

Ex 23.1

Q1

(i) Angle made with positive x axis is $\frac{-\pi}{4}$.

$$\therefore m = \tan \theta = \tan\left(\frac{-\pi}{4}\right) = -1$$

(ii) Angle made with positive x axis is $\frac{2\pi}{3}$

$$\therefore m = \tan \theta = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

(iii) Angle made with positive x axis is $\frac{3\pi}{4}$

$$\therefore m = \tan \theta = \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -1$$

(iv) Angle made with positive x axis is $\frac{\pi}{3}$

$$\therefore m = \tan \theta = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

Q2

(i) $(-3, 2)$ and $(1, 4)$

$$\text{slope of line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{1 - (-3)} = \frac{2}{4} = \frac{1}{2}$$

(ii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

$$\text{slope of line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \frac{2}{t_2 + t_1}$$

(iii) $(3, -5)$ and $(1, 2)$

$$\text{slope of line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{1 - 3} = \frac{7}{-2} = -\frac{7}{2}$$

Q3(i)

Slope of line joining (5,6) and (2,3)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{2 - 5} = \frac{-3}{-3} = 1$$

Slope of line joining (9,-2) and (6,-5)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-2)}{6 - 9} = \frac{-5 + 2}{-3} = 1$$

$$\text{Here } m_1 = m_2$$

∴ The two lines are parallel.

Q3(ii)

Slope of line joining (-1,1) and (9,5)

$$m_1 = \frac{5 - 1}{9 - (-1)} = \frac{4}{10} = \frac{2}{5}$$

Slope of line joining (3,-5) and (8,-3)

$$m_2 = \frac{-3 - (-5)}{8 - 3} = \frac{-3 + 5}{5} = \frac{2}{5}$$

$$\text{Here } m_1 = m_2$$

∴ The two lines are parallel

Q3(iii)

Slope of line joining (6,3) and (1,1)

$$m_1 = \frac{1 - 3}{1 - 6} = \frac{-2}{-5} = \frac{2}{5}$$

Slope of line joining (-2,5) and (2,-5)

$$m_2 = \frac{-5 - 5}{2 - (-2)} = \frac{-10}{4} = \frac{-5}{2}$$

$$\text{Here } m_1 \times m_2 = \frac{2}{5} \times \frac{-5}{2} = -1$$

∴ The lines are perpendicular to each other.

Q3(iv)

Slope of line joining (3,15) and (16,6)

$$m_1 = \frac{6 - 15}{16 - 3} = \frac{-9}{13}$$

Slope of line joining (-5,3) and (8,2)

$$m_2 = \frac{2 - 3}{8 - (-5)} = \frac{-1}{13}$$

Here, neither $m_1 = m_2$ nor $m_1 \times m_2 = -1$

$\therefore$ The lines are neither parallel nor perpendicular.

Q4

(i) Line bisects first quadrant.

$$\Rightarrow \text{Angle between line and positive direction of } x\text{-axis} = \frac{90^\circ}{2} = 45^\circ$$

Slope of line (m) = $\tan \theta$

$$m = \tan 45^\circ$$

$$m = 1$$

(ii) Line makes angle of 30° with the positive direction of y -axis.

= Angle between line and positive side of axis = $90^\circ + 30^\circ$

$$\theta^\circ = 120^\circ$$

$$m = \tan 120^\circ$$

$$m = -\sqrt{3}$$

Q5(i)

A (4,8), B (5,12) and C (9,28)

$$\text{slope of } AB = \frac{12 - 8}{5 - 4} = \frac{4}{1} = 4$$

$$\text{slope of } BC = \frac{28 - 12}{9 - 5} = \frac{16}{4} = 4$$

$$\text{slope of } CA = \frac{8 - 28}{4 - 9} = \frac{-20}{-5} = 4$$

Since all 3 line segments have the same slope, they are parallel.

Since they have a common point B, they are collinear.

Q5(ii)

$A(16, -18), B(3, -6)$ and $C(-10, 6)$

$$\text{slope of } AB = \frac{-6 - (-18)}{3 - 16} = \frac{12}{-13}$$

$$\text{slope of } BC = \frac{6 - (-6)}{-10 - 3} = \frac{12}{-13}$$

$$\text{slope of } CA = \frac{6 - (-18)}{-10 - 16} = \frac{12}{-13}$$

Since all 3 line segments have the same slope and share a common vertex B, they are collinear.

Q6

Slope of line joining $(-1, 4)$ and $(0, 6)$ is

$$m_1 = \frac{6 - 4}{0 - (-1)} = 2$$

Slope of line joining $(3, y)$ and $(2, 7)$ is

$$m_2 = \frac{7 - y}{2 - 3} = y - 7$$

Since the two lines are parallel $m_1 = m_2$

$$\Rightarrow 2 = y - 7$$

$$\Rightarrow y = 9$$

Q7

(i) If slope = $\tan \theta = 0 \Rightarrow \theta = 0$

When the slope of a line is zero then the line is parallel to x-axis.

(ii) If the slope is positive then $\tan \theta = \text{positive} \Rightarrow \theta = \text{acute}$

Thus the line makes an acute angle $\left(0 < \theta < \frac{\pi}{2}\right)$ with the positive x-axis.

(iii) When the slope is negative then $\tan \theta = \text{negative} \Rightarrow \theta$ is obtuse

Thus the line makes an obtuse angle $\left(\theta > \frac{\pi}{2}\right)$ with the positive x-axis.

Q8

Slope of line joining $(2, -3)$ and $(-5, 1)$

$$m_1 = \frac{1 - (-3)}{-5 - 2} = \frac{4}{-7}$$

Slope of line joining $(7, -1)$ and $(0, 3)$

$$m_2 = \frac{3 - (-1)}{0 - 7} = \frac{4}{-7}$$

Since $m_1 = m_2$, the two lines are parallel.

Q9

Slope of line joining $(2, -5)$ and $(-2, 5)$ is

$$m_1 = \frac{5 - (-5)}{-2 - 2} = \frac{-5}{2}$$

Slope of line joining $(6, 3)$ and $(1, 1)$

$$m_2 = \frac{1 - 3}{1 - 6} = \frac{2}{5}$$

$$m_1 \times m_2 = \frac{-5}{2} \times \frac{2}{5} = -1$$

$\therefore$ The two lines are perpendicular to each other.

