SCALAR, OR DOT, PRODUCT OF VECTORS (XII, R. S. AGGARWAL)

EXERCISE 23 [Pg. No.: 1030]

1. Find $\vec{a}.\vec{b}$ when

(i) $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=3\hat{i}-4\hat{j}-2\hat{k}$ (ii) $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=-2\hat{j}+4\hat{k}$

(iii) $\vec{a}=\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{k}$

Sol. **(i)** We have $\vec{a}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-4\hat{j}-2\hat{k}\right)$

$\Rightarrow \vec{a}.\vec{b}=\left(\hat{i}-2\hat{j}+\hat{k}\right).\left(3\hat{i}-4\hat{j}-2\hat{k}\right)=(3+8-2)=9$

(ii) We have $\vec{a}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{b}=\left(-2\hat{j}+4\hat{k}\right)$

$\Rightarrow \vec{a}.\vec{b}=\left(\hat{i}+2\hat{j}+3\hat{k}\right).\left(-2\hat{j}+4\hat{k}\right)=(-4+12)=8$

(iii) We have $\vec{a}=\left(\hat{i}-\hat{j}+5\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-2\hat{k}\right)$

$\Rightarrow \vec{a}.\vec{b}=\left(\hat{i}-\hat{j}+5\hat{k}\right).\left(3\hat{i}-2\hat{k}\right)=(3-10)=-7$

2. Find the value of λ for which $\vec{a}$ and $\vec{b}$ are perpendicular, wh

(i) $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$ (ii) $\vec{a}=3\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b}=-\lambda\hat{i}+3\hat{j}+3\hat{k}$

(iii) $\vec{a}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\lambda\hat{k}$ (iv) $\vec{a}=3\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=-5\hat{j}+\lambda\hat{k}$

Sol. **(i)** We have $\vec{a}=\left(2\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{b}=\left(4\hat{i}-2\hat{j}+2\hat{k}\right)$

$\vec{a}\perp\vec{b} \Rightarrow \vec{a}.\vec{b}=0 \Rightarrow \left(2\hat{i}+\lambda\hat{j}+\hat{k}\right).\left(4\hat{i}-2\hat{j}+2\hat{k}\right)=0$

$\Rightarrow 8-2\lambda+2=0 \Rightarrow 10-2\lambda=0 \Rightarrow 2\lambda=10 \therefore \lambda=5$

(ii) We have $\vec{a}=\left(3\hat{i}-\hat{j}+4\hat{k}\right)$ and $\vec{b}=\left(-\lambda\hat{i}+3\hat{j}+3\hat{k}\right)$

$\vec{a}\perp\vec{b} \Rightarrow \vec{a}.\vec{b}=0 \Rightarrow \left(3\hat{i}-\hat{j}+4\hat{k}\right).\left(-\lambda\hat{i}+3\hat{j}+3\hat{k}\right)=0$

$\Rightarrow -3\lambda-3+12=0 \Rightarrow -3\lambda+9=0 \Rightarrow 3\lambda=9 \therefore \lambda=3$

(iii) We have $\vec{a}=\left(2\hat{i}+4\hat{j}-\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-2\hat{j}+\lambda\hat{k}\right)$

$\vec{a}\perp\vec{b} \Rightarrow \vec{a}.\vec{b}=0 \Rightarrow \left(2\hat{i}+4\hat{j}-\hat{k}\right).\left(3\hat{i}-2\hat{j}+\lambda\hat{k}\right)=0$

$\Rightarrow (6-8-\lambda)=0 \Rightarrow -2-\lambda=0 \therefore \lambda=-2$

(iv) We have $\vec{a}=\left(3\hat{i}+2\hat{j}-5\hat{k}\right)$ and $\vec{b}=\left(-5\hat{j}+\lambda\hat{k}\right)$

$\vec{a}\perp\vec{b} \Rightarrow \vec{a}.\vec{b}=0 \Rightarrow \left(3\hat{i}+2\hat{j}-5\hat{k}\right).\left(-5\hat{j}+\lambda\hat{k}\right)=0 \Rightarrow -10-5\lambda=0 \therefore \lambda=-2$

3. (i) If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, show that $\left(\vec{a}+\vec{b}\right)$ is perpendicular to $\left(\vec{a}-\vec{b}\right)$.

(ii) If $\vec{a}=\left(5\hat{i}-\hat{j}-3\hat{k}\right)$ and $\vec{b}=\left(\hat{i}+3\hat{j}-5\hat{k}\right)$ then show that $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are orthogonal.

Sol. **(i)** We have $\vec{a}=\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)$

$$\left(\vec{a}+\vec{b}\right)=\left(\hat{i}+2\hat{j}-3\hat{k}\right)+\left(3\hat{i}-\hat{j}+2\hat{k}\right)=\left(4\hat{i}+\hat{j}-\hat{k}\right)$$

$$\left(\vec{a}-\vec{b}\right)=\left(\hat{i}+2\hat{j}-3\hat{k}\right)-\left(3\hat{i}-\hat{j}+2\hat{k}\right)=\left(-2\hat{i}+3\hat{j}-5\hat{k}\right)$$

$$\left(\vec{a}+\vec{b}\right)\perp\left(\vec{a}-\vec{b}\right)\Rightarrow\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=\left(4\hat{i}+\hat{j}-\hat{k}\right)\left(-2\hat{i}+3\hat{j}-5\hat{k}\right)=-8+3+5=-8+8=0$$

Hence, $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are perpendicular to each other.

