

Ex 20.1

Q1

(i) $4, -2, 1, -\frac{1}{2}, \dots$

$$\frac{t_n}{t_{n-1}} = r = \text{common ratio}$$

---(i)

$$\frac{t_2}{t_1} = \frac{-2}{4} = \frac{-1}{2}$$

$$\frac{t_3}{t_2} = \frac{1}{-2} = \frac{-1}{2}$$

(ii) $\frac{-2}{3}, -6, -54, \dots$

Using (i)

$$\frac{t_2}{t_1} = \frac{-6}{\frac{-2}{3}} = \frac{18}{2} = 9$$

$$\frac{t_3}{t_2} = \frac{-54}{-6} = 9$$

$\therefore r = 9$

(iii) $a, \frac{3a^2}{4}, \frac{9a^3}{16}, \dots$

Using (i)

$$\frac{t_3}{t_2} = \frac{\frac{9a^3}{16}}{\frac{3a^2}{4}} = \frac{9a^3}{16} \times \frac{4}{3a^2} = \frac{3a}{4}$$

$$\frac{t_2}{t_1} = \frac{\frac{3a^2}{4}}{a} = \frac{3a^2}{4a}$$

$\therefore r = \frac{3}{4}a$

$$(iii) a, \frac{3a^2}{4}, \frac{9a^3}{16}, \dots$$

Using (i)

$$\frac{t_2}{t_1} = \frac{\frac{9a^3}{16}}{\frac{3a^2}{4}} = \frac{9a^3}{16} \times \frac{4}{3a^2} = \frac{3a}{4}$$

$$\frac{t_2}{t_1} = \frac{\frac{3a^2}{4}}{a} = \frac{3a^2}{4a}$$

$$\therefore r = \frac{3}{4}a$$

$$(iv) \frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \dots$$

Using (i)

$$\frac{t_2}{t_1} = \frac{\frac{2}{9}}{\frac{1}{3}} = \frac{2}{3}$$

$$\frac{t_2}{t_1} = \frac{\frac{3}{1}}{\frac{2}{1}} = \frac{3}{2}$$

$$\therefore r = \frac{2}{3}$$

Q2

$$a_n = \frac{2}{3^n}, n \in \mathbb{N}$$

Put $n = 1, 2, 3, \dots$ because n is natural number

$$\frac{2}{3}, \frac{2}{3^2}, \frac{2}{3^3}, \dots$$

$$\frac{t_3}{t_2} = \frac{\frac{2}{3^3}}{\frac{2}{3^2}} = \frac{1}{3}$$

$$\frac{t_2}{t_1} = \frac{\frac{2}{3^2}}{\frac{2}{3}} = \frac{1}{3}$$

Ratio of consecutive terms is solve

$\therefore \frac{1}{3}$ is common ratio, Hence it is G.P. $\forall n \in \mathbb{N}$.



Q3

(i) 9th term of G.P. 1, 4, 16, 64, ...

$$t_1 = 1 = a$$

$$t_2 = 4$$

Because it is G.P.

$$\frac{t_2}{t_1} = \text{common ratio} = r$$

$$r = \frac{4}{1} = 4$$

$$t_n = ar^{n-1}$$

$$t_9 = ar^{8} = 1(4)^8 = 65536$$

(ii) 10th term of G.P. $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \dots$

$$a = -\frac{3}{4}$$

Because it is G.P.

$$r = \frac{t_2}{t_1} = \frac{\frac{1}{2}}{-\frac{3}{4}} = -\frac{2}{3}$$

$$t_n = ar^{n-1}$$

$$t_{10} = ar^9 = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^9 = \frac{1}{2}\left(\frac{2}{3}\right)^8$$

(iv) 12th term of G.P. $\frac{1}{2^3 \sqrt{x}}, 2\sqrt{x}, 2^3 \sqrt{x^3}, \dots$

$$a = \frac{1}{2^3 \sqrt{x}}$$

$$r = \frac{t_2}{t_1} = \frac{t_2}{t_1} = \frac{2\sqrt{x}}{\frac{1}{2^3 \sqrt{x}}} = 2^4 \sqrt{x^4}$$

$$t_n = ar^{n-1}$$

$$t_{12} = ar^{11}$$

$$= \left(\frac{1}{2^3 \sqrt{x}}\right)\left(2^4 \sqrt{x^4}\right)^{11}$$

$$= (2x)^{11}$$

(v) 7th term of G.P. $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \dots$

$$r = \frac{t_2}{t_1} = \frac{t_2}{t_1} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$$

$$t_n = ar^{n-1}$$

$$t_7 = \left(\sqrt{3}\right)\left(\frac{1}{3}\right)^6$$

(vi) 10th term of G.P. $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \dots$

$$a = \sqrt{2}$$

$$r = \frac{t_2}{t_1} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$$

$$t_n = ar^{n-1}$$

$$t_{10} = ar^9$$

$$= \left(\sqrt{2}\right)\left(\frac{1}{2}\right)^9$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)^9$$

Q4

$$\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots, 162$$

n^{th} term from the end

$$a_n = l \left(\frac{1}{r} \right)^{n-1}$$

$$l = 162, r = \text{common ratio} = \frac{t_2}{t_1}$$

$$= \frac{\frac{2}{9}}{\frac{2}{27}} = 3$$

$$n = 4$$

$$t_4 = (162) \left(\frac{1}{3} \right)^3$$

$$= \frac{162}{27}$$

$$= 6$$

Q5

$$0.004, 0.02, 0.1, \dots \text{ is } 12.5$$

Here,

$$a = 0.004, \quad t_n = 12.5$$

$$r = \frac{t_2}{t_1} = \frac{0.02}{0.004} = 5$$

$$t_n = ar^{n-1}$$

$$12.5 = (0.004)(5)^{n-1}$$

$$\frac{12.5}{0.004} = (5)^{n-1}$$

$$\frac{125 \times 100}{4} = 5^{n-1}$$

$$5^5 = 5^{n-1}$$

$$= n - 1$$

$$n = 6$$

Q6

$$\sqrt{2}, \frac{1}{\sqrt{2}}, \dots, \frac{1}{512\sqrt{2}}$$

$$t_n = ar^{n-1}$$

$$a = \sqrt{2}, r = \frac{t_n}{t_{n-1}} = \frac{t_2}{t_1} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$$

$$t_n = \frac{1}{512\sqrt{2}}, n = ?$$

$$t_n = ar^{n-1}$$

$$\frac{1}{512\sqrt{2}} = \left(\sqrt{2}\right)\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{512 \times \sqrt{2} \times \sqrt{2}} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1}$$

$$10 = (n-1)$$

$$n = 11$$

∴ term is 11th.

Q6(i)

$$2, 2\sqrt{2}, 4, \dots \text{ is } 128$$

$$a = 2, r = \frac{t_n}{t_{n-1}} = \frac{2\sqrt{2}}{2} = \sqrt{2}, n = ?$$

$$t_n = 128$$

Also,

$$t_n = ar^{n-1}$$

$$128 = (2)\left(\sqrt{2}\right)^{n-1}$$

$$\frac{128}{2} = \left(\sqrt{2}\right)^{n-1}$$

$$64 = \left(\sqrt{2}\right)^{n-1}$$

$$(2)^6 = \left(\sqrt{2}\right)^{n-1}$$

$$\rightarrow 12 = n - 1$$

$$n = 13$$

∴ 12th term is 128.

Q6(ii)

$$\sqrt{3}, 3, 3\sqrt{3}, \dots, 729$$

$$a = \sqrt{3}, r = \frac{t_n}{t_{n-1}}, n = ?, t_n = 729$$

Now,

$$t_n = ar^{n-1}$$

$$729 = (\sqrt{3})(r)^{n-1}$$

Now,

$$r = \frac{t_2}{t_1} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$729 = (\sqrt{3})(\sqrt{3})^{n-1}$$

$$729 = (\sqrt{3})^n$$

$$(3)^6 = (\sqrt{3})^n$$

$$(\sqrt{3})^{12} = (\sqrt{3})^n$$

$$\Rightarrow n = 12$$

$\therefore$ 12th term is 729.

Q6(iii)

$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, \frac{1}{19683}$$

$$a = \frac{1}{3}, r = \frac{t_n}{t_{n-1}} = \frac{t_2}{t_1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}, t_n = \frac{1}{19683}, n = ?$$

Now,

$$t_n = ar^{n-1}$$

$$\frac{1}{19683} = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

$$\left(\frac{1}{3}\right)^9 = \left(\frac{1}{3}\right)^n$$

$$\Rightarrow n = 9$$

$\therefore$ 9th term of G.P is $\frac{1}{19683}$.

Q7

$$18, -12, 8, \dots \text{ is } \frac{512}{729}$$

$$a = 18, n = ?, t_n = \frac{512}{729}, r = \frac{t_{n-1}}{t_n}$$

$$r = \frac{t_2}{t_1} = \frac{-12}{18} = -\frac{2}{3}$$

Also,

$$t_n = ar^{n-1}$$

$$\frac{512}{729} = (18) \left(\frac{-2}{3} \right)^{n-1}$$

$$\frac{2^9}{3^6} \times \frac{1}{2 \times 3^2} = \left(\frac{-2}{3} \right)^{n-1}$$

$$\left(\frac{2}{3} \right)^8 = (-1)^{n-1} \left(\frac{2}{3} \right)^{n-1}$$

$$n = 9$$

Q8

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \dots, \frac{1}{4374}$$

$$a = \frac{1}{2}, l = \frac{1}{4374}, r = \frac{t_n}{t_{n-1}} = \frac{t_2}{t_1} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

Term from the end is

$$a_n = l \left(\frac{1}{r} \right)^{n-1}$$

$$t_4 = \left(\frac{1}{4374} \right) (3)^{n-1}$$

$$= \frac{1}{4374} \times 3^3$$

$$= \frac{1}{162}$$

$\therefore$ 4th term from the end is $\frac{1}{162}$.

Q9

$$t_4 = 27$$

$$t_7 = 729$$

We know that $t_n = ar^{n-1}$

$$t_4 = ar^3 = 27$$

$$t_7 = ar^6 = 729$$

Now,

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = r^3 = \frac{729}{27}$$

$$r^3 = \left(\frac{9}{3}\right)^3$$

$$r^3 = 3^3$$

$$r = 3$$

$$t_4 = ar^3 = 27$$

$$a(3)^3 = 27$$

$$a(27) = 27$$

$$a = 1$$

Now G.P is $a, ar, ar^2, \dots$

$$1, 3, 9, \dots$$

Q10

$$t_7 = 81$$

$$t_5 = 40$$

We know that $t_n = ar^{n-1}$

a = first term

r = common ratio

n = number of terms

$$t_7 = ar^6 = 81$$

$$r^2 = 3$$

$$r = 2$$

Also,

$$t_5 = 40$$

$$ar^4 = 40$$

$$a(2)^4 = 40$$

$$a = \frac{40}{16} = 3$$

∴ G.P is $a, ar, ar^2, \dots$

$$3, 6, 12, \dots$$

Q11

5, 10, 20, ... n term

1200, 640, 320, ... n terms.

Let t_n be the general term of first G.P and t_n' be general term of second G.P whose n th terms are equal

a for first G.P = 5

a for second G.P = 1200

$$r \text{ for first G.P} = \frac{10}{5} = 2$$

$$r \text{ for second G.P} = \frac{640}{1200} = \frac{1}{2}$$

$$t_n = ar^{n-1}$$

Applying and equating for both G.P's

$$5 \cdot 2^{n-1} = 1200 \left(\frac{1}{2}\right)^{n-1}$$

$$12^{n-1} = \frac{1200}{5} \left(\frac{1}{2}\right)^{n-1}$$

$$= 256 \left(\frac{1}{2}\right)^{n-1}$$

$$= 2^3 \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{12^{n-1}}{256} = \left(\frac{1}{2}\right)^{n-1} = 2^{n-1} = 2^{n-1}$$

$$\Rightarrow 2n = 10$$

$$n = 5$$

Q12

We have

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + ca)p + (b^2 + c^2 + d^2) \leq 0$$

$$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leq 0$$

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$$

This is only possible when

$$ap - b = 0 \Rightarrow p = \frac{b}{a}$$

$$bp - c = 0 \Rightarrow p = \frac{c}{b}$$

$$cp - d = 0 \Rightarrow p = \frac{d}{c}$$

Thus

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Hence a, b, c and d are in G.P

Q13

$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$, two show that a, b, c, d are in G.P

$\Rightarrow$ to show $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ --- (i)

Now,

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} \text{ and } \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

Cross multiplying

$$(a+bx)(b-cx) = (b+cx)(a-bx)$$

$$ab - acx + b^2x - bcx^2 = ab - b^2x + acx - bcx^2$$

Cancelling ab and $-bcx^2$ on both sides

$$-acx + b^2x = -b^2x + acx$$

$$x(b^2 - ac) = -x(b^2 - ac)$$

$$2b^2x = 2acx$$

$$2b^2 = 2ac = b^2 = ac$$

From (i) $b^2 = ac$

Also,

$$\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}, \text{ cross multiplying}$$

$$c^2x - cdx^2 + bc - bdx = bc + bdx - c^2x - cdx^2$$

$$2c^2x = 2bdx$$

From (i) $c^2 = bd$

Hence, a, b, c, d are in G.P.

