

VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

EXERCISE 22 [Pg. No.: 1008]

1. Write down the magnitude of each of the following vectors:

(i) $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$

(ii) $\vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

(iii) $\vec{c} = \left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \right)$

(iv) $\vec{d} = (\sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k})$

Sol. (i) Let $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$; $|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (5)^2} = \sqrt{1+4+25} = \sqrt{30}$

(ii) Let $\vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$; $|\vec{b}| = \sqrt{(5)^2 + (-4)^2 + (-3)^2} = \sqrt{25+16+9} = 5\sqrt{2}$

(iii) Let $\vec{c} = \left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \right)$

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{\frac{1+1+1}{3}} = \sqrt{\frac{3}{3}} = \sqrt{1} = 1$$

(iv) Let $\vec{d} = (\sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k})$; $|\vec{d}| = \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2 + (-\sqrt{5})^2} = \sqrt{2+3+5} = \sqrt{10}$

2. Find a unit vector in the direction of the vector

(i) $(3\hat{i} + 4\hat{j} - 5\hat{k})$

(ii) $(3\hat{i} - 2\hat{j} + 6\hat{k})$

(iii) $(\hat{i} + \hat{k})$

(iv) $(2\hat{i} + \hat{j} + 2\hat{k})$

Sol. (i) Let $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{3^2 + 4^2 + (-5)^2} \Rightarrow |\vec{a}| = \sqrt{50} \Rightarrow |\vec{a}| = 5\sqrt{2}$$

Unit vector along $\vec{a}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Rightarrow \hat{a} = \frac{1}{5\sqrt{2}}(3\hat{i} + 4\hat{j} - 5\hat{k})$

(ii) Let, $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{3^2 + (-2)^2 + 6^2} \Rightarrow |\vec{a}| = \sqrt{49} \Rightarrow |\vec{a}| = 7$$

Unit vector along $\vec{a}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Rightarrow \hat{a} = \frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$

(iii) Let, $\vec{a} = \hat{i} + \hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{1^2 + 1^2} \Rightarrow |\vec{a}| = \sqrt{2}$$

Unit vector along $\vec{a}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

(iv) Let, $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{2^2 + 1^2 + 2^2} = 3$$

VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Rightarrow \hat{a} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3} \Rightarrow \hat{a} = \frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$$

3. If $\vec{a} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector

Sol. Given:- $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$$\Rightarrow \lambda\vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$$

$\therefore \lambda\vec{a}$ is a unit vector

$$\therefore |\lambda\vec{a}| = 1$$

$$\Rightarrow \sqrt{(2\lambda)^2 + (-4\lambda)^2 + (5\lambda)^2} = 1 \Rightarrow 4\lambda^2 + 16\lambda^2 + 25\lambda^2 = 1 \Rightarrow 45\lambda^2 = 1 \Rightarrow \lambda^2 = \frac{1}{45} \Rightarrow \lambda = \pm \frac{1}{\sqrt{45}}$$

$$\Rightarrow \lambda = \pm \frac{1}{3\sqrt{5}}$$

4. If $\vec{a} = (-\hat{i} + \hat{j} - \hat{k})$ and $\vec{b} = (2\hat{i} - \hat{j} + 2\hat{k})$ then find the unit vector in the direction of $(\vec{a} + \vec{b})$

Sol. $\vec{a} = -\hat{i} + \hat{j} - \hat{k}$

$$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{a} + \vec{b} = (-\hat{i} + \hat{j} - \hat{k}) + (2\hat{i} - \hat{j} + 2\hat{k})$$

$$= \hat{i} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{Unit vector } \vec{a} + \vec{b} \text{ is, } \hat{r} = \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \Rightarrow \hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$$

5. If $\vec{a} = (3\hat{i} + \hat{j} - 5\hat{k})$ and $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ then find a unit vector in the direction of $(\vec{a} - \vec{b})$

Sol. $\vec{a} = 3\hat{i} + \hat{j} - 5\hat{k}$

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Now, } \vec{a} - \vec{b} = (3\hat{i} + \hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})$$

$$= 2\hat{i} - \hat{j} - 4\hat{k}$$

$$\text{Unit vector in the direction of } \vec{a} - \vec{b} \text{ is, } \hat{r} = \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|}$$

$$= \frac{2\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{2^2 + (-1)^2 + (-4)^2}} = \frac{1}{\sqrt{21}}(2\hat{i} - \hat{j} - 4\hat{k})$$