(ii) We have $\vec{a}=\left(5\hat{i}-\hat{j}-3\hat{k}\right)$ and $\vec{b}=\left(\hat{i}+3\hat{j}-5\hat{k}\right)$

$$\left(\vec{a}-\vec{b}\right)=\left(5\hat{i}-\hat{j}-3\hat{k}\right)-\left(\hat{i}+3\hat{j}-5\hat{k}\right)=\left(4\hat{i}-4\hat{j}+2\hat{k}\right)$$

$$\left(\vec{a}+\vec{b}\right)=\left(5\hat{i}-\hat{j}-3\hat{k}\right)+\left(\hat{i}+3\hat{j}-5\hat{k}\right)=6\hat{i}+2\hat{j}-8\hat{k}$$

$$\left(\vec{a}+\vec{b}\right)\perp\left(\vec{a}-\vec{b}\right)\Rightarrow\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=\left(6\hat{i}+2j-8\hat{k}\right).\left(4\hat{i}-4\hat{j}+2\hat{k}\right)$$

$=24-8-16=24-24=0$. Hence, $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are orthogonal.

4. if $\vec{a}=\left(\hat{i}-\hat{j}+7\hat{k}\right)$ and $\vec{b}=\left(5\hat{i}-\hat{j}+\lambda\hat{k}\right)$ then find the value of λ so that $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are orthogonal vectors

Sol. $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$

$\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$

Now, $\vec{a}+\vec{b}=\hat{i}-2\hat{j}+(7+\lambda)\hat{k}$

and, $\vec{a}-\vec{b}=-4\hat{i}+(7-\lambda)\hat{k}$

$\because\left(\vec{a}+\vec{b}\right)$ and $\vec{a}-\vec{b}$ are orthogonal vectors.

$$\Rightarrow\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=0\Rightarrow-24+(7+\lambda)\cdot(7-\lambda)=0\Rightarrow-24+49-\lambda^2=0\Rightarrow\lambda^2=25\Rightarrow\lambda=\pm5$$

5. Show that the vectors $\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$, $\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$ and $\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$ are mutually perpendicular unit vectors.

Sol. Let, $\vec{a}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$, $\vec{b}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$ & $\vec{c}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$

$$\vec{a}\perp\vec{b}\Rightarrow\vec{a}.\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right).\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$$

$$=\frac{1}{49}\left[\left(2\hat{i}+3\hat{j}+6\hat{k}\right).\left(3\hat{i}-6\hat{j}+2\hat{k}\right)\right]=\frac{1}{49}(6-18+12)=\frac{1}{49}\times0=0$$

$$\vec{b}\perp\vec{c}\Rightarrow\vec{b}.\vec{c}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right).\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

$$=\frac{1}{49}\left[\left(3\hat{i}-6\hat{j}+2\hat{k}\right).\left(6\hat{i}+2\hat{j}-3\hat{k}\right)\right]=\frac{1}{49}(18-12-6)=\frac{1}{49}\times0=0$$

$$\vec{c}\perp\vec{a}\Rightarrow\vec{c}.\vec{a}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right).\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

$$=\frac{1}{49}\left[\left(6\hat{i}+2\hat{j}-3\hat{k}\right).\left(2\hat{i}+3\hat{j}+6\hat{k}\right)\right]=\frac{1}{49}(12+6-18)=\frac{1}{49}\times0=0$$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Hence, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other.

6. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and is such that $\vec{d}.\vec{c} = 21$.

Sol. Let $\vec{a} = \left(4\hat{i} + 5\hat{j} - \hat{k}\right), \vec{b} = \left(\hat{i} - 4\hat{j} + 5\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j} - \hat{k}\right)$, Let $\vec{d} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\vec{d} \perp \vec{a} \Rightarrow \vec{d}.\vec{a} = 0 \Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(4\hat{i} + 5\hat{j} - \hat{k}\right) = 0 \Rightarrow 4x + 5y - z = 0$ ...(i)

$\vec{d} \perp \vec{b} \Rightarrow \vec{d}.\vec{b} = 0 \Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(\hat{i} - 4\hat{j} + 5\hat{k}\right) = 0 \Rightarrow x - 4y + 5z = 0$ ...(ii)

$\vec{d}.\vec{c} = 21 \Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(3\hat{i} + \hat{j} - \hat{k}\right) = 21 \Rightarrow 3x + y - z = 21$ ...(iii)

Solving equation (i) and (ii), then we get, $21x + 21y = 0 \Rightarrow x + y = 0$ ...(iv)

Again solving equation (ii) and (iii), then we get, $16x + y = 105$ ...(v)

Again solving equation (iv) and (v), then we get $x = 7,\ y = -7$

Putting the value of x and y in equation (i), then

$4x + 5y - z = 0 \Rightarrow z = 4x + 5y = 4(7) + 5(-7) = 28 - 35 \therefore z = -7$

Hence, $\vec{d} = 7\hat{i} - 7\hat{j} - 7\hat{k} = 7\left(\hat{i} - \hat{j} - \hat{k}\right)$

7. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Find the projection of (i) $\vec{a}$ on $\vec{b}$, and (ii) $\vec{b}$ on $\vec{a}$