Q14

We have

$$a_5 = p$$

$$a_8 = q$$

$$a_{11} = s$$

We have to show that

$$\Rightarrow \frac{q^2}{p} = \frac{s}{q}$$

$$\text{Now, } q = ar^7$$

$$p = ar^4$$

$$s = ar^{10}$$

$$\therefore \frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow \frac{ar^7}{ar^4} = \frac{ar^{10}}{ar^7}$$

$$\Rightarrow r^3 = r^3$$

Hence proved.

Q15

Let a be the first term

then $a = -3$

Now we have

$$a_4 = (a_2)^2$$

$$\Rightarrow ar^3 = (ar)^2$$

$$\Rightarrow ar^3 = a^2r^2$$

$$\Rightarrow r = a = -3$$

$$\therefore a_7 = ar^6 = (-3)^7 = -2187$$

Q16

Let the first term is a and the common ratio is r .

Then

$$ar^2 = 24 \dots\dots (1)$$

$$\text{and } ar^5 = 192 \dots\dots (2)$$

(2) $\div$ (1), we get

$$\frac{ar^5}{ar^2} = \frac{192}{24}$$

$$r^3 = 8$$

$$r = 2$$

Now

$$ar^2 = 24$$

$$a \cdot 2^2 = 24$$

$$a = 6$$

Thus the 10th term will be: $ar^9 = 6 \cdot 2^9 = 3072$

Q17

n th term of GP = ar^{n-1}

p th term = $1 = ar^{p-1}$

q th term = $9 = ar^{q-1}$

$$\frac{q}{p} = \frac{r^{q-1}}{r^{p-1}}$$

$$r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

$$a = p \left(\frac{p}{q}\right)^{\frac{1-q}{p-q}}$$

$$p+q \text{ th term} = p \left(\frac{p}{q}\right)^{\frac{1-q}{p-q}} \left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}}$$

$$= p \left(\frac{q}{p}\right)^{\frac{1-q+p+q-1}{p-q}}$$

$$= p \left(\frac{q}{p}\right)^{\frac{p}{p-q}}$$

$$= \frac{p^{\frac{p}{p-q}}}{q^{\frac{p}{p-q}}}$$

$$= \frac{p^{\frac{p}{p-q}}}{p^{\frac{p}{p-q}}}$$

$$= \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

Ex 20.2

Q1

Let the three number in G.P be $\frac{a}{r}, a, ar$

$$\text{Sum of these numbers} = \frac{a}{r} + a + ar = 65$$

3375 = Product of these numbers

$$3375 = \left(\frac{a}{r}\right)(a)(ar) = a^3$$

$$a^3 = (5)^3 \times (3)^3 = (15)^3$$

$$\Rightarrow a = 15$$

$$a\left(\frac{1}{r} + 1 + r\right) = 65$$

$$15\left(\frac{1}{r} + 1 + r\right) = \frac{65}{15} = \frac{13}{3}$$

$$\frac{1 + r + r^2}{r} = \frac{13}{3}$$

$$3 + 3r + 3r^2 = 13r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$r = 3, \frac{1}{3} \quad r = \frac{1}{3} \text{ or } r = 3$$

∴ G.P. is a, ar, ar^2

∴ G.P. is 45, 15, 5 or 5, 15, 45

Q2

Let the three numbers be a, ar, ar^2 in G.P., where a is first term and r is the common ratio.

Then,

$$a + ar + ar^2 = 38$$

$$a(1 + r + r^2) = 38 \quad \text{---(i)}$$

and

$$(a)(ar)(ar)^2 = 1728$$

$$a^3 r^3 = 1728 = 4^3 3^3 = (12)^3$$

$$a^3 = \frac{12^3}{r^3} \Rightarrow \frac{12}{r} = a$$

Putting $a = \frac{12}{r}$ in (i)

$$\frac{12}{r}(1 + r + r^2) = 38$$

$$12 + 12r + 12r^2 = 38r$$

$$12r^2 - 26r + 12 = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(3r - 3) - 2(3r - 3) = 0$$

$$r = \frac{3}{2}, \frac{2}{3}$$

$$a = \frac{12}{\frac{3}{2}} = 8 \text{ or } \frac{12}{\frac{2}{3}} = 18$$

∴ G.P. is 8, 12, 18.

Q3

Let the first three terms of G.P. are $\frac{a}{r}$, a , ar

Here,

$$\frac{a}{r} + a + ar = \frac{13}{12} \quad \text{---(i)}$$

and $\frac{a}{r} \times a \times ar = -1$

$$\Rightarrow a^3 = -1$$

$$\Rightarrow a = -1$$

Put $a = -1$ in equation (i),

$$\frac{-1}{r} + (-1) - r = \frac{13}{12}$$

$$\Rightarrow -1 - r - r^2 = \frac{13}{12}r$$

$$\Rightarrow -12 - 12r - 12r^2 = 13r$$

$$\Rightarrow 12r^2 + 12r + 13r + 12 = 0$$

$$\Rightarrow 12r^2 + 25r + 12 = 0$$

$$\Rightarrow 12r^2 + 16r + 9r + 12 = 0$$

$$\Rightarrow 4r(3r + 4) + 3(3r + 4) = 0$$

$$\Rightarrow (4r + 3)(3r + 4) = 0$$

$$r = \frac{-3}{4}, \frac{-4}{3}$$

So,

Required G.P. is, $\frac{4}{3}, -1, \frac{3}{4}, \dots$

or $\frac{3}{4}, -1, \frac{4}{3}, \dots$

Q4

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$ then product of these numbers $\left(\frac{a}{r}\right)(a)(ar)$

$$\Rightarrow a^3 = 125 = 5^3$$

$$a = 5$$

Also, sum of these products in pair

$$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar) = 87 \frac{1}{2} = \frac{195}{2}$$

$$\frac{a^2}{r} + a^2r + a^2 = a^2 \left(\frac{1}{r} + r + 1\right)$$

$$= (5)^2 \left(\frac{1+r^2+r}{r}\right) = \frac{195}{2}$$

$$1+r^2+r = \left(\frac{195}{2 \times 25}\right)r$$

$$2(1+r^2+r) = \frac{39}{5}r$$

$$10 + 10r^2 + 10r = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r-5) - 2(2r-5) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

$\therefore$ G.P. is $\frac{a}{r}, a, ar$

$$10, 5, \frac{5}{2}, \dots \text{ or } \frac{5}{2}, 5, 10, \dots$$

Q5

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$

then product of them is $\left(\frac{a}{r}\right)(a)(ar) = 21$ --- (i)

$$= \frac{a}{r}(1+r+r^2) = 21$$

and sum of their squares

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = a^2 \frac{(1+r^2+r^4)}{r^2} = 189$$
 --- (ii)

Now,

$$a(1+r+r^2) = 21r$$
 --- (iii)

Then, $a^2(1+r+r^2)^2 = 441r^2$ [squaring]

$$a^2(1+r^2+r^4) + 2a^2r(1+r+r^2) = 441r$$

$$189r^2 + 2ar \times 21r = 441r^2$$

Dividing both sides by $21r^2$

$$9 + 2a = 21$$

$$2a = 21 - 9 = 12$$

$$a = 6 \Rightarrow a = 6$$

Putting in (iii)

$$6(1+r+r^2) = 21r$$

$$6 + 6r + 6r^2 - 21r = 0$$

$$6r^2 - 15r + 6 = 0$$

$$6r^2 - 12r - 3r + 6 = 0$$

$$\Rightarrow 6r(r-2) - 3(r-2) = 0$$

$$r = 2, \frac{1}{2}$$

$\therefore$ G.P. is 3, 6, 12 or 12, 6, 3.

Q6

Let the numbers are: $\frac{a}{r}$, a and ar .

Then

$$\frac{a}{r} + a + ar = 14$$

Again the numbers $a+1$, $ar+1$ and ar^2-1 are in A.P, therefore

$$2(a+1) = (ar-1) + \left(\frac{a}{r}+1\right)$$

$$2(a+1) = ar + \frac{a}{r}$$

$$2(a+1) = 14 - a$$

$$3a = 12$$

$$a = 4$$

Now we have

$$\frac{4}{r} + 4 + 4r = 14$$

$$2 - 5r + 2r^2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2, \frac{1}{2}$$

Thus the numbers are: 2, 4, 8 or 8, 4, 2.

Q7

Let the number in G.P. are $\frac{a}{r}$, a , ar

So,

$$\frac{a}{r} \times a \times ar = 216$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

And also given,

$$\frac{a}{r} + 2, a + 8, ar + 6 \text{ are in A.P.}$$

$$2(a + 8) = \left(\frac{a}{r} + 2\right) + (ar + 6)$$

$$\Rightarrow 2(6 + 8) = \left(\frac{6 + 2r}{r}\right) + 6r + 6$$

$$\Rightarrow 28r = 6 + 2r + 6r^2 + 6r$$

$$\Rightarrow 6r^2 - 20r + 6 = 0$$

$$\Rightarrow 6r^2 - 18r - 2r + 6 = 0$$

$$\Rightarrow 6r(r - 3) - 2(r - 3) = 0$$

$$\Rightarrow (r - 3)(6r - 2) = 0$$

$$r = 3, r = \frac{1}{3}$$

So,

Required G.P. is 18, 6, 2, ...

or, 2, 6, 18, ...

Q8

Let three numbers in G.P. be $\frac{a}{r}$, a , ar

Here,

$$\frac{a}{r} \times a \times ar = 729$$

$$\Rightarrow a^3 = 729$$

$$\Rightarrow a = 9$$

And,

$$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(\frac{a}{r} \times ar\right) = 819$$

$$\Rightarrow \frac{81}{r} + 81r + 81 = 819$$

$$\Rightarrow \frac{9}{r} + 9r + 9 = 91$$

$$\Rightarrow 9 + 9r^2 - 9r - 91r$$

$$\Rightarrow 9r^2 - 81r - 81 = 0$$

$$\Rightarrow 9r^2 - 81r - 81 = 0$$

$$\Rightarrow 9r(r - 9) - 81 = 0$$

$$r = 11, \frac{1}{9}$$

So, the required G.P. are

$$81, 9, 1$$

$$\text{or, } \frac{1}{81}, \frac{1}{9}, 1$$

Q9

Let the numbers are $\frac{a}{r}$, a and ar . Then we have

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

And

$$\frac{a}{r} \cdot a \cdot ar = 1$$

$$a^3 = 1$$

$$a = 1$$

Now we have

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$1 + r + r^2 = \frac{39}{10}r$$

$$r^2 - \frac{29}{10}r + 1 = 0$$

$$10r^2 - 29r + 10 = 0$$

$$(2r - 5)(5r - 2) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

Thus the numbers are: either $\frac{2}{5}, 1, \frac{5}{2}$ or $\frac{5}{2}, 1, \frac{2}{5}$.