6. If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (2\hat{i} + 4\hat{j} + 9\hat{k})$ then find a unit vector parallel to $(\vec{a} + \vec{b})$

Sol. $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$$\vec{b} = 2\hat{i} + 4\hat{j} + 9\hat{k}$$

$$\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} + 4\hat{j} + 9\hat{k})$$

$$= 3\hat{i} + 6\hat{j} + 6\hat{k}$$

$$\therefore |\vec{a} + \vec{b}| = \sqrt{3^2 + 6^2 + 6^2} = 9$$

Unit vector parallel to $\vec{a} + \vec{b}$ is, $\vec{r} = \pm \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$

$$= \pm \frac{3\hat{i} + 6\hat{j} + 6\hat{k}}{9} = \pm \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

7. Find a vector of magnitude 9 units in the direction of the vector $(-2\hat{i} + \hat{j} + 2\hat{k})$

Sol. Let, $\vec{a} = -2\hat{i} + \hat{j} + 2\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{(-2)^2 + 1^2 + 2^2} \Rightarrow |\vec{a}| = \sqrt{4 + 1 + 4} \Rightarrow |\vec{a}| = 3$$

A vector of magnitude 9 units in the direction of $\vec{a}$ is, $\vec{r} = 9 \cdot \frac{\vec{a}}{|\vec{a}|}$

$$= \frac{9}{3}(-2\hat{i} + \hat{j} + 2\hat{k}) = -6\hat{i} + 3\hat{j} + 6\hat{k}$$

8. Find a vector magnitude 8 units in the direction of the vector $(5\hat{i} - \hat{j} + 2\hat{k})$.

Sol. Let, $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} \Rightarrow |\vec{a}| = \sqrt{30}$$

A vector of magnitude 8 units in the direction at $\vec{a}$ is $\vec{r} = 8 \cdot \frac{\vec{a}}{|\vec{a}|}$

$$\Rightarrow \vec{r} = 8 \cdot \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} = \frac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$$

9. Find a vector of magnitude 21 units in the direction of the vector $(2\hat{i} - 3\hat{j} + 6\hat{k})$

Sol. Let, $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{2^2 + (-3)^2 + 6^2} \Rightarrow |\vec{a}| = 7$$

A vector of magnitude 21 units in the direction at $\vec{a}$ is, $\vec{r} = 21 \cdot \frac{\vec{a}}{|\vec{a}|}$

$$\Rightarrow \vec{r} = 21 \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} \Rightarrow \vec{r} = 6\hat{i} - 9\hat{j} + 18\hat{k}$$

VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

10. If $\vec{a} = (\hat{i} - 2\hat{j})$, $\vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$, find $(\vec{a} + \vec{b} + \vec{c})$

Sol. $\vec{a} + \vec{b} + \vec{c} = (\hat{i} - 2\hat{j}) + (2\hat{i} - 3\hat{j}) + 2\hat{i} + 3\hat{k} = 5\hat{i} - 5\hat{j} + 3\hat{k}$

11. If $A(-2, 1, 2)$ and $B(2, -1, 6)$ are two given points, find a unit vector in the direction of $\overrightarrow{AB}$

Sol. Let, O be the origin,

Now, P.V. of A = $\overrightarrow{OA} = -2\hat{i} + \hat{j} + 2\hat{k}$

P.V. of B = $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 6\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (-2\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} - \hat{j} + 6\hat{k}) = -4\hat{i} + 2\hat{j} - 4\hat{k}$

Unit vector along $\overrightarrow{AB}$ is, $\widehat{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$\Rightarrow \widehat{AB} = \frac{-4\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{(-4)^2 + 2^2 + (-4)^2}} \Rightarrow \widehat{AB} = \frac{1}{6}(-4\hat{i} + 2\hat{j} - 4\hat{k}) \Rightarrow \widehat{AB} = \frac{1}{3}(-2\hat{i} + \hat{j} - 2\hat{k})$