Sol. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k} \Rightarrow |\vec{a}| = \sqrt{(2)^2 + (3)^2 + (2)^2} = \sqrt{4 + 9 + 4} = \sqrt{17}$

$\vec{b} = \hat{i} + 2\hat{j} + \hat{k} \Rightarrow |\vec{b}| = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$

$\vec{a}.\vec{b} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right).\left(\hat{i} + 2\hat{j} + \hat{k}\right) = 2.1 + 3.2 + 2.1 = 2 + 6 + 2 = 10$

(i) $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} = \dfrac{10}{\sqrt{6}} = \dfrac{10}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{10\sqrt{6}}{6} = \dfrac{5\sqrt{6}}{3}$

(ii) $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|} = \dfrac{10}{\sqrt{17}} = \dfrac{10\sqrt{17}}{17}$

8. Find the projection of $\left(8\hat{i} + \hat{j}\right)$ in the direction of $\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$.

Sol. Let $\vec{a} = \left(8\hat{i} + \hat{j}\right), \vec{b} = \left(\hat{i} + 2\hat{j} - 2\hat{k}\right) \Rightarrow |\vec{b}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\Rightarrow \vec{a}.\vec{b} = \left(8\hat{i} + \hat{j}\right).\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) = 8 + 2 = 10 \quad \therefore$ Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} = \dfrac{10}{3}$.

9. Write the projection of vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$ along the vector $\hat{j}$

Sol. Projection of $(\hat{i} + \hat{j} + \hat{k})$

on $\hat{j} = \dfrac{(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{j}}{|\hat{j}|} = \dfrac{1}{1} = 1$

10. (i) Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\cdot\vec{b}=8$ and $\vec{b}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$

(ii) Write the projection of the vector $\left(\hat{i}+\hat{j}\right)$ on the vector $\left(\hat{i}-\hat{j}\right)$

Sol. (i) Projection of $\vec{a}$ on $\vec{b}$

$$=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{8}{\sqrt{2^2+6^2+3^2}}=\frac{8}{7}$$

(ii) Projection of $\hat{i}+\hat{j}$ on $\hat{i}-\hat{j}$

$$=\frac{\left(\hat{i}+\hat{j}\right)\cdot\left(\hat{i}-\hat{j}\right)}{|\hat{i}-\hat{j}|}=\frac{1-1}{\sqrt{2}}=0$$

11. Find the angle between the vectors $\vec{a}$ and $\vec{b}$, when

(i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$ (ii) $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$

(iii) $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$

Sol. (i) we have $\vec{a}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)$, $\vec{b}=\left(3\hat{i}-2\hat{j}+\hat{k}\right)$, $|\vec{a}|=\sqrt{(1)^2+(-2)^2+(3)^2}=\sqrt{1+4+9}=\sqrt{14}$

$\left|\vec{b}\right|=\sqrt{(3)^2+(-2)^2+(1)^2}=\sqrt{9+4+1}=\sqrt{14}$, $\vec{a}.\vec{b}=\left(\hat{i}-2\hat{j}+3\hat{k}\right).\left(3\hat{i}-2\hat{j}+\hat{k}\right)=3+4+3=10$

Now, $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\Rightarrow\cos\theta=\frac{10}{\sqrt{14}.\sqrt{14}}\Rightarrow\cos\theta=\frac{10}{14}\Rightarrow\cos\theta=\frac{5}{7}\Rightarrow\theta=\cos^{-1}\left(\frac{5}{7}\right)$

Hence required angle is $\cos^{-1}\left(\frac{5}{7}\right)$

(ii) We have $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$

$$|\vec{a}|=\sqrt{(3)^2+(1)^2+(2)^2}=\sqrt{9+1+4}=\sqrt{14}$$

$$\left|\vec{b}\right|=\sqrt{(2)^2+(-2)^2+(4)^2}=\sqrt{4+4+16}=\sqrt{24}$$

$$\vec{a}.\vec{b}=\left(3\hat{i}+\hat{j}+2\hat{k}\right).\left(2\hat{i}-2\hat{j}+4\hat{k}\right)=6-2+8=12$$

$$=\frac{12}{\sqrt{2\times2\times2\times3\times2\times7}}=\frac{12}{4\times\sqrt{3}\times\sqrt{7}}=\frac{3}{\sqrt{3}\times\sqrt{7}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{3\sqrt{7}}=\sqrt{\frac{3}{7}}$$

Hence required angle is $\cos^{-1}\left(\sqrt{\frac{3}{7}}\right)$

(iii) We have $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$

$$|\vec{a}|=\sqrt{(1)^2+(-1)^2}=\sqrt{1+1}=\sqrt{2};\quad\left|\vec{b}\right|=\sqrt{(1)^2+(1)^2}=\sqrt{2}$$

$$\vec{a}.\vec{b}=\left(\hat{i}-\hat{j}\right).\left(\hat{j}+\hat{k}\right)=-1$$

Now, $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{-1}{\sqrt{2}.\sqrt{2}}\Rightarrow\cos\theta=\frac{-1}{2}\Rightarrow\cos\theta=\cos120°\quad\therefore\theta=120°$

Hence required angle is $\frac{2\pi}{3}$.

12. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ then calculate the angle between $(2\vec{a}+\vec{b})$ and $(\vec{a}+2\vec{b})$

Sol. We have $\vec{a}=(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})$

(i) $2\vec{a}+\vec{b}=2(\hat{i}+2\hat{j}-3\hat{k})+(3\hat{i}-\hat{j}+2\hat{k})=(2\hat{i}+4\hat{j}-6\hat{k})+(3\hat{i}-\hat{j}+2\hat{k})=(5\hat{i}+3\hat{j}-4\hat{k})$

and $\vec{a}+2\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})+2(3\hat{i}-\hat{j}+2\hat{k})=(\hat{i}+2\hat{j}-3\hat{k})+(6\hat{i}-2\hat{j}+4\hat{k})=(7\hat{i}+\hat{k})$

Now, $|2\vec{a}+\vec{b}|=\sqrt{(5)^2+(3)^2+(-4)^2}=\sqrt{25+9+16}=\sqrt{50}$

$|\vec{a}+2\vec{b}|=\sqrt{(7)^2+(1)^2}=\sqrt{49+1}=\sqrt{50}$

Now, $\cos\theta=\dfrac{(2\vec{a}+\vec{b}).(\vec{a}+2\vec{b})}{|2\vec{a}+\vec{b}|\,|\vec{a}+2\vec{b}|}\Rightarrow\cos\theta=\dfrac{(5\hat{i}+3\hat{j}-4\hat{k}).(7\hat{i}+\hat{k})}{|\sqrt{50}|\,|\sqrt{50}|}$

$\Rightarrow\cos\theta=\dfrac{35-4}{50}\Rightarrow\cos\theta=\dfrac{31}{50}\quad\therefore\theta=\cos^{-1}\left(\dfrac{31}{50}\right)$

Hence required angle is $\cos^{-1}\left(\dfrac{31}{50}\right)$

13. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$, find $|\vec{x}|$.

Sol. We have, $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8\Rightarrow\vec{x}.(\vec{x}+\vec{a})-\vec{a}.(\vec{x}+\vec{a})=8\Rightarrow\vec{x}.\vec{x}+\vec{x}.\vec{a}-\vec{a}.\vec{x}=8$

$\Rightarrow|\vec{x}|^2-|\vec{a}|^2=8\quad\Rightarrow|\vec{x}|^2=8+|\vec{a}|^2\quad\Rightarrow|\vec{x}|^2=8+1\quad[\because|\vec{a}|=1]$

$\Rightarrow|\vec{x}|^2=9\quad\Rightarrow|\vec{x}|=\sqrt{9}\quad\Rightarrow|\vec{x}|=3$

14. Find the angles which the vector $\vec{a}=3\hat{i}-6\hat{j}+2\hat{k}$ makes with the coordinate axes.

Sol. Let $\vec{a}=3\hat{i}-6\hat{j}+2\hat{k}$

$|\vec{a}|=\sqrt{(3)^2+(-6)^2+(2)^2}=\sqrt{9+36+4}=\sqrt{49}=7$

$\Rightarrow\vec{a}.\hat{i}=|\vec{a}||\hat{i}|\cos\theta\quad\Rightarrow\cos\theta=\dfrac{\vec{a}.\hat{i}}{|\vec{a}|\,|\hat{i}|}\quad\Rightarrow\cos\theta=\dfrac{(3\hat{i}-6\hat{j}+2\hat{k}).\hat{i}}{7}$

$\Rightarrow\cos\theta=\dfrac{3}{7}\quad\Rightarrow\theta=\cos^{-1}\left(\dfrac{3}{7}\right)$

$\vec{a}.\hat{j}=|\vec{a}||\hat{j}|\cos\theta\quad\Rightarrow\cos\theta=\dfrac{\vec{a}.\hat{j}}{|\vec{a}||\hat{j}|}\quad\Rightarrow\cos\theta=\dfrac{(3\hat{i}-6\hat{j}+2\hat{k}).\hat{j}}{7}$

$\Rightarrow\cos\theta=\dfrac{-6}{7}\quad\Rightarrow\theta=\cos^{-1}\left(\dfrac{-6}{7}\right)\quad\Rightarrow\theta=\cos^{-1}\left(\dfrac{6}{7}\right)$

$\vec{a}.\hat{k}=|\vec{a}||\hat{k}|\cos\theta\quad\Rightarrow\cos\theta=\dfrac{\vec{a}.\hat{k}}{|\vec{a}||\hat{k}|}\quad\Rightarrow\cos\theta=\dfrac{(3\hat{i}-6\hat{j}+2\hat{k}).\hat{k}}{7}$

$\Rightarrow\cos\theta=\dfrac{2}{7}\quad\Rightarrow\theta=\cos^{-1}\left(\dfrac{2}{7}\right)$

15. Show that the vector $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right)$ is equally inclined to the coordinate axes.