Ex 20.3

Q1

2, 6, 18, ... to 7 term

$$a = 2, r = \frac{6}{2} = 3, n = 7$$

$$S_n = a \frac{(r^n - 1)}{r - 1}$$

$$\begin{aligned} S_7 &= 2 \frac{(3^7 - 1)}{3 - 1} = \frac{2}{2} (3^7 - 1) \\ &= 2187 - 1 = 2186 \end{aligned}$$

1, 3, 9, 27, ... to 8 terms

$$a = 1, r = \frac{3}{1} = 3, n = 8$$

$$S_n = a \frac{(r^n - 1)}{r - 1}$$

$$S_8 = 1 \frac{(3^8 - 1)}{3 - 1} = 3280$$

$1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \dots$, 9 terms

$$a = 1, r = \frac{\frac{-1}{2}}{1} = \frac{-1}{2}, n = 9$$

$$S_n = a \frac{(r^n - 1)}{r - 1}$$

$$S_9 = 1 \frac{\left(\frac{-1}{2}\right)^9 - 1}{\frac{-1}{2} - 1}$$

$$= \frac{\frac{-1}{2} - 1}{\frac{-1}{2} - 1}$$

$$= \frac{-1 - 512}{-1 - 2}$$

$$= \frac{512}{-1 - 2}$$

$$= \frac{-513}{-3}$$

$$= \frac{171}{256}$$

$$(a^2 - b^2), (a - b), \left(\frac{a-b}{a+b}\right), \dots n \text{ terms}$$

$$a = a^2 - b^2, r = \frac{a-b}{a^2 - b^2} = \frac{1}{a+b}, n = n$$

$$S_n = a \frac{(1 - r^n)}{1 - r} \quad [\because r < 1]$$

$$\begin{aligned} S_n &= (a^2 - b^2) \frac{\left(1 - \frac{1}{(a+b)^n}\right)}{1 - \frac{1}{a+b}} \\ &= \frac{(a-b) \left((a+b)^n - 1\right)}{(a+b)^{-1} (a+b)^n (a+b) - 1} \\ &= \frac{a-b}{(a+b)^n} \frac{(a+b)^n - 1}{(a+b) - 1} \end{aligned}$$

$$4, 2, 1, \frac{1}{2}, \dots 10 \text{ terms}$$

$$a = 4, r = \frac{2}{4} = \frac{1}{2}, n = 10$$

$$\begin{aligned} S_n &= a \frac{(1 - r^n)}{1 - r} \\ &= 4 \frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}} \\ &= 8 \left(1 - \frac{1}{2^{10}}\right) \\ &= 8 \left(1 - \frac{1}{1024}\right) \end{aligned}$$

Q2

$0.15 + 0.015 + 0.0015 + \dots$ upto 8 terms

$= 15(0.1 + 0.01 + 0.001 + \dots)$ upto 8 terms

$$= 15\left(\frac{1}{10} + \frac{1}{100} + \dots\right)$$

$$r = \frac{1}{10}, a = \frac{1}{10}$$

$$Sum = 15 \left(\frac{\frac{1}{10} \left(1 - \frac{1}{10^8}\right)}{1 - \frac{1}{10}} \right)$$

$$= \frac{5}{3} \left(1 - \frac{1}{10^8}\right)$$

Here the first term of the series is $a = \sqrt{2}$ and the common ratio is $r = \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$

Thus the sum of the G.P up to 8th terms is:

$$S_8 = \frac{a(1-r^8)}{1-r} = \frac{\sqrt{2} \left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}} = 2\sqrt{2} \left(1 - \frac{1}{256}\right) = \frac{255\sqrt{2}}{128}$$

$\frac{2}{5} - \frac{1}{3} + \frac{1}{2} - \frac{3}{4} + \dots$ to 5 terms.

$$a = \frac{2}{5}, r = \frac{-1/3}{2/5} = \frac{-1}{3} \times \frac{5}{2} = \frac{-5}{6}, n = 5$$

$$S_5 = a \frac{(1-r^5)}{1-r}$$

$$= \frac{2}{5} \frac{\left(1 - \left(\frac{-5}{6}\right)^5\right)}{1 - \left(\frac{-5}{6}\right)}$$

$$= \frac{2}{5} \frac{\left(1 + \frac{243}{32}\right)}{1 + \frac{5}{6}}$$

$$= \frac{2}{5} \frac{(275)}{32} \times \frac{6}{11}$$

$$= \frac{55}{72}$$

$$\begin{aligned}
 & (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots \\
 &= \frac{1}{x-y} \{ (x^2-y^2) + (x^3-y^3) + \dots \text{to } \infty \} \dots \left[\therefore \frac{x^n-y^n}{x-y} = x^{n-1} + x^{n-2}y + \dots + y^{n-1} \right] \\
 &= \frac{1}{x-y} \{ (x^2+x^3+\dots \text{to } \infty) - (y^2+y^3+\dots \text{to } \infty) \} \\
 &= \frac{1}{x-y} \left\{ \frac{x^2}{1-x} - \frac{y^2}{1-y} \right\} \\
 &= \frac{1}{x-y} \left\{ \frac{x^2 - x^2y - y^2 + xy^2}{(1-x)(1-y)} \right\} \\
 &= \frac{x+y-xy}{(1-x)(1-y)}
 \end{aligned}$$

The series can be written as:

$$3 \left(\frac{1}{5} + \frac{1}{5^3} + \frac{1}{5^5} + \dots n \text{ terms} \right) + 4 \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots n \text{ terms} \right)$$

For the first part $a = \frac{1}{5}$ and the common ratio $r = \frac{1}{5^2} = \frac{1}{25}$

Thus the sum is:

$$\begin{aligned}
 3 \left(\frac{1}{5} + \frac{1}{5^3} + \frac{1}{5^5} + \dots n \text{ terms} \right) &= 3 \cdot \frac{\frac{1}{5} \left(1 - \left(\frac{1}{25} \right)^n \right)}{1 - \frac{1}{25}} \\
 &= \frac{5}{8} \left(1 - \frac{1}{5^{2n}} \right)
 \end{aligned}$$

For the second part $a = \frac{1}{25}$ and common ratio $r = \frac{1}{25}$ then

$$\begin{aligned}
 4 \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots n \text{ terms} \right) &= 4 \cdot \frac{\frac{1}{25} \left(1 - \left(\frac{1}{25} \right)^n \right)}{1 - \frac{1}{25}} \\
 &= \frac{1}{6} \left(1 - \frac{1}{5^{2n}} \right)
 \end{aligned}$$

Thus the sum is:

$$\frac{3}{5} + \frac{4}{5^3} + \frac{3}{5^5} + \dots 2n \text{ terms} = \frac{5}{8} \left(1 - \frac{1}{5^{2n}} \right) + \frac{1}{6} \left(1 - \frac{1}{5^{2n}} \right)$$

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \dots + \frac{a}{(1+i)^n}$$

$$a = \frac{a}{1+i}, \quad r = \frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}} = \frac{1}{1+i}$$

$$\begin{aligned} S_n &= a \frac{(1-r^n)}{1-r} \\ &= \frac{a}{1+i} \frac{\left(1 - \left(\frac{1}{1+i}\right)^n\right)}{1 - \frac{1}{1+i}} \\ &= \frac{a}{1+i} \times \frac{1+i}{(-i)} \left(1 - (1+i)^{-n}\right) \\ &= -ai \left(1 - (1+i)^{-n}\right) \end{aligned}$$

Re writing the sequence and sum we get,

$$\text{Sum} = 1 - a + a^2 - a^3 + a^4 - a^5 + \dots$$

Here, $r = -a$ and first term = 1

$$\text{Sum} = \frac{[1 - (-a)^n]}{1+a}$$

Here the first term of the G.P is $a = x^3$ and the common ratio is $r = \frac{x^5}{x^3} = x^2$

Thus the sum of the G.P is:

$$x^3 + x^5 + x^7 + \dots \text{ to } n \text{ terms} = \frac{x^3 \left((x^2)^n - 1 \right)}{x^2 - 1} = \frac{x^3 (x^{2n} - 1)}{x^2 - 1}$$

Here the first term of the G.P is $a = \sqrt{7}$ and the common ratio is $r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

Thus the sum of the G.P is:

$$\sqrt{7} + \sqrt{21} + 3\sqrt{7} + \dots \text{ to } n \text{ terms} = \frac{\sqrt{7} \left((\sqrt{3})^n - 1 \right)}{\sqrt{3} - 1} = \frac{\sqrt{7} \left(3^{\frac{n}{2}} - 1 \right)}{\sqrt{3} - 1}$$

Q3

$$\begin{aligned}
 & \sum_{n=1}^{11} (2 + 3^n) \\
 &= (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + \dots + (2 + 3^{11}) \\
 &= 2 \times 11 + 3^1 + 3^2 + 3^3 + \dots + 3^{11} \\
 &= 22 + \frac{3(3^{11} - 1)}{(3 - 1)} \\
 &= 22 + \frac{3(3^{11} - 1)}{2} \\
 &= \frac{44 + 3(177147 - 1)}{2} \\
 &= \frac{44 + 3(177146)}{2} \\
 &= 265741
 \end{aligned}$$

So,

$$\sum_{n=1}^{11} (2 + 3^n) = 265741$$

$$\sum_{k=1}^n (2^k - 3^{k-1})$$

$$\begin{aligned}
 &= (2 + 3^0) + (2^2 + 3) + (2^3 + 3^2) + \dots + (2^n + 3^{n-1}) \\
 &= (2 + 2^2 + 2^3 + \dots + 2^n) + (3^0 + 3^1 + 3^2 + \dots + 3^{n-1}) \\
 &= S_p + S_m
 \end{aligned}$$

$$S_p \rightarrow a = 2, n = n, r = \frac{2^2}{2} = 2$$

$$S_p = \frac{a(2^n - 1)}{r - 1} = \frac{2(2^n - 1)}{2 - 1} = 2(2^n - 1)$$

$$\text{Also, } S_m = S_{n-1}$$

$$a = 1, r = 3, n = n - 1$$

$$S_{n-1} = \frac{1(3^{n-1} - 1)}{3 - 1} = \frac{1}{2}(3^n - 1)$$

$$\begin{aligned}
 \therefore \sum_{k=1}^n (2^k - 3^{k-1}) &= 2(2^n - 1) + \frac{1}{2}(3^n - 1) \\
 &= \frac{1}{2}[2^{n+2} + 3^n - 4 - 1] \\
 &= \frac{1}{2}[2^{n+2} + 3^n - 5]
 \end{aligned}$$

$$\sum_{n=2}^{10} 4^n$$

$$= 4^2 + 4^3 + 4^4 + \dots + 4^{10}$$

$$a = 4^2, r = \frac{4^3}{4} = 4, n = 9$$

$$S_{10} = \frac{a(r^9 - 1)}{1 - r}$$

$$= \frac{4^2(4^9 - 1)}{4 - 1}$$

$$= \frac{1}{3}[4^{11} - 16]$$

$$= \frac{16}{3}[4^9 - 1]$$

Q4

$$5 + 55 + 555 + \dots n \text{ terms}$$

Taking 5 common from each term.

$$5[1 + 11 + 111 + \dots n \text{ terms}]$$

Dividing and multiplying by 9

$$= \frac{5}{9}[9 + 99 + 999 + \dots n \text{ terms}]$$

$$= \frac{5}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots n \text{ terms}]$$

$$= \frac{5}{9}[(10 + 10^2 + 10^3 + \dots n \text{ terms}) - n] \text{ this is G.P.}$$

$$\text{So, } S_n = \frac{a(r^n - 1)}{r - 1}$$

$$a = 10, r = 10, n = n$$

$$= \frac{5}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{5}{9 \times 9} (10^{n+1} - 10 - 9n)$$

$$= \frac{5}{81} (10^{n+1} - 9n - 10)$$



Now we have

$$\begin{aligned}
 7 + 77 + 777 + \dots \text{ to } n \text{ terms} &= 7[1 + 11 + 111 + \dots \text{ to } n \text{ terms}] \\
 &= \frac{7}{9}[9 + 99 + 999 + \dots \text{ to } n \text{ terms}] \\
 &= \frac{7}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \text{ to } n \text{ terms}] \\
 &= \frac{7}{9}[10 + 10^2 + 10^3 + \dots \text{ to } n \text{ terms}] - \frac{7}{9}(1 + 1 + 1 + \dots \text{ to } n \text{ terms}) \\
 &= \frac{7}{9} \cdot \frac{10(10^n - 1)}{10 - 1} - \frac{7n}{9} \\
 &= \frac{7}{81}(10^{n+1} - 9n - 10)
 \end{aligned}$$

$9 + 99 + 999 + \dots n \text{ term}$

This can be written as

$$\begin{aligned}
 &= (10 - 1) + (100 - 1) + (1000 - 1) + \dots n \text{ term} \\
 &= (10 + 10^2 + 10^3 + \dots n \text{ term}) - n
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow S_n &= \frac{a(r^n - 1)}{r - 1}, \quad a = 10, \quad r = 10, \quad n = n \\
 &= \frac{10(10^n - 1)}{10 - 1} - n \\
 &= \frac{10}{9}(10^n - 1) - n \\
 &= \frac{1}{9}[10^{n+1} - 10 - 9n] \\
 &= \frac{1}{9}[10^{n+1} - 9n - 10]
 \end{aligned}$$

