$I = \int_0^{\sqrt{3}} \frac{1}{(1+2v^2)} dv$$

$$I = \frac{1}{\sqrt{2}} \left[\tan^{-1}(\sqrt{2}v) \right]_0^{\sqrt{3}}$$

$$I = \frac{1}{\sqrt{2}} \left[\tan^{-1} \left(\sqrt{\frac{2}{3}} \right) \right]$$

Definite Integrals Ex 20.2 Q59

$$I = \int_{\frac{1}{3}}^1 \frac{(x - x^3)^{\frac{1}{3}}}{x^4} dx$$

$$I = \int_{\frac{1}{3}}^1 \frac{\left(\frac{1}{x^3} - 1\right)^{\frac{1}{3}}}{x^3} dx$$

$$\text{Let } \frac{1}{x^3} - 1 = t$$

$$\frac{-2}{x^3} dx = dt$$

$$x = \frac{1}{3} \Rightarrow t = 8 \text{ and } x = 1 \Rightarrow t = 0$$

$$I = -\frac{1}{2} \int_8^0 (t)^{\frac{1}{3}} dt$$

$$I = -\frac{1}{2} \left[\frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right]_8^0$$

$$I = -\frac{1}{2} [0 - 12]$$

$$I = 6$$

Definite Integrals Ex 20.2 Q60

$$\int \sec^2 x \frac{\tan^2 x}{\tan^6 x + 2 \tan^3 x + 1} dx$$

$$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$$

$$\int \frac{u^2}{u^6 + 2u^3 + 1} du$$

$$v = u^3 \rightarrow \frac{dv}{du} = 3u^2$$

$$\frac{1}{3} \int \frac{1}{v^2 + 2v + 1} dv$$

$$\frac{1}{3} \int \frac{1}{(v+1)^2} dv$$

$$-\frac{1}{3(v+1)}$$

$$-\frac{1}{3(u^3+1)}$$

$$-\frac{1}{3(\tan^3 x + 1)}$$

$$\left\{ -\frac{1}{3(\tan^3 x + 1)} \right\}_0^{\frac{\pi}{4}}$$

$$\left\{ -\frac{1}{6} + \frac{1}{3} \right\}$$

$$\frac{1}{6}$$

Definite Integrals Ex 20.2 Q61

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x (1 - \cos^2 x)} \tan^2 x \cos^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x \sin^2 x} \sin^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \sin^3 x dx$$

$$\cos x = t \rightarrow -\sin x = \frac{dt}{dx}$$

$$-\int_1^0 \sqrt{t} (1-t^2) dt$$

$$\int_0^1 (\sqrt{t} - t^{\frac{5}{2}}) dt$$

$$\left[\frac{2t^{\frac{3}{2}}}{3} - \frac{2t^{\frac{7}{2}}}{7} \right]_0^1$$

$$\frac{2}{3} - \frac{2}{7}$$

$$\frac{8}{21}$$

Definite Integrals Ex 20.2 Q62

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^{n-1}} dx$$

$$\text{Let } \cos \frac{x}{2} + \sin \frac{x}{2} = t$$

$$\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) dx = 2dt$$

$$x = 0 \Rightarrow t = 1 \text{ and } x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$$

$$I = \int_1^{\sqrt{2}} \frac{2}{(t)^{n-1}} dt$$

$$I = \left[\frac{2t^{-n+2}}{-n+2} \right]_1^{\sqrt{2}}$$

$$I = \frac{2}{2-n} \left[(\sqrt{2})^{2-n} - 1 \right]$$

$$I = \frac{2}{2-n} \left[2^{1-\frac{n}{2}} - 1 \right]$$

Ex 20.3



Definite Integrals Ex 20.3 Q1(i)

We have,

$$\begin{aligned} & \int_1^4 f(x) dx \\ &= \int_1^2 (4x + 3) dx + \int_2^4 (3x + 5) dx \\ &= \left[\frac{4x^2}{2} + 3x \right]_1^2 + \left[\frac{3x^2}{2} + 5x \right]_2^4 \\ &= \left[\left(\frac{16}{2} + 6 \right) - \left(\frac{4}{2} + 3 \right) \right] + \left[\left(\frac{48}{2} + 20 \right) - \left(\frac{12}{2} + 10 \right) \right] \\ &= [14 - 5] + [44 - 16] \\ &= 9 + 28 \\ &= 37 \end{aligned}$$

Definite Integrals Ex 20.3 Q1(ii)

We have,

$$\begin{aligned} & \int_0^9 f(x) dx \\ &= \int_0^{\frac{\pi}{2}} \sin x dx + \int_{\frac{\pi}{2}}^3 1 dx + \int_3^9 e^{x-3} dx \\ &= [-\cos x]_0^{\frac{\pi}{2}} + [x]_{\frac{\pi}{2}}^3 + [e^{x-3}]_3^9 \\ &= \left[-\cos \frac{\pi}{2} + \cos 0 \right] + \left[3 - \frac{\pi}{2} \right] + [e^{9-3} - e^{3-3}] \\ &= [0 + 1] + \left[3 - \frac{\pi}{2} \right] + [e^6 - e^0] \\ &= 0 + 1 + 3 - \frac{\pi}{2} + e^6 - e^0 \\ &= 1 + 3 - \frac{\pi}{2} + e^6 - 1 \\ &= 3 - \frac{\pi}{2} + e^6 \end{aligned}$$

Definite Integrals Ex 20.3 Q1(iii)

We have,

$$\begin{aligned} & \int_1^4 f(x) dx \\ &= \int_1^3 (7x + 3) dx + \int_3^4 8x dx \\ &= \left[\frac{7x^2}{2} + 3x \right]_1^3 + \left[\frac{8x^2}{2} \right]_3^4 \\ &= \left[\left(\frac{7 \times 9}{2} + 3 \times 3 \right) - \left(\frac{7 \times 1}{2} + 3 \times 1 \right) \right] + \left[\left(\frac{8 \times 16}{2} - \frac{8 \times 9}{2} \right) \right] \\ &= \left[\frac{63}{2} + 9 - \frac{7}{2} - 3 \right] + [64 - 36] \\ &= 34 + 28 \\ &= 62 \end{aligned}$$

Definite Integrals Ex 20.3 Q2

We have,

$$\begin{aligned} & \int_{-4}^4 |x + 2| dx \\ &= \int_{-4}^{-2} -(x + 2) dx + \int_{-2}^4 (x + 2) dx \\ &= - \left[\frac{x^2}{2} + 2x \right]_{-4}^{-2} + \left[\frac{x^2}{2} + 2x \right]_{-2}^4 \\ &= - \left[\left(\frac{4}{2} - 4 \right) - \left(\frac{16}{2} - 8 \right) \right] + \left[\left(\frac{16}{2} + 8 \right) - \left(\frac{4}{2} - 4 \right) \right] \\ &= - [(-2) - (0)] + [(16) - (-2)] \\ &= -[-2] + [16 + 2] \\ &= 2 + 18 \\ &= 20 \end{aligned}$$

$$\therefore \int_{-4}^4 |x + 2| dx = 20$$

Definite Integrals Ex 20.3 Q3

We have,

$$\begin{aligned}
 & \int_{-3}^3 |x+1| dx \\
 &= \int_{-3}^{-1} -(x+1) dx + \int_{-1}^3 (x+1) dx \\
 &= \left[-\frac{x^2}{2} + x \right]_{-3}^{-1} + \left[\frac{x^2}{2} + x \right]_{-1}^3 \\
 &= -\left[\left(\frac{1}{2} - 1 \right) - \left(\frac{9}{2} - 3 \right) \right] + \left[\left(\frac{9}{2} + 3 \right) - \left(\frac{1}{2} - 1 \right) \right] \\
 &= -\left[\left(-\frac{1}{2} \right) - \left(1\frac{1}{2} \right) \right] + \left[\left(7\frac{1}{2} \right) - \left(-\frac{1}{2} \right) \right] \\
 &= -\left[-\frac{1}{2} - 1\frac{1}{2} \right] + \left[7\frac{1}{2} + \frac{1}{2} \right] \\
 &= [-2] + [8] \\
 &= 2 + 8 \\
 &= 10
 \end{aligned}$$

$$\therefore \int_{-3}^3 |x+1| dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\begin{aligned}
 & \int_{-1}^1 |2x+1| dx \\
 &= \int_{-1}^{-\frac{1}{2}} -(2x+1) dx + \int_{-\frac{1}{2}}^1 (2x+1) dx \\
 &= \left[-\frac{2x^2}{2} + x \right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2} + x \right]_{-\frac{1}{2}}^1 \\
 &= -\left[\left(\frac{2}{8} - \frac{1}{2} \right) - \left(\frac{2}{2} - 1 \right) \right] + \left[\left(\frac{2}{2} + 1 \right) - \left(\frac{2}{8} - \frac{1}{2} \right) \right] \\
 &= -\left[\left(\frac{1}{4} - \frac{1}{2} \right) - (1 - 1) \right] + \left[(1 + 1) - \left(\frac{1}{4} - \frac{1}{2} \right) \right] \\
 &= -\left[-\frac{1}{4} \right] + \left[2 + \frac{1}{4} \right] \\
 &= \frac{1}{4} + 2 + \frac{1}{4} \\
 &= 2\frac{1}{2}
 \end{aligned}$$

$$\therefore \int_{-1}^1 |2x+1| dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

$$\begin{aligned}
 & \text{(i)} \\
 & \int_{-2}^2 |2x + 3| dx \\
 &= \int_{-2}^{-\frac{3}{2}} -(2x + 3) dx + \int_{-\frac{3}{2}}^2 (2x + 3) dx \\
 &= \left[-\frac{2x^2}{2} - 3x \right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2} + 3x \right]_{-\frac{3}{2}}^2 \\
 &= -\left[\frac{2 \times 9}{2} - \frac{9}{2} \right] - \left(\frac{2 \times 4}{2} - 6 \right) + \left[\left(\frac{2 \times 4}{2} + 6 \right) - \left(\frac{2 \times 9}{2} - \frac{9}{2} \right) \right] \\
 &= -\left[\left(\frac{18}{2} - \frac{9}{2} \right) - \left(\frac{8}{2} - 6 \right) \right] + \left[\left(\frac{8}{2} + 6 \right) - \left(\frac{18}{2} - \frac{9}{2} \right) \right] \\
 &= -\left[\left(\frac{9}{2} - \frac{9}{2} \right) - (-2) \right] + \left[(10) - \left(\frac{9}{2} - \frac{9}{2} \right) \right] \\
 &= \left[-\frac{9}{4} + 2 \right] + \left[10 + \frac{9}{4} \right] \\
 &= \frac{9}{4} - 2 + 10 + \frac{9}{4} \\
 &\Rightarrow 8\frac{9}{2} \\
 &= 12\frac{1}{2}
 \end{aligned}$$

$$\therefore \int_{-2}^2 |2x + 3| dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$\begin{aligned}
 f(x) &= |x^2 - 3x + 2| \\
 &= |(x - 1)(x - 2)| \\
 &= \begin{cases} x^2 - 3x + 2 & 0 \leq x \leq 1 \\ -(x^2 - 3x + 2) & 1 \leq x \leq 2 \end{cases}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 & \int_0^2 |x^2 - 3x + 2| dx \\
 &= \int_0^1 (x^2 - 3x + 2) dx + \int_1^2 -(x^2 - 3x + 2) dx \\
 &= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_1^2 \\
 &= \left[\frac{1}{3} - \frac{3}{2} + 2 - 0 \right] - \left[\frac{8}{3} - \frac{12}{2} + 4 - \frac{1}{3} + \frac{3}{2} + 2 \right] \\
 &= \left[\frac{1}{6} \right] - \left[-\frac{5}{6} \right] \\
 &= \frac{1}{6} + \frac{5}{6} \\
 &= 1
 \end{aligned}$$

$$\therefore \int_0^2 |x^2 - 3x + 2| dx = 1$$

Definite Integrals Ex 20.3 Q7

$$\begin{aligned}
 \int_0^3 |3x-1| dx &= \int_0^{\frac{1}{3}} -(3x-1) dx + \int_{\frac{1}{3}}^3 (3x-1) dx \\
 &= -\left[\frac{3x^2}{2} - x\right]_0^{\frac{1}{3}} + \left[\frac{3x^2}{2} - x\right]_{\frac{1}{3}}^3 \\
 &= -\left[\left(\frac{3}{9 \times 2} - \frac{1}{3}\right) - (0)\right] + \left[\left(\frac{3 \times 9}{2} - 3\right) - \left(\frac{3}{9 \times 2} - \frac{1}{3}\right)\right] \\
 &= -\left[\left(\frac{1}{6} - \frac{1}{3}\right)\right] + \left[\left(\frac{27}{2} - 3\right) - \left(\frac{1}{6} - \frac{1}{3}\right)\right] \\
 &= -\left[\left(-\frac{1}{6}\right)\right] + \left[\left(10\frac{1}{2}\right) - \left(-\frac{1}{6}\right)\right] \\
 &= -\left[\left(-\frac{1}{6}\right)\right] + \left[10\frac{1}{2} + \frac{1}{6}\right] \\
 &= \frac{1}{6} + 10\frac{1}{2} + \frac{1}{6} \\
 &= \frac{1}{3} + \frac{21}{2} = \frac{2+63}{6} = \frac{65}{6} \\
 &= \frac{65}{6}
 \end{aligned}$$