Q10

$$\text{Slope of } AB = \frac{2 - 4}{1 - 0} = -2$$

$$\text{Slope of } BC = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$

$$\text{slope of } AB \times \text{slope of } BC = -2 \times \frac{1}{2} = -1$$

$$\therefore \text{Angle between } AB \text{ and } BC = \frac{\pi}{2}$$

$\therefore ABC$ are the vertices of a right angled triangle.



Q11

Here $A(-4, -1), B(-2, -4), C(4, 0), D(2, 3)$

$$\text{Slope of } AB = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{-4 + 1}{-2 + 4}$$

$$M_{AB} = \frac{-3}{2}$$

$$\text{Slope of } BC = \frac{0 + 4}{4 + 2}$$

$$M_{BC} = \frac{2}{3}$$

$$\text{Slope of } AD = \frac{3 + 1}{2 + 4}$$

$$M_{AD} = \frac{2}{3}$$

$$\text{Slope of } CD = \frac{3 - 0}{2 - 4}$$

$$M_{CD} = \frac{-3}{2}$$

$$\Rightarrow M_{AB} = M_{CD} \text{ and } M_{BC} = M_{AD}$$

$$\Rightarrow AB \parallel CD \text{ and } BC \parallel AD$$

$$M_{AB} \times M_{BC} = \frac{-3}{2} \times \frac{2}{3}$$

$$M_{AB} \times M_{BC} = -1$$

$$\Rightarrow AB \perp BC$$

$$M_{BC} \times M_{CD} = \frac{2}{3} \times \frac{-3}{2}$$

$$M_{BC} \times M_{CD} = -1$$

$$\Rightarrow BC \perp CD$$

Thus,

$$AB \parallel CD \text{ and } BC \parallel AD$$

$$AB \perp BC, BC \perp CD, CD \perp AD$$

$$\Rightarrow ABCD \text{ is a rectangle}$$

Q12

If 3 points lie on a line (ie they are collinear) lines joining these point have the same slope

$\therefore$ slope of AP = slope of PB = slope of BA

$$\Rightarrow \frac{b-0}{a-h} = \frac{k-b}{0-a} = \frac{k-0}{0-h} \dots\dots\dots (i)$$

$$\Rightarrow \frac{k-b}{0-a} = \frac{k-0}{0-h}$$

$$\Rightarrow -kh + bh = -ka$$

$$\Rightarrow -1 + \frac{b}{k} = \frac{-a}{h} \quad (\text{dividing by } kh)$$

$$\Rightarrow \frac{a}{h} + \frac{b}{k} = 1$$

Hence Proved



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Q13

Let $m_1 = x$, $m_2 = 2x$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\frac{1}{3} = \left| \frac{x - 2x}{1 + 2x^2} \right|$$

Case I:

$$\frac{1}{3} = \frac{x - 2x}{1 + 2x^2}$$

$$2x^2 + 1 = -3x$$

$$2x^2 + 3x + 1 = 0$$

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x + 1) + 1(x + 1) = 0$$

$$(x + 1)(2x + 1) = 0$$

$$x = -1, -\frac{1}{2}$$

Case II:

$$\frac{1}{3} = \left(\frac{-x}{1 + 2x^2} \right)$$

$$\frac{1}{3} = \frac{x}{1 + 2x^2}$$

$$2x^2 + 1 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

$$(x - 1)(2x - 1) = 0$$

$$x = 1, \frac{1}{2}$$

Slope of other line is

$$1, \frac{1}{2} \text{ or } -1, -\frac{1}{2}$$

Q14

$$\text{Slope of } AB = \frac{97 - 92}{1995 - 1985} = \frac{5}{10} = \frac{1}{2}$$

Population (p) in 2010 can be calculated using the slope of AC:

$$\text{Slope of } AC = \frac{p - 92}{2010 - 1985} = \frac{p - 92}{25} = \frac{1}{2} = \text{Slope of } AB$$

$$\Rightarrow p - 92 = \frac{25}{2}$$

$$\Rightarrow 2p - 184 = 25$$

$$\Rightarrow 2p = 209$$

$$\Rightarrow p = \frac{209}{2}$$

$$\therefore p = 104.50 \text{ crores}$$

Q15

Let $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$ be a quadrilateral.

$$\text{slop of } AB = \frac{0 - (-1)}{4 - (-2)} = \frac{1}{6}$$

$$\text{slop of } BC = \frac{3 - 0}{3 - 4} = -3$$

$$\text{slop of } CD = \frac{3 - 2}{3 - (-3)} = \frac{1}{6}$$

$$\text{slop of } DA = \frac{2 - (-1)}{-3 - (-2)} = -3$$

we observe that slope of opposite side of the quadrilateral $ABCD$ are equal.
Hence the quadrilateral $ABCD$ is a parallelogram.

Q16

Slope of the line segment joining the points $(3, -1)$ and $(4, -2)$ is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{4 - 3} = \frac{-2 + 1}{4 - 3} = \frac{-1}{1} = -1$$

Slope of x axis is 0

$$\Rightarrow m_2 = 0$$

If θ is the angle between x-axis and the line segment then

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{-1 - 0}{1 + (-1)(0)} \right| \\ &= \frac{-1}{1} = -1 \end{aligned}$$

$$\therefore \theta = 135^\circ$$

Q17

The slope of the line joining $(-2, 6)$ and $(4, 8)$ is

$$m_1 = \frac{8 - 6}{4 - (-2)} = \frac{2}{6} = \frac{1}{3}$$

The slope of the line joining $(8, 12)$ and $(x, 24)$ is

$$m_2 = \frac{24 - 12}{x - 8} = \frac{12}{x - 8}$$

Since the lines are perpendicular to each other

$$m_1 \times m_2 = -1$$

$$\frac{1}{3} \times \frac{12}{x - 8} = -1$$

$$\Rightarrow 4 = 8 - x$$

$$\Rightarrow x = 4$$

Q18

The given points are $A(x, -1)$, $B(2, 1)$ and $C(4, 5)$

It is given that the points are collinear. So, the area of the triangle that they form must be zero.

Hence,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \quad \text{--- (i)}$$

Putting the value of (x_1, y_1) , (x_2, y_2) , (x_3, y_3) in (i)

$$x(1 - 5) + (2)(5 - (-1)) + 4(-1 - 1) = 0$$

$$-4x + 2(5 + 1) + 4(-2) = 0$$

$$-4x + 12 - 8 = 0$$

$$-4x = -12 + 8$$

$$-4x = -4$$

$$x = 1$$

Q19

Slope of the line segment joining the points $(3, -1)$ and $(4, -2)$ is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{4 - 3} = \frac{-2 + 1}{4 - 3} = \frac{-1}{1} = -1$$

Slope of x axis is 0

$$\Rightarrow m_2 = 0$$

If θ is the angle between x-axis and the line segment then

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{-1 - 0}{1 + (-1)(0)} \right| \\ &= \frac{-1}{1} = -1 \end{aligned}$$

$$\therefore \theta = 135^\circ$$

Q20

Let the vertices be $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$, $D(-3, 2)$.