Sol. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$$|\vec{a}|=\sqrt{(1)^2+(1)^2+(1)^2}=\sqrt{3}$$

$$\Rightarrow \vec{a}.\hat{i}=|\vec{a}||\hat{i}|\cos\theta \quad \Rightarrow \cos\theta=\frac{\vec{a}.\hat{i}}{|\vec{a}||\hat{i}|} \quad \Rightarrow \cos\theta=\frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\hat{i}}{\sqrt{3}.1}$$

$$\Rightarrow \cos\theta=\frac{1}{\sqrt{3}} \quad \Rightarrow \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \vec{a}.\hat{j}=|\vec{a}||\hat{j}|\cos\theta \quad \Rightarrow \cos\theta=\frac{\vec{a}.\hat{j}}{|\vec{a}||\hat{j}|} \quad \Rightarrow \cos\theta=\frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\hat{j}}{\sqrt{3}.1}$$

$$\Rightarrow \cos\theta=\frac{1}{\sqrt{3}} \quad \Rightarrow \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \vec{a}.\hat{k}=|\vec{a}||\hat{k}|\cos\theta \quad \Rightarrow \cos\theta=\frac{\vec{a}.\hat{k}}{|\vec{a}||\hat{k}|} \quad \Rightarrow \cos\theta=\frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\hat{k}}{\sqrt{3}.1}$$

$$\Rightarrow \cos\theta=\frac{1}{\sqrt{3}} \quad \Rightarrow \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right). \text{ Hence } \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

16. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle $\frac{\pi}{4}$ and x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis

Sol. Direction cosines of $\vec{a}$ are,

$\ell=\cos\frac{\pi}{4}$, $m=\cos\frac{\pi}{2}$

and $n=\cos\theta$

$\because\ \ell^2+m^2+n^2=1$

$$\Rightarrow \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{2}+\cos^2\theta=1 \Rightarrow \frac{1}{2}+0+\cos^2\theta=1 \Rightarrow \cos^2\theta=1-\frac{1}{2} \Rightarrow \cos^2\theta=\frac{1}{2} \Rightarrow \cos\theta=\pm\frac{1}{\sqrt{2}}$$

for acute angle, $\cos\theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$

17. Find the angle between vectors $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$, if $\vec{a}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}+\hat{j}-2\hat{k}\right)$.

Sol. $\vec{a}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)$, $\vec{b}=\left(3\hat{i}+\hat{j}-2\hat{k}\right) \quad \Rightarrow \left(\vec{a}+\vec{b}\right)=\left(2\hat{i}-\hat{j}+3\hat{k}\right)+\left(3\hat{i}+\hat{j}-2\hat{k}\right)=\left(5\hat{i}+\hat{k}\right)$

$$\Rightarrow \left(\vec{a}-\vec{b}\right)=\left(2\hat{i}-\hat{j}+3\hat{k}\right)-\left(3\hat{i}+\hat{j}-2\hat{k}\right)=\left(-\hat{i}-2\hat{j}+5\hat{k}\right)$$

$$\Rightarrow \left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=\left(5\hat{i}-\hat{k}\right)\cdot\left(-\hat{i}-2\hat{j}+5\hat{k}\right)=-5+5=0 \quad \Rightarrow \left|\vec{a}+\vec{b}\right|=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right)=\sqrt{(-1)^2+(-2)^2+(5)^2}=\sqrt{1+4+25}=\sqrt{30}$$

$$\cos\theta=\frac{\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right|} \quad \Rightarrow \cos\theta=\frac{0}{\left(\sqrt{20}\right)\left(\sqrt{30}\right)} \quad \Rightarrow \cos\theta=0 \quad \Rightarrow \cos\theta=\cos\frac{\pi}{2} \quad \therefore \theta=\frac{\pi}{2}$$

18. Express the vector $\vec{a}=(6\hat{i}-3\hat{j}-6\hat{k})$ as sum of two vectors such that one is parallel to the vector $\vec{b}=(\hat{i}+\hat{j}+\hat{k})$ and the other is perpendicular to $\vec{b}$

Sol. Let, $\vec{a}=\vec{r}+\vec{s}$ such that, $\vec{r}\parallel\vec{b}$ and $\vec{s}\perp\vec{b}$

$\because \vec{r}\parallel\vec{b}$

$\Rightarrow \vec{r}=r\vec{b}$, where r is some non-zero real number. $\Rightarrow \vec{r}=(r\hat{i}+r\hat{j}+r\hat{k})$

$\because \vec{s}\perp\vec{b}=\vec{s}\cdot\vec{b}=0$

Now, $\vec{a}=\vec{r}+\vec{s}$

$\Rightarrow \vec{a}\cdot\vec{b}=\vec{r}\cdot\vec{b}+\vec{s}\cdot\vec{b}\Rightarrow(6\hat{i}-3\hat{j}-6\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=(r+r+r)+0$

$\Rightarrow 6-3-6=3r\Rightarrow r=-1\therefore \vec{r}=-\hat{i}-\hat{j}-\hat{k}$

Now, $\vec{a}=\vec{r}+\vec{s}$

$\Rightarrow \vec{s}=\vec{a}-\vec{r}\Rightarrow\vec{s}=6\hat{i}-3\hat{j}-6\hat{k}+\hat{i}+\hat{j}+\hat{k}\Rightarrow\vec{s}=7\hat{i}-2\hat{j}-5\hat{k}$

Hence, $\vec{a}=(-\hat{i}-\hat{j}-\hat{k})+(7\hat{i}-2\hat{j}-5\hat{k})$

19. Proved that $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2\Leftrightarrow\vec{a}\perp\vec{b}$, where $\vec{a}\neq\vec{0}$ and $\vec{b}\neq\vec{0}$.