$0.5 + 0.55 + 0.555 + \dots \text{ to } n$

$= 5 \times 0.1 + 5 \times 0.11 + 5 \times 0.111 + \dots$

$$= \frac{5}{9} \left\{ \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \right\}$$

$$= \frac{5}{9} \left\{ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \dots \right\}$$

$$= \frac{5}{9} \left\{ n \left(\frac{1}{10} + \frac{1}{10^2} + \dots + \frac{1}{10^n} \right) \right\}$$

$$= \frac{5}{9} \left[n - \frac{1}{10} \left\{ 1 - \left(\frac{1}{10} \right)^n \right\} \right]$$

$$= \frac{5}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) \right]$$

$$\begin{aligned}
 &0.6 + 0.66 + 0.666 + \dots \text{ to } n \\
 &= 6 \times 0.1 + 6 \times 0.11 + 6 \times 0.111 + \dots \\
 &= \frac{6}{9} \left\{ \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots + \dots \right\} \\
 &= \frac{6}{9} \left\{ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \dots + \dots \right\} \\
 &= \frac{6}{9} \left\{ n - \left(\frac{1}{10} + \frac{1}{10^2} + \dots + \frac{1}{10^n} \right) \right\} \\
 &= \frac{6}{9} \left[n - \frac{1}{10} \left\{ 1 - \left(\frac{1}{10} \right)^n \right\} \right] \\
 &= \frac{6}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) \right]
 \end{aligned}$$

Q5

Here,

$$3, \frac{3}{2}, \frac{3}{4}, \dots \text{ is a G.P.}$$

$$\text{and } S_n = \frac{3069}{512}, a = 3, r = \frac{1}{2}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\frac{3069}{512} = \frac{3 \left(1 - \left(\frac{1}{2} \right)^n \right)}{1 - \frac{1}{2}}$$

$$\frac{3069}{512} = \frac{3(2^n - 1)}{2^n \times \frac{1}{2}}$$

$$\frac{1023}{512} = \frac{2(2^n - 1)}{2^n}$$

$$1023 \cdot 2^n = 1024 \cdot 2^n - 1024$$

$$1024 = 2^n$$

$$\Rightarrow 2^{10} = 2^n$$

$$\Rightarrow n = 10$$

Q6

$$2 + 6 + 18 + \dots$$

$$S_n = 728$$

Now,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$a = 2, r = \frac{6}{2} = 3$$

$$728 = \frac{2(3^n - 1)}{3 - 1}$$

$$728 = \frac{2(3^n - 1)}{2} = (3^n - 1)$$

$$728 + 1 = 3^n$$

$$729 = 3^n$$

$$(3)^6 = 3^n$$

$$\Rightarrow n = 6$$

Q7

$$\sqrt{3}, 3, 3\sqrt{3}, \dots$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$a = \sqrt{3}, r = \frac{3}{\sqrt{3}} = \sqrt{3}, S_n = 39 + 13\sqrt{3}$$

Putting into formula

$$39 + 13\sqrt{3} = \frac{\sqrt{3}[(\sqrt{3})^n - 1]}{\sqrt{3} - 1}$$

$$39 + 13\sqrt{3} = \frac{(\sqrt{3})^{n+1} - \sqrt{3}}{\sqrt{3} - 1}$$

$$(39 + 13\sqrt{3})(\sqrt{3} - 1) = (\sqrt{3})^{n+1} - \sqrt{3}$$

$$39\sqrt{3} - 39 + 39 - 13\sqrt{3} = (\sqrt{3})^{n+1} - \sqrt{3}$$

$$26\sqrt{3} + \sqrt{3} = (\sqrt{3})^{n+1}$$

$$(27\sqrt{3})^1 = (\sqrt{3})^{n+1}$$

$$(\sqrt{3})^6 (\sqrt{3})^1 = (\sqrt{3})^{n+1}$$

$$7 = n + 1$$

$$\Rightarrow n = 6$$

Q8

3, 6, 12, ... n 381

$$a = 3, r = \frac{6}{3} = 2, n = ? S_n = 381$$

We know that

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$381 = \frac{3(2)^n - 1}{2 - 1}$$

$$\frac{381}{3} = 2^n - 1$$

$$127 = 2^n - 1$$

$$128 = 2^n$$

$$2^7 = 2^n$$

$$n = 7$$

Q9

$r = 3$, last term is 486

Sum of terms $s = S_n = 728$, $a = ?$

We know that

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$728 = \frac{a(3^n - 1)}{3 - 1}$$

Also, $t_n = ar^{n-1}$

$$t_n = 486$$

$$\therefore 486 = a(3)^{n-1}$$

$$a(3^{n-1}) = 3^5 \times 2$$

$$3^{n-1} = 3^5$$

$$n = 6$$

and $a = 2$

Q10

Let Sum of first three terms $= a + ar + ar^2$

$$\begin{aligned} \text{The ratio} &= \frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5} \\ &= \frac{1 + r + r^2}{1 + r + r^2 + r^3 + r^4 + r^5} \\ &= \frac{1 + r + r^2}{1 + r + r^2 + r^3(1 + r + r^2)} \quad \dots\dots\dots (1) \end{aligned}$$

$$\text{Let } A = 1 + r + r^2 \quad \dots\dots\dots (2)$$

$$\text{Ratio} = \frac{A}{A + r^3 A} = \frac{125}{152}$$

$$\frac{1}{1 + r^3} = \frac{125}{152}$$

$$152 = 125 + 125 r^3$$

$$r^3 = \frac{27}{125}$$

$$r = \frac{3}{5}$$

Q11

$$t_4 = \frac{1}{27}, t_7 = \frac{1}{729}, t_n = ar^{n-1}$$

Where $t_n = n^{\text{th}}$ term, r = common difference, n = number of terms.

$$t_4 = ar^3 = \frac{1}{27} \quad \dots\dots (i)$$

$$t_7 = ar^6 = \frac{1}{729} \quad \dots\dots (ii)$$

Dividing (ii) by (i), we get

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = r^3 = \frac{27}{729} = \frac{1}{27}, \quad r = \frac{1}{3}$$

$$\text{Sum of } n \text{ terms } = S_n = \frac{a(1 - r^n)}{1 - r} \quad \dots\dots (iii)$$

$$\text{When, } r = \frac{1}{3}, t_4 = ar^3 = \frac{1}{27}$$

$$a \left(\frac{1}{3} \right)^3 = \frac{1}{27}$$

$$a = 1$$

$$\text{Substituting } a = 1, r = \frac{1}{3}, n(i)$$

$$S_n = \frac{1 \left(1 - \left(\frac{1}{3} \right)^n \right)}{1 - \frac{1}{3}}$$

$$= \frac{1 - \left(\frac{1}{3} \right)^n}{\frac{2}{3}}$$

$$= \frac{3}{2} \left(1 - \left(\frac{1}{3} \right)^n \right)$$

Q12

$$\begin{aligned} & \sum_{n=1}^{10} \left\{ \left(\frac{1}{2} \right)^{n-1} + \left(\frac{1}{5} \right)^{n+1} \right\} \\ &= \sum_{n=1}^{10} \left(\frac{1}{2} \right)^{n-1} + \sum_{n=1}^{10} \left(\frac{1}{5} \right)^{n+1} \\ &= 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \dots \\ &= \frac{(1 - \frac{1}{2^{10}})}{1 - \frac{1}{2}} + \frac{\frac{1}{5}(1 - \frac{1}{5^{10}})}{1 - \frac{1}{5}} \\ &= \frac{2^{10} - 1}{2^9} + \frac{5^{10} - 1}{5^{11}} \end{aligned}$$

Q13

Fifth term of series is

$$ar^{4} = 81 \dots \dots \dots (1)$$

Second term of series is

$$ar = 24 \dots \dots \dots (2)$$

Dividing (2) by (1) we get,

$$\frac{ar}{ar^4} = \frac{24}{81} = \frac{8}{27}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

Substituting r in (2), we get,

$$a = \frac{24 \times 2}{3} = 16$$

$$Sum = \frac{16 \left[\left(\frac{3}{2} \right)^8 - 1 \right]}{\frac{3}{2} - 1}$$

$$= \frac{16 [3^8 - 2^8]}{2^7}$$

$$= \frac{6305}{8}$$

**Q14** S_1 = sum of n terms, S_1 = sum of $2n$ terms, S_1 = sum of $3n$ terms.Then, $S_1^2 + S_2^2$

$$\begin{aligned}
 &= (S_n)^2 + (S_{2n})^2 \\
 &= \left(\frac{a(1-r^n)}{1-r} \right)^2 + \left(\frac{a(1-r^{2n})}{1-r} \right)^2 \\
 &= \frac{a^2}{(1-r)^2} \left[(1-r^n)^2 + (1-r^{2n})^2 \right] \\
 &= \frac{a^2}{(1-r)^2} [1 + r^{2n} - 2r^n + 1 + r^{4n} - 2r^{2n}] \\
 &= \frac{a^2}{(1-r)^2} [2 - r^{2n} - 2r^n + r^{4n}] \quad \text{--- (i)}
 \end{aligned}$$

Also, $S_1(S_2 + S_3)$

$$\begin{aligned}
 &= \frac{a(1-r^n)}{1-r} \left(\frac{a(1-r^{2n})}{1-r} + \frac{a(1-r^{3n})}{1-r} \right) \\
 &= \frac{a^2}{(1-r)^2} [(1-r^n)(1-r^{2n}) + (1-r^n)(1-r^{3n})] \\
 &= \frac{a^2}{(1-r)^2} [1 - r^{2n} - r^n + r^{3n} - r^{3n} - r^n + 1 + r^{4n}] \\
 &= \frac{a^2}{(1-r)^2} [2 - r^{2n} - 2r^n + r^{4n}] \quad \text{--- (ii)}
 \end{aligned}$$

(i) = (ii) Hence, $S_1^2 + S_2^2 = S_1(S_2 + S_3)$ **Q15** $S_1, S_2, \dots, S_n$ are the sums of n terms of G.P. $a = 1, r = 1, 2, 3, \dots, n$ Then, $S_1 + S_2 + 2S_3 + 3S_4 + \dots + (n-1)S_n$

$$\begin{aligned}
 &= \frac{1(1^n-1)}{1-1} + \frac{1(2^n-1)}{2-1} + \frac{2(3^n-1)}{3-1} + \dots + (n-1)1\left(\frac{1^n-1}{1-1}\right) \\
 &= 2^n - 1 + 2 \cdot 3^n - 1 + 3 \cdot 4^n - 1 + \dots \\
 &= 2^n + 3^n + 4^n + \dots + n^n
 \end{aligned}$$

Q16

Let the G.P. be $2n, 2, 2n+4, \dots$

$$\text{Then, } S_n = \frac{a(r^n - 1)}{r - 1}, \quad a = 2n, \quad r = 2$$

$$\therefore S_n = \frac{2n(2^n - 1)}{2 - 1} = 2n^{n+1} - 2n$$

Then the G.P. of odd term

$$a_1 + a_3 + a_5 + \dots + a_{2n-1}$$

According to the question

Sum of all terms = 5 (sum of terms occupying the odd places)

$$a_1 + a_3 + a_5 + \dots + a_{2n-1} = 5(a_1 + a_3 + a_5 + \dots + a_{2n-1})$$

$$a + ar + ar^2 + \dots + ar^{2n-1} = 5(a + ar^2 + ar^4 + \dots + ar^{2n-2})$$

$$\frac{a(1 - r^{2n})}{1 - r} = 5 \left(\frac{a(1 - (r^2)^n)}{1 - r^2} \right)$$

$$\frac{a}{1 - r} \text{ is cancelled on both side}$$

$$1 - r^{2n} = \frac{5(1 - r^{2n})}{1 - r}$$

$$1 + r - r^{2n} - r^{2n+1} = 5 - 5r^{2n}$$

$$r^{2n+1} - 4r^{2n} - r + 4 = 0$$

$$r^{2n}(r - 4) - 1(r - 4) = 0$$

$$r^{2n} = 1, \quad r = 4$$

$$\Rightarrow r = 4$$

Q17

$$\text{Given } \sum_{n=1}^{100} a_{2n} = \alpha$$

$$\Rightarrow a_2 + a_4 + a_6 + \dots + a_{200} = \alpha \quad \text{---(i)}$$

$$\text{also, } \sum_{n=1}^{100} a_{2n-1} = \beta$$

$$\Rightarrow a_1 + a_3 + a_5 + \dots + a_{199} = \beta \quad \text{---(ii)}$$

Sum of G.P.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$a - ar^n = r^2, \quad n = 100$$

$$ar = ar^3 + ar^5 + \dots + ar^{199} = \alpha$$

$$ar \frac{(1 - r^{100})}{1 - r^2} = \alpha \quad \text{---(iii)}$$

$$a + ar^2 + ar^4 + \dots + ar^{198} = \beta$$

$$a \frac{(1 - (r^2)^{100})}{1 - r^2} = \beta \quad \text{---(iv)}$$

$$r(\beta) = \alpha$$

$$r = \frac{\alpha}{\beta} \quad \text{[From (v) and (vi)]}$$

Q18

Let the series be $a_1 + a_2 + a_3 + \dots + a_{2n}$

It is given that $a_1 = 1, a_2 = a, a_3 = ac, a_4 = a^2c, a_5 = a^2c^2, \dots$

$\therefore$ Sum of $2n$ term

$$a_1 + a_2 + a_3 + \dots + a_{2n}$$

$$= 1 + a + ac + a^2c + a^2c^2 + \dots + 2n \text{ term}$$

$$= (1+a) + ac(1+a) + a^2c^2(1+a) + \dots + n \text{ term}$$

$$= (1+a) \frac{(1-(ac)^n)}{1-ac}$$

$$= (a+1) \frac{((ac)^n - 1)}{ac - 1}$$

Q19

Sum of first n term of G.P.