Using slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, we get:

$$\text{Slope of } AB (m_1) = \frac{0 - (-1)}{4 - (-2)} = \frac{1}{6}$$

$$\text{Slope of } CD (m_2) = \frac{2 - 3}{-3 - 3} = \frac{-1}{-6} = \frac{1}{6}$$

$$\Rightarrow m_1 = m_2 \Rightarrow AB \parallel CD$$

Also

$$\text{Slope of } AD (m_3) = \frac{2 - (-1)}{-3 - (-2)} = \frac{3}{-1} = -3$$

$$\text{Slope of } BC (m_4) = \frac{3 - 0}{3 - 4} = \frac{3}{-1} = -3$$

$$\Rightarrow m_3 = m_4 \Rightarrow AD \parallel BC$$

Hence, ABCD is a parallelogram.



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Q21

Let $ABCD$ be the given quadrilateral

E is mid point of AB

F is mid point of BC

G is mid point of CD

H is mid point of AD

Using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\text{Coordinates of } E = \left(\frac{4+1}{2}, \frac{1+7}{2}\right) = \left(\frac{5}{2}, 4\right)$$

$$\text{Coordinates of } F = \left(\frac{1+6}{2}, \frac{7+0}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right)$$

$$\text{Coordinates of } G = \left(\frac{-6+1}{2}, \frac{0+9}{2}\right) = \left(\frac{-5}{2}, \frac{9}{2}\right)$$

$$\text{Coordinates of } H = \left(\frac{-1+4}{2}, \frac{-9+1}{2}\right) = \left(\frac{3}{2}, -4\right)$$

Now, $EFGH$ is parallelogram if diagonals EG and FH have the same mid-point.

$$\text{Coordinates of mid-point of } EG = \left(\frac{5-5}{2}, \frac{4-\frac{9}{2}}{2}\right) = \left(\frac{0}{2}, \frac{-\frac{1}{2}}{2}\right) = \left(0, -\frac{1}{4}\right)$$

$$\text{Coordinates of mid-point of } FH = \left(\frac{-5+3}{2}, \frac{\frac{7}{2}-8}{2}\right) = \left(\frac{-2}{2}, \frac{-\frac{9}{2}}{2}\right) = \left(-1, -\frac{9}{4}\right)$$

$\Rightarrow EFGH$ is parallelogram

Ex 23.2

Q1

Let the equation of the line be:

$$y - y_1 = m(x - x_1)$$

Now,

$$m = 0 \quad [\because \text{Parallel lines have equal slopes, the slope of } x\text{-axis is } 0]$$

$$(x_1, y_1) = (3, -5)$$

$$\therefore y - y_1 = m(x - x_1)$$

$$y - (-5) = 0(x - 3)$$

$$y + 5 = 0$$

Q2

The slope of x -axis is 0, any line perpendicular to it will have

$$\text{slope} = \frac{-1}{0}$$

Also the required line is passing through the point $(-2, 0)$

(because it is given it has x -intercept is -2)

The required equation of line is

$$y - y_1 = m(x - x_1)$$

$$\text{where } m = \frac{-1}{0}, (x_1, y_1) = (-2, 0)$$

$$y - 0 = \frac{-1}{0}(x - (-2))$$

$$y - 0 = \frac{-1}{0}(x + 2)$$

$$-(x + 2) = 0$$

$$x + 2 = 0$$

$$x = -2$$

Q3

The slope of x -axis is 0

Any line parallel to x -axis will also have the same slope.

therefore $m = 0$

Also line has y -intercept, ie. $(0, b)$

$$\Rightarrow (0, -2) \Rightarrow (x_1, y_1)$$

The required equation of the line is $y - y_1 = m(x - x_1)$

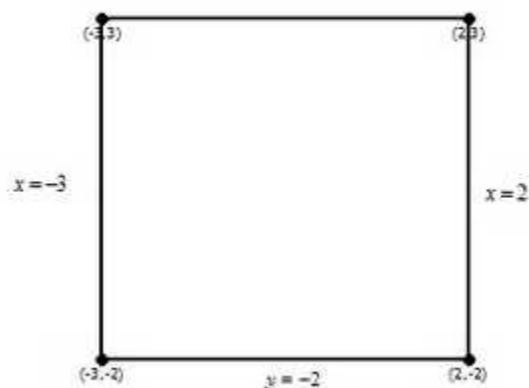
$$y - (-2) = 0(x - 0)$$

$$y + 2 = 0$$

$$y = -2$$

Q4

The figure with the lines $x = -3, x = 2, y = -2, y = 3$ is as follows:



From the figure, the co-ordinates of the vertices of the square are $(2, 3), (-3, 3), (-3, -2), (2, -2)$.

Q5

Slope of a line parallel to x -axis = 0

Since the line passes through $(4, 3)$,

The required equation of the line parallel to x -axis is

$$y - y_1 = m(x - x_1)$$

$$y - (3) = 0(x - 4)$$

$$y - 3 = 0$$

$$y = 3$$

Slope of a line perpendicular to x -axis = $-\frac{1}{0}$

The required equation of the line perpendicular to x -axis is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{-1}{0}(x - 4)$$

$$x - 4 = 0$$

$$x = 4$$

Q6

Let $x = \lambda$ be the line equidistant from

$x = -2$ and $x = 6$

$$\text{so } \left| \frac{-2-\lambda}{\sqrt{1}} \right| = \left| \frac{\lambda-6}{\sqrt{1}} \right|$$

$$-2-\lambda = \lambda-6$$

$$4 = 2\lambda$$

$$\therefore \lambda = 2$$

$\therefore$ The line equidistant from $x = -2$ and $x = 6$ is $x = 2$

Q7

A line which is equidistant from two other lines, must have the same slope.

The slope of $y = 10$ and $y = -2$ is 0, ie line parallel to x-axis.