$$\therefore \int_0^3 |3x-1| dx = \frac{65}{6}$$

Definite Integrals Ex 20.3 Q8

$$\begin{aligned}
 \int_{-6}^6 |x+2| dx &= \int_{-6}^{-2} -(x+2) dx + \int_{-2}^6 (x+2) dx \\
 &= -\left[\frac{x^2}{2} + 2x\right]_{-6}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^6 \\
 &= -\left[\left(\frac{4}{2} + 2(-2)\right) - \left(\frac{36}{2} - 12\right)\right] + \left[\left(\frac{36}{2} + 12\right) - \left(\frac{4}{2} - 4\right)\right] \\
 &= -[(2-4) - (18-12)] + [(18+12) - (2-4)] \\
 &= -[-8] + [30+2] \\
 &= 8+32 \\
 &= 40
 \end{aligned}$$

$$\therefore \int_{-6}^6 |x+2| dx = 40$$

Definite Integrals Ex 20.3 Q9

$$\begin{aligned}
 \int_{-2}^2 |x+1| dx &= \int_{-2}^{-1} -(x+1) dx + \int_{-1}^2 (x+1) dx \\
 &= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^2 \\
 &= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} - 2\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right] \\
 &= -\left[\left(-\frac{1}{2}\right) - 0\right] + \left[4 + \frac{1}{2}\right] \\
 &= \frac{1}{2} + 4\frac{1}{2} \\
 &= 5
 \end{aligned}$$

$$\therefore \int_{-2}^2 |x+1| dx = 5$$

Definite Integrals Ex 20.3 Q10

$$\begin{aligned}
 \int_1^2 |x-3| dx &= \int_1^2 -(x-3) dx \quad [x-3 < 0 \text{ for } 1 < x < 2] \\
 &= -\left[\frac{x^2}{2} - 3x\right]_1^2 \\
 &= -\left[\left(\frac{4}{2} - 6\right) - \left(\frac{1}{2} - 3\right)\right] \\
 &= -\left[(-4) - \left(-2\frac{1}{2}\right)\right] \\
 &= -\left[-4 + 2\frac{1}{2}\right] \\
 &= -\left[-\frac{3}{2}\right] \\
 &= \frac{3}{2}
 \end{aligned}$$

$$\therefore \int_1^2 |x-3| dx = \frac{3}{2}$$

Definite Integrals Ex 20.3 Q11

$$\begin{aligned}
 &\int_0^{\frac{\pi}{2}} |\cos 2x| dx \\
 &= \int_0^{\frac{\pi}{4}} -\cos 2x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x dx \\
 &= \left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\sin \frac{\pi}{2} - \sin 0\right] + \frac{1}{2} \left[\sin \pi - \sin \frac{\pi}{2}\right] \\
 &= \frac{1}{2} [1] + \frac{1}{2} [1] \\
 &= \frac{1}{2} + \frac{1}{2} \\
 &= 1
 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} |\cos 2x| dx = 1$$

Definite Integrals Ex 20.3 Q12

$$\begin{aligned}
 \int_0^{2\pi} |\sin x| dx &= \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} -\sin x dx \\
 &= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} \\
 &= [1+1] + [1+1] \\
 \int_0^{2\pi} |\sin x| dx &= 4
 \end{aligned}$$

Definite Integrals Ex 20.3 Q13

$$\begin{aligned}
 &\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x| dx \\
 &= \int_{-\frac{\pi}{4}}^0 -\sin x dx + \int_0^{\frac{\pi}{4}} \sin x dx \\
 &= [\cos x]_{-\frac{\pi}{4}}^0 + [-\cos x]_0^{\frac{\pi}{4}} \\
 &= \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right) \\
 &= (2 - \sqrt{2})
 \end{aligned}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x| dx = 2 - \sqrt{2}$$



Definite Integrals Ex 20.3 Q14

We have,

$$I = \int_2^8 |x - 5| dx$$

We have,

$$|x - 5| = \begin{cases} x - 5 & \text{if } x \in (5, 8) \\ -(x - 5) & \text{if } x \in (2, 5) \end{cases}$$

Hence,

$$\begin{aligned} I &= \int_2^5 -(x - 5) dx + \int_5^8 (x - 5) dx \\ &= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8 \\ &= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right] \\ &= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right] \\ &= \frac{25}{2} - 8 - 8 + \frac{25}{2} \\ &= 25 - 16 = 9 \end{aligned}$$

$$\therefore \int_2^8 |x - 5| dx = 9$$

Definite Integrals Ex 20.3 Q15

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin |x| + \cos |x|\} dx$$

$$\text{Let } f(x) = \sin |x| + \cos |x|$$

$$\text{Then, } f(x) = f(-x)$$

$\therefore f(x)$ is an even function.

$$\text{So, } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin |x| + \cos |x|\} dx = 2 \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2 [\cos x + \sin x]_0^{\frac{\pi}{2}} = 4$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin |x| + \cos |x|\} dx = 4$$

Definite Integrals Ex 20.3 Q16

$$I = \int_0^4 |x - 1| dx$$

It can be seen that, $(x - 1) \leq 0$ when $0 \leq x \leq 1$ and $(x - 1) \geq 0$ when $1 \leq x \leq 4$

$$\begin{aligned} I &= \int_0^1 |x - 1| dx + \int_1^4 |x - 1| dx & \left(\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right) \\ &= \int_0^1 -(x - 1) dx + \int_1^4 (x - 1) dx \\ &= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4 \\ &= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1 \\ &= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 \\ &= 5 \end{aligned}$$

Definite Integrals Ex 20.3 Q17



$$\begin{aligned}
 \text{Let } I &= \int_1^4 \{|x-1| + |x-2| + |x-4|\} dx \\
 &= \int_1^2 \{(x-1) - (x-2) - (x-4)\} dx + \int_2^4 \{(x-1) + (x-2) - (x-4)\} dx \\
 &= \int_1^2 \{(x-1-x+2-x+4)\} dx + \int_2^4 \{(x-1+x-2-x+4)\} dx \\
 &= \int_1^2 (5-x) dx + \int_2^4 (x+1) dx \\
 &= \left[5x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} + x \right]_2^4 \\
 &= \left[10 - 2 - 5 + \frac{1}{2} \right] + [8 + 4 - 2 - 2] \\
 &= \frac{7}{2} + 8 \\
 I &= \frac{23}{2}
 \end{aligned}$$

Definite Integrals Ex 20.3 Q18

We have,

$$I = \int_{-5}^0 (|x| + |x+2| + |x+5|) dx = \int_{-5}^0 |x| dx + \int_{-5}^0 |x+2| dx + \int_{-5}^0 |x+5| dx$$

$$\begin{aligned}
 \Rightarrow I &= \int_{-5}^0 -x dx + \int_{-5}^{-2} -(x+2) dx + \int_{-2}^0 (x+2) dx + \int_{-5}^0 (x+5) dx \\
 &= \left[-\frac{x^2}{2} \right]_{-5}^0 + \left[-\frac{x^2}{2} - 2x \right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x \right]_{-2}^0 + \left[\frac{x^2}{2} + 5x \right]_{-5}^0 \\
 &= \left[0 + \frac{25}{2} \right] - \left[\frac{4}{2} - 4 - \frac{25}{2} + 10 \right] + \left[0 + 0 - \frac{4}{2} + 4 \right] + \left[0 + 0 - \frac{25}{2} + 25 \right] \\
 &= \frac{25}{2} - \left[8 - \frac{25}{2} \right] + [2] + \left[25 - \frac{25}{2} \right] \\
 &= \frac{25}{2} - 8 + \frac{25}{2} + 2 + 25 - \frac{25}{2} \\
 &= 19 + \frac{25}{2} = 31\frac{1}{2} \\
 I &= \frac{63}{2}
 \end{aligned}$$

Definite Integrals Ex 20.3 Q19

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$$

$$|x-4| = \begin{cases} x-4, & x \geq 4 \\ 4-x, & x < 4 \end{cases}$$

Splitting the limits of the integral, we get

$$\begin{aligned}
 &\int_0^4 (|x| + |x-2| + |x-4|) dx \\
 &= \int_0^2 (|x| + |x-2| + |x-4|) dx + \int_2^4 (|x| + |x-2| + |x-4|) dx \\
 &= \int_0^2 (x+2-x+4-x) dx + \int_2^4 (x+x-2+4-x) dx \\
 &= \int_0^2 (6-x) dx + \int_2^4 (2+x) dx \\
 &= \left[6x - \frac{x^2}{2} \right]_0^2 + \left[2x + \frac{x^2}{2} \right]_2^4 \\
 &= [12-2] + [16-6] \\
 &= 10+10 \\
 &= 20
 \end{aligned}$$

Definite Integrals Ex 20.3 Q20



$$\begin{aligned} & \int_{-1}^2 |x+1| dx + \int_{-1}^2 |x| dx + \int_{-1}^2 |x-1| dx \\ & \int_{-1}^2 (x+1) dx - \int_{-1}^0 x dx + \int_0^2 x dx - \int_{-1}^1 (x-1) dx + \int_1^2 (x-1) dx \\ & \left\{ \frac{x^2}{2} + x \right\}_{-1}^2 - \left\{ \frac{x^2}{2} \right\}_{-1}^0 + \left\{ \frac{x^2}{2} \right\}_0^2 - \left\{ \frac{x^2}{2} - x \right\}_{-1}^1 + \left\{ \frac{x^2}{2} - x \right\}_1^2 \\ & \left\{ (4) - \left(-\frac{1}{2}\right) \right\} - \left\{ -\frac{1}{2} \right\} + \{2\} - \left\{ \left(-\frac{1}{2}\right) - \left(\frac{3}{2}\right) \right\} + \left\{ (0) - \left(-\frac{1}{2}\right) \right\} \\ & \left\{ 4 + \frac{1}{2} \right\} + \left\{ \frac{1}{2} \right\} + \{2\} + \{2\} + \left\{ \frac{1}{2} \right\} \\ & \frac{19}{2} \end{aligned}$$

Definite Integrals Ex 20.3 Q21

$$\int_{-2}^0 x e^{-x} dx + \int_0^2 x e^x dx$$

For

$$\int_{-2}^0 x e^{-x} dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^{-x}, g = x$$

$$f = -e^{-x}, g' = 1$$

$$\int_{-2}^0 x e^{-x} dx = \left\{ -x e^{-x} \right\}_{-2}^0 + \int_{-2}^0 e^{-x} dx$$

$$\int_{-2}^0 x e^{-x} dx = \left\{ -x e^{-x} - e^{-x} \right\}_{-2}^0$$

$$\int_{-2}^0 x e^{-x} dx = \left\{ (-1) - (2e^2 - e^2) \right\}$$

$$\int_{-2}^0 x e^{-x} dx = \left\{ -1 - e^2 \right\}$$

For

$$\int_0^2 x e^x dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^x, g = x$$

$$f = e^x, g' = 1$$

$$\int_0^2 x e^x dx = \left\{ x e^x \right\}_0^2 - \int_0^2 e^x dx$$

$$\int_0^2 x e^x dx = \left\{ x e^x - e^x \right\}_0^2$$

$$\int_0^2 x e^x dx = 2e^2 - e^2 + 1$$

$$\int_0^2 x e^x dx = e^2 + 1$$

Hence answer is,

$$\int_{-2}^2 x e^{|x|} dx = -1 - e^2 + e^2 + 1 = 0$$

Definite Integrals Ex 20.3 Q22

$$\begin{aligned}
 & -\int_{-\frac{\pi}{4}}^0 \sin^2 x dx + \int_0^{\frac{\pi}{2}} \sin^2 x dx \\
 & \sin^2 x = \frac{1 - \cos 2x}{2} \\
 & -\int_{-\frac{\pi}{4}}^0 \frac{1 - \cos 2x}{2} dx + \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx \\
 & -\frac{1}{2} \left\{ x - \frac{\sin 2x}{2} \right\}_{-\frac{\pi}{4}}^0 + \frac{1}{2} \left\{ x - \frac{\sin 2x}{2} \right\}_0^{\frac{\pi}{2}} \\
 & -\frac{1}{2} \left\{ -\left(-\frac{\pi}{4} + \frac{1}{2}\right) \right\} + \frac{1}{2} \left\{ \frac{\pi}{2} \right\} \\
 & \left\{ -\frac{\pi}{8} + \frac{1}{4} \right\} + \left\{ \frac{\pi}{4} \right\} \\
 & \frac{\pi}{8} + \frac{1}{4} \\
 & \frac{\pi + 2}{8}
 \end{aligned}$$

Definite Integrals Ex 20.3 Q23

$$\begin{aligned}
 & \int_0^{\frac{\pi}{2}} \cos^2 x dx - \int_{\frac{\pi}{2}}^{\pi} \cos^2 x dx \\
 & \cos^2 x = \frac{1 + \cos 2x}{2} \\
 & \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} dx - \int_{\frac{\pi}{2}}^{\pi} \frac{1 + \cos 2x}{2} dx \\
 & \frac{1}{2} \left\{ x + \frac{\sin 2x}{2} \right\}_0^{\frac{\pi}{2}} - \frac{1}{2} \left\{ x + \frac{\sin 2x}{2} \right\}_{\frac{\pi}{2}}^{\pi} \\
 & \frac{\pi}{4} - \frac{\pi}{4} \\
 & 0
 \end{aligned}$$