12. Find the direction ratios and the direction cosines of the vector $\vec{a} = (5\hat{i} - 3\hat{j} + 4\hat{k})$.

Sol. Let $\vec{a} = (5\hat{i} - 3\hat{j} + 4\hat{k}) \Rightarrow |\vec{a}| = \sqrt{(5)^2 + (-3)^2 + (4)^2} = \sqrt{25 + 9 + 16} = \sqrt{50} = 5\sqrt{2}$

Hence, the required direction ratios are $(5, -3, 4)$

Direction cosines are $\left(\frac{5}{5\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}\right)$

13. Find the direction ratios and the direction cosines of the vector joining the points $A(2, 1, -2)$ and $B(3, 5, -4)$.

Sol. Let, $\vec{A} = (2\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{B} = (3\hat{i} + 5\hat{j} - 4\hat{k})$

$\overrightarrow{AB} = \text{P.V. of B} - \text{P.V. of A}$

$= (3\hat{i} + 5\hat{j} - 4\hat{k}) - (2\hat{i} + \hat{j} - 2\hat{k}) = (\hat{i} + 4\hat{j} - 2\hat{k})$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{(1)^2 + (4)^2 + (-2)^2} = \sqrt{1 + 16 + 4} = \sqrt{21}$

Hence, the required direction ratios are $(1, 4, -2)$

Direction cosines are $\left(\frac{1}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}\right)$

14. Show that the points A, B and C having position vectors $(\hat{i} + 2\hat{j} + 7\hat{k})$, $(2\hat{i} + 6\hat{j} + 3\hat{k})$ and $(3\hat{i} + 10\hat{j} - 3\hat{k})$ respectively, are collinear

Sol. $\overrightarrow{AB} = \text{P.V. of B} - \text{P.V. of A}$

$= (2\hat{i} + 6\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k}) = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = \text{P.V. of C} - \text{P.V. of B}$

$$= (3\hat{i} + 10\hat{j} - 1\hat{k}) - (2\hat{i} + 6\hat{j} + 3\hat{k}) = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\therefore \overrightarrow{AB} = \overrightarrow{BC} \Rightarrow \overrightarrow{AB} \parallel \overrightarrow{BC}$$

Here, B is common Hence, A, B & C are collinear.

15. The position vectors of the points A, B and C are $(2\hat{i} + \hat{j} - \hat{k})$, $(3\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 4\hat{j} - 3\hat{k})$ respectively. Show that the points A, B and C are collinear

Sol. $\overrightarrow{AB}$ = P.V. of B - P.V. at A

$$= (3\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i} - 3\hat{j} + 2\hat{k}$$

$\overrightarrow{BC}$ = P.V. of C - P.V. at B

$$= (\hat{i} + 4\hat{j} - 3\hat{k}) - (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= -2\hat{i} + 6\hat{j} - 4\hat{k}$$

$$\text{Hence, } \overrightarrow{BC} = -2 \cdot \overrightarrow{AB}$$

$$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{BC}$$

Here B is common

$\therefore$ A, B & C are collinear

16. if the position vectors of the vertices A, B and C of a $\triangle ABC$ be $(\hat{i} + 2\hat{j} + 3\hat{k})$, $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(3\hat{i} + \hat{j} + 2\hat{k})$ respectively, prove that $\triangle ABC$ is equilateral

Sol. $\overrightarrow{AB}$ = P.V. at B - P.V. at A

$$= (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + \hat{j} - 2\hat{k}$$

$\overrightarrow{BC}$ = P.V. at C - P.V. at B

$$= (3\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k}) = \hat{i} - 2\hat{j} + \hat{k}$$

$\overrightarrow{AC}$ = P.V. at C - P.V. at A

$$= (3\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = (2\hat{i} - \hat{j} - \hat{k})$$

$$\text{Here, } |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{AC}| = \sqrt{6}$$

Hence, $\triangle ABC$ is an equilateral triangle

17. Show that the points A, B and C having position vectors $(3\hat{i} - 4\hat{j} - 4\hat{k})$, $(2\hat{i} - \hat{j} + \hat{k})$ and $(\hat{i} - 3\hat{j} - 5\hat{k})$ respectively, form the vertices of a right-angled triangle