The required line is also parallel to $y = 10$ and $y = -2$

$$\therefore m = 0$$

Also, the required line will pass from the mid-point of the line joining $(0, -2)$ and $(0, 10)$

Coordinates of this point will be $(0, \frac{10-2}{2}) = (0, \frac{8}{2}) = (0, 4)$

$\therefore$ The equation of the require line is:

$$y-4=0(x-x_1)$$

$$\Rightarrow y = 4$$

Ex 23.3

Q1

The equation of the line having slope m and y -intercept $(0, c)$ is given by:

$$y = mx + c$$

Now, $m = \tan(150^\circ) = \frac{-1}{\sqrt{3}}$

and

y -intercept is $(0, 2)$

The required equation of line is

$$y = mx + c$$

$$\Rightarrow y = \frac{-x}{\sqrt{3}} + 2$$

$$\Rightarrow \sqrt{3}y - 2\sqrt{3} + x = 0$$

$$\Rightarrow x + \sqrt{3}y = 2\sqrt{3}$$



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Q2

(i) With slope 2 and y intercept 3

$m = 2$, point is $(0, 3)$

The required equation of line is

$$y = mx + c$$

$$\Rightarrow y = 2x + 3$$

(ii) slope = $-\frac{1}{3}$, y intercept = $(0, -4)$

$$m = -\frac{1}{3}, c = -4$$

The required equation of line is $y = mx + c$

$$\Rightarrow y = -\frac{1}{3}x - 4$$

$$\Rightarrow 3y + x = -12$$

(iii) $m = -2$, $c = -3$

The required equation of line is

$$y - y_1 = m(x - x_1)$$

Since the line cuts the x -axis at $(-3, 0)$ with slope -2 , we have,

$$y - 0 = -2(x + 3)$$

$$\Rightarrow y = -2x - 6$$

$$\Rightarrow 2x + y + 6 = 0$$

Q3

The given lines are $x = 0$, $y = 0$.

The equation of the bisectors of the angles between $x = 0$ and $y = 0$ are:

$$\frac{x}{\sqrt{(1)^2 + (0)^2}} = \pm \frac{y}{\sqrt{(0)^2 + (1)^2}}$$

$$x = \pm y$$

$$x \pm y = 0$$

Q4

$$\theta = \tan^{-1} 3 \Rightarrow m = \tan \theta = 3$$

Intercept in negative direction of y -axis is $(0, -4)$

Hence, required equation of line is

$$y = mx + c$$

$$\Rightarrow y = 3x - 4$$

Q5

Here, y intercept, $c = -4$

The required line is parallel to line joining $(2, -5)$ and $(1, 2)$

Let m be the slope of the required line, then

$$m = \text{slope of } (2, -5) \text{ and } (1, 2)$$

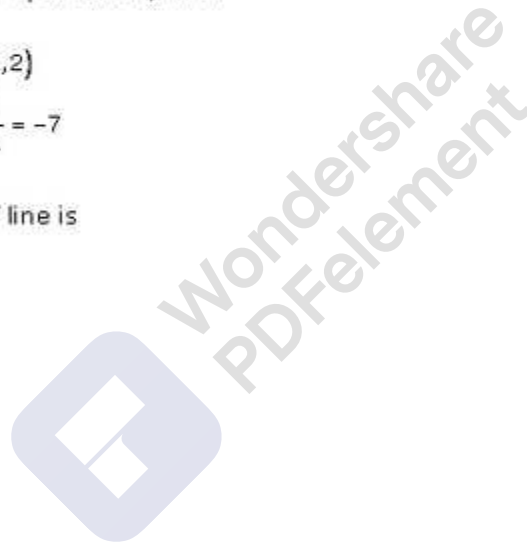
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{1 - 2} = \frac{7}{-1} = -7$$

$\therefore$ the required equation of line is

$$y = mx + c$$

$$y = -7x - 4$$

$$7x + y + 4 = 0$$



Q6

The required equation of line is $y = mx + c$

Here, $c = 3$

Let m be slope of the required line.

Then,

$m \times \text{slope of given line} = -1$

$$\text{Slope of given line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{3 - 4} = \frac{3}{-1} = -3$$

$$\Rightarrow m = \frac{1}{3}$$

So, the required equation is:

$$y = mx + c$$

$$y = \frac{1}{3}x + 3$$

$$x - 3y + 9 = 0$$

Q7

The required equation of line is $y = mx + c$

Here, $c = -3$

Let m be slope of the required line.

Then,

$m \times \text{slope of given line} = -1$

$$\text{Slope of line joining } (4, 3) \text{ and } (-1, 1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{-1 - 4} = \frac{-2}{-5} = \frac{2}{5}$$

$$\Rightarrow m = -\frac{5}{2}$$

So, the required equation is:

$$y = mx + c$$

$$y = -\frac{5}{2}x - 3$$

$$y + 3 = \frac{-5x}{2}$$

$$2y + 5x + 6 = 0$$



Q8

The required equation of line is

$$y - y_1 = m(x - x_1)$$

$$\text{where } m = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{point is } (x_1, y_1) = (0, 2)$$

$$\Rightarrow y - 2 = \frac{1}{\sqrt{3}}(x - 0)$$

$$x - \sqrt{3}y + 2\sqrt{3} = 0$$



Ex 23.4

Q1

Let the required equation of the line be

$$y - y_1 = m(x - x_1)$$

Now,

$$m = \text{slope} = -3$$

$$(x_1, y_1) = (6, 2)$$

$$\therefore y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 2 = -3(x - 6)$$

$$\Rightarrow y - 2 = -3x + 18$$

$$\Rightarrow 3x + y = +20$$

$$\Rightarrow 3x + y - 20 = 0$$

$\therefore$ The equation of the given line is $3x + y - 20 = 0$.

Q2

Let the required equation of the line be

$$y - y_1 = m(x - x_1)$$

Now,

The line is inclined at an angle of 45° with x-axis

$$\therefore m = \tan 45^\circ = 1$$

$$(x_1, y_1) = (-2, 3)$$

$$\therefore y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 3 = 1(x - (-2))$$

$$\Rightarrow y - 3 = x + 2$$

$$\Rightarrow x - y = -5$$

$\therefore$ Equation of required line is $x - y + 5 = 0$

Q3

The required equation of the line is

$$y - y_1 = m(x - x_1)$$

$(x_1, y_1) = (0, 0)$ and slope is m

Therefore, $y - y_1 = m(x - x_1)$

$$y - 0 = m(x - 0)$$

$$y = mx$$

Q4

The required equation of the line is

$$y - y_1 = m(x - x_1)$$

Since the line makes an angle 75° with x -axis

$$m = \tan 75^\circ = 3.73$$

$$(x_1, y_1) = (2, 2\sqrt{3})$$

Therefore, $y - y_1 = m(x - x_1)$

$$y - 2\sqrt{3} = (2 + \sqrt{3})(x - 2)$$

$$y - 2\sqrt{3} = (2 + \sqrt{3})x - 7.46$$

$$(2 + \sqrt{3})x - y - 4 = 0$$

Q5

$$\text{Let } \sin \theta = \frac{3}{4}$$

Then,

$$\Rightarrow m = \text{slope} = \tan \theta = \frac{3}{4}$$

The equation of straight line with slope m and passing through $(1, 2)$ is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{4}(x - 1)$$

$$4y - 8 = 3x - 3$$

$$3x - 4y = -5$$

$$3x - 4y + 5 = 0$$



Q6

The required equation of the line is

$$y - y_1 = m(x - x_1)$$

Since the line makes an angle 60° with the positive direction of y axis, it makes 30° with the positive direction of x axis.