Sol. We have, $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}+\vec{b}|^2=|a|^2+|b|^2$

If $|a|^2+|b|^2+2\vec{a}.\vec{b}=|\vec{a}|^2+|\vec{b}|^2$, i.e., if $2\vec{a}.\vec{b}=0$ if $\vec{a}.\vec{b}=0$, i.e., $\vec{a}$ and $\vec{b}$ are at right angle.

20. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, find the angle between $\vec{a}$ and $\vec{b}$.

Sol. We have, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}\quad\Rightarrow(\vec{a}+\vec{b})(\vec{a}+\vec{b})=(-\vec{c})(-\vec{c})$

$\Rightarrow |\vec{a}|^2+\vec{a}.\vec{b}+\vec{b}.\vec{a}+|\vec{b}|^2=|\vec{c}|^2\quad\Rightarrow|\vec{a}|^2+2(\vec{a}.\vec{b})+|\vec{b}|^2=|\vec{c}|^2\quad\Rightarrow|\vec{a}|+2(|\vec{a}||\vec{b}|\cos\theta)+|\vec{b}|=|\vec{c}|^2$

$\Rightarrow (3)^2+2(2.5\cos\theta)+(5)^2=(7)^2\quad\Rightarrow9+30\cos\theta+25=49$

$\Rightarrow 30\cos\theta=49-34\quad\Rightarrow\cos\theta=\frac{15}{30}\quad\Rightarrow\cos\theta=\frac{1}{2}\quad\Rightarrow\theta=\cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \theta=\cos^{-1}\left(\cos\frac{\pi}{3}\right)\quad\Rightarrow\theta=\frac{\pi}{3}$. Required angle is $\frac{\pi}{3}=60^\circ$.

21. Find the angle between $\vec{a}$ and $\vec{b}$, when

(i) $|\vec{a}|=2, |\vec{b}|=1$ and $\vec{a}\cdot\vec{b}=\sqrt{3}$ (ii) $|\vec{a}|=|\vec{b}|=\sqrt{2}$ and $\vec{a}\cdot\vec{b}=-1$

Sol. Let θ be the angle between $\vec{a}$ and $\vec{b}$.

(i) $\theta=\cos^{-1}\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\quad\Rightarrow\theta=\cos^{-1}\frac{\sqrt{3}}{2\times1}\Rightarrow\theta=\frac{\pi}{6}$

(ii) $\theta = \cos^{-1}\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \theta = \cos^{-1}\dfrac{-1}{\sqrt{2}\cdot\sqrt{2}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-1}{2}\right) \Rightarrow \theta = \pi - \cos^{-1}\dfrac{1}{2} \Rightarrow \theta = \pi - \dfrac{\pi}{3} = 2\dfrac{\pi}{3}.$

22. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, find $|\vec{a}-\vec{b}|$.

Sol. We have, $|\vec{a}| = 2,\ |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$

$\Rightarrow |\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} = (2)^2 + (3)^2 - 2(4) = 4+9-8 = 5 \Rightarrow |\vec{a}-\vec{b}|^2 = 5 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{5}$

23. If $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$ find $|\vec{a}|$ and $|\vec{b}|$

Sol. $\because \left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right) = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8 \Rightarrow \{8|\vec{b}|\}^2 - |\vec{b}|^2 = 8\ \{\because |\vec{a}| = 8|\vec{b}|$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8 \Rightarrow 63\cdot|\vec{b}|^2 = 8 \Rightarrow |\vec{b}|^2 = \dfrac{8}{63} \Rightarrow |\vec{b}| = \dfrac{2\sqrt{2}}{\sqrt{63}}$

$\therefore\ |\vec{a}| = \dfrac{16\sqrt{2}}{\sqrt{63}}$

24. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ then prove that

(i) $\cos\dfrac{\theta}{2} = \dfrac{1}{2}|\hat{a}+\hat{b}|$ (ii) $\tan\dfrac{\theta}{2} = \dfrac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$

Sol. (i) We know that, $|\hat{a}+\hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}\cdot\hat{b}$

$\Rightarrow |\hat{a}+\hat{b}|^2 = 1+1+2\cos\theta \Rightarrow |\hat{a}+\hat{b}|^2 = 2(1+\cos\theta) \Rightarrow |\hat{a}+\hat{b}|^2 = 2\cdot2\cos^2\dfrac{\theta}{2} \Rightarrow |\hat{a}+\hat{b}| = 2\cos\dfrac{\theta}{2}$

$\Rightarrow \cos\dfrac{\theta}{2} = \dfrac{1}{2}|\hat{a}+\hat{b}|$ Proved

(ii) We have, $\dfrac{|\hat{a}+\hat{b}|^2}{|\hat{a}-\hat{b}|^2} = \dfrac{|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}\cdot\hat{b}}{|\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a}\cdot\hat{b}}$

$\Rightarrow \dfrac{|\hat{a}+\hat{b}|^2}{|\hat{a}-\hat{b}|^2} = \dfrac{1+1+2\cos\theta}{1+1-2\cos\theta} \to \dfrac{|\hat{a}+\hat{b}|^2}{|\hat{a}-\hat{b}|^2} = \dfrac{2(1+\cos\theta)}{2(1-\cos\theta)} \Rightarrow \dfrac{|\hat{a}+\hat{b}|^2}{|\hat{a}-\hat{b}|^2} = \dfrac{2\cos^2\theta/2}{2\sin^2\theta/2}$

$\Rightarrow \dfrac{|\hat{a}+\hat{b}|}{|\hat{a}-\hat{b}|} = \cot\dfrac{\theta}{2} \Rightarrow \tan\dfrac{\theta}{2} = \dfrac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$

25. If $\vec{a} = (5\hat{i}-\hat{j}+7\hat{k})$ and $\vec{b} = (\hat{i}-\hat{j}-\lambda\hat{k})$, find the value of λ for which $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are orthogonal.