Sol. $\overrightarrow{AB}$ = P.V. at B - P.V. at A

$$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k}) = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$\overrightarrow{BC}$ = P.V. at C - P.V. at B

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} - 2\hat{j} - 6\hat{k}$$

VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

$\overrightarrow{AC} = \text{P.V. at C} - \text{P.V. at A}$

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k}) = -2\hat{i} + \hat{j} - \hat{k}$$

$$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{35}$$

$$\Rightarrow |\overrightarrow{AB}|^2 = 35$$

$$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$$

$$\Rightarrow |\overrightarrow{BC}|^2 = 41$$

$$|\overrightarrow{AC}| = \sqrt{(-2)^2 + 1^2 + (-1)^2} = \sqrt{6}$$

$$\Rightarrow |\overrightarrow{AC}|^2 = 6$$

$\therefore |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |\overrightarrow{BC}|^2 \quad \therefore \triangle ABC \text{ is a right angled triangle at A.}$

18. Using vector method, show that the points $A(1, -1, 0)$, $B(4, -3, 1)$ and $C(2, -4, 5)$ are the vertices of a right-angled triangle

Sol. Given: $\rightarrow A(1, -1, 0)$, $B(4, -3, 1)$ and $C(2, -4, 5)$ are the vertices of a triangle.

$$\begin{aligned} \text{Here, } \overrightarrow{AB} &= (4-1)\hat{i} + (-3+1)\hat{j} + (1-0)\hat{k} \\ &= 3\hat{i} - 2\hat{j} + \hat{k} \end{aligned}$$

$$\overrightarrow{BC} = (2-4)\hat{i} + (-4+3)\hat{j} + (5-1)\hat{k} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$$\overrightarrow{AC} = (2-1)\hat{i} + (-4+1)\hat{j} + (5-0)\hat{k} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\text{Here, } |\overrightarrow{AB}|^2 = 9 + 4 + 1 = 14$$

$$\text{and, } |\overrightarrow{BC}|^2 = 4 + 1 + 16 = 21 \text{ and, } |\overrightarrow{AC}|^2 = 1 + 9 + 25 = 35$$

$\therefore |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AC}|^2 \quad \therefore \triangle ABC \text{ is a right angle triangle.}$

19. Find the position vector of the point which divides the join of the points $(2\vec{a} - 3\vec{b})$ and $(3\vec{a} - 2\vec{b})$ (i) internally and (ii) externally in the ratio 2 : 3

Sol. Let, Position vector of A is $\vec{a}_1 = 2\vec{a} - 3\vec{b}$

and, position vector of B is $\vec{a}_2 = 3\vec{a} - 2\vec{b}$

(i) Let, R divides AB internally in the ratio $m_1:m_2 = 2 : 3$

$$\text{P.V. at R} = \frac{m_1\vec{a}_2 + m_2\vec{a}_1}{m_1 + m_2}$$

$$= \frac{2(3\vec{a} - 2\vec{b}) + 3(2\vec{a} - 3\vec{b})}{2 + 3} = \frac{12\vec{a} - 13\vec{b}}{5} = \frac{12}{5}\vec{a} - \frac{13}{5}\vec{b}$$

(ii) Let, S divides AB externally in the ratio 2 : 3

$$\text{P.V. of R} = \frac{2\vec{a}_2 - 3\vec{a}_1}{2 - 3}$$

$$= \frac{2(3\vec{a} - 2\vec{b}) - 3(2\vec{a} - 3\vec{b})}{-1} = -\left(6\vec{a} - 4\vec{b} - 6\vec{a} + 9\vec{b}\right) = -5\vec{b}$$

20. The position vectors of two points A and B are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively. Find the position vector of a point C which divides AB externally in the ratio 1 : 2. Also, show that A is the mid-point of the line segment CB