$$\therefore m = \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ (angle with y-axis)}$$

A point on the line is $(x_1, y_1) = (3, -2)$

Therefore, the equation of the line is:

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{1}{\sqrt{3}}(x - 3)$$

$$x - \sqrt{3}y - 3 - 2\sqrt{3} = 0$$



Q7

Equation of the line passing through (x_1, y_1)

and making angle θ with the x-axis is,

$$(y - y_1) = \tan \theta (x - x_1)$$

For the first line: $(x_1, y_1) = (0, 2), \theta = \frac{\pi}{3}$

$$(y - y_1) = \tan \theta (x - x_1)$$

$$(y - 2) = \left(\tan \frac{\pi}{3} \right) (x - 0)$$

$$y - 2 = \sqrt{3}x$$

$$\sqrt{3}x - y + 2 = 0$$

For the second line: $(x_1, y_1) = (0, 2), \theta = \frac{2\pi}{3}$

$$(y - y_1) = \tan \theta (x - x_1)$$

$$(y - 2) = \left(\tan \frac{2\pi}{3} \right) (x - 0)$$

$$y - 2 = -\sqrt{3}x$$

$$\sqrt{3}x + y - 2 = 0$$

The line parallel to $\sqrt{3}x - y + 2 = 0$

and cutting y-axis at a distance of 2 units below the origin.

$$y = \sqrt{3}x - 2$$

$$\sqrt{3}x + y - 2 = 0$$

The line parallel to $\sqrt{3}x + y - 2 = 0$

and cutting y-axis at a distance of 2 units below the origin.

$$y = -\sqrt{3}x - 2$$

$$\sqrt{3}x + y + 2 = 0$$

Q8

If a line is equally inclined to axis, then

$$\theta = 45^\circ \quad \text{or } \theta = 135^\circ \Rightarrow m = \tan \theta = \pm 1$$

Since, y intercept, $c = 5$

$\therefore$ We get the solution of the line as:

$$y = mx + c$$

$$y = \pm 1x + 5$$

$$y - x = 5 \text{ or } y + x = 5$$

Q9

The line passes through the point (2,0).

Also its inclination to y-axis is 135° .

That is, the inclination of the given line with the x-axis is $180^\circ - 135^\circ$.

That is, the slope of the given line is 45°

The equation of the line having slope 'm' and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

Therefore, the required equation is

$$y - 0 = \tan 45^\circ (x - 2)$$

$$\Rightarrow y = 1 \times (x - 2)$$

$$\Rightarrow y = x - 2$$

$$\Rightarrow x - y - 2 = 0$$

Q10

The coordinates of the point which divides the join of the points (2,3) and (-5,8) in the ratio 3:4 is given by (x, y) where,

$$x = \frac{lx_2 + mx_1}{l+m} = \frac{3(-5) + 4(2)}{3+4} = \frac{-15+8}{7} = \frac{-7}{7}$$

$$y = \frac{ly_2 + my_1}{l+m} = \frac{3(8) + 4(3)}{3+4} = \frac{24+12}{7} = \frac{36}{7}$$

$$\text{Slope of the line joining the points (2,3) and (-5,8)} = \frac{8-3}{-5-2} = \frac{5}{-7} = \frac{-5}{7}$$

$$\therefore \text{Slope of line perpendicular to line} = m = \frac{7}{5}$$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - \frac{36}{7} = \frac{7}{5} \left(x - \left(\frac{-7}{7} \right) \right)$$

$$49x - 35y + 229 = 0$$



Q11

Let the perpendicular drawn from $P(4,1)$ on line joining $A(2,-1)$ and $B(6,5)$ divide in the ratio $k:1$ at the point R .

Using section formula, coordinates of R are:

$$x = \frac{6k+2}{k+1} \text{ and } y = \frac{5k-1}{k+1} \quad \text{---(1)}$$

PR is perpendicular to AB

$$\therefore (\text{slope of } PR) \times (\text{slope of } AB) = -1$$

$$\Rightarrow \left(\frac{y-1}{x-4} \right) \times \left(\frac{5-(-1)}{6-2} \right) = -1$$

$$\Rightarrow \frac{\frac{5k-1}{k+1}-1}{\frac{6k+2}{k+1}-4} \times \frac{6}{4} = -1$$

$$\Rightarrow \frac{5k-1-k-1}{6k+2-4k-4} = \frac{-4}{6}$$

$$\Rightarrow \frac{4k-2}{2k-2} = \frac{-2}{3}$$

$$\Rightarrow 3(2k-1) = -2(k-1)$$

$$\Rightarrow 6k-3 = -2k+2$$

$$\Rightarrow 8k = 5$$

$$\Rightarrow k = \frac{5}{8}$$

ratio is $5:8$

$\therefore R$ divides AB in the ratio $5:8$

Q12

AD, BE and CF are the three altitudes of the triangle

We know,

$$\text{Slope of } AD \times \text{Slope of } BC = -1; \quad AD \text{ passes through } A(2, -2)$$

$$\text{Slope of } BE \times \text{Slope of } AC = -1; \quad AD \text{ passes through } B(1, 1)$$

$$\text{Slope of } CF \times \text{Slope of } AB = -1; \quad AD \text{ passes through } C(-1, 0)$$

$$\text{Slope of } BC = \frac{0 - 1}{-1 - 1} = \frac{-1}{-2} = \frac{1}{2} \quad \Rightarrow \text{Slope of } AD = -2$$

$$\text{Slope of } AC = \frac{0 - (-2)}{-1 - 2} = \frac{2}{-3} = -\frac{2}{3} \quad \Rightarrow \text{Slope of } BE = \frac{3}{2}$$

$$\text{Slope of } AB = \frac{1 + 2}{1 - 2} = \frac{3}{-1} = -3 \quad \Rightarrow \text{Slope of } CF = \frac{1}{3}$$

So, for AD , we have

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - (-2) = -2(x - 2)$$

$$\Rightarrow y + 2 = -2x + 4$$

$$\Rightarrow 2x + y - 2 = 0$$

And, for BE , we have

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 1 = \frac{3}{2}(x - 1)$$

$$\Rightarrow 2y - 3x + 1 = 0$$

And, for CF , we have

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 0 = \frac{1}{3}(x + 1)$$

$$\Rightarrow x - 3y + 1 = 0$$

Q13

The right bisector PQ of AB bisects AB at C and is perpendicular to AB .