Sol. $\vec{a}+\vec{b} = 5\hat{i}-\hat{j}+7\hat{k}+\hat{i}-\hat{j}-\lambda\hat{k} = 6\hat{i}-2\hat{j}+(7-\lambda)\hat{k}$

$\vec{a}-\vec{b} = (5\hat{i}-\hat{j}+7\hat{k})-(\hat{i}-\hat{j}-\lambda\hat{k}) = 5\hat{i}-\hat{j}+7\hat{k}-\hat{i}+\hat{j}+\lambda\hat{k} \Rightarrow \vec{a}-\vec{b} = 4\hat{i}+(7+\lambda)\hat{k}$

$(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b}) \Rightarrow (\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0 \Rightarrow \{6\hat{i}-2\hat{j}+(7-\lambda)\hat{k}\}.\{4\hat{i}+(7+\lambda)\hat{k}\} = 0$

$\Rightarrow 24+(7+\lambda)(7-\lambda) = 0 \Rightarrow 24+49-\lambda^2 = 0 \Rightarrow \lambda^2 = 73 \Rightarrow \lambda = \pm\sqrt{73}$

26. If $\vec{AB}-\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ and the coordinates of A are $(0,-2,-1)$, find the coordinates of B

Sol. Let, Coordinates of
B are (α, β, γ)

Now, $\vec{AB}=(\alpha-1)\hat{i}+(\beta+2)\hat{j}+(\gamma+1)\hat{k}$

$\Rightarrow 3\hat{i}-\hat{j}+2\hat{k}=\alpha\hat{i}+(\beta+2)\hat{j}+(\gamma+1)\hat{k}$

$\therefore \alpha=3, \beta+2=-1$ and $\gamma+1=2 \Rightarrow \alpha=3. \beta=-3$ & $\gamma=1$

Hence, Co-ordinates of B are (3, - 3, 1).

27. If $A(2,3,4), B(5m4m-1), C(3,6,2)$ and $D(1,2,0)$ be four points show that $\vec{AB}$ is perpendicular to $\vec{CD}$

Sol. Here, $\vec{AB}=(5-2)\hat{i}+(4-3)\hat{j}+(-1-4)\hat{k}=3\hat{i}+\hat{j}-5\hat{k}$

and $\vec{CD}=(1-3)\hat{i}+(2-6)\hat{j}+(0-2)\hat{k}=-2\hat{i}-4\hat{j}-2\hat{k}$

Now, $\vec{AB}\cdot\vec{CD}=3\times(-2)+1\times(-4)+(-5)\times(-2)$

$=-6-4+10=0 \therefore \vec{AB}\perp\vec{CD}$

28. Find the value of λ for which the vectors $\left(2\hat{i}+\lambda\hat{j}+3\hat{k}\right)$ and $\left(3\hat{i}+2\hat{j}-4\hat{k}\right)$ are perpendicular to each other

Sol. $\because (2\hat{i}+\lambda\hat{j}+3\hat{k}) \perp (3\hat{i}+2\hat{j}-4\hat{k})$

$\Rightarrow (2\hat{i}+\lambda\hat{j}+3\hat{k})\cdot(3\hat{i}+2\hat{j}-4\hat{k})=0 \Rightarrow 6+2\lambda-12=0 \Rightarrow 2\lambda-6=0 \Rightarrow 2\lambda=6 \Rightarrow \lambda=3$

29. Show that the vectors $\vec{a}=\left(3\hat{i}-2\hat{j}+\hat{k}\right), \vec{b}=\left(\hat{i}-3\hat{j}+5\hat{k}\right)$ and $\vec{c}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)$ form a right angled triangle

Sol. $\vec{b}+\vec{c}=(\hat{i}-3\hat{j}+5\hat{k})+(2\hat{i}+\hat{j}-4\hat{k})$

$=3\hat{i}-2\hat{j}+\hat{k}=\vec{a} \quad \because \vec{b}+\vec{c}=\vec{a}$

$\Rightarrow \vec{a}, \vec{b}$ and $\vec{c}$ from a triangle

Now, $\vec{a}\cdot\vec{c}=(3\hat{i}-2\hat{j}+\hat{k})\cdot(2\hat{i}+\hat{j}-4\hat{k})=6-2-4=0 \Rightarrow \vec{a}\perp\vec{c}$

Hence, $\vec{a}, \vec{b}$ and $\vec{c}$ from a right angled triangle.