Definite Integrals Ex 20.3 Q24

$$\begin{aligned}
 & \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \sin |x| + \cos |x|) dx \\
 & = \int_{-\frac{\pi}{4}}^0 (-2 \sin x + \cos x) dx + \int_0^{\frac{\pi}{2}} (2 \sin x + \cos x) dx \\
 & = [2 \cos x + \sin x]_{-\frac{\pi}{4}}^0 + [-2 \cos x + \sin x]_0^{\frac{\pi}{2}} \\
 & = 2 + 0 - 0 + 1 + 0 + 1 + 2 - 0 \\
 & = 6
 \end{aligned}$$

Definite Integrals Ex 20.3 Q25

$$\begin{aligned}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^{-1}(\sin x) dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x dx + \int_{\frac{\pi}{2}}^{\pi} (\pi - x) dx \\&\Rightarrow \left\{ \frac{x^2}{2} \right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \left\{ \pi x - \frac{x^2}{2} \right\}_{\frac{\pi}{2}}^{\pi} \\&\Rightarrow \left\{ (\pi^2 - \frac{\pi^2}{2}) - \left(\frac{\pi^2}{2} - \frac{\pi^2}{8} \right) \right\} \\&\Rightarrow \left\{ \frac{\pi^2}{2} - \frac{3\pi^2}{8} \right\} \\&\Rightarrow \frac{\pi^2}{8}\end{aligned}$$

Definite Integrals Ex 20.3 Q27

$[x]=0$ for 0
and $[x]=1$ for 1
Hence

$$\begin{aligned}\int_0^1 0 + \int_1^2 2x dx \\&= \left\{ x^2 \right\}_1^2 \\&= 3\end{aligned}$$

Definite Integrals Ex 20.3 Q18

$$\begin{aligned}\int_0^{2\pi} \cos^{-1}(\cos x) dx \\&= -\int_0^{\pi} \cos^{-1}(\cos x) dx + \int_{\pi}^{2\pi} \cos^{-1}(\cos x) dx \\&= -\int_0^{\pi} x dx + \int_{\pi}^{2\pi} x dx \\&= -\left[\frac{x^2}{2} \right]_0^{\pi} + \left[\frac{x^2}{2} \right]_{\pi}^{2\pi} \\&= -\frac{\pi^2}{2} + \frac{4\pi^2}{2} - \frac{\pi^2}{2} \\&= \pi^2\end{aligned}$$

Definite Integrals Ex 20.3 Q33

$$\text{Let } I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx \quad \text{--- (i)}$$

$$\text{We know that } \int_a^b f(x) = \int_a^b f(a+b-x) dx$$

Then

$$\begin{aligned}I &= \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(a+b-(a+b-x))} dx \\I &= \int_a^b \frac{f(a+b-x)}{f(a+b-x) f(x)} dx \quad \text{--- (ii)}\end{aligned}$$

Adding (i) & (ii)

$$\begin{aligned}2I &= \int_a^b \frac{f(x) + f(a+b-x)}{f(x) + f(a+b-x)} dx \\2I &= \int_a^b dx \\I &= \left[x \right]_a^b \\I &= \frac{1}{2} [b-a] \\I &= \frac{b-a}{2}\end{aligned}$$



Ex 20.4A

Definite Integrals Ex 20.4A Q1

We know

$$\int_0^{2\pi} f(x) dx = \int_0^{2\pi} f(2\pi - x) dx$$

Hence

$$\int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{\sin(2\pi - x)}}{e^{\sin(2\pi - x)} + e^{-\sin(2\pi - x)}} dx$$

We know

$$\sin(2\pi - x) = -\sin x$$

$$\int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

If

$$I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$$

Then also

$$I = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

Hence

$$2I = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx + \int_0^{2\pi} \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_0^{2\pi} dx$$

$$2I = 2\pi$$

$$I = \pi$$

Definite Integrals Ex 20.4A Q2

We know

$$\int_0^{2\pi} f(x) dx = \int_0^{2\pi} f(2\pi - x) dx$$

Hence

$$\int_0^{2\pi} \log(\sec x + \tan x) dx = \int_0^{2\pi} \log(\sec(2\pi - x) + \tan(2\pi - x)) dx$$

$$\int_0^{2\pi} \log(\sec x + \tan x) dx = \int_0^{2\pi} \log(\sec x - \tan x) dx$$

If

$$I = \int_0^{2\pi} \log(\sec x + \tan x) dx$$

Then

$$I = \int_0^{2\pi} \log(\sec x - \tan x) dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x) dx + \int_0^{2\pi} \log(\sec x - \tan x) dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x) + \log(\sec x - \tan x) dx$$

$$2I = \int_0^{2\pi} \log(\sec^2 x - \tan^2 x) dx$$

$$2I = \int_0^{2\pi} \log(1) dx$$

$$2I = 0$$

$$I = 0$$



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Definite Integrals Ex 20.4A Q3

We know

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\pi}{2}-x)}}{\sqrt{\tan(\frac{\pi}{2}-x)} + \sqrt{\cot(\frac{\pi}{2}-x)}} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

So

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} + \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q4

We know

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)} + \sqrt{\cos(\frac{\pi}{2}-x)}} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q5



We know

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Hence

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2(-x)}{1+e^{-x}} dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}} dx$$

If

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} dx$$

Then

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}} dx$$

So

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x) \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1 + e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1 + e^x) \tan^2 x}{1 + e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

We know

If $f(x)$ is even

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

If $f(x)$ is odd

$$\int_{-a}^a f(x) dx = 0$$

Here

$$f(x) = \tan^2 x$$

$f(x)$ is even, hence

$$I = \int_0^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 dx$$

$$I = \left(\tan x - x \right) \Big|_0^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q6

We know

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

Hence

$$\int_{-a}^a \frac{1}{1 + a^x} dx = \int_{-a}^a \frac{1}{1 + a^{-x}} dx$$

If

$$I = \int_{-a}^a \frac{1}{1 + a^x} dx$$

Then

$$I = \int_{-a}^a \frac{1}{1 + a^{-x}} dx$$

So

$$2I = \int_{-a}^a \frac{1}{1 + a^x} + \frac{1}{1 + a^{-x}} dx$$

$$2I = \int_{-a}^a \frac{1}{1 + a^x} + \frac{a^x}{1 + a^x} dx$$

$$2I = \int_{-a}^a 1 dx$$

$$2I = 2a$$

$$I = a$$

Definite Integrals Ex 20.4A Q7

We know

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Hence

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

If

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} dx$$

Then

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

So

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{1}{1+e^{-\tan x}} dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{e^{\tan x}}{1+e^{\tan x}} dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 dx$$

$$2I = \frac{2\pi}{3}$$

$$I = \frac{\pi}{3}$$

Definite Integrals Ex 20.4A Q8

We know

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Hence

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(-x)}{1+e^{-x}} dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}} dx$$

If

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx$$

Then

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}} dx$$



So

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{\cos^2 x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{e^x \cos^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+e^x) \cos^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$$

$$I = \frac{1}{4} \left\{ x + \frac{\sin 2x}{2} \right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$I = \frac{1}{4} \left\{ \left(\frac{\pi}{2} \right) - \left(-\frac{\pi}{2} \right) \right\}$$

$$I = \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q9

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11} - 3x^9 + 5x^7 - x^5 + 1}{\cos^2 x} dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11} - 3x^9 + 5x^7 - x^5}{\cos^2 x} + \frac{1}{\cos^2 x} dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11} - 3x^9 + 5x^7 - x^5}{\cos^2 x} dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$$

If $f(x)$ is even

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

If $f(x)$ is odd

$$\int_{-a}^a f(x) dx = 0$$

Here

$$\frac{x^{11} - 3x^9 + 5x^7 - x^5}{\cos^2 x} \text{ is odd and}$$

 $\sec^2 x$ is even. Hence

$$0 + 2 \int_0^{\frac{\pi}{4}} \sec^2 x dx$$

$$2 \{ \tan x \}_0^{\frac{\pi}{4}}$$

$$2$$

Definite Integrals Ex 20.4A Q10

$$I = \int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}} + (a+b-x)^{\frac{1}{n}}} dx$$

$$I = \int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}} + x^{\frac{1}{n}}} dx$$

$$2I = \int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}} + (a+b-x)^{\frac{1}{n}}} dx + \int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}} + x^{\frac{1}{n}}} dx$$

$$2I = \int_a^b \frac{x^{\frac{1}{n}} + (a+b-x)^{\frac{1}{n}}}{x^{\frac{1}{n}} + (a+b-x)^{\frac{1}{n}}} dx$$

$$I = \frac{1}{2} \int_a^b dx$$

$$I = \frac{b-a}{2}$$

Definite Integrals Ex 20.4A Q11

We have,

$$I = \int_0^{\frac{\pi}{2}} (2 \log \cos x - \log \sin 2x) dx$$

$$= \int_0^{\frac{\pi}{2}} (\log \cos^2 x - \log \sin 2x) dx$$

$$= \int_0^{\frac{\pi}{2}} \log \frac{\cos^2 x}{\sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \log \frac{\cos^2 x}{2 \sin x \cdot \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \log \frac{\cos x}{2 \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} (\log \cos x - \log \sin x - \log 2) dx$$

$$= \int_0^{\frac{\pi}{2}} \log \cos x dx - \int_0^{\frac{\pi}{2}} \log \sin x dx - \int_0^{\frac{\pi}{2}} \log 2$$

$$\text{We know that } \int_0^{\frac{\pi}{2}} \log \cos x dx = \int_0^{\frac{\pi}{2}} \log \sin x dx \quad - (i)$$

Hence from equation (i)

$$I = - \int_0^{\frac{\pi}{2}} \log 2 = - \frac{\pi}{2} \log 2$$

Definite Integrals Ex 20.4A Q12



$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(1)$$

It is known that, $\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow 2I = \int_0^a 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Definite Integrals Ex 20.4A Q13

$$\text{Let } I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx \quad \dots(i)$$

We know that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

So,

$$I = \int_0^5 \frac{\sqrt[4]{(5-x)+4}}{\sqrt[4]{(5-x)+4} + \sqrt[4]{9-(5-x)}} dx$$

$$I = \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x} + \sqrt[4]{4+x}} dx \quad \dots(ii)$$

Adding (i) & (ii)

$$2I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx + \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x} + \sqrt[4]{4+x}} dx$$

$$2I = \int_0^5 \frac{\sqrt[4]{x+4} + \sqrt[4]{9-x}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx$$

$$2I = \int_0^5 1 dx$$

$$2I = [x]_0^5$$

$$I = \frac{1}{2} [5 - 0] = \frac{5}{2}$$

$$\therefore \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx = \frac{5}{2}$$

Definite Integrals Ex 20.4A Q14



$$\text{Let } I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx \quad \text{--- (i)}$$

$$\text{We know that } \int_0^a f(x) = \int_0^a f(a-x)$$

Hence,

$$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx \quad \text{--- (ii)}$$

Adding (i) & (ii)

$$2I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$$

$$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$$

$$2I = \int_0^7 dx$$

$$2I = [x]_0^7$$

$$I = \frac{7}{2}$$

Definite Integrals Ex 20.4A Q15

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{--- (i)}$$

$$\text{We know that } \int_a^b f(x) = \int_a^b f(a+b-x) dx$$

Hence,

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{--- (ii)}$$

Adding (i) & (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$2I = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q16



$$\begin{aligned}
 I &= \int_a^b x f(x) dx \\
 I &= \int_a^b (a+b-x) f(a+b-x) dx \\
 I &= \int_a^b (a+b-x) f(x) dx \dots \dots \dots [\because f(a+b-x) = f(x)] \\
 I &= \int_a^b (a+b) f(x) dx - \int_a^b f(x) dx \\
 I &= (a+b) \int_a^b f(x) dx - I \\
 2I &= (a+b) \int_a^b f(x) dx \\
 I &= \frac{(a+b)}{2} \int_a^b f(x) dx \\
 \therefore \int_a^b x f(x) dx &= \frac{(a+b)}{2} \int_a^b f(x) dx
 \end{aligned}$$

Ex 20.4B

Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1 + \tan x} = \frac{1}{1 + \frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \quad \dots (I)$$

So,

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right] \\
 &= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \quad \dots (II)
 \end{aligned}$$

Hence, adding (I) & (II)

$$\begin{aligned}
 2I &= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \\
 &= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx \\
 &= \int_0^{\frac{\pi}{2}} dx
 \end{aligned}$$

$$\begin{aligned}
 2I &= [x]_0^{\frac{\pi}{2}} \\
 2I &= \left[\frac{\pi}{2} - 0 \right] \Rightarrow I = \frac{\pi}{4}
 \end{aligned}$$

Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1 + \cot x} = \frac{1}{1 + \frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1 + \cot x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \quad \text{--- (I)}$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \quad \text{--- (II)}$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q3



We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} = \frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\cos x}{\sin x}} + \sqrt{\frac{\sin x}{\cos x}}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} \cdot \sqrt{\cos x}} \times \frac{\sqrt{\sin x} \cdot \sqrt{\cos x}}{\cos x + \sin x} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \quad \text{--- (I)}$$

So,

$$B \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \quad \text{--- (II)}$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q4

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots (1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q5