The co-ordinates of C are $= \left(\frac{3-1}{2}, \frac{4+2}{2} \right) = (1, 3)$

And slope of $PQ = \frac{-1}{\text{slope of } AB} = \frac{-1}{2-4} (-1-3) = \frac{4}{-2} = -2$

The equation of PQ is

$$(y - 3) = -2(x - 1)$$

$$y - 3 = -2x + 2$$

$$y + 2x = 5$$

Q14

The line passes through the point $(-3, 5)$

So $(x_1, y_1) = (-3, 5)$

The line is perpendicular to the line joining $(2, 5)$ and $(-3, 6)$.

$$\Rightarrow m = \frac{-1}{\text{slope of line joining } (2, 5) \text{ and } (-3, 6)} = \frac{-1}{\frac{y_2 - y_1}{x_2 - x_1}} = \frac{-1}{\frac{6 - 5}{-3 - 2}} = \frac{-1}{\frac{-1}{5}} = 5$$

$$\therefore m = 5$$

Hence, equation of straight line is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 5(x - (-3))$$

$$y - 5 = 5x + 15$$

$$5x - y + 20 = 0$$

Q15

The right bisector PQ of AB bisects AB at C and is also perpendicular to AB.

$$\text{Slope of } AB = \frac{3-0}{2-1} = 3$$

Now,

$$(\text{slope of } AB) \times (\text{slope of } PQ) = -1$$

$$\therefore \text{slope of } PQ = \frac{-1}{3}$$

$$\text{Co-ordinates of } C \text{ are } = \left(\frac{1+2}{2}, \frac{3+0}{2} \right) = \left(\frac{3}{2}, \frac{3}{2} \right)$$

$\therefore$ Equation of right bisector PQ is

$$\left(y - \frac{3}{2} \right) = \frac{-1}{3} \left(x - \frac{3}{2} \right)$$

$$6y - 9 = -2x + 3$$

$$x + 3y = 6$$

Q16

Equation of the line passing through (x_1, y_1)

and making angle θ with the x-axis is,

$$(y - y_1) = \tan \theta (x - x_1)$$

Here $(x_1, y_1) = (1, 2)$, angle with y-axis is 30°

$\therefore$ angle with x-axis is $\theta = 90^\circ - 30^\circ = 60^\circ$

$$(y - y_1) = \tan \theta (x - x_1)$$

$$(y - 2) = (\tan 60^\circ)(x - 1)$$

$$y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\sqrt{3}x - y + 2 - \sqrt{3} = 0$$

Ex 23.5

Q1(i)

Here,

$$(x_1, y_1) = (0, 0)$$

$$(x_2, y_2) = (2, -2)$$

The equation of the given straight line is:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - 0 = \frac{-2 - 0}{2 - 0} (x - 0)$$

$$\Rightarrow y = \frac{-2x}{2}$$

$$\Rightarrow y = -x$$

$\therefore$ The equation of the line joining the points $(0, 0)$ and $(2, -2)$ is $y = -x$

Q1(ii)

$$\text{Let } A(a, b) = (x_1, y_1)$$

$$B(a + c \sin \alpha, b + c \cos \alpha) = (x_2, y_2)$$

Then equation of line AB is

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - b = \frac{b + c \cos \alpha - b}{a + c \sin \alpha - a} (x - a)$$

$$\Rightarrow y - b = \frac{c \cot \alpha}{c \sin \alpha} (x - a)$$

$$\Rightarrow y - b = \cot \alpha (x - a)$$

$\therefore$ The equation of the line joining the points (a, b) and $(a + c \sin \alpha, b + c \cos \alpha)$ is $y - b = \cot \alpha (x - a)$

Q1(iii)

Let $A(a, -a)$ be (x_1, y_1)

$B(b, 0)$ be (x_2, y_2)

Then equation of line AB is

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - (-a) = \frac{0 - (-a)}{b - 0} (x - 0)$$

$$\Rightarrow y + a = \frac{a}{b} (x - 0)$$

$$\Rightarrow ax - by = ab$$

$\therefore$ The equation of the line joining the points $(0, -a)$ and $(b, 0)$ is $ax - by = ab$

Q1(iv)

Let $A(a, b)$ be (x_1, y_1)

$B(a+b, a-b)$ be (x_2, y_2)

Then equation of line AB is

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - b = \frac{a - b - b}{a + b - a} (x - a)$$

$$\Rightarrow y - b = \frac{a - 2b}{b} (x - a)$$

$$\Rightarrow by - b^2 = ax - a^2 - 2bx + 2ba$$

$$\Rightarrow (a - 2b)x - by + b^2 - a^2 + 2ab = 0$$

$\therefore$ The equation of the line joining the points (a, b) and $(a+b, a-b)$ is $(a - 2b)x - by + b^2 - a^2 + 2ab = 0$

Q1(v)

Let $A(x_1, y_1)$ be $\left(at_1, \frac{a}{t_1}\right)$

$B(x_2, y_2)$ be $\left(at_2, \frac{a}{t_2}\right)$

Then equation of line AB is

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - \frac{a}{t_1} = \frac{\frac{a}{t_2} - \frac{a}{t_1}}{at_2 - at_1} (x - at_1)$$

$$\Rightarrow y - \frac{a}{t_1} = \frac{a(t_1 - t_2)}{at_1t_2(t_2 - t_1)} (x - at_1)$$

$$\Rightarrow y - \frac{a}{t_1} = \frac{-1}{t_1t_2} (x - at_1)$$

$$\Rightarrow t_1t_2y + x = a(t_1 + t_2)$$

$\therefore$ The equation of the line joining the points $\left(at_1, \frac{a}{t_1}\right)$ and $\left(at_2, \frac{a}{t_2}\right)$ is $t_1t_2y + x = a(t_1 + t_2)$