Sol. Let, P.V. of A is $\vec{a}_1 = 2\vec{a} + \vec{b}$

and, P.V of B is $\vec{a}_2 = \vec{a} - 3\vec{b}$

∴ Since, C divides AB externally in the ratio 1 : 2 ∴ Position vector of C is,

$$\vec{r} = \frac{1 \times \vec{a}_2 - 2 \times \vec{a}_1}{1 - 2}$$

$$\Rightarrow \vec{r} = \frac{(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{-1} \Rightarrow \vec{r} = -\{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}\} \Rightarrow \boxed{\vec{r} = 3\vec{a} + 5\vec{b}}$$

$$\text{Now, } \frac{\text{P.V. of B} + \text{P.V of C}}{2} = \frac{\vec{a}_2 + \vec{r}}{2} = \frac{\vec{a} - 3\vec{b} + 3\vec{a} + 5\vec{b}}{2} = \frac{4\vec{a} + 2\vec{b}}{2}$$

$$= 2\vec{a} + \vec{b} = \text{P.V. of A} \quad \text{Hence, A is the mid-point of BC.}$$

21. Find the position vector of a point R which divides the line joining $A(-2, 1, 3)$ and $B(3, 5, 2)$ in the ratio 2:1 (i) internally (ii) externally.

Sol. Here $\vec{a} = -2\hat{i} + \hat{j} + 3\hat{k}$ & $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

(i) When R divided PQ internally in the ratio 2:1

$$\text{Then, P.V. of R} = \frac{(m\vec{b} + n\vec{a})}{(m+n)} = \frac{2(3\hat{i} + 5\hat{j} - 2\hat{k}) + 1(-2\hat{i} + \hat{j} + 3\hat{k})}{2+1}$$

$$\frac{6\hat{i} + 10\hat{j} - 4\hat{k} - 2\hat{i} + \hat{j} + 3\hat{k}}{3} = \frac{4\hat{i} + 11\hat{j} - \hat{k}}{3} = \frac{4}{3}\hat{i} + \frac{11}{3}\hat{j} - \frac{1}{3}\hat{k} \quad \text{i.e., } \left(\frac{4}{3}, \frac{11}{3}, -\frac{1}{3}\right)$$

(ii) When R divided PQ externally in the ratio 2:1

$$\text{Then position vector of R} = \frac{(m\vec{b} - n\vec{a})}{(m-n)} = \frac{2(3\hat{i} + 5\hat{j} - 2\hat{k}) - 1(-2\hat{i} + \hat{j} + 3\hat{k})}{2-1}$$

$$= \frac{(6\hat{i} + 10\hat{j} - 4\hat{k}) - (-2\hat{i} + \hat{j} + 3\hat{k})}{1} = (8\hat{i} + 9\hat{j} - 7\hat{k}) \quad \text{i.e., } (8, 9, -7)$$

22. Find the position vector of the midpoint of the vector joining the points $A(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $B(\hat{i} + 4\hat{j} - 2\hat{k})$.

Sol. The position vector of A and B are given by

$$\vec{a} = (3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ and } \vec{b} = (\hat{i} + 4\hat{j} - 2\hat{k})$$

$$\begin{aligned} \therefore \text{P.V. of midpoint of AB} &= \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\{(3\hat{i} + 2\hat{j} + 6\hat{k}) + (\hat{i} + 4\hat{j} - 2\hat{k})\} \\ &= \frac{1}{2}(4\hat{i} + 6\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 2\hat{k}) \end{aligned}$$

VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

23. If $\overrightarrow{AB} = (2\hat{i} + \hat{j} - 3\hat{k})$ and $A(1, 2, -1)$ is the given point, find the coordinates of B

Sol. Let, Co-ordinates of B are (α, β, γ)

Now, $\overrightarrow{AB} = \text{P.V. of B} - \text{P.V. of A}$

$$\Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) \Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = (\alpha - 1)\hat{i} + (\beta - 2)\hat{j} + (\gamma + 1)\hat{k}$$

$$\Rightarrow \alpha - 1 = 2, \beta - 2 = 1 \text{ \& } \gamma + 1 = -3 \Rightarrow \alpha = 3, \beta = 3 \text{ \& } \gamma = -4$$

Hence, Co-ordinates of B are (3, 3, -4)

24. Write a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 0) and (4, 5, 6) respectively

Sol. $\overrightarrow{PQ} = (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k}$

$$\Rightarrow \overrightarrow{PQ} = 3\hat{i} + 2\hat{j} + 6\hat{k} \Rightarrow |\overrightarrow{PQ}| = \sqrt{3^2 + 2^2 + 6^2}$$

$$\text{Unit vector along } \overrightarrow{PQ}, \hat{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ Ans.}$$