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \quad \text{--- (i)}$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n \left(\frac{\pi}{2} - x \right)}{\sin^n \left(\frac{\pi}{2} - x \right) + \cos^n \left(\frac{\pi}{2} - x \right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} dx \quad \text{--- (II)}$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x + \cos^n x}{\sin^n x + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{--- (i)}$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos \left(\frac{\pi}{2} - x \right)}}{\sqrt{\cos \left(\frac{\pi}{2} - x \right)} + \sqrt{\sin \left(\frac{\pi}{2} - x \right)}} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{--- (ii)}$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q7



$$\text{Let } I = \int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$$

$$\begin{aligned} \text{Let } x &= a \sin \theta \\ dx &= a \cos \theta d\theta \end{aligned}$$

$$\text{Now, } x = 0 \Rightarrow \theta = 0$$

$$x = a \Rightarrow \theta = \frac{\pi}{2}$$

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{a \cos \theta d\theta}{a \sin \theta + a \cos \theta} \\ &= \int_0^{\frac{\pi}{2}} \frac{\cos \theta}{\sin \theta + \cos \theta} \quad \text{--- (i)} \end{aligned}$$

So,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right] \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin \theta}{\cos \theta + \sin \theta} \quad \text{--- (ii)} \end{aligned}$$

Adding (i) & (ii) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos \theta + \sin \theta}{\cos \theta + \sin \theta} d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta$$

$$2I = \frac{1}{2} [\theta]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q8

$$\text{Put } x = \tan \theta$$

$$\Rightarrow dx = \sec^2 \theta d\theta$$

$$\text{If } x = 0, \theta = 0$$

$$\text{If } x = \infty, \theta = \frac{\pi}{2}$$

$$\begin{aligned} \therefore I &= \int_0^{\frac{\pi}{2}} \frac{\log x}{1+x^2} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\log(\tan \theta) \sec^2 \theta d\theta}{1+\tan^2 \theta} \end{aligned}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log(\tan \theta) d\theta \quad \text{--- (i)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \tan\left(\frac{\pi}{2} - \theta\right) d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \cot(\theta) d\theta \quad \text{--- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} (\log \tan \theta + \log \cot \theta) d\theta$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 \times dx = \int_0^{\frac{\pi}{2}} 0 \times dx = 0$$

$$\Rightarrow I = 0$$

Definite Integrals Ex 20.4B Q9

Let $x = \tan \theta$

$$\Rightarrow dx = \sec^2 \theta d\theta$$

If $x = 0, \theta = 0$

If $x = 1, \theta = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log(1+\tan \theta) d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \tan \left(\frac{\pi}{4} - \theta \right) \right\} d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{1 - \tan \theta}{1 + \tan \theta} \right\} d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left(\frac{2}{1 + \tan \theta} \right) d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} (\log 2 - \log(1 + \tan \theta)) d\theta$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{4}} \log 2 \times d\theta = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

Definite Integrals Ex 20.4B Q10

$$I = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$$

Let,

$$\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow x = A(1+x^2) + (Bx+C)(1+x)$$

Equating coefficients, we get

$$A+B=0 \Rightarrow A=-B$$

$$B+C=1 \Rightarrow -2A=1$$

$$A+C=0 \Rightarrow A=-C$$

$$\therefore A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$

So,

$$I = \int_0^{\infty} \left(\frac{-\frac{1}{2}}{1+x} + \frac{1}{2} \frac{x+1}{x^2+1} \right) dx$$

$$= \int_0^{\infty} -\frac{1}{2} \frac{dx}{1+x} + \frac{1}{2} \int_0^{\infty} \frac{x}{x^2+1} dx + \frac{1}{2} \int_0^{\infty} \frac{dx}{1+x^2}$$

$$= \left[-\frac{1}{2} \log|1+x| + \frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x \right]_0^{\infty}$$

$$= 0 + 0 + \frac{\pi}{4} + 0 - 0 - 0$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q11



We have,

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$$

$$I = \int_0^{\pi} \frac{x \left(\frac{\sin x}{\cos x} \right)}{\left(\frac{1}{\cos x} \right) \left(\frac{1}{\sin x} \right)} dx$$

$$I = \int_0^{\pi} x \sin^2 x dx \quad \dots (i)$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 x dx \quad \dots (ii)$$

Add (i) and (ii), we get

$$2I = \int_0^{\pi} (\pi) \sin^2 x dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi} = \frac{\pi}{2} [\pi - 0 - 0 + 0] = \frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q12

$$\text{Let } I = \int_0^{\pi} x \sin x \cdot \cos^4 x dx \quad \dots (i)$$

So,

$$I = \int_0^{\pi} (\pi - x) \sin (\pi - x) \cdot \cos^4 (\pi - x) dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\pi} (\pi - x) \sin x \cdot \cos^4 x dx$$

$$= \int_0^{\pi} \pi \sin x \cdot \cos^4 x dx - \int_0^{\pi} x \sin x \cdot \cos^4 x dx$$

So from equation (i)

$$I = \int_0^{\pi} \pi \sin x \cdot \cos^4 x dx - I$$

$$2I = \pi \int_0^{\pi} \sin x \cdot \cos^4 x dx$$

$$\text{Let } t = \cos x$$

$$dt = -\sin x dx$$

As,

$$x = 0 \quad t = 1$$

$$x = \pi \quad t = -1$$

Hence

$$2I = \pi \int_{-1}^{+1} t^4 dt = \pi \left[\frac{t^5}{5} \right]_{-1}^{+1} = \pi \left[\frac{1}{5} + \frac{1}{5} \right]$$

$$I = \frac{\pi}{5}$$

Definite Integrals Ex 20.4B Q13

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$$\text{Let } I = \int_0^{\pi} x \sin^3 x \, dx$$

$$= \int_0^{\pi} (\pi - x) \sin^3 (\pi - x) \, dx \quad \left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx \right]$$

$$= \int_0^{\pi} \pi \sin^3 x \, dx - \int_0^{\pi} x \sin^3 x \, dx$$

$$\therefore I = \int_0^{\pi} \pi \sin^3 x \, dx - I$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \sin^3 x \, dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{3 \sin x - \sin 3x}{4} \, dx$$

$$= \frac{\pi}{4} \int_0^{\pi} (3 \sin x - \sin 3x) \, dx$$

$$= \frac{\pi}{4} \left[-3 \cos x + \frac{\cos 3x}{3} \right]_0^{\pi}$$

$$= \frac{\pi}{4} \left[\left(-3 \cos \pi + \frac{\cos 3\pi}{3} \right) - \left(-3 \cos 0 + \frac{\cos 0}{3} \right) \right]$$

$$= \frac{\pi}{4} \left[\left(3 - \frac{1}{3} \right) - \left(-3 + \frac{1}{3} \right) \right]$$

$$= \frac{\pi}{4} \left[3 - \frac{1}{3} + 3 - \frac{1}{3} \right]$$

$$= \frac{\pi}{4} \left[6 - \frac{2}{3} \right]$$

$$= \frac{\pi}{4} \times \frac{16}{3} = \frac{4\pi}{3}$$

$$\therefore I = \frac{2\pi}{3}$$

Definite Integrals Ex 20.4B Q14



We have,

$$I = \int_0^{\pi} x \log \sin x \, dx = \int_0^{\pi} (\pi - x) \log \sin(\pi - x) \, dx$$

$$I = \pi \int_0^{\pi} \log \sin(x) \, dx - \int_0^{\pi} x \log \sin x \, dx$$

$$2I = \pi \int_0^{\pi} \log \sin x \, dx$$

Since $f(x) = f(-x)$, $f(x)$ is an even function.

$$\therefore 2I = 2\pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx \quad \dots(i)$$

$$\Rightarrow I = \pi \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx \quad \dots(ii)$$

Now adding (i) & (ii) we get

$$2I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \pi \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \sin x \cdot \cos x \, dx$$

$$\Rightarrow 2I = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{2 \sin x \cdot \cos x}{2}\right) \, dx = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2x}{2}\right) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \pi \int_0^{\frac{\pi}{2}} \log 2 \, dx \quad \dots(iii)$$

$$\text{Now let } I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx$$

Putting $2x = t$ we get

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \frac{dt}{2} = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt = \frac{1}{2} \times 2 \int_0^{\frac{\pi}{2}} \log \sin t \, dt = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx = I$$

So from (iii) we get

$$2I = I - \pi \frac{\pi}{2} \log 2$$

$$I = -\frac{\pi}{2} \log 2$$

Definite Integrals Ex 20.4B Q15

$$\text{Let } I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$$

$$= \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \sin x} dx$$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$I = \int_0^{\pi} \frac{\pi \sin x}{1 + \sin x} dx - \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x} \times \frac{(1 - \sin x)}{(1 - \sin x)} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x - \sin^2 x}{1 + \sin^2 x} dx$$

$$2I = \pi \int_0^{\pi} \frac{(\sin x - \sin^2 x)}{\cos^2 x} dx$$

$$2I = \pi \int_0^{\pi} (\tan x \cdot \sec x - \tan^2 x) dx$$

$$2I = \pi \int_0^{\pi} [\tan x \cdot \sec x - (\sec^2 x - 1)] dx$$

$$2I = \pi \int_0^{\pi} (\sec x \cdot \tan x - \sec^2 x + 1) dx$$

$$2I = \pi \int_0^{\pi} (\sec x \cdot \tan x - \sec^2 x + 1) dx$$

$$2I = \pi [\sec x - \tan x + x]_0^{\pi}$$

$$2I = \pi [(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)]$$

$$2I = \pi [(-1 - 0 + \pi) - (1 - 0 + 0)]$$

$$2I = \pi (\pi - 1 - 1)$$

$$I = \frac{\pi}{2} (\pi - 2)$$

$$\therefore \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx = \pi \left(\frac{\pi}{2} - 1 \right)$$

Definite Integrals Ex 20.4B Q16

We have

$$I = \int_0^{\pi} \frac{x \, dx}{1 + \cos \alpha \sin x} \quad \text{--- (i)}$$

$$\therefore \int_0^{\pi} f(x) \, dx = \int_0^{\pi} f(\pi - x) \, dx$$

$$I = \int_0^{\pi} \frac{(\pi - x) \, dx}{1 + \cos \alpha \sin(\pi - x)} = \int_0^{\pi} \frac{(\pi - x) \, dx}{1 + \cos \alpha \sin x} \quad \text{--- (ii)}$$

Adding (i) & (ii) we get

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \cos \alpha \sin x} \, dx$$

$$\text{Substituting } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$2I = \pi \int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} \cdot 2 \cos \alpha \cdot \tan \frac{x}{2}} \, dx = \pi \int_0^{\pi} \frac{\sec^2 \frac{x}{2} \, dx}{1 - \cos^2 \alpha + \left(\cos \alpha \cdot \tan \frac{x}{2} \right)^2} \, dx$$

$$\text{Let } \tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} \, dx = dt$$

When $x = 0$ $t = 0$

$$\pi \Rightarrow t = \alpha$$

$$\begin{aligned} 2I &= \int_0^{\alpha} \frac{dt}{(1 + \cos^2 \alpha) + (\cos \alpha + t)^2} \, dx = 2\pi \cdot \frac{1}{\sqrt{1 + \cos^2 \alpha}} \left[\tan^{-1} \left(\frac{\cos \alpha + 1}{\sqrt{1 + \cos^2 \alpha}} \right) \right]_0^{\alpha} \\ &= \frac{2\pi}{\sin \alpha} \left[\frac{\pi}{2} - \tan^{-1} \cot \alpha \right] \\ &= \frac{2\pi}{\sin \alpha} \left[\cot^{-1}(\cot \alpha) \right] \\ &= \frac{2\pi}{\sin \alpha} \alpha \end{aligned}$$

$$\Rightarrow I = \frac{\pi \alpha}{\sin \alpha}$$

Definite Integrals Ex 20.4B Q17

$$\text{Let } I = \int_0^{\pi} x \cos^2 x \, dx$$

$$I = \int_0^{\pi} (\pi - x) \cos^2(\pi - x) \, dx \quad \left[\because \int_0^{\pi} f(x) \, dx = \int_0^{\pi} f(\pi - x) \, dx \right]$$

$$I = \pi \int_0^{\pi} \cos^2 x \, dx - \int_0^{\pi} x \cos^2 x \, dx$$

$$2I = \pi \int_0^{\pi} \cos^2 x \, dx$$

$$= \pi \int_0^{\pi} \left(\frac{1 + \cos 2x}{2} \right) \, dx \quad \text{Since } \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= \frac{\pi}{2} \int_0^{\pi} (1 + \cos 2x) \, dx$$

$$= \frac{\pi}{2} \left[x + \left(-\frac{\sin 2x}{2} \right) \right]_0^{\pi}$$

$$\therefore 2I = \frac{\pi}{2} \left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2} \right]$$

$$\Rightarrow 2I = \frac{\pi}{2} [\pi - 0 - 0 + 0]$$

$$I = \frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q18



$$\begin{aligned}
 I &= \int_{\pi/6}^{\pi/3} \frac{1}{1 + \cot^{3/2} x} dx \\
 I &= \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx \\
 I &= \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} \left(\frac{\pi}{2} - x \right)}{\sin^{3/2} \left(\frac{\pi}{2} - x \right) + \cos^{3/2} \left(\frac{\pi}{2} - x \right)} dx = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2} (x)}{\cos^{3/2} (x) + \sin^{3/2} (x)} dx \\
 \therefore 2I &= \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx + \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2} (x)}{\cos^{3/2} (x) + \sin^{3/2} (x)} dx \\
 2I &= \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx \\
 I &= \frac{1}{2} \int_{\pi/6}^{\pi/3} dx \\
 I &= \frac{\pi}{12}
 \end{aligned}$$