Q1(vi)

Let $A(x_1, y_1)$ be $(a \cos \alpha, a \sin \alpha)$ $B(x_2, y_2)$ be $(a \cos \beta, a \sin \beta)$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - a \sin \alpha = \frac{a \sin \beta - a \sin \alpha}{a \cos \beta - a \cos \alpha} (x - a \cos \alpha)$$

$$\Rightarrow y - a \sin \alpha = \frac{a \left(-2 \sin \left(\frac{\beta - \alpha}{2} \right) \right) \cos \beta \left(\frac{\beta + \alpha}{2} \right)}{a \left(-2 \sin \frac{\beta - \alpha}{2} \right) \sin \left(\frac{\beta + \alpha}{2} \right)} (x - a \cos \alpha)$$

$$\Rightarrow y - a \sin \alpha = \frac{\cos \left(\frac{\alpha + \beta}{2} \right)}{\sin \left(\frac{\alpha + \beta}{2} \right)} (x - a \cos \alpha)$$

$$\Rightarrow x \cos \left(\frac{\alpha + \beta}{2} \right) + y \sin \frac{\alpha + \beta}{2} = a \cos \frac{\alpha - \beta}{2}$$

 $\therefore$ The equation of the line joining the points $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$ is

$$x \cos \left(\frac{\alpha + \beta}{2} \right) + y \sin \frac{\alpha + \beta}{2} = a \cos \frac{\alpha - \beta}{2}$$

Q2(i)

Let $A(1, 4), B(2, -3), C(-1, -2)$

Then equation of AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 4 = \frac{-3 - 4}{2 - 1} (x - 1)$$

$$y - 4 = \frac{-7}{1} (x - 1)$$

$$7x + y = 11$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2} (x - x_2)$$

$$y - (-3) = \frac{-2 - (-3)}{-1 - 2} (x - 2)$$

$$y + 3 = \frac{1}{-3} (x - 2)$$

$$x + 3y + 7 = 0$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1} (x - x_1)$$

$$y - 4 = \frac{-2 - 4}{-1 - 1} (x - 1)$$

$$y - 4 = 3(x - 1)$$

$$y - 3x = 1$$



Q2(ii)

Let $A(0, 1), B(2, 0), C(-1, -2)$

then equation of side AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{0 - 1}{2 - 0} (x - 0)$$

$$y - 1 = \frac{-1}{2} (x)$$

$$x + 2y = 2$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2} (x - x_2)$$

$$y - 0 = \frac{-2 - 0}{-1 - 2} (x - 2)$$

$$y = \frac{2}{3} (x - 2)$$

$$2x - 3y = 4$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1} (x - x_1)$$

$$y - 1 = \frac{-2 - 1}{-1 - 0} (x - 0)$$

$$y - 1 = 3 (x - 0)$$

$$y - 3x = 1$$



Q3

$$\begin{aligned} \text{Let } A(-1, 6) &\text{ be } (x_1, y_1) \\ B(-3, -9) &\text{ be } (x_2, y_2) \\ C(5, -8) &\text{ be } (x_3, y_3) \end{aligned}$$

Median is a line segment which joins a vertex to the mid-point of the side opposite to it.
Let D, E and F be the mid points of sides AB, BC, and CA.

Then, using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ we can find the coordinates of D, E and F as:

$$D = \left(\frac{-3+5}{2}, \frac{-9+6}{2}\right) = \left(1, -\frac{3}{2}\right)$$

$$E = \left(\frac{-1+5}{2}, \frac{6-8}{2}\right) = (2, -1)$$

$$F = \left(\frac{-1-3}{2}, \frac{6-9}{2}\right) = \left(-2, -\frac{3}{2}\right)$$

Equation of median AD is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-6 = \frac{-\frac{3}{2}-6}{1-(-1)}(x+1) = \frac{-\frac{15}{2}}{2}(x+1) \quad \left[A(-1, 6), D\left(1, -\frac{3}{2}\right)\right]$$

$$29x + 4y + 5 = 0$$

Equation of median BE is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-9) = \frac{-1-(-9)}{2-(-3)}(x-(-3)) \quad \left[B(-3, -9), E(2, -1)\right]$$

$$y+9 = \frac{8}{5}(x+3)$$

$$5y+45 = 8x+24$$

$$8x-5y-21 = 0$$

Equation of median CF is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-8) = \frac{-3-(-8)}{-2-5}(x-5) \quad \left[C(5, -8), F\left(-2, -\frac{3}{2}\right)\right]$$

$$y+8 = \frac{-3+8}{-2-5}(x-5)$$

$$y+8 = \frac{-13}{-7}(x-5)$$

$$13x+14y+47 = 0$$



Q4

The rectangle ABCD will have diagonals AC and BD

AC passes through A(a,b) and C(a',b').

Thus equation of AC is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - b}{b' - b} = \frac{x - a}{a' - a}$$

$$\Rightarrow (y - b)(a' - a) = (x - a)(b' - b)$$

$$\Rightarrow y(a' - a) - a'b + ab = x(b' - b) - ab' + ab$$

$$\Rightarrow y(a' - a) = x(b' - b) - ab' + a'b$$

$$\Rightarrow y(a' - a) - x(b' - b) = a'b - ab'$$

BD passes through B(a',b) and D(a,b').

Thus equation of BD is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - b}{b' - b} = \frac{x - a'}{a - a'}$$

$$\Rightarrow (y - b)(a - a') = (x - a')(b' - b)$$

$$\Rightarrow -y(a' - a) - ab + a'b = x(b' - b) - a'b' + a'b$$

$$\Rightarrow a'b' - ab = x(b' - b) + y(a' - a)$$

$$\Rightarrow x(b' - b) + y(a' - a) = a'b' - ab$$

**Q5**

Equation of BC

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{0 - 1}{2 - 0} (x - 0) \quad [\because B(0, 1), C(2, 0)]$$

$$2y - 2 = -x$$

$$x + 2y = 2$$

D is mid point of BC

So,

$$D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 2}{2}, \frac{1 + 0}{2} \right) = \left(1, \frac{1}{2} \right)$$

$\therefore$ Equation of the median AD :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - (-2) = \frac{\frac{1}{2} - (-2)}{1 - (-1)} (x - (-1)) = \frac{5}{2} (x + 1) \quad [\because A(-1, -2), D(1, \frac{1}{2})]$$

$$4y + 8 = 5x + 5$$

$$5x - 4y - 3 = 0$$

Q6

The equation of the line passing through points $(-2, -2)$ and $(8, 2)$ is

$$y + 2 = \frac{2 + 2}{8 + 2} (x + 2)$$

$$2x - 5y - 6 = 0$$

Clearly, $(3, 0)$ satisfies this equation which means that the line passing through $(-2, -2)$ and $(8, 2)$

also passes through $(3, 0)$.