30. Three vertices of a triangle are $A(0,-1,-2), B(3,1,4)$ and $C(5,7,1)$. show that it is a right angled triangle. Also, find its other two angles

Sol. $\vec{AB}=(3-0)\hat{i}+(1+1)\hat{j}+(4+2)\hat{k}$

$=3\hat{i}+2\hat{j}+6\hat{k}$

$\vec{BC}=(5-3)\hat{i}+(7-1)\hat{j}+(1-4)\hat{k}=2\hat{i}+6\hat{j}-3\hat{k}$

$\vec{AC}=(5-0)\hat{i}+(7+1)\hat{j}+(1+2)\hat{k}=5\hat{i}+8\hat{j}+3\hat{k}$

Now, $\vec{AB}\cdot\vec{BC}=6+12-18=0 \therefore \vec{AB}\perp\vec{BC}$

Hence, ΔABC is a right angled triangle.

Now, $\angle A = \cos^{-1}\dfrac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AC}|}$

$= \cos^{-1}\dfrac{15+16+18}{\sqrt{3^2+2^2+6^2}\sqrt{5^2+8^2+3^2}} = \cos^{-1}\dfrac{49}{7\times\sqrt{98}} = \cos^{-1}\dfrac{49}{49\times\sqrt{2}} = \cos^{-1}\dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

$\because \angle B = \dfrac{\pi}{2}$ and $\angle A = \dfrac{\pi}{4}$ $\quad \Rightarrow \angle C = \pi - \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4}$

31. If the position vectors of the vertices A, B and C of ΔABC be $(1,2,3),(-1,0,0)$ and $(0,1,2)$ respectively then find $\angle ABC$

Sol. Let O be the origin,

Position vector of A,

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

Position vector of B,

$\overrightarrow{OB} = \hat{i}$

Position vector of C, $\quad \overrightarrow{OC} = \hat{j} + 2\hat{k}$

Now, $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ & $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \hat{i} + \hat{j} + 2\hat{k}$

$\angle B = \cos^{-1}\dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$\Rightarrow \angle B = \cos^{-1}\dfrac{2+2+6}{\sqrt{4+4+9}\sqrt{1+1+4}} \Rightarrow \angle B = \cos^{-1}\dfrac{10}{\sqrt{17}\sqrt{6}} \Rightarrow \angle B = \cos^{-1}\dfrac{10}{\sqrt{102}}$

32. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|\vec{a}+\vec{b}| = \sqrt{3}$, find $(2\vec{a}-5\vec{b})\cdot(3\vec{a}+\vec{b})$

Sol. Given:$\rightarrow$ $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{a}+\vec{b}| = \sqrt{3}$

$\because |\vec{a}+\vec{b}| = \sqrt{3} \Rightarrow |\vec{a}+\vec{b}|^2 = 3$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 3 \Rightarrow 1+1+2\,\vec{a}\cdot\vec{b} = 3 \Rightarrow \vec{a}\cdot\vec{b} = \dfrac{1}{2}$

Now, $\left(2\vec{a}-5\vec{b}\right)\cdot\left(2\vec{a}+\vec{b}\right)$

$= 6|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} - 15\vec{a}\cdot\vec{b} - 5|\vec{b}|^2 = 6 - 13\vec{a}\cdot\vec{b} - 5 = 1 - 13\times\dfrac{1}{2} = \dfrac{2-13}{2} = -\dfrac{11}{2}$

33. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$ then prove that vector $(2\vec{a}+\vec{b})$ is perpendicular to vector $\vec{b}$

Sol. $\because |\vec{a}+\vec{b}| = |\vec{a}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 \Rightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{b}\cdot\vec{b} + 2\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{b}\cdot\left(2\vec{a}+\vec{b}\right) = 0$

Hence, $\vec{b} \perp r\ 2\vec{a}+\vec{b}$ Proved

34. If $\vec{a}=(3\hat{i}-\hat{j})$ and $\vec{b}=(2\hat{i}+\hat{j}-3\hat{k})$ then express $\vec{b}$ in the form $\vec{b}=(\vec{b_1}+\vec{b_2})$, where $\vec{b_1} \parallel \vec{a}$ and $\vec{b_1} \perp \vec{a}$

Sol. Let, $\vec{b_1} = x\vec{a}$ $\{\because \vec{b_1} \parallel \vec{a}$

$\Rightarrow \vec{b_1} = 3x\hat{i} - x\hat{j}$

Now, $\vec{b} = \vec{b_1} + \vec{b_2}$

$\Rightarrow \vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{b_1} + \vec{a}\cdot\vec{b_2} \Rightarrow \vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{b_1} + +0 \{\because \vec{a} \perp \vec{b_2} \Rightarrow \vec{a}\cdot\vec{b_2} = 0$

$\Rightarrow \left(3\hat{i}-\hat{j}\right)\cdot\left(2\hat{i}+\hat{j}-3\hat{k}\right) = \left(3\hat{i}-\hat{j}\right)\cdot\left(3x\hat{i}-x\hat{j}\right)$

$\Rightarrow 6-1 = 9x + x \Rightarrow 5 - 10x \Rightarrow x = \frac{1}{2} \therefore \vec{b_1} = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

Now, $\vec{b_2} = \vec{b} - \vec{b_1}$

$\Rightarrow \vec{b_2} = \left(2\hat{i}+\hat{j}-3\hat{k}\right) - \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) \Rightarrow \vec{b_2} = \left(2-\frac{3}{2}\right)\hat{i} + \left(1+\frac{1}{2}\hat{j}\right) - 3\hat{k} \Rightarrow \vec{b_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Hence, $\vec{b_1} = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$ $\vec{b_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$