Definite Integrals Ex 20.4B Q19

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x + \cot^7 x} dx \\
 I &= \int_0^{\frac{\pi}{2}} \frac{\tan^7 \left(\frac{\pi}{2} - x \right)}{\tan^7 \left(\frac{\pi}{2} - x \right) + \cot^7 \left(\frac{\pi}{2} - x \right)} dx \\
 I &= \int_0^{\frac{\pi}{2}} \frac{\cot^7 x}{\tan^7 x + \cot^7 x} dx
 \end{aligned}$$

Hence

$$\begin{aligned}
 2I &= \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x + \cot^7 x} + \frac{\cot^7 x}{\tan^7 x + \cot^7 x} dx \\
 2I &= \int_0^{\frac{\pi}{2}} 1 dx \\
 2I &= \frac{\pi}{2} \\
 I &= \frac{\pi}{4}
 \end{aligned}$$

Definite Integrals Ex 20.4B Q20

$$\begin{aligned}
 I &= \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx \\
 I &= \int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2-x)} + \sqrt{10-(8+2-x)}} dx \\
 I &= \int_2^8 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}} dx \\
 2I &= \int_2^8 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}} + \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx \\
 2I &= \int_2^8 1 dx \\
 2I &= 6 \\
 I &= 3
 \end{aligned}$$

Definite Integrals Ex 20.4B Q21



$$\begin{aligned}\int_0^{\pi} x \sin x \cos^2 x dx &= \int_0^{\pi} (\pi - x) \sin(\pi - x) \cos^2(\pi - x) dx \\ \int_0^{\pi} x \sin x \cos^2 x dx &= \int_0^{\pi} (\pi - x) \sin x \cos^2 x dx \\ \int_0^{\pi} x \sin x \cos^2 x dx &= \int_0^{\pi} \pi \sin x \cos^2 x dx - \int_0^{\pi} x \sin x \cos^2 x dx \\ 2 \int_0^{\pi} x \sin x \cos^2 x dx &= \int_0^{\pi} \pi \sin x \cos^2 x dx \\ \int_0^{\pi} x \sin x \cos^2 x dx &= \frac{\pi}{2} \int_0^{\pi} \sin x \cos^2 x dx\end{aligned}$$

Now

$$\int_0^{\pi} \sin x \cos^2 x dx$$

Let $\cos x = t$

$$\sin x dx = -dt$$

$$-\int_1^{-1} t^2 dt$$

$$\int_{-1}^1 t^2 dt$$

$$\left\{ \frac{t^3}{3} \right\}_{-1}^1$$

$$\frac{2}{3}$$

$$\therefore \int_0^{\pi} x \sin x \cos^2 x dx = \frac{\pi}{2} \times \frac{2}{3} = \frac{\pi}{3}$$

Definite Integrals Ex 20.4B Q22

We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cdot \cos x}{\sin^4 x + \cos^4 x} dx \quad \text{--- (i)}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \cos x \cdot \sin x}{\cos^4 x + \sin^4 x} dx \quad \text{--- (ii)}$$

Adding (i) & (ii)

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x}{\cos^4 x + \sin^4 x} dx$$

$$2I = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{2 \sin x \cdot \cos x}{\cos^4 x + \sin^4 x} dx$$

$$\text{Let } t = \sin^2 x$$

$$\Rightarrow 2I = \frac{\pi}{4} \int_0^1 \frac{1}{(1-t)^2 + t^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \times 2 \left[\tan^{-1}(2t-1) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{8} \left[\frac{\pi}{4} + \frac{\pi}{4} \right] = \frac{\pi^2}{16}$$

Definite Integrals Ex 20.4B Q23



$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx$$

$$f(-x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(-x) \, dx$$

$$= - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx$$

$$\text{Here } f(x) = -f(-x)$$

Hence $f(x)$ is odd function.

So,

$$I = 0$$

Definite Integrals Ex 20.4B Q24

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \, dx = 2 \int_0^{\frac{\pi}{2}} \sin^4 x \, dx \quad [\because \sin^4 x \text{ is an even function}]$$

$$= 2 \int_0^{\frac{\pi}{2}} (\sin^2 x)^2 \, dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 2x}{2} \right)^2 \, dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 2x)^2 \, dx$$

$$= \frac{1}{2} \left[\int_0^{\frac{\pi}{2}} (1 + \cos^2 2x - 2 \cos 2x) \, dx \right]$$

$$= \frac{1}{2} \left[\int_0^{\frac{\pi}{2}} \left(1 - 2 \cos 2x + \frac{1 + \cos 4x}{2} \right) \, dx \right]$$

$$= \frac{1}{4} \left[\int_0^{\frac{\pi}{2}} (3 - 4 \cos 2x + \cos 4x) \, dx \right]$$

$$= \frac{1}{4} \left[3x - \frac{4 \sin 2x}{2} + \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} \left[\left\{ \frac{3\pi}{2} - 2 \sin \pi + \frac{1}{4} \sin 2\pi \right\} - \{0 - 0 + 0\} \right]$$

$$= \frac{1}{4} \left[\frac{3\pi}{2} - 0 + 0 \right] = \frac{1}{4} \times \frac{3\pi}{2}$$

$$= \frac{3\pi}{8}$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \, dx = \frac{3\pi}{8}$$

Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^1 \log \left(\frac{2-x}{2+x} \right) \, dx$$

$$\text{Since, } \log \left\{ \frac{2-(-x)}{2+(-x)} \right\} = -\log \left(\frac{2-x}{2+x} \right) \therefore \text{ This is an odd function.}$$

Hence,

$$I = 0$$



Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$

$\sin^2 x$ is even function.

Hence,

$$\begin{aligned} I &= 2 \int_0^{\frac{\pi}{4}} \sin^2 x \, dx = 2 \int_0^{\frac{\pi}{4}} \left(\frac{1 - \cos 2x}{2} \right) dx = \frac{2}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}} = \frac{1}{2} \left[\frac{2\pi}{4} - \sin \frac{\pi}{2} - 0 + \sin 0 \right] \\ &= \frac{1}{2} \left[\frac{2\pi}{4} - 1 \right] \\ &= \frac{\pi}{4} - \frac{1}{2} \end{aligned}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.4B Q27

$$\begin{aligned} I &= \int_0^{\pi} \log(1 - \cos x) \, dx \\ &= \int_0^{\pi} \log \left(2 \sin^2 \frac{x}{2} \right) dx \\ &= \int_0^{\pi} \log 2 \, dx + \int_0^{\pi} \log \sin^2 \frac{x}{2} \, dx \\ &= \int_0^{\pi} \log 2 \, dx + 2 \int_0^{\pi} \log \sin \frac{x}{2} \, dx \end{aligned}$$

$$I = \log 2 [x]_0^{\pi} + 4 \int_0^{\frac{\pi}{2}} \log \sin t \, dt \quad \left[\text{Put } t = \frac{x}{2} \Rightarrow dt = \frac{1}{2} dx \right]$$

$$I = \pi \log 2 + 4I_1 \quad \dots (i)$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \, dt \quad \dots (ii)$$

$$= \int_0^{\frac{\pi}{2}} \log \cos t \, dt \quad \dots (iii)$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \cdot \cos t \, dt = \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin 2t}{2} \right) dt = \int_0^{\frac{\pi}{2}} \log \sin 2t \, dt - \frac{\pi}{2} \log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2} \log 2$$

$$\Rightarrow I_1 = -\frac{\pi}{2} \log 2 \quad \dots (iv)$$

Putting the value from (iv) to (i)

$$I = \pi \log 2 + 4 \left(-\frac{\pi}{2} \log 2 \right) = \pi \log 2 - 2\pi \log 2 = -\pi \log 2$$

$$I = -\pi \log 2$$

Definite Integrals Ex 20.4B Q28



We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log \left(\frac{2 - \sin x}{2 + \sin x} \right) dx$$

$$\text{Let } f(x) = \log \left(\frac{2 - \sin x}{2 + \sin x} \right)$$

Then,

$$f(-x) = \log \left(\frac{2 - \sin(-x)}{2 + \sin(-x)} \right) = -\log \left(\frac{2 - \sin x}{2 + \sin x} \right) = -f(x)$$

Thus, $f(x)$ is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log \left(\frac{2 - \sin x}{2 + \sin x} \right) dx = 0$$

Definite Integrals Ex 20.4B Q29

$$I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$$

$$I = 0 + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx, \dots \dots \left[\because \frac{2x}{1 + \cos^2 x} \text{ is an odd function} \right]$$

$$I = 2 \int_0^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx, \dots \dots \left[\because \frac{2x \sin x}{1 + \cos^2 x} \text{ is an even function} \right]$$

$$I = 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$I = 2\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx, \dots \dots \left[\because \int_0^a x f(x) dx = \frac{a}{2} \int_0^a f(x) dx \right]$$

Put $\cos x = t$ then $-\sin x dx = dt$

$$I = -2\pi \int_1^{-1} \frac{1}{1 + t^2} dt$$

$$I = -2\pi [\tan^{-1} t]_1^{-1}$$

$$I = \pi^2$$

Definite Integrals Ex 20.4B Q30

$$I = \int_{-\pi}^{\pi} \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) d\theta$$

$$\text{Let } f(\theta) = \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right)$$

$$f(-\theta) = \log \left(\frac{a - \sin(-\theta)}{a + \sin(-\theta)} \right) = -\log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) = -f(\theta)$$

$$\therefore f(\theta) = \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) \text{ is an odd function.}$$

$$\therefore I = \int_{-\pi}^{\pi} \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) d\theta = 0$$

Definite Integrals Ex 20.4B Q31

$$I = \int_{-2}^2 \frac{3x^3 + 2|x| + 1}{x^2 + |x| + 1} dx$$

$$I = \int_{-2}^2 \frac{3x^3}{x^2 + |x| + 1} dx + \int_{-2}^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx$$

$$I = 0 + \int_{-2}^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx \dots \dots \dots \left[\because \frac{3x^3}{x^2 + |x| + 1} \text{ is an odd function} \right]$$

$$I = 2 \int_0^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx \dots \dots \dots \left[\because \frac{2|x| + 1}{x^2 + |x| + 1} \text{ is an even function} \right]$$

$$I = 2 \left[\log(x^2 + |x| + 1) \right]_0^2$$

$$I = 2 [\log(4 + 2 + 1) - \log(1)]$$

$$I = 2 \log_e(7)$$

Definite Integrals Ex 20.4B Q32

$$I = \int_{-\pi/2}^{\pi/2} \{ \sin^2(3\pi + x) + (\pi + x)^3 \} dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \{ \sin^2(2\pi + u) + (u)^3 \} du$$

$$I = \int_{-\pi/2}^{\pi/2} \{ \sin^2(u) + (u)^3 \} du$$

$$I = \left[\frac{1}{2} \left(u - \frac{1}{2} \sin(2u) \right) + \frac{u^4}{4} \right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

Definite Integrals Ex 20.4B Q33

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

$$I = \int_0^2 (2-x)\sqrt{x} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$= \int_0^2 \left\{ 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right\} dx$$

$$= \left[2 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$$

$$= \left[\frac{4}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} \right]_0^2$$

$$= \frac{4}{3} (2)^{\frac{3}{2}} - \frac{2}{5} (2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Definite Integrals Ex 20.4B Q34

$$\text{Let } I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx$$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right) dx$$

$$= \int_0^1 \log(1-x) dx - \int_0^1 \log(x) dx$$

$$\text{Applying the property, } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\text{Thus, } I = \int_0^1 \log(1-(1-x)) dx - \int_0^1 \log(x) dx$$

$$= \int_0^1 \log(1-1+x) dx - \int_0^1 \log(x) dx$$

$$= \int_0^1 \log(x) dx - \int_0^1 \log(x) dx$$

$$= 0$$

Definite Integrals Ex 20.4B Q35

$$I = \int_{-1}^1 |x \cos \pi x| dx$$

$$\text{Let } f(x) = |x \cos \pi x|$$

$$f(-x) = |-x \cos(-\pi x)| = |-x \cos(\pi x)| = |x \cos \pi x| = f(x)$$

$$\therefore I = \int_{-1}^1 |x \cos \pi x| dx = 2 \int_0^1 |x \cos \pi x| dx$$

Now,

$$f(x) = |x \cos \pi x| = \begin{cases} x \cos \pi x, & \text{if } 0 \leq x \leq \frac{1}{2} \\ -x \cos \pi x, & \text{if } \frac{1}{2} < x \leq 1 \end{cases}$$

$$\therefore I = 2 \int_0^1 |x \cos \pi x| dx$$

$$\Rightarrow I = 2 \left[\int_0^{\frac{1}{2}} x \cos \pi x dx + \int_{\frac{1}{2}}^1 -x \cos \pi x dx \right]$$

$$\Rightarrow I = 2 \left\{ \left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right]_0^{\frac{1}{2}} - \left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right]_{\frac{1}{2}}^1 \right\}$$

$$\Rightarrow I = 2 \left\{ \left[\frac{1}{2\pi} - \frac{1}{\pi^2} \right] - \left[-\frac{1}{\pi^2} - \frac{1}{2\pi} \right] \right\}$$

$$\Rightarrow I = \frac{2}{\pi}$$

Definite Integrals Ex 20.4B Q36

$$I = \int_0^{\pi} \left(\frac{x}{1 + \sin^2 x} + \cos^7 x \right) dx$$

$$I = \int_0^{\pi} \left(\frac{\pi - x}{1 + \sin^2(\pi - x)} + \cos^7(\pi - x) \right) dx$$

$$I = \int_0^{\pi} \left(\frac{\pi - x}{1 + \sin^2 x} - \cos^7 x \right) dx$$

$$2I = \int_0^{\pi} \left(\frac{\pi}{1 + \sin^2 x} \right) dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \sin^2 x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + 2 \tan^2 x} \sec^2 x dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \frac{1}{1 + 2 \tan^2 x} \sec^2 x dx, \dots \dots \dots \left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a - x) = f(x) \right]$$