$$= a + a_2 + a_3 + \dots + a_n$$

$$= a + ar + ar^2 + \dots + ar^{n-1}$$

$$[\because t_n = ar^{n-1}] \text{ --- (i)}$$

Also sum of term from

$$(n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term is}$$

$$= a_{n+1} + a_{n+2} + \dots + a_{2n}$$

$$= ar^n + ar^{n+1} + \dots + ar^{2n-1}$$

--- (ii)

Ratio of (i) and (ii) is

$$= \frac{a + ar + ar^2 + \dots + ar^{n-1}}{ar^n + ar^{n+1} + \dots + ar^{2n-1}}$$

$$[\because S_n = \frac{a(1-r^n)}{1-r}]$$

$$= \frac{a(1-r^n)}{1-r}$$

$$= \frac{ar^n(1-r^n)}{1-r}$$

$$= \frac{1}{r^n}$$

Q20

Given,

a, b are roots of the equation $x^2 - 3x + p = 0$

$$\Rightarrow a + b = 3, ab = p$$

and c, d are roots of the equation $x^2 - 12x + q = 0$

$$\Rightarrow c + d = 12, cd = q$$

Let $b = ar$, $c = ar^2$ and $d = ar^3$, then $a + b = 3$ and $c + d = 12$

$$a(1+r) = 3 \text{ and } ar^2(1+r) = 12$$

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$$\Rightarrow r = 2$$

$$\text{and } a(r+1) = 3$$

$$\Rightarrow a = 1$$

$$p = ab$$

$$= a \times ar$$

$$p = 2$$

$$q = cd$$

$$= ar^2 \times ar^3$$

$$= 2^5$$

$$a = 32$$

$$\frac{q+p}{q-p} = \frac{32+2}{32-2}$$

$$= \frac{34}{30}$$

$$= \frac{17}{15}$$

$$(q+p) : (q-p) = 17 : 15$$

Q21

$$\text{Sum} = \frac{3069}{512} = \frac{3(1 - \frac{1}{2^n})}{\frac{1}{2}}$$

$$1 - \frac{1}{2^n} = \frac{3069}{512 \times 6} = \frac{1023}{512 \times 2}$$

$$1 - \frac{1023}{1024} = \frac{1}{2^n}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$$n = 10$$



Q22

To find number of ancestors, we will find the sum of $2, 2^2, 2^3, \dots$

$$\text{Number of ancestors} = \frac{2(2^{10} - 1)}{2 - 1}$$

$$= 2(1024 - 1)$$

$$= 2 \times 1023$$

$$= 2046$$



Ex 20.4

Q1

$$S_{\infty} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots$$

$$\Rightarrow a = 1, r = -\frac{1}{3}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1 + \frac{1}{3}}$$

$$S_{\infty} = \frac{3}{4}$$

$$S_{\infty} = 0 - 4\sqrt{2} - 4 + \dots$$

$$\Rightarrow a = 0, r = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{0}{1 - \frac{1}{\sqrt{2}}}$$

$$= \frac{0\sqrt{2}}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

$$= \frac{0(2+\sqrt{2})}{2-1}$$

$$S_{\infty} = 0(2+\sqrt{2})$$

$$S_{\infty} = \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{1}{5^4} + \dots$$

$$= \left(\frac{2}{5} + \frac{2}{5^3} + \dots \right) + \left(\frac{3}{5^2} + \frac{1}{5^4} + \dots \right)$$

$$S_{\infty} = S'_{\infty} + S''_{\infty}$$

For

$$S'_{\infty} = \frac{2}{1-r}$$

$$= \frac{\frac{2}{5}}{1 - \frac{1}{5}}$$

$$= \frac{2}{5} \times \frac{5}{24}$$

$$S'_{\infty} = \frac{5}{12}$$

$$S''_{\infty} = \frac{3}{1 - \frac{1}{5}}$$

$$= \frac{3}{5} \times \frac{5}{24}$$

$$= \frac{3}{24}$$

$$S_{\infty} = S'_{\infty} + S''_{\infty}$$

$$= \frac{5}{12} + \frac{3}{24}$$

$$= \frac{13}{24}$$

$$S_{\infty} = \frac{13}{24}$$

Wondershare
PDFelement

MillionStars.edu
 Think, Learn & Practice



This infinite G.P has first term $a = 10$ and common ratio $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinite G.P will be:

$$\begin{aligned} 10 - 9 + 8.9 - 7.29 + \dots \infty &= \frac{a}{1-r} \quad [\text{Since } |r| < 1] \\ &= \frac{10}{1 - (-0.9)} \\ &= \frac{10}{1.9} \\ &= \frac{100}{19} \end{aligned}$$

The G.P can be written as follows:

$$\begin{aligned} \frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \dots \infty &= \left(\frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \dots \infty \right) + \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots \infty \right) \\ &= \frac{\frac{1}{3}}{1 - \frac{1}{3^2}} + \frac{\frac{1}{5^2}}{1 - \frac{1}{5^2}} \\ &= \frac{\frac{1}{3}}{\frac{8}{9}} + \frac{\frac{1}{25}}{\frac{24}{25}} \\ &= \frac{10}{24} \\ &= \frac{5}{12} \end{aligned}$$

Q2

$$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \dots \infty$$

$$= 9^{\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty\right)}$$

$$= 9^{\left(\frac{\frac{1}{3}}{1 - \frac{1}{3}}\right)}$$

$$\left[\text{Using } S_{\infty} = \frac{a}{1-r} \right]$$

$$= 9^{\left(\frac{1}{3} \times \frac{3}{2}\right)}$$

$$= 9^{\frac{1}{2}}$$

$$= 3$$

So,

$$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \dots \infty = 3$$

MillionStars edu
Think, Learn & Practice

Q3

$$\begin{aligned}
 & 2^{\frac{1}{4}}, 4^{\frac{1}{8}}, 8^{\frac{1}{16}}, 16^{\frac{1}{32}}, \dots, \infty \\
 &= 2^{\frac{1}{4}}, 2^{\frac{2}{8}}, 2^{\frac{3}{16}}, 2^{\frac{4}{32}}, \dots, \infty \\
 &= \left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots, \infty \right) \\
 &= 2 \\
 &= 2^5 \dots \dots \dots (1) \\
 S &= \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots, \infty \\
 S &= \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots, \infty \right) 2 \\
 \frac{S}{2} &= \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots, \infty \\
 &= \frac{\frac{1}{4}}{1 - \frac{1}{2}} \\
 &= \frac{1}{4} \times \frac{2}{1} \\
 S &= \frac{1}{2} \\
 S &= 1
 \end{aligned}$$

Thus $2^{\frac{1}{4}}, 4^{\frac{1}{8}}, 8^{\frac{1}{16}}, 16^{\frac{1}{32}}, \dots, \infty = 2^1 = 2$

Q4

$$S_p = 1 + r^p + r^{2p} + \dots + \infty$$

$$S_p = \frac{1}{1 - r^p}$$

$$s_p = 1 - r^p + r^{2p} + \dots + \infty$$

$$s_p = \frac{1}{1 + r^p}$$

Now,

$$S_p + s_p = \frac{1}{1 - r^p} + \frac{1}{1 + r^p}$$

$$= \frac{2}{1 - r^{2p}}$$

$$S_p + s_p = 2 \times S_{2p}$$

Q5

Here,

$$a = 4$$

$$A_3 - a_5 = \frac{31}{81}$$

$$ar^2 - ar^4 = \frac{32}{81}$$

$$r^2 4(1 - r^2) = \frac{32}{81}$$

$$r^2(1 - r^2) = \frac{8}{81}$$

Let $r^2 = A$

$$A(1 - A) = \frac{8}{81}$$

$$A - A^2 = \frac{8}{81}$$

$$81A - 81A^2 = 8$$

$$81A^2 - 81A + 8 = 0$$

$$A = \frac{81 \pm \sqrt{(81)^2 - 4 \times 81 \times 8}}{81 \times 2}$$

$$= \frac{81 \pm \sqrt{6561 - 2592}}{162}$$

$$= \frac{81 \pm \sqrt{3969}}{162}$$

$$= \frac{81 \pm 63}{162}$$

$$= \frac{81 + 63}{162} \text{ or } \frac{81 - 63}{162}$$

$$= \frac{144}{162} \text{ or } \frac{18}{162}$$

$$r^2 = \frac{8}{9} \text{ or } \frac{1}{9}$$

$$r = \pm \frac{2\sqrt{2}}{3} \text{ or } \pm \frac{1}{3}$$

Since it is a decreasing G.P.

$$r = \frac{2\sqrt{2}}{3}, \frac{1}{3}$$

$$S_{\infty} = \frac{4}{1 - \frac{2\sqrt{2}}{3}} \text{ and } S_{\infty} = \frac{4}{1 - \frac{1}{3}}$$

$$S_{\infty} = \frac{12}{3 - 2\sqrt{2}}, 6$$

Q6

$$a = 1$$

$$a_n = a_{n+1} + a_{n+2} + a_{n+3} + \dots$$

$$ar^{n-1} = ar^n + ar^{n+1} + ar^{n+2} + \dots$$

$$ar^{n-1} = ar^n (1 + r + r^2 + \dots \infty)$$

$$1 = r \left(\frac{1}{1-r} \right)$$

$$1 - r = r$$

$$1 = 2r$$

$$r = \frac{1}{2}$$

$$\text{G.P. is } 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$

Q7

$$a + ar = 5$$

$$a(1+r) = 5 \dots \dots (1)$$

$$a_n = 3(a_{n+1} + a_{n+2} + a_{n+3} + \dots)$$

$$ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + \dots)$$

$$ar^{n-1} = 3ar^n (1 + r + r^2 + \dots \infty)$$

$$1 = 3r \left(\frac{1}{1-r} \right)$$

$$1 - r = 3r$$

$$1 = 4r$$

$$r = \frac{1}{4}$$

$$a(1+r) = 5$$

$$a \left(\frac{5}{4} \right) = 5$$

$$a = 4$$

$$\text{G.P. is } 4, 1, \frac{1}{4}, \frac{1}{16}, \dots$$



Q8

$$\begin{aligned}0.125125125\dots\dots\dots &= 0.\overline{125} \\&= 0.125 + 0.000125 + 0.000000125 + \dots\dots \\&= \frac{125}{10^3} + \frac{125}{10^6} + \frac{125}{10^9} + \dots\dots \\&= \frac{125}{10^3} \left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \dots\dots \right) \\&= \frac{125}{10^3} \left(\frac{1}{1 - \frac{1}{1000}} \right) \\&= \frac{125}{1000} \left(\frac{1000}{999} \right) \\0.125125125\dots\dots\dots &= \frac{125}{999}\end{aligned}$$

Q9

$$\begin{aligned}0.4\overline{23} &= 0.4 + 0.0232323\dots\dots\dots \\&= 0.4 + 0.023 + 0.00023 + \dots\dots\dots \\&= 0.4 + \frac{23}{10^3} + \frac{23}{10^5} + \dots\dots\dots \\&= 0.4 + \frac{23}{10^3} \left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \dots\dots\dots \right) \\&= 0.4 + \frac{23}{1000} \left(\frac{1}{1 - \frac{1}{100}} \right) \\&= 0.4 + \frac{23}{1000} \left(\frac{100}{99} \right) \\&= \frac{4}{10} + \frac{23}{990} \\&= \frac{396 + 23}{990} \\0.4\overline{23} &= \frac{419}{990}\end{aligned}$$

Q10

Let a be first term and r be common ratio of G.P. Here,

$$\begin{aligned}\frac{a_n}{(a_{n+1} + a_{n+2} + \dots \infty)} &= \frac{ar^{n-1}}{ar^n + ar^{n+1} + \dots} \\&= \frac{ar^{n-1}}{ar^n (1 + r + r^2 + \dots \infty)} \\&= \frac{ar^{n-1}}{ar^n \left(\frac{1}{1-r} \right)} \\&= \left(\frac{1-r}{r} \right)\end{aligned}$$

Since r is a constant, so

$$\left(\frac{a_n}{a_{n+1} + a_{n+2} + \dots \infty} \right) = k \text{ (constant)}$$