Let $\tan x = v$

$$dv = \sec^2 x dx$$

$$\Rightarrow I = \pi \int_0^{\infty} \frac{1}{1 + 2v^2} dv$$

$$\Rightarrow I = \pi \left[\frac{\tan^{-1}(\sqrt{2}v)}{\sqrt{2}} \right]_0^{\infty}$$

$$\Rightarrow I = \pi \left[\frac{\pi}{2\sqrt{2}} \right]$$

$$\Rightarrow I = \frac{\pi^2}{2\sqrt{2}}$$

Definite Integrals Ex 20.4B Q37





$$I = \int_0^{\pi} \frac{x}{1 + \cos \alpha \sin x} dx$$

Then,

$$I = \int_0^{\pi} \frac{(\pi - x)}{1 + \cos \alpha \sin(\pi - x)} dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1 + \cos \alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \cos \alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1 + \tan^2\left(\frac{x}{2}\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right) + 2\cos \alpha \tan\left(\frac{x}{2}\right)\right)} dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 2\cos \alpha \tan\left(\frac{x}{2}\right) + 1} dx$$

Put $\tan\left(\frac{x}{2}\right) = t$ then $\sec^2\left(\frac{x}{2}\right) dx = 2dt$

$x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$$I = \frac{\pi}{2} \int_0^{\infty} \frac{2}{t^2 + 2t \cos \alpha + 1} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t + \cos \alpha)^2 + (1 - \cos^2 \alpha)} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t + \cos \alpha)^2 + \sin^2 \alpha} dt$$

$$I = \frac{\pi}{\sin \alpha} \left[\tan^{-1} \left(\frac{t + \cos \alpha}{\sin \alpha} \right) \right]_0^{\infty}$$

$$I = \frac{\pi \alpha}{\sin \alpha}$$

Definite Integrals Ex 20.4B Q38

We know

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

Also here

$$f(x) = f(2\pi - x)$$

So

$$I = \int_0^{2\pi} \sin^{100} x \cos^{101} x dx = 2 \int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

$$I = 2 \int_0^{\pi} \sin^{100} (\pi - x) \cos^{101} (\pi - x) dx$$

$$I = -2 \int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

Hence

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4B Q39

$$I = \int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a \sin\left(\frac{\pi}{2} - x\right) + b \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx$$

$$2I = (a+b) \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{(a+b)}{2} \int_0^{\pi/2} 1 dx$$

$$I = \frac{(a+b)\pi}{4}$$

Definite Integrals Ex 20.4B Q40

We have,

$$I = \int_0^{2a} f(x) dx$$

Then

$$I = \int_0^a f(x) dx + \int_a^{2a} f(x) dx$$

$$I = \int_0^a f(x) dx + I_1$$

$$\text{where, } I_1 = \int_a^{2a} f(x) dx$$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^a f(x) dx = \int_a^0 f(2a-t) (-dt) = - \int_a^0 f(2a-t) dt$$

$$I_1 = \int_0^a f(2a-t) dt = \int_0^a f(2a-x) dx$$

$$\therefore I = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

$$I = \int_0^a f(x) dx + \int_0^a f(x) dx = 2 \int_0^a f(x) dx \quad [f(2a-x) = f(x)]$$

Hence Proved.

Definite Integrals Ex 20.4B Q41



We have,

$$I = \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx$$

$$I = \int_0^a f(x) dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$$t = a, \quad x = a$$

$$t = 2a, \quad x = 0$$

$$I_1 = \int_0^a f(x) dx = \int_a^{2a} f(2a - t)(-dt)$$

$$= -\int_a^{2a} f(2a - t) dt$$

$$I_1 = \int_0^a f(2a - t) dt = \int_0^a f(2a - x) dx$$

$$I = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$I = \int_0^a f(x) dx + \int_0^a f(x) dx \quad [\because f(2a - x) = f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x) dx = 0$$

Definite Integrals Ex 20.4B Q42

(i) We have,

$$I = \int_{-a}^a f(x^2) dx$$

Clearly $f(x^2)$ is an even function.

So,

$$\int_{-a}^a f(x^2) dx = 2 \int_0^a f(x^2) dx$$

$$I = 2 \int_0^a f(x^2) dx$$

(ii) We have,

$$I = \int_{-a}^a xf(x^2) dx$$

Clearly, $xf(x^2)$ is odd function.

So, $I = 0$

$$\therefore \int_{-a}^a xf(x^2) dx = 0$$

Definite Integrals Ex 20.4B Q43



We have from LHS,

$$I = \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx \quad \dots(i)$$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$$\therefore \int_0^{2a} f(x) dx = - \int_a^0 f(2a - t) dt$$

$$\Rightarrow \int_0^{2a} f(x) dx = \int_0^a f(2a - t) dt$$

$$\Rightarrow \int_0^{2a} f(x) dx = \int_0^a f(2a - x) dx$$

Substituting $\int_0^{2a} f(x) dx = \int_0^a f(2a - x) dx$ in (i)

we get,

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$\Rightarrow \int_0^{2a} f(x) dx = \int_0^a \{f(x) + f(2a - x)\} dx$$

Definite Integrals Ex 20.4B Q44

$$I = \int_a^b xf(x) dx$$

$$\Rightarrow I = \int_a^b (a + b - x) f(a + b - x) dx$$

$$\Rightarrow I = \int_a^b (a + b - x) f(x) dx \dots \dots \dots [\text{Given that } f(a + b - x) = f(x)]$$

$$\Rightarrow I = \int_a^b (a + b) f(x) dx - \int_a^b xf(x) dx$$

$$\Rightarrow I = \int_a^b (a + b) f(x) dx - I$$

$$\Rightarrow 2I = \int_a^b (a + b) f(x) dx$$

$$\Rightarrow I = \frac{a+b}{2} \int_a^b f(x) dx$$

Definite Integrals Ex 20.4B Q45

We have,

$$I = \int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

Let $x = -t$ then $dx = -dt$

$x = -a \Rightarrow t = a$

$x = 0 \Rightarrow t = 0$

$$\therefore \int_{-a}^0 f(x) dx = \int_a^0 f(-t) (-dt) = - \int_a^0 f(-t) dt$$

$$\Rightarrow \int_{-a}^0 f(x) dx = \int_0^a f(-t) dt$$

$$\Rightarrow \int_{-a}^0 f(x) dx = \int_0^a f(-x) dx$$

$$\therefore \int_{-a}^a f(x) dx = \int_0^a f(-x) dx + \int_0^a f(x) dx$$

Hence,

$$\int_{-a}^a f(x) dx = \int_0^a \{f(-x) + f(x)\} dx$$

Proved

Definite Integrals Ex 20.4B Q46

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$$I = \int_0^{\pi} x f(\sin x) dx$$

$$I = \int_0^{\pi} (\pi - x) f(\sin(\pi - x)) dx$$

$$I = \int_0^{\pi} (\pi - x) f(\sin x) dx$$

$$2I = \int_0^{\pi} \pi f(\sin x) dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$$





Ex 20.5

Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 3$ and $f(x) = (x+4)$

$$h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$\Rightarrow I = \int_0^3 (x+4) dx$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [4 + (h+4) + (2h+4) + \dots + ((n-1)h+4)]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [4n + h(1+2+3+\dots+(n-1))]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h \left[4n + h \left(\frac{n(n-1)}{2} \right) \right] \quad \left[\because h \rightarrow 0 \text{ \& } h = \frac{3}{n} \Rightarrow n \rightarrow \infty \right]$$

$$\Rightarrow I = \lim_{n \rightarrow \infty} \frac{3}{n} \left[4n + \frac{3}{n} \left(\frac{n^2-1}{2} \right) \right]$$

$$\Rightarrow I = \lim_{n \rightarrow \infty} 12 + \frac{9}{2} \left(1 - \frac{1}{n} \right)$$

$$= 12 + \frac{9}{2} = \frac{33}{2}$$

$$\therefore \int_0^3 (x+4) dx = \frac{33}{2}$$

Definite Integrals Ex 20.5 Q2



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$

$$\Rightarrow h = \frac{2}{n} \quad \& \quad f(x) = x + 3$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x+3) dx \\ \Rightarrow I &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ \Rightarrow I &= \lim_{h \rightarrow 0} h [3 + (h+3) + (2h+3) + (3h+3) + \dots + (n-1)h + 3] \\ &= \lim_{h \rightarrow 0} h [3n + h(1+2+3+\dots+(n-1))] \\ &= \lim_{h \rightarrow 0} h \left[3n + h \frac{n(n-1)}{2} \right] \\ \therefore h &= \frac{2}{n} \quad \& \quad \text{if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[3n + \frac{2}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[6 + \frac{2}{n} n^2 \left(1 - \frac{1}{n} \right) \right] \\ &= 6 + 2 = 8 \\ \therefore \int_0^2 (x+3) dx &= 8 \end{aligned}$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 3$ and $f(x) = 3x - 2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_1^3 (3x-2) dx \\ \Rightarrow I &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [1 + \{3(1+h) - 2\} + \{3(1+2h) - 2\} + \dots + \{3(1+(n-1)h) - 2\}] \\ &= \lim_{h \rightarrow 0} h [n + 3h(1+2+3+\dots+(n-1))] \\ &= \lim_{h \rightarrow 0} h \left[n + 3h \frac{n(n-1)}{2} \right] \\ \therefore h &= \frac{2}{n} \quad \therefore \text{if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ \therefore \lim_{n \rightarrow \infty} \frac{2}{n} \left[n + \frac{6}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[2 + \frac{6}{n^2} n^2 \left(1 - \frac{1}{n} \right) \right] \\ &= \lim_{n \rightarrow \infty} 2 + 6 = 8 \\ \therefore \int_1^3 (3x-2) dx &= 8 \end{aligned}$$

Definite Integrals Ex 20.5 Q4



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where $h = \frac{b-a}{n}$

Here $a = -1$, $b = 1$ and $f(x) = x + 3$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_{-1}^1 (x+3) dx \\ I &= \lim_{h \rightarrow 0} h [f(-1) + f(-1+h) + f(-1+2h) + \dots + f(-1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [2 + (2+h) + (2+2h) + \dots + \{(n-1)h+2\}] \\ &= \lim_{h \rightarrow 0} h [2n + h(1+2+3+\dots)] \\ &= \lim_{h \rightarrow 0} h \left[2n + h \frac{n(n-1)}{2} \right] \quad \left[\because h = \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[2n + \frac{2}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} 4 + \frac{2n^2}{n^2} \left(1 - \frac{1}{n} \right) \\ &= 4 + 2 = 6 \end{aligned}$$

$$\therefore \int_{-1}^1 (x+3) dx = 6$$

Definite Integrals Ex 20.5 Q5

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where $h = \frac{b-a}{n}$

Here $a = 0$, $b = 5$

and $f(x) = (x+1)$

$$\therefore h = \frac{5}{n} \Rightarrow nh = 5$$

Thus, we have,

$$\begin{aligned} I &= \int_0^5 (x+1) dx \\ I &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f\{(n-1)h\}] \\ &= \lim_{h \rightarrow 0} h [1 + (h+1) + (2h+1) + \dots + \{(n-1)h+1\}] \\ &= \lim_{h \rightarrow 0} h [n + h(1+2+3+\dots+(n-1))] \\ \therefore h &= \frac{5}{n} \text{ and if } h \rightarrow 0, n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{5}{n} \left[n + \frac{5}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} 5 + \frac{25}{2n^2} n^2 \left(1 - \frac{1}{n} \right) \\ &= 5 + \frac{25}{2} \end{aligned}$$

$$\therefore \int_0^5 (x+1) dx = \frac{35}{2}$$

Definite Integrals Ex 20.5 Q6

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We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1$, $b = 3$

and $f(x) = (2x + 3)$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_1^3 (2x + 3) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [2 + 3 + \{2(1+h) + 3\} + \{2(1+2h) + 3\} + \dots + \{2(1+(n-1)h) + 3\}] \\ &= \lim_{h \rightarrow 0} h [5 + (5+2h) + (5+4h) + \dots + 5 + 2(n-1)h] \\ &= \lim_{h \rightarrow 0} h [5n + 2h(1+2+3+\dots+(n-1))] \\ \therefore h &= \frac{2}{n} \text{ and if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ \therefore \lim_{n \rightarrow \infty} \frac{2}{n} \left[5n + \frac{4n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[10 + \frac{4n(n-1)}{n^2} \right] = 14 \\ \therefore \int_1^3 (2x + 3) dx &= 14 \end{aligned}$$

Definite Integrals Ex 20.5 Q7

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 3$, $b = 5$

and $f(x) = (2-x)$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_3^5 (2-x) dx \\ &= \lim_{h \rightarrow 0} h [f(3) + f(3+h) + f(3+2h) + \dots + f(3+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [(2-3) + \{2-(3+h)\} + \{2-(3+2h)\} + \dots + \{2-(3+(n-1)h)\}] \\ &= \lim_{h \rightarrow 0} h [-1 + (-1-h) + (-1-2h) + \dots + (-1-(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [-n - h(1+2+\dots+(n-1))] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ \therefore \lim_{n \rightarrow \infty} \frac{2}{n} \left[-n - \frac{2n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} -2 - \frac{2}{n^2} n^2 \left(1 - \frac{1}{n} \right) = -2 - 2 = -4 \\ \therefore \int_3^5 (2-x) dx &= -4 \end{aligned}$$

Definite Integrals Ex 20.5 Q8



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$ and $f(x) = (x^2 + 1)$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x^2 + 1) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ &= \lim_{h \rightarrow 0} h [1 + (h^2 + 1) + (2h)^2 + 1 + \dots + \{(n-1)h\}^2 + 1] \\ &= \lim_{h \rightarrow 0} h [n + h^2 \{1 + 2^2 + 3^2 + \dots + (n-1)^2\}] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ \therefore \lim_{n \rightarrow \infty} \frac{2}{n} \left[n + \frac{4}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 2 + \frac{4}{3n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 2 + \frac{4}{3} \times 2 = \frac{14}{3} \\ \therefore \int_0^2 (x^2 + 1) dx &= \frac{14}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 2$ and $f(x) = x^2$

$$\therefore h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$\begin{aligned} I &= \int_1^2 x^2 dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [1 + (1+h)^2 + (1+2h)^2 + \dots + \{1+(n-1)h\}^2] \\ &= \lim_{h \rightarrow 0} h [1 + \{1+2h+h^2\} + \{1+2 \times 2h+2 \times 2h^2\} + \dots + \{1+2 \times (n-1)h + (1-n)^2 h^2\}] \\ &= \lim_{h \rightarrow 0} h [n + 2h \{1+2+3+\dots+(n-1)\} + h^2 \{1^2+2^2+3^2+\dots+(n-1)^2\}] \\ \therefore h &= \frac{1}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{2}{n} \frac{n(n-1)}{2} + \frac{1}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 1 + \frac{n^2}{n^2} \left(1 - \frac{1}{n} \right) + \frac{1}{6n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 1 + 1 + \frac{2}{6} = \frac{7}{3} \\ \therefore \int_1^2 x^2 dx &= \frac{7}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q10



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 2$, $b = 3$ and $f(x) = 2x^2 + 1$

$$\therefore h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$\begin{aligned} I &= \int_2^3 (2x^2 + 1) dx \\ &= \lim_{h \rightarrow 0} h [f(2) + f(2+h) + f(2+2h) + \dots + f(2+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[(2 \times 2^2 + 1) + (2(2+h)^2 + 1) + (2(2+2h)^2 + 1) + \dots + (2(2+(n-1)h)^2 + 1) \right] \\ &= \lim_{h \rightarrow 0} h \left[9n + 8h(1+2+3+\dots) + 2h^2(1^2+2^2+3^2+\dots) \right] \\ \therefore h &= \frac{1}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[9n + \frac{8n(n-1)}{2} + \frac{2n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 9 + \frac{4}{n^2} n^2 \left(1 - \frac{1}{n} \right) + \frac{1}{3n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 9 + 4 + \frac{2}{3} = \frac{41}{3} \\ \therefore \int_2^3 (2x^2 + 1) dx &= \frac{41}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 2$ and $f(x) = x^2 - 1$

$$\therefore h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$\begin{aligned} I &= \int_1^2 (x^2 - 1) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[(1^2 - 1) + ((1+h)^2 - 1) + ((1+2h)^2 - 1) + \dots + ((1+(n-1)h)^2 - 1) \right] \\ &= \lim_{h \rightarrow 0} h \left[0 + 2h(1+2+3+\dots) + h^2(1^2+2^2+3^2+\dots) \right] \\ \therefore h &= \frac{1}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{2n(n-1)}{2} + \frac{1n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^2} n^2 \left(1 - \frac{1}{n} \right) + \frac{1}{6n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 1 + \frac{2}{6} = \frac{4}{3} \\ \therefore \int_1^2 (x^2 - 1) dx &= \frac{4}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$ and $f(x) = x^2 + 4$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x^2 + 4) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f(0 + (n-1)h)] \\ &= \lim_{h \rightarrow 0} h [4(h^2 + 4) + \{(2h)^2 + 4\} + \dots + \{(n-1)h^2 + 4\}] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[4n + \frac{4}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 8 + \frac{4}{3n^2} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 8 + \frac{4 \times 2}{3} = \frac{32}{3} \end{aligned}$$

$$\therefore \int_0^2 (x^2 + 4) dx = \frac{32}{3}$$

Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 4$ and $f(x) = x^2 - x$

$$h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$\begin{aligned} I &= \int_1^4 (x^2 - x) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[\{1^2 - 1\} + \{(1+h)^2 - (1+h)\} + \{(1+2h)^2 - (1+2h)\} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[0 + \{h + h^2\} + \{2h + (2h)^2\} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[h + \{1+2+3+\dots+(n-1)\} + h^2 \{1+2^2+3^2+\dots+(n-1)^2\} \right] \\ \therefore h &= \frac{3}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\frac{3}{n} \frac{n(n-1)}{2} + \frac{9}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} \frac{9}{n^2} n^2 \left(1 - \frac{1}{n} \right) + \frac{3}{2n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= \frac{9}{2} + 3 = \frac{27}{2} \\ \therefore \int_1^4 (x^2 - x) dx &= \frac{27}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q14



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 1$ and $f(x) = 3x^2 + 5x$

$$h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$\begin{aligned} I &= \int_0^1 (3x^2 + 5x) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f(0+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [0 + (3h^2 + 5h) + (3(2h)^2 + 5(2h)) + \dots] \\ &= \lim_{h \rightarrow 0} h [3h^2 (1 + 2^2 + 3^2 + \dots + (n-1)^2) + 5h (1 + 2 + 3 + \dots + (n-1))] \\ \therefore h &= \frac{1}{n} \text{ if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{3}{n^2} \frac{n(n-1)(2n-1)}{6} + \frac{5}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3}{n^3} \frac{n^3 \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right)}{6} + \frac{5}{2n^2} n^2 \left(1 - \frac{1}{n}\right) \\ &= \frac{3 \times 2}{6} + \frac{5}{2} = \frac{7}{2} \\ \therefore \int_0^1 (3x^2 + 5x) dx &= \frac{7}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{Where } h = \frac{b-a}{n}$$

Here

$$a = 0, b = 2 \text{ and } f(x) = e^x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$\begin{aligned} I &= \int_0^2 e^x dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ &= \lim_{h \rightarrow 0} h [1 + e^h + e^{2h} + \dots + e^{(n-1)h}] \\ &= \lim_{h \rightarrow 0} h \left[\frac{(e^h)^n - 1}{e^h - 1} \right] \\ &= \lim_{h \rightarrow 0} h \left[\frac{e^{nh} - 1}{e^h - 1} \right] \\ &= \lim_{h \rightarrow 0} h \left[\frac{e^2 - 1}{e^h - 1} \right] \quad [nh = 2] \\ &= \lim_{h \rightarrow 0} \left[\frac{e^2 - 1}{\frac{e^h - 1}{h}} \right] \\ &= e^2 - 1 \end{aligned}$$

Definite Integrals Ex 20.5 Q16



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where $h = \frac{b-a}{n}$

Here, $a = a$, $b = b$ and $f(x) = e^x$

$$\therefore h = \frac{b-a}{n} \Rightarrow nh = b-a$$

Thus, we have,

$$\begin{aligned} I &= \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [e^a + e^{a+h} + e^{a+2h} + \dots + e^{a+(n-1)h}] \\ &= \lim_{h \rightarrow 0} h e^a [1 + e^h + e^{2h} + e^{3h} + \dots + e^{(n-1)h}] \\ &= \lim_{h \rightarrow 0} h e^a [1 + e^h + (e^h)^2 + (e^h)^3 + \dots + (e^h)^{n-1}] \\ &= \lim_{h \rightarrow 0} h e^a \left\{ \frac{(e^h)^n - 1}{e^h - 1} \right\} \quad \left[\because a + ar + ar^2 + \dots + ar^{n-1} = a \left\{ \frac{r^n - 1}{r - 1} \right\} \text{ if } r > 1 \right] \\ &= \lim_{h \rightarrow 0} h e^a n \left\{ \frac{e^{nh} - 1}{nh} \right\} \times \left(\frac{h}{e^h - 1} \right) \quad \left[\because \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1 \quad \& \quad nh = b-a \right] \\ \therefore \lim_{h \rightarrow 0} (e^{b-a} - 1) &= e^b - e^a \end{aligned}$$

$$\therefore \int_a^b e^x dx = e^b - e^a$$

Definite Integrals Ex 20.5 Q17

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}.$$

Since we have to find $\int_a^b \cos x dx$

We have, $f(x) = \cos x$

$$\therefore I = \int_a^b \cos x dx$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [\cos a + \cos(a+h) + \cos(a+2h) + \dots + \cos(a+(n-1)h)]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h \left[\frac{\cos\left(a + (n-1)\frac{h}{2}\right) \sin\frac{nh}{2}}{\sin\frac{h}{2}} \right] = \lim_{h \rightarrow 0} h \left[\frac{\cos\left(a + \frac{nh}{2} - \frac{h}{2}\right) \sin\frac{nh}{2}}{\sin\frac{h}{2}} \right]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h \left[\frac{\cos\left(a + \frac{b-a}{2} - \frac{h}{2}\right) \sin\left(\frac{b-a}{2}\right)}{\sin\frac{h}{2}} \right] \quad [\because nh = b-a]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} \left[\frac{\frac{h}{2}}{\sin\frac{h}{2}} \times 2 \cos\left(\frac{a+b}{2} - \frac{h}{2}\right) \sin\left(\frac{b-a}{2}\right) \right]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} \left(\frac{\frac{h}{2}}{\sin\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} 2 \cos\left(\frac{a+b}{2} - \frac{h}{2}\right) \sin\left(\frac{b-a}{2}\right) = 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{b-a}{2}\right)$$

$$\Rightarrow I = \sin b - \sin a \quad [\because 2 \cos A \sin B = \sin(A+B) - \sin(A-B)]$$

Definite Integrals Ex 20.5 Q18

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We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where $h = \frac{b-a}{n}$

Here, $a = 0$, $b = \frac{\pi}{2}$ and $f(x) = \sin x$

$$\therefore h = \frac{\frac{\pi}{2} - 0}{n} = \frac{\pi}{2n} \quad nh = \frac{2}{\pi}$$

Thus, we have,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \sin x \, dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f(0+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [\sin 0 + \sin h + \sin 2h + \dots + \sin(n-1)h] \\ &= \lim_{h \rightarrow 0} h \left[\frac{\sin\left(\frac{nh}{2} - \frac{h}{2}\right) \times \sin \frac{nh}{2}}{\sin \frac{h}{2}} \right] \\ &= \lim_{h \rightarrow 0} h \left[\frac{\sin\left(\frac{\pi}{4} - \frac{h}{2}\right) \times \sin \frac{\pi}{4}}{\sin \frac{h}{2}} \right] \\ &\left[\because \lim_{h \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \quad \therefore \lim_{h \rightarrow 0} \frac{h}{\sin \frac{h}{2}} \left[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right] \\ &= 2 \times \frac{1}{2} = 1 \\ \therefore \int_0^{\frac{\pi}{2}} \sin x \, dx &= 1 \end{aligned}$$

Definite Integrals Ex 20.5 Q19



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = \frac{\pi}{2}$ and $f(x) = \cos x$

$$\therefore h = \frac{\frac{\pi}{2} - 0}{n} = \frac{\pi}{2n} \quad nh = \frac{2}{\pi}$$

Thus, we have,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \cos x \, dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f(0+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [\cos 0 + \cos h + \cos 2h + \dots + \cos (n-1)h] \\ &= \lim_{h \rightarrow 0} h \left[\frac{\cos \left(\frac{nh}{2} - \frac{h}{2} \right) \times \cos \frac{nh}{2}}{\cos \frac{h}{2}} \right] \\ &= \lim_{h \rightarrow 0} h \left[\frac{\cos \left(\frac{\pi}{4} - \frac{h}{2} \right) \times \cos \frac{\pi}{4}}{\cos \frac{h}{2}} \right] \\ &\left[\therefore \lim_{h \rightarrow 0} \frac{\cos \theta}{\theta} = 1 \right] \quad \therefore \lim_{h \rightarrow 0} \frac{h}{\cos \frac{h}{2}} \left[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right] \\ &= 2 \times \frac{1}{2} = 1 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos x \, dx = 1$$

Definite Integrals Ex 20.5 Q20

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1$, $b = 4$ and $f(x) = 3x^2 + 2x$

$$\begin{aligned} I &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(a+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[(3+2) + \{3(1+h)^2 + 2(1+h)\} + \{3(1+2h)^2 + 2(1+2h)\} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[5 + 8h(1+2+3+\dots) + 3h^2(1+2^2+3^2+\dots) \right] \\ \therefore h &= \frac{3}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[5n + \frac{24n(n-1)}{2} + \frac{27n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 15 + \frac{36}{n^2} n^2 \left(1 - \frac{1}{n} \right) + \frac{27}{2n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 15 + 36 + 27 = 78 \\ \therefore \int_1^4 (3x^2 + 2x) dx &= 78 \end{aligned}$$