Such that $k = \left(\frac{1-r}{r} \right)$

Q11

$$\begin{aligned}0.\bar{3} &= 0.3333\dots \\&= 0.3 + 0.03 + 0.003 + \dots \\&= \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \dots \\&= \frac{3}{10} \left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots \right) \\&= \frac{3}{10} \left(\frac{1}{1 - \frac{1}{10}} \right) \\&= \frac{3}{10} \times \frac{10}{9} \\&= \frac{3}{9} \\0.\bar{3} &= \frac{1}{3}\end{aligned}$$



$$\begin{aligned}
 0.\overline{231} &= 0.231231231... \\
 &= 0.231 + 0.000231 + 0.000000231 + \dots \\
 &= \frac{231}{10^3} + \frac{231}{10^6} + \frac{231}{10^9} + \dots \\
 &= \frac{231}{10^3} \left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \dots \right) \\
 &= \frac{231}{1000} \left(\frac{1}{1 - \frac{1}{1000}} \right) \\
 0.\overline{231} &= \frac{231}{999}
 \end{aligned}$$

$$\begin{aligned}
 3.5\overline{2} &= 3 + 0.52222... \\
 &= 3 + 0.5 + 0.02 + 0.002 + 0.0002 + \dots \\
 &= 3 + 1 \left(\frac{2}{10^2} + \frac{2}{10^3} + \frac{2}{10^4} + \dots \right) \\
 &= 3 + 1 \left(\frac{2}{10^2} \left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots \right) \right) \\
 &= \frac{36}{10} + \frac{2}{100} \left(\frac{1}{1 - \frac{1}{10}} \right) \\
 &= \frac{36}{10} + \frac{2}{100} \times \left(\frac{10}{9} \right) \\
 &= \frac{36}{10} + \frac{2}{90} \\
 &= \frac{315 + 2}{90} \\
 3.5\overline{2} &= \frac{317}{90}
 \end{aligned}$$

The rational number can be written as:

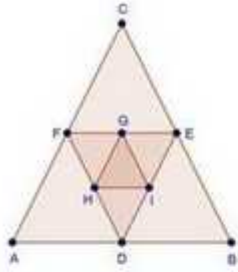
$$\begin{aligned}
 0.\overline{68} &= 0.6 + 0.08 + 0.008 + 0.0008 + \dots \\
 &= \frac{6}{10} + 8 \left[0.01 + 0.001 + 0.0001 + \dots \right] \\
 &= \frac{6}{10} + 8 \left[\frac{1}{100} + \frac{1}{1000} + \dots \right]
 \end{aligned}$$

This is an infinite GP with first term $\frac{1}{100}$ and common ratio $\frac{1}{10}$

$$\begin{aligned}
 &= \frac{6}{10} + 8 \cdot \frac{1}{100} \cdot \frac{1}{1 - \frac{1}{10}} \\
 &= \frac{6}{10} + \frac{4}{45} \\
 &= \frac{31}{45}
 \end{aligned}$$



Q12



Side of triangle = 18 cm.

$$AD = BD = 9 \text{ cm.}$$

$$DE = BD = 9 \text{ cm.}$$

$$GI = IF = \frac{9}{2} \text{ cm.}$$

Sides of the triangles are 18, 9, $\frac{9}{2}$

(i) sum of perimeters of the equilateral triangle = $\left(54 + 27 + \frac{27}{2} + \dots \right)$

$$= \frac{54}{1 - \frac{1}{2}}$$

$$= 54 \times 2$$

Perimeter = 108 cm.

(ii) sum of area of equilateral triangle

$$= \left[\frac{\sqrt{3}}{4} (18)^2 + \frac{\sqrt{3}}{4} (9)^2 + \frac{\sqrt{3}}{4} \left(\frac{9}{2} \right)^2 + \dots \right]$$

$$= \frac{\sqrt{3}}{4} \left[324 + 81 + \frac{81}{4} + \dots \right]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{324}{1 - \frac{1}{4}} \right]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{324 \times 4}{3} \right]$$

$$= \sqrt{3} (108)$$

Q13

$$S = a + ar + ar^2 + ar^3 + \dots$$

$$S = \frac{a}{1-r} \quad \text{--- (1)}$$

$$S_1 = a^2 + a^2r^2 + a^2r^4 + a^2r^6 + \dots$$

$$S_1 = \frac{a^2}{1-r^2} \quad \text{--- (2)}$$

$$S^2 = \frac{a^2}{(1-r)^2}$$

$$S^2 = \frac{S_1(1-r^2)}{(1-r^2)}$$

$$(1-r)S^2 = S_1(1+r)$$

$$S^2 - S^2r = S_1 + S_1r$$

$$S_1r + S^2r = S^2 - S_1$$

$$r = \frac{S^2 - S_1}{S_1 + S^2}$$

Put r in equation (1)

$$S(1-r) = a$$

$$a = S \left[1 - \frac{S^2 - S_1}{S^2 + S_1} \right]$$

$$a = S \left[\frac{S^2 + S_1 - S^2 + S_1}{S^2 + S_1} \right]$$

$$a = \frac{2SS_1}{S^2 + S_1}$$

Ex 20.5

Q1

Here,

a, b, c are in G.P.

$$b^2 = ac \quad \text{---(i)}$$

Now,

$$2 \log b = \log b^2$$

$$= \log ac$$

$$2 \log b = \log a + \log c$$

$$\log b - \log a = \log c - \log b$$

$$\Rightarrow \log a, \log b, \log c \text{ are in A.P.}$$

Q2

Here,

a, b, c are in G.P., so

$$b^2 = ac$$

$$\frac{2}{\log_b m} = 2 \log_m b$$

$$= \log_m b^2$$

$$= \log_m ac$$

$$= \log_m a + \log_m c$$

$$\frac{2}{\log_b m} = \frac{1}{\log_a m} + \frac{1}{\log_c m}$$

$$\Rightarrow \frac{1}{\log_b m} - \frac{1}{\log_a m} = \frac{1}{\log_c m} - \frac{1}{\log_b m}$$

$$\Rightarrow \frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m} \text{ are in A.P.}$$

Q3

Here,

a, b, c are in A.P.

$$2b = a + c \quad \text{--- (i)}$$

and a, b, d are in G.P., so

$$b^2 = ad \quad \text{--- (ii)}$$

Now,

$$\begin{aligned} (a - b)^2 &= a^2 + b^2 - 2ab \\ &= a^2 + ad - a(a + c) \end{aligned}$$

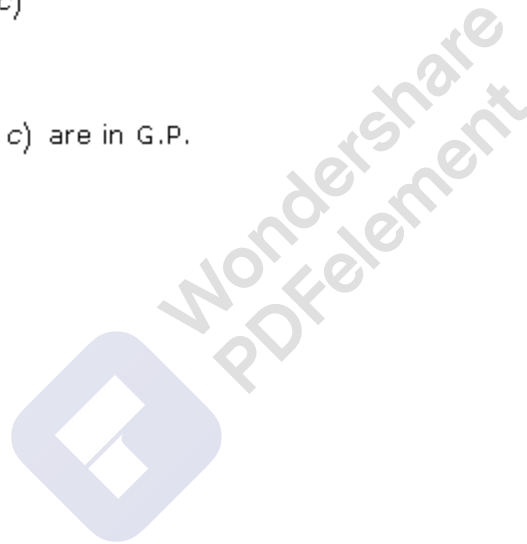
Using equation (i) and (ii)

$$\begin{aligned} &= a^2 + ad - a^2 - ac \\ &= ad - ac \end{aligned}$$

$$(a - b)^2 = a(d - c)$$

$$\frac{(a - b)}{a} = \frac{(d - c)}{(a - b)}$$

$\Rightarrow a, (a - b), (d - c)$ are in G.P.



Q4

Here, Let R be common ratio,

a_p, a_q, a_r, a_s of AP are in GP

$$\begin{aligned} R &= \frac{a_q}{a_p} = \frac{a_r}{a_q} \\ &= \frac{a_q - a_r}{a_p - a_q} \quad (\text{Ratio property}) \\ &= \frac{[a + (q-1)d] - [a + (r-1)d]}{[a + (p-1)d] - [a + (q-1)d]} \\ &= \frac{(q-r)d}{(p-q)d} \\ R &= \frac{q-r}{p-q} \text{----- (1)} \end{aligned}$$

Now,

$$\begin{aligned} R &= \frac{a_r}{a_q} = \frac{a_s}{a_r} \\ &= \frac{a_r - a_s}{a_q - a_r} \quad (\text{Ratio property}) \\ &= \frac{[a + (r-1)d] - [a + (s-1)d]}{[a + (q-1)d] - [a + (r-1)d]} \\ &= \frac{(r-s)d}{(q-r)d} \\ R &= \frac{r-s}{q-r} \text{----- (2)} \end{aligned}$$

From equation as (1) and (2)

$$\frac{q-r}{p-q} = \frac{r-s}{p-r}$$

$\Rightarrow (p-q), (q-r), (r-s)$ are in GP

Q5

$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

$$\frac{2}{2b} = \frac{1}{(a+b)} + \frac{1}{(b+c)}$$

$$\frac{1}{b} = \frac{b+c+a+b}{(a+b)(b+c)}$$

$$\frac{1}{b} = \frac{2b+c+a}{ab+ac+b^2+bc}$$

$$ab+ac+b^2+bc = 2b^2+bc+ba$$

$$b^2+ac = 2b^2$$

$$b^2 = ac$$

So,

a, b, c are in G.P.

Q6

$$x^a = x^{\frac{b}{2}} z^{\frac{b}{2}} = z^c = \lambda \text{ (say)}$$

$$x = \lambda^{\frac{1}{a}}, z = \lambda^{\frac{1}{c}}$$

$$x^{\frac{b}{2}} \times z^{\frac{b}{2}} = \lambda$$

$$\lambda^{\frac{1}{a} \left(\frac{b}{2} \right)} \times \lambda^{\frac{b}{2} \times \frac{1}{c}} = \lambda$$

$$\lambda^{\frac{b}{2a} + \frac{b}{2c}} = \lambda^1$$

$$\frac{b}{2a} + \frac{b}{2c} = 1$$

$$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

Q7

$k + 9, k - 6, 4$ are in G.P.

$$(k - 6)^2 = (k + 9)4$$

$$k^2 + 36 - 12k = 4k + 36$$

$$k^2 - 16k = 0$$

$$k(k - 16) = 0$$

$$k = 0, k = 16$$

Q8

Let $a - d, a, a + d$ be numbers in A.P.

Here,

$$a - d + a + a + d = 15$$

$$3a = 15$$

$$a = 5$$

Find

$[(5 - d) + 1], (5 + 3), [(5 + d) + 9]$ are in G.P.

$\Rightarrow (6 - d), 8, (14 + d)$ are in G.P.

$$(8)^2 = (6 - d)(14 + d)$$

$$64 = 84 + 6d - 14d - d^2$$

$$d^2 + 8d - 20 = 0$$

$$(d + 10)(d - 2) = 0$$

$$d = 2, -10$$

So,

Numbers are 3, 5, 7 or 15, 5, -5

Q9

Let three numbers in A.P. be $a - d, a, a + d$

Then,

$$a - d + a + a + d = 21$$

$$3a = 21$$

$$a = 7$$

And

$(7 - d)^2, (7 - 1), (7 + d)^2$ are in G.P.

$(7 - d)^2, 6, (7 + d)^2$ are in G.P.

$$(6)^2 = (7 - d)(7 + d)$$

$$36 = 49 - d^2 - 49 + d^2$$

$$d^2 + d^2 - 20 = 0$$

$$(d - 2)(d - 2) = 0$$

$$d = 2, -2$$

So,

Numbers are 5, 7, 9 or 13, 7, 1.

Q10

Here,

a, b, c are in A.P.

Let $a = A - d$, $b = A$, $c = A + d$

Here,

$$a + b + c = 18$$

$$A - d + A + A + d = 18$$

$$3A = 18$$

$$A = 6$$

And,

$(a + 4)$, $(b + 4)$, $(c + 36)$ are in G.P.

$(6 - d + 4)$, $(6 + 4)$, $(6 + d + 36)$ are in G.P.

$(10 - d)$, (10) , $(42 + d)$ are in G.P.

$$(10)^2 = (10 - d)(42 + d)$$

$$100 = 420 + 10d - 42d - d^2$$

$$d^2 + 32d - 320 = 0$$

$$(d + 40)(d - 8) = 0$$

$$d = -40, 8$$

So,

Numbers of $-2, 6, 14$ or $46, 6, -34$.

Q11

Let numbers are a, ar, ar^2

$$a + ar + ar^2 = 56 \text{ ----- (1)}$$

$(a-1), (ar-7), (ar^2-21)$ are in AP

$$\Rightarrow 2(ar-7) = a-1 + ar^2 - 21$$

$$= (ar^2 + a) - 22$$

$$2ar - 14 = (56 - ar) - 22$$

[using equation (1)]

$$2ar - 14 = 34 - ar$$

$$3ar = 48$$

$$ar = 16 \text{ ----- (2)}$$

$$a = \frac{16}{r}$$

Put a in equation (1),

$$\frac{16 + 16r + 16r^2}{r} = 56$$

$$16 + 16r + 16r^2 = 56r$$

$$16r^2 - 40r + 16 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2, \frac{1}{2}$$

Put r in equation (2),

$$ar = 16$$

$$\text{for } r = \frac{2}{a} = 8$$

$$\text{for } r = \frac{1}{2}, a = 32$$

thus, there numbers are

$$8, 16, 32$$

in both cases.

Q12

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$a(b^2 + c^2) - c(a^2 + b^2)$$

$$a(a^2r^2 + a^2r^4) - ar^2(a^2 + a^2r^2)$$

$$a^3r^2(1 + r^2) - a^3r^2(1 + r^2)$$

$$\text{LHS} = \text{RHS}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\text{LHS} = a^2b^2c^2 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} \right)$$

$$= a^2 \times a^2r^2 \times a^2r^4 \left(\frac{1}{a^3} + \frac{1}{a^3r^3} + \frac{1}{a^3r^6} \right)$$

$$= a^6r^6 \left(\frac{r^6 + r^3 + 1}{a^3r^6} \right)$$

$$= a^3(r^6 + r^3 + 1)$$

$$= a^3 + a^3r^3 + a^3r^6$$

$$= a^3 + (ar)^3 + (ar^2)^3$$

$$= a^3 + b^3 + c^3$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\text{LHS} = \frac{(a + b + c)^2}{a^2 + b^2 + c^2}$$

$$= \frac{(a + ar + ar^2)^2}{a^2 + a^2r^2 + a^2r^4}$$

$$= \frac{a^2(1 + r + r^2)^2}{a^2(1 + r^2 + r^4)}$$

$$= \frac{a^2(1 + r + r^2)^2}{a^2[(1 - 2r^2 + r^4) - r^2]}$$

$$= \frac{a^2(1 + r + r^2)^2}{a^2[(1 - r^2 - r)(1 + r^2 + r)]}$$

$$= \frac{a(1 + r + r^2)}{a(1 + r^2 - r)}$$

$$= \frac{a - ar + ar^2}{a - ar^2 - ar}$$

$$= \frac{a - b + c}{a - b + c}$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned} \text{LHS} &= \frac{1}{a^2 - b^2} + \frac{1}{b^2} \\ &= \frac{1}{a^2 - a^2r^2} + \frac{1}{a^2r^2} \\ &= \frac{1}{a^2} \left[\frac{1}{1-r^2} + \frac{1}{r^2} \right] \\ &= \frac{1}{a^2} \left[\frac{r^2 + 1 - r^2}{(1-r^2)r^2} \right] \\ &= \frac{1}{a^2} \left[\frac{1}{r^2 - r^4} \right] \\ &= \frac{1}{(ar)^2 - (ar^2)^2} \\ &= \frac{1}{b^2 - c^2} \\ &= \text{RHS} \\ \therefore \text{LHS} &= \text{RHS} \end{aligned}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned} \text{LHS} &= (a + 2b + 2c)(a - 2b + 2c) \\ &= (a + 2ar + 2ar^2)(a - 2ar + 2ar^2) \\ &= a^2(1 + 2r + 2r^2)(1 - 2r + 2r^2) \\ &= a^2[(1 + 2r^2)^2 - (2r)^2] \\ &= a^2[1 + 4r^4 + 4r^2 - 4r^2] \\ &= a^2[1 + 4r^4] \\ &= a^2 + 4(ar^2)^2 \\ &= a^2 + 4c^2 \\ &= \text{RHS} \\ \therefore \text{LHS} &= \text{RHS} \end{aligned}$$



Q13

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$$

$$\frac{a(ar) - (ar^2)(ar^3)}{a^2r^2 - a^2r^4} = \frac{a + ar^2}{ar}$$

$$\frac{a^2r - a^2r^5}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{a^2r(1 - r^4)}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{1 + r^2}{r} = \frac{1 + r^2}{r}$$

$$\text{LHS} = \text{RHS}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$$

$$\Rightarrow (a + ar + ar^2 + ar^3)^2 = (a + ar)^2 + 2(ar + ar^2)^2 + (ar^2 + ar^3)^2$$

$$\Rightarrow a^2(1 + r + r^2 + r^3)^2 = a^2[(1 + r)^2 + 2(r + r^2)^2 + (r^2 + r^3)^2]$$

$$\Rightarrow (1 + r + r^2 + r^3)^2 = 1 + r^2 + 2r + 2(r^2 + r^4 + 2r^3) + r^4 + r^6 + 2r^5$$

$$\Rightarrow (1 + r + r^2 + r^3 + r + r^2 + r^3 + r^4 + r^2 + r^3 + r^4 + r^5 + r^3 + r^4 + r^5 + r^6) \\ = (1 + r^2 + 2r + 2r^2 + 2r^4 + 4r^3 + r^4 + r^6 + 2r^5)$$

$$\Rightarrow (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1) = (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1)$$

$$\text{LHS} = \text{RHS}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$(b + c)(b + d) = (c + a)(c + d)$$

$$\Rightarrow (ar + ar^2)(ar + ar^3) = (ar^2 + a)(ar^2 + ar^3)$$

$$\Rightarrow a^2(r + r^2)(r + r^3) = a^2(r^2 + 1)(r^2 + r^3)$$

$$\Rightarrow r^2(1 + r)(1 + r^2) = r^2(1 + r^2)(1 + r)$$

$$\therefore \text{LHS} = \text{RHS}$$



$$\Rightarrow a, b, c \text{ are in G.P.} \quad \text{---(i)}$$

$$b^2 = ac$$

$$(b^2)^2 = (ac)^2$$

$$\Rightarrow a^2, b^2, c^2 \text{ are in G.P.}$$

$$a, b, c \text{ are in G.P.}$$

$$a, b = ar, c = ar^2$$

$$(b^3)^2 = a^3 c^3$$

$$((ar)^3)^2 = a^3 (ar^2)^3$$

$$a^6 r^6 = a^3 (a^3 r^6)$$

$$a^6 r^6 = a^6 r^6$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (b^3)^2 = a^3 c^3$$

So,

$$a^3, b^3, c^3 \text{ are in G.P.}$$

$$a, b, c \text{ are in G.P.}$$

$$a, b = ar, c = ar^2$$

$$(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$$

$$(a \times ar + ar \times ar^2)^2 = (a^2 + (ar)^2)((ar)^2 + (ar^2)^2)$$

$$(a^2 r + a^2 r^3)^2 = (a^2 + a^2 r^2)(a^2 r^2 + a^2 r^4)$$

$$a^4 (r + r^3)^2 = a^4 (1 + r^2)(r^2 + r^4)$$

$$a^4 r^2 (1 + r^2)^2 = a^4 (1 + r^2) r^2 (1 + r^2)$$

$$a^4 r^2 (1 + r^2)^2 = a^4 r^2 (1 + r^2)^2$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$$

$$\Rightarrow (a^2 + b^2), (ab + bc), (b^2 + c^2) \text{ are in G.P.}$$

Q14

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$$

$$(a^2r^2 + a^2r^4)^2 = (a^2 + a^2r^2)(a^2r^4 + a^2r^6)$$

$$a^4(r^2 + r^4)^2 = a^2(1 + r^2)a^2r^4(1 + r^2)$$

$$a^4r^4(1 + r^2)^2 = a^4r^4(1 + r^2)^2$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$$

$$\Rightarrow (a^2 + b^2), (b^2 + c^2), (c^2 + d^2) \text{ are in G.P.}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$$

$$(a^2r^2 - a^2r^4)^2 = (a^2 - a^2r^2)(a^2r^4 - a^2r^6)$$

$$a^4(r^2 - r^4)^2 = a^2(1 - r^2)a^2r^4(1 - r^2)$$

$$a^4r^4(1 - r^2)^2 = a^4r^4(1 - r^2)^2$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$$

$$\Rightarrow (a^2 - b^2), (b^2 - c^2), (c^2 - d^2) \text{ are in G.P.}$$



a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$\left(\frac{1}{b^2 + c^2}\right)^2 = \left(\frac{1}{a^2 + b^2}\right)\left(\frac{1}{c^2 + d^2}\right)$$

$$\left(\frac{1}{a^2r^2 + a^2r^4}\right)^2 = \left(\frac{1}{a^2 + a^2r^2}\right)\left(\frac{1}{a^2r^4 + a^2r^6}\right)$$

$$\frac{1}{a^4(r^2 + r^4)^2} = \frac{1}{a^2(1 + r^2)} \times \frac{1}{a^2(r^4 + r^6)}$$

$$\frac{1}{a^4r^4(1 + r^2)^2} = \frac{1}{a^2r^4(1 + r^2)(1 + r^2)}$$

$$\frac{1}{a^4r^4(1 + r^2)^2} = \frac{1}{a^2r^4(1 + r^2)^2}$$

LHS = RHS

$$\Rightarrow \left(\frac{1}{b^2 + c^2}\right)^2 = \left(\frac{1}{a^2 + b^2}\right)\left(\frac{1}{c^2 + d^2}\right)$$

$$\Rightarrow \left(\frac{1}{a^2 + b^2}\right), \left(\frac{1}{b^2 + c^2}\right), \left(\frac{1}{c^2 + d^2}\right) \text{ are in G.P.}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$

$$(a^2r + a^2r^3 + a^2r^5)^2 = (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$$

$$a^4(r + r^3 + r^5)^2 = a^2(1 + r^2 + r^4)a^2r^2(1 + r^2 + r^4)$$

$$a^4r^2(1 + r^2 + r^4)^2 = a^4r^2(1 + r^2 + r^4)^2$$

LHS = RHS

$$\Rightarrow (ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$

$$\Rightarrow (a^2 + b^2 + c^2), (ab + bc + cd), (b^2 + c^2 + d^2) \text{ are in G.P.}$$

Q15

$(a-b), (b-c), (c-a)$ are in G.P.

$$(b-c)^2 = (a-b)(c-a)$$

$$b^2 + c^2 - 2bc = ac - a^2 - bc + ab$$

$$b^2 + c^2 + a^2 = ac + bc + ab \quad \text{---(i)}$$

Now,

$$\begin{aligned} (a+b+c)^2 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ &= ac + bc + ab + 2ab + 2bc + 2ca \end{aligned}$$

Using equation (i)

$$= 3ab + 3bc + 3ca$$

$$(a+b+c)^2 = 3(ab+bc+ca)$$

Q16

$$a, b, c \text{ are in A.P.} \quad \Rightarrow 2b = a + c \quad \text{---(i)}$$

$$b, c, d \text{ are in G.P.} \quad \Rightarrow c^2 = bd \quad \text{---(ii)}$$

$$\frac{1}{c}, \frac{1}{d}, \frac{1}{e} \text{ are in A.P.} \quad \Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e} \quad \text{---(iv)}$$

We need to prove that

a, b, c are in G.P.

$$\Rightarrow c^2 = ae$$

Now,

$$c^2 = bd = 2b \times \frac{d}{2}$$

$$\Rightarrow c^2 = (a+c) \times \frac{ce}{c+e} \quad \left[\because \frac{2}{d} = \frac{e+c}{ce} \right]$$

$$\Rightarrow c^2 = \frac{(a+c)ce}{c+e}$$

$$\Rightarrow c^2(c+e) = ace + c^2e$$

$$\Rightarrow c^3 + c^2e = ace + c^2e$$

$$\Rightarrow c^3 = ace$$

$$\Rightarrow c^2 = ae$$

Hence proved.