Definite Integrals Ex 20.5 Q21



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = 3x^2 - 2$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (3x^2 - 2) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f(0+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [-2 + (3h^2 - 2) + (3(2h)^2 - 2) + \dots] \\ &= \lim_{h \rightarrow 0} h [-2h + 3h^2(1 + 2^2 + 3^2 + \dots)] \\ \therefore h &= \frac{2}{n} \text{ if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[-2n + \frac{12}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} -4 + \frac{4}{n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) = -4 + 8 = 4 \\ \therefore \int_0^2 (3x^2 - 2) dx &= 4 \end{aligned}$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x^2 + 2) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(2h) + \dots + f(0+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [2 + (h^2 + 2) + ((2h)^2 + 2) + \dots] \\ &= \lim_{h \rightarrow 0} h [2h + h^2(1 + 2^2 + 3^2 + \dots + (n-1)^2)] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[2n + \frac{4}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} 4 + \frac{4}{3n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \\ &= 4 + \frac{8}{3} = \frac{20}{3} \\ \therefore \int_0^2 (x^2 + 2) dx &= \frac{20}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 4$, and $f(x) = x + e^{2x}$

$$\therefore h = \frac{4-0}{n} = \frac{4}{n}$$

$$\begin{aligned} \Rightarrow \int_0^4 (x + e^{2x}) dx &= (4-0) \lim_{n \rightarrow \infty} \frac{1}{n} [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [(0 + e^0) + (h + e^{2h}) + (2h + e^{2 \cdot 2h}) + \dots + \{(n-1)h + e^{2(n-1)h}\}] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [1 + (h + e^{2h}) + (2h + e^{4h}) + \dots + \{(n-1)h + e^{2(n-1)h}\}] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [\{h + 2h + 3h + \dots + (n-1)h\} + \{1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h}\}] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[h \{1 + 2 + \dots + (n-1)\} + \left(\frac{e^{2hn} - 1}{e^{2h} - 1} \right) \right] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{h(n-1)n}{2} + \left(\frac{e^{2hn} - 1}{e^{2h} - 1} \right) \right] \\ &= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{4}{n} \cdot \frac{(n-1)n}{2} + \left(\frac{e^8 - 1}{e^{\frac{8}{n}} - 1} \right) \right] \\ &= 4(2) + 4 \lim_{n \rightarrow \infty} \left(\frac{e^{\frac{8}{n}} - 1}{\frac{8}{n}} \right) \left(\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right) \\ &= 8 + \frac{4 \cdot (e^8 - 1)}{8} \\ &= 8 + \frac{e^8 - 1}{2} \\ &= \frac{15 + e^8}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where $h = \frac{b-a}{n}$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x^2 + x) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f((n-1)h)] \\ &= \lim_{h \rightarrow 0} h [0 + (h^2 + h) + \{(2h)^2 + 2h\} + \dots] \\ &= \lim_{h \rightarrow 0} h [\{h^2(1 + 2^2 + 3^2 + \dots + (n-1)^2)\} + h\{1 + 2 + 3 + \dots + (n-1)\}] \\ \therefore h &= \frac{2}{n} \quad \& \text{ if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[\frac{4}{n^2} \frac{n(n-1)(2n-1)}{6} + \frac{2}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4}{3n^3} n^3 \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) + \frac{2}{n^2} n^2 \left(1 - \frac{1}{n}\right) \\ &= \frac{8}{3} + 2 = \frac{14}{3} \\ \therefore \int_0^2 (x^2 + x) dx &= \frac{14}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q25



We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2x + 1$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$\begin{aligned} I &= \int_0^2 (x^2 + 2x + 1) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f(0 + (n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[1 + (h^2 + 2h + 1) + \{(2h)^2 + 2 \times 2h + 1\} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[n + h^2 (1 + 2^2 + 3^2 + \dots + (n-1)^2) + 2h (1 + 2 + 3 + \dots + (n-1)) \right] \\ \therefore h &= \frac{2}{n} \text{ if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[n + \frac{4}{n^2} \frac{n(n-1)(2n-1)}{6} + \frac{4}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} 2 + \frac{4}{3n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) + \frac{4}{n^2} n^2 \left(1 - \frac{1}{n} \right) \\ &= 2 + \frac{8}{3} + 4 = \frac{26}{3} \\ \therefore \int_0^2 (x^2 + 2x + 1) dx &= \frac{26}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q26

We have,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 3$ and $f(x) = 2x^2 + 3x + 5$

$$\therefore h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$\begin{aligned} I &= \int_0^3 (2x^2 + 3x + 5) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[5 + (2h^2 + 3h + 5) + \{2(2h)^2 + 3 \times 2h + 5\} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[5n + 2h^2 (1 + 2^2 + 3^2 + \dots + (n-1)^2) + 3h (1 + 2 + 3 + \dots + (n-1)) \right] \\ \therefore h &= \frac{3}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[5n + \frac{18}{n^2} \frac{n(n-1)(2n-1)}{6} + \frac{9}{n} \frac{n(n-1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} 15 + \frac{9}{n^3} n^3 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) + \frac{27}{2n^2} n^2 \left(1 - \frac{1}{n} \right) \\ &= 15 + 18 + \frac{27}{2} = \frac{93}{2} \\ \therefore \int_0^3 (2x^2 + 3x + 5) dx &= \frac{93}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q27

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It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a = a$, $b = b$, and $f(x) = x$

$$\begin{aligned} \therefore \int_a^b x dx &= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [a + (a+h) + \dots + (a+(n-1)h)] \\ &= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[\underset{n \text{ times}}{a + a + a + \dots + a} + (h + 2h + 3h + \dots + (n-1)h) \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [na + h(1 + 2 + 3 + \dots + (n-1))] \\ &= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[na + h \left\{ \frac{(n-1)(n)}{2} \right\} \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[na + \frac{n(n-1)h}{2} \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \frac{n}{n} \left[a + \frac{(n-1)h}{2} \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \left[a + \frac{(n-1)h}{2} \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \left[a + \frac{(n-1)(b-a)}{2n} \right] \\ &= (b-a) \lim_{n \rightarrow \infty} \left[a + \frac{\left(1 - \frac{1}{n}\right)(b-a)}{2} \right] \\ &= (b-a) \left[a + \frac{(b-a)}{2} \right] \\ &= (b-a) \left[\frac{2a + b - a}{2} \right] \\ &= \frac{(b-a)(b+a)}{2} \\ &= \frac{1}{2}(b^2 - a^2) \end{aligned}$$

Definite Integrals Ex 20.5 Q28



$$\text{Let } I = \int_0^5 (x+1) dx$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x) = (x+1)$

$$\Rightarrow h = \frac{5-0}{n} = \frac{5}{n}$$

$$\begin{aligned} \therefore \int_0^5 (x+1) dx &= (5-0) \lim_{n \rightarrow \infty} \frac{1}{n} \left[f(0) + f\left(\frac{5}{n}\right) + \dots + f\left((n-1)\frac{5}{n}\right) \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[1 + \left(\frac{5}{n} + 1\right) + \dots + \left\{ 1 + \left(\frac{5(n-1)}{n}\right) \right\} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(1 + 1 + 1 \dots 1\right) + \left[\frac{5}{n} + 2 \cdot \frac{5}{n} + 3 \cdot \frac{5}{n} + \dots + (n-1) \frac{5}{n}\right] \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5}{n} \{1 + 2 + 3 \dots (n-1)\} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5}{n} \cdot \frac{(n-1)n}{2} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5(n-1)}{2} \right] \\ &= 5 \lim_{n \rightarrow \infty} \left[1 + \frac{5}{2} \left(1 - \frac{1}{n}\right) \right] \\ &= 5 \left[1 + \frac{5}{2} \right] \\ &= 5 \left[\frac{7}{2} \right] \\ &= \frac{35}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q29



It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 2$, $b = 3$, and $f(x) = x^2$

$$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$$

$$\begin{aligned} \therefore \int_2^3 x^2 dx &= (3-2) \lim_{n \rightarrow \infty} \frac{1}{n} \left[f(2) + f\left(2 + \frac{1}{n}\right) + f\left(2 + \frac{2}{n}\right) + \dots + f\left(2 + (n-1)\frac{1}{n}\right) \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[(2)^2 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \dots + \left(2 + \frac{(n-1)}{n}\right)^2 \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[2^2 + \left\{ 2^2 + \left(\frac{1}{n}\right)^2 + 2 \cdot 2 \cdot \frac{1}{n} \right\} + \dots + \left\{ (2)^2 + \frac{(n-1)^2}{n^2} + 2 \cdot 2 \cdot \frac{(n-1)}{n} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(2^2 + \dots + 2^2 \right) + \left\{ \left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2 \right\} + 2 \cdot 2 \cdot \left\{ \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{(n-1)}{n} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{1}{n^2} \{ 1^2 + 2^2 + 3^2 + \dots + (n-1)^2 \} + \frac{4}{n} \{ 1 + 2 + \dots + (n-1) \} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{1}{n^2} \left\{ \frac{n(n-1)(2n-1)}{6} \right\} + \frac{4}{n} \left\{ \frac{n(n-1)}{2} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{n \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right)}{6} + \frac{4n-4}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[4 + \frac{1}{6} \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) + 2 - \frac{2}{n} \right] \\ &= 4 + \frac{2}{6} + 2 \\ &= \frac{19}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q30

We have

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} \sum_{k=0}^{n-1} f(a+kh)h$$

Where $h = \frac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x) = x^2 + x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$\begin{aligned} I &= \int_1^3 (x^2 + x) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[(1^2 + 1) + \{ (1+h)^2 + (1+h) \} + \{ (1+2h)^2 + (1+2h) \} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[(1^2 + (1+h)^2 + (1+2h)^2 + \dots) + \{ 1 + (1+h) + (1+2h) + \dots \} \right] \\ &= \lim_{h \rightarrow 0} h \left[(n + 2h(1+2+3+\dots)) + h^2(1+2^2+3^2+\dots) + (n + h(1+2+3+\dots)) \right] \\ &= \lim_{h \rightarrow 0} h \left[(2n + 3h(1+2+3+\dots + (n-1))) + h^2(1+2^2+3^2+\dots + (n-1)^2) \right] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[2n + \frac{9}{n} \frac{n(n-1)}{2} + \frac{9}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \frac{38}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q31



We have

$$\int_a^b f(x) = dx \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

Where $h = \frac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x) = x^2 - x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$\begin{aligned} I &= \int_0^2 (x^2 - x) dx \\ &= \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n-1)h)] \\ &= \lim_{h \rightarrow 0} h [\{(0)^2 - (0)\} + \{(h)^2 - (h)\} + \{(2h)^2 - (2h)\} + \dots] \\ &= \lim_{h \rightarrow 0} h [\{(h)^2 + (2h)^2 + \dots\} - \{(h) + (2h) + \dots\}] \\ &= \lim_{h \rightarrow 0} h [h^2 (1 + 2^2 + 3^2 + \dots + (n-1)^2) - h \{1 + 2 + 3 + \dots + (n-1)\}] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{h \rightarrow 0} h \left[\frac{9}{n^2} \frac{n(n-1)(2n-1)}{6} - \frac{9}{n} \frac{n(n-1)}{2} \right] \\ &= \frac{2}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q32

We have

$$\int_a^b f(x) = dx \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

Where $h = \frac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x) = 2x^2 + 5x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$\begin{aligned} I &= \int_1^3 (2x^2 + 5x) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h [(2+5) + \{2(1+h)^2 + 5(1+h)\} + \{2(1+2h)^2 + 5(1+2h)\} + \dots] \\ &= \lim_{h \rightarrow 0} h [7n + 9h(1+2+3+\dots) + 2h^2(1+2^2+3^2+\dots)] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{h \rightarrow 0} h \left[7n + \frac{18}{n} \frac{n(n-1)}{2} + \frac{8}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \frac{112}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q33

Given,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)],$$

$$\text{where } h = \frac{b-a}{n}$$

$$\text{Here, } f(x) = 3x^2 + 1, \quad a = 1, \quad b = 3. \text{ Therefore, } h = \frac{3-1}{n} = \frac{2}{n}$$

$$\therefore I = \int_1^3 (3x^2 + 1) dx$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [3(1)^2 + 1 + 3(1+h)^2 + 1 + 3(1+2h)^2 + 1 + \dots + 3(1+(n-1)h)^2 + 1]$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h [3n + n + 6h(1+2+3+\dots+(n-1)) + 3h^2(1^2 + 2^2 + \dots + (n-1)^2)]$$

$$\Rightarrow I = \lim_{n \rightarrow \infty} \frac{2}{n} \left[4n + \frac{12}{n} (1+2+3+\dots+(n-1)) + 3 \times \frac{4}{n^2} (1^2 + 2^2 + \dots + (n-1)^2) \right]$$

$$\Rightarrow I = \lim_{n \rightarrow \infty} \left[8 + \frac{24}{n^2} \times \frac{n(n-1)}{2} + \frac{24}{n^3} \times \frac{(n-1)(n)(2n-1)}{6} \right]$$

$$\Rightarrow I = \lim_{n \rightarrow \infty} \left[8 + 12 \left(1 - \frac{1}{n} \right) + 4 \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) \right]$$

$$\Rightarrow I = 8 + 12 + 4 \times 2 = 28$$