Q17

a, b, c are in A.P.

$$\Rightarrow 2b = a + c \text{ ----- (1)}$$

a, x, b are in GP

$$\Rightarrow x^2 = ab \text{ ----- (2)}$$

b, y, c are in G.P.

$$\Rightarrow y^2 = bc \text{ ----- (3)}$$

Now

$$2b^2 = x^2 + y^2$$

$$= (ab) + (bc) \quad [\text{Using (2) and (3)}]$$

$$2b^2 = b(a + c)$$

$$2b^2 = b(2b) \quad [\text{Using (1)}]$$

$$2b^2 = 2b^2$$

$$LHS = RHS$$

$$\Rightarrow 2b^2 = x^2 + y^2$$

$$\Rightarrow x^2, b^2, y^2 \text{ are in A.P.}$$

Q18

a, b, c are in A.P.

$$\Rightarrow 2b = a + c \text{ ----- (1)}$$

a, b, d are in GP

$$\Rightarrow b^2 = ad \text{ ----- (2)}$$

Now

$$(a - b)^2 = a(d - c)$$

[Using (2)]

$$a^2 - 2ab = -ac$$

$$a^2 - 2ab = ab - ac$$

$$a(a - b) = a(b - c)$$

$$a - b = a - c$$

$$2b = a + c$$

$$a + c = a + c,$$

[Using equation (1)]

$$LHS = RHS$$

$$\Rightarrow a, (a - b), (d - c) \text{ are in G.P.}$$

Q19

a, b, c are in G.P.

$$a, b = ar, \quad c = ar^2$$

$$\frac{a^2 + ab + b^2}{bc + ca + ab} = \frac{b + a}{c + b}$$

$$\frac{a^2 + a(ar) + a^2r^2}{(ar)(ar^2) + (ar^2)a + a(ar)} = \frac{ar + a}{ar^2 + ar}$$

$$\frac{a^2(1 + r + r^2)}{a^2(r^3 + r^2 + r)} = \frac{a(1 + r)}{a(r^2 + r)}$$

$$\frac{1 + r + r^2}{r(1 + r + r^2)} = \frac{1 + r}{r(1 + r)}$$

$$\frac{1}{r} = \frac{1}{r}$$

$$LHS = RHS$$

so,

$$\frac{a^2 + ab + b^2}{bc + ca + ab} = \frac{b + a}{c + b}$$

Q20

Let r be the common ratio of G.P.

$$a, \quad b = ar, \quad c = ar^2$$

$$a + b + c = xb$$

$$a + ar + ar^2 = x(ar)$$

$$a(1 + r + r^2) = xar$$

$$r^2 + (1 - x)r + 1 = 0$$

Here, r is real, so

$$D \geq 0$$

$$(1 - x)^2 - 4(1)(1) \geq 0$$

$$1 + x^2 - 2x - 4 \geq 0$$

$$x^2 - 2x - 3 \geq 0$$

$$(x - 3)(x + 1) \geq 0$$

$$\Rightarrow \quad x < -1 \text{ or } x > 3$$

Q21

Let the 4th term be ar^3

10th term be ar^9

16th term be ar^{15}

$$ar^9 = \sqrt{(ar^3)(ar^{15})} = ar^9$$

$\therefore$ 4th, 10th, 16th terms are also in GP

Hence Proved

Q22

Let the A.P. be $A, A + D, A + 2D, \dots$ and G.P. be $x, xR, xR^2, \dots$ then

$$a = A + (p-1)D, b = A + (q-1)D, c = A + (r-1)D$$

$$\Rightarrow a - b = (p-q)D, b - c = (q-r)D, c - a = (r-p)D$$

$$\text{Also } a = xR^{p-1}, b = xR^{q-1}, c = xR^{r-1}$$

$$\text{Hence } a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = (xR^{p-1})^{(q-r)D} \cdot (xR^{q-1})^{(r-p)D} \cdot (xR^{r-1})^{(p-q)D}$$

$$= x^{(q-r+r-p+p-q)D} \cdot R^{[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]D}$$

$$= x^0 \cdot R^0 = 1 \cdot 1 = 1$$



Ex 20.6

Q1

6 Geometric means between 27 and $\frac{1}{81}$

Let $G_1, G_2, G_3, G_4, G_5, G_6$ be 6 geometric means between $a = 27$ and $b = \frac{1}{81}$.

Then, $27, G_1, G_2, G_3, G_4, G_5, G_6, \frac{1}{81}$ is a G.P. with common ratio r given by

$$\begin{aligned} r &= \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \\ &= \left(\frac{\frac{1}{81}}{27}\right)^{\frac{1}{6+1}} = \left(\frac{1}{81 \times 27}\right)^{\frac{1}{7}} = \left(\frac{1}{3^7}\right)^{\frac{1}{7}} \end{aligned}$$

$$\therefore G_1 = ar = 27 \left(\frac{1}{3}\right) = 9$$

$$G_2 = ar^2 = 27 \times \frac{1}{9} = 3$$

$$G_3 = ar^3 = 27 \times \frac{1}{27} = 1$$

$$G_4 = ar^4 = 27 \times \frac{1}{27 \times 3} = \frac{1}{3}$$

$$G_5 = ar^5 = 27 \times \frac{1}{3^5} = \frac{1}{9}$$

$$G_6 = ar^6 = 27 \times \frac{1}{3^6} = \frac{1}{27}$$

Hence, $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ are 6 geometric means between 27 and $\frac{1}{81}$.

Q2

5 Geometric means between 16 and $\frac{1}{4}$

Let G_1, G_2, G_3, G_4, G_5 , be five geometric means between 16 and $\frac{1}{4}$.

$16, G_1, G_2, G_3, G_4, G_5, \frac{1}{4}$ is a G.P. with $a = 16, b = \frac{1}{4}$.

Then,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{1}{4}}{16}\right)^{\frac{1}{5+1}} = \left(\frac{1}{256}\right)^{\frac{1}{6}} = \frac{1}{2}$$

$$\therefore G_1 = ar = 16 \times \frac{1}{2} = 8$$

$$G_2 = ar^2 = 16 \times \frac{1}{4} = 4$$

$$G_3 = ar^3 = 16 \times \frac{1}{8} = 2$$

$$G_4 = ar^4 = 16 \times \frac{1}{16} = 1$$

$$G_5 = ar^5 = 16 \times \frac{1}{2^5} = \frac{1}{2}$$

Hence, $8, 4, 2, 1, \frac{1}{2}$ are five geometric means between 16 and $\frac{1}{4}$.

**Q3**

5 Geometric means between $\frac{32}{9}$ and $\frac{81}{2}$

Let G_1, G_2, G_3, G_4, G_5 , be five geometric means between $\frac{32}{9}$ and $\frac{81}{2}$.

Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P. with $a = \frac{32}{9}, b = \frac{81}{2}$.

Then,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{81}{2}}{\frac{32}{9}}\right)^{\frac{1}{5+1}} = \left(\frac{81}{2} \times \frac{9}{32}\right)^{\frac{1}{6}} = \left(\frac{3^6}{2^6}\right)^{\frac{1}{6}} = \frac{3}{2}$$

Thus, $G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$

$$G_2 = ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$$

$$G_3 = ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$$

$$G_4 = ar^4 = \frac{32}{9} \times \frac{3^4}{2^4} = 2 \times 9 = 18$$

$$G_5 = ar^5 = \frac{32}{9} \times \frac{3^5}{2^5} = 27$$

Hence, $\frac{16}{3}, 8, 12, 18, 27$ are five geometric means between $\frac{32}{9}$ and $\frac{81}{2}$.

Q4

(i) 2 and 8

Geometric means between a and $b = \sqrt{ab}$

---(i)

Here, $a = 2, b = 8$

$$\therefore \text{Geometric means} = \sqrt{2 \times 8} = \sqrt{16} = 4$$

(ii) a^3b and ab^3

Using (i)

$$a = a^3b, b = ab^3$$

$$\text{Geometric means} = \sqrt{a^3b \times ab^3} = \sqrt{a^4b^4} = a^2b^2$$

(iii) -8 and -2

Using (ii)

$$a = -8, b = -2$$

$$\text{Geometric means} = \sqrt{-8 \times -2} = \sqrt{16} = 4, -4$$

**Q5**

a is geometric means between 2 and $\frac{1}{4}$.

$$\begin{aligned}\text{Then, } a &= \sqrt{2 \times \frac{1}{4}} \\ a &= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}\end{aligned}$$

Q6

Let the first term of a GP is a and common ratio of the series is r .

The $(n+2)$ th term is ar^{n+1} .

The GM of a and ar^{n+1} will be:

$$G_1 = \sqrt{a \cdot ar^{n+1}} = (a^2 r^{n+1})^{\frac{1}{2}}$$

Now the n GM in between a and ar^{n+1} are:

$$ar, ar^2, \dots, ar^n$$

Therefore the product of n GM will be:

$$\begin{aligned}ar \times ar^2 \times \dots \times ar^n &= a^n r^{1+2+3+\dots+n} \\ &= a^n r^{\frac{n(n+1)}{2}} \\ &= (a^2 r^{n+1})^{\frac{n}{2}} \\ &= G_1^n\end{aligned}$$

Hence it is proved.

Q7

Given,

$$A.M = 25$$

$$G.M = 20$$

$$\text{Now, } A.M = \frac{a+b}{2} = 25$$

$$\text{and, } G.M = \sqrt{ab} = 20$$

$$a+b = 50, ab = 400$$

$$\begin{aligned}(a-b) &= \sqrt{(a+b)^2 - 4ab} \\ &= \sqrt{(50)^2 - 1600} \\ &= \sqrt{2500 - 1600} \\ &= \pm 30\end{aligned}$$

$$a-b = \pm 30$$

$$\frac{a-b}{a+b} = \frac{30}{50}$$

$$\frac{a-b}{a+b} = \frac{3}{5}$$

$$a-b = 40$$

$$\text{Also, } 2b = 20$$

$$b = 10$$

$\therefore$ The numbers are 40, 10

Q8

A.M. between two numbers a and b ($a > b$) is $\frac{a+b}{2}$

Also, geometric mean between 2 numbers is $\sqrt{ab}$

Given,

$$A.M = 2G.M$$

$$\Rightarrow \frac{a+b}{2} = 2\sqrt{ab}$$

$$a+b = 4\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$$

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{2+1}{2-1} = \frac{3}{1}$$

[By componendo and dividendo]

$$\frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{(\sqrt{3})^2}{(1)^2}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{3}}{1}$$

By componendo and dividendo, we get

$$\frac{(\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b}) - (\sqrt{a} - \sqrt{b})} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{a}{b} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3} - 1)^2} = \frac{3 + 1 + 2\sqrt{3}}{3 + 1 - 2\sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$\frac{a}{b} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

Thus, $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.

Q9

Let A.M = A between a and b

G.M = G_1 and G_2 between a and b

$$\Rightarrow A = \frac{a+b}{2}$$

a, G_1, G_2, b is G.P. with common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$$G_1 = ar = a \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$G_2 = ar^2 = a \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$$

Now,

$$G_1^2 = a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}$$

$$G_2^2 = a^{\frac{2}{3}}b^{\frac{4}{3}}$$

Then,

$$\begin{aligned} \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} &= \frac{a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}}{a^{\frac{1}{3}}b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}}b^{\frac{4}{3}}}{a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}} \\ &= a^{2-\frac{2}{3}-\frac{1}{3}}b^{\frac{2}{3}-\frac{2}{3}} + a^{\frac{2}{3}-2+\frac{2}{3}}b^{\frac{4}{3}-\frac{2}{3}} \\ &= a^{\frac{3}{3}}b^0 + a^0b \\ &= a + b \\ &= 2a \\ &= \text{RHS} \end{aligned}$$

Q10

A.M. of root of quadratic equation is A .

G.M. of root of quadratic equation is G .

Then,

$$\frac{a+b}{2} = A, \quad F = \sqrt{ab}$$

The equation having a and b as roots of quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (a+b)x + ab = 0$$

$$x^2 - 2Ax + G^2 = 0$$

Q11

Let a, b be the numbers.

$$a + b = 6 \text{ (G.M of } a, b)$$

$$a + b = 6\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying components and dividends,

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\left(\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} \right)^2 = \frac{4}{2}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{2}}{1}$$

Again applying components and dividends,

$$\frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\left(\frac{a}{b} \right) = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2$$

$$= \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = (3+2\sqrt{2}) : (3-2\sqrt{2})$$

Q12

Let quadratic equation be $(x - \alpha)(x - \beta) = 0$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Roots are α, β

Here,

$$\frac{\alpha + \beta}{2} = 8, \sqrt{\alpha\beta} = 5$$

$$\alpha + \beta = 16, \alpha\beta = 25$$

$\therefore$ Required quadratic equation is,

$$x^2 - 16x + 25 = 0$$



Q13

The AM and GM of a and b will be:

$$\frac{a+b}{2} = 10 \Rightarrow a+b = 20 \quad \dots\dots(1)$$

$$\sqrt{ab} = 8 \Rightarrow ab = 64$$

Now

$$\begin{aligned} a-b &= \sqrt{(a+b)^2 - 4ab} \\ &= \sqrt{20^2 - 4 \cdot 64} \\ &= \sqrt{400 - 256} \\ &= \sqrt{144} \end{aligned}$$

$$a-b = 12 \quad \dots\dots(2)$$

Adding (1) and (2)

$$2a = 32$$

$$a = 16$$

From (1)

$$b = 20 - 16 = 4$$

Thus the numbers are $a = 16$ and $b = 4$.