Hence three points are collinear.

Q7

Let AB be the line segment

Let P be any point which divides the line segment in the ratio $2:3$

then using section formula

$$x = \frac{lx_2 + mx_1}{l+m}, y = \frac{ly_2 + my_1}{l+m}$$

where $l:m :: 2:3$

$$\Rightarrow x = \frac{2(8) + 3(3)}{2+3} = \frac{16+9}{5} = \frac{25}{5} = 5$$

$$y = \frac{2(9) + 3(-1)}{2+3} = \frac{18-3}{5} = \frac{15}{5} = 3$$

Now P must lie on the line, where P is $(5,3)$

$$y - x + 2 = 0$$

$$\Rightarrow 3 - (5) + 2 = 0$$

$$-2 + 2 = 0$$

$$0 = 0$$

Hence, Proved

Q8

The line that bisects the distance between the points $A(a, b)$, $B(a', b')$ and between $C(-a, b)$, $D(a' - b')$ means a line passing through the mid-point of AB and CD

$$\text{mid point of } AB \text{ is } \left(\frac{a+a'}{2}, \frac{b+b'}{2} \right)$$

$$\text{mid point of } CD \text{ is } \left(\frac{-a+a'}{2}, \frac{b-b'}{2} \right)$$

$$\text{Equation is } y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - \left(\frac{b+b'}{2} \right) = \frac{\left(\frac{b-b'}{2} \right) - \left(\frac{b+b'}{2} \right)}{\left(\frac{-a+a'}{2} \right) - \left(\frac{a+a'}{2} \right)} \left(x - \left(\frac{a+a'}{2} \right) \right)$$

$$y - \left(\frac{b+b'}{2} \right) = \frac{\frac{b}{2} - \frac{b'}{2} - \frac{b}{2} - \frac{b'}{2}}{\frac{-a}{2} + \frac{a'}{2} - \frac{a}{2} - \frac{a'}{2}} \left(x - \left(\frac{a+a'}{2} \right) \right)$$

$$y - \left(\frac{b+b'}{2} \right) = \frac{+b'}{a} \left(x - \left(\frac{a+a'}{2} \right) \right)$$

$$2ay - 2b'x = ab - a'b'$$

Q9

In what ratio is the line joining the points $(2, 3)$ and $(4, -5)$ divided by the line passing through the points $(6, 8)$ and $(-3, -2)$.

Let the equation of line AB joining the points $(6, 8)$ and $(-3, -2)$ be

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 8 = \frac{-2 - 8}{-3 - 6}(x - 6)$$

$$y - 8 = \frac{10}{9}(x - 6)$$

$$9y - 10x = 12 \quad \text{---(1)}$$

Suppose the line joining $(2, 3)$ and $(4, -5)$ is divided by the line $9y - 10x = 12$ in the ratio $k : 1$ at the point (x, y) , then

$$x = \frac{k(4) + 1(2)}{k + 1}, y = \frac{k(-5) + 1(3)}{k + 1}$$

Substituting in equation (i), we get:

$$\frac{9(5k + 3)}{k + 1} - 10\left(\frac{4k + 2}{k + 1}\right) = 12$$

$$\Rightarrow -45k + 27 - 40k - 20 = 12k + 12$$

$$\Rightarrow 97k = 5$$

$$\Rightarrow k = \frac{5}{97}$$

Q10

The quadrilateral $ABCD$ has diagonals AC and BD .
The required equation is

Since, $A(-2, 6)$, $C(10, 4)$, the equation for AC is:

$$y - 6 = \frac{4 - 6}{10 - (-2)}(x - (-2)) \quad \left[y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \right]$$

$$y - 6 = \frac{-12}{6}(x + 2)$$

$$y - 6 = \frac{-(x + 2)}{6}$$

$$6y - 36 = -x - 2$$

$$x + 6y - 34 = 0$$

Since, $B(1, 2)$, $D(7, 8)$, the equation for BD is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{8 - 2}{7 - 1}(x - 1)$$

$$y - 2 = \frac{6}{6}(x - 1)$$

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$



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Q11

$$L_1 = 124.942, C_1 = 20$$

$$L_2 = 125.134, C_2 = 110$$

Equation of line passing through

(L_1, C_1) and (L_2, C_2)

$$L - L_1 = \left(\frac{L_2 - L_1}{C_2 - C_1} \right) (C - C_1)$$

$$L - 124.942 = \left(\frac{125.134 - 124.942}{110 - 20} \right) (C - 20)$$

$$L - 124.942 = \frac{0.192}{90} (C - 20)$$

$$L - 124.942 = \frac{192}{90000} (C - 20)$$

$$L - 124.942 = \frac{4}{1875} (C - 20)$$

$$L = \frac{4}{1875} C + 124.942 - 4 \times \frac{20}{1875}$$

$$\Rightarrow L = \frac{4}{1875} C + 124.899$$

Q12

Assuming x be the price per litre and y be the quantity of the milk sold at this price.

So, the line representing the relationship passes through $(14, 980)$ and $(16, 1220)$.

So its equation is

$$y - 980 = \frac{1220 - 980}{16 - 14} (x - 14)$$

$$y - 980 = 120(x - 14)$$

$$120x - y - 700 = 0$$

$$\text{When } x = 17, 120 \times 17 - y - 700 = 0$$

$$y = 1340$$

Q13

Let AD be the bisector of $\angle A$

Then, $BD : DC = AB : AC$

Now,

$$|AB| = \sqrt{(4-0)^2 + (3-0)^2} = 5$$

$$|AC| = \sqrt{(4-2)^2 + (3-3)^2} = 2$$

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{2}$$

$\Rightarrow D$ divides BC in the ratio $5 : 2$

So, coordinates of D are $\left(\frac{5 \times 2 + 0}{5 + 2}, \frac{5 \times 3 + 0}{5 + 2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

$\therefore$ The equation of AD is

$$y - 3 = \left(\frac{\frac{15}{7} - 3}{\frac{10}{7} - 4}\right)(x - 4)$$

$$y - 3 = \left(\frac{15 - 21}{10 - 28}\right)(x - 4)$$

$$\Rightarrow y - 3 = \frac{1}{3}(x - 4)$$

$$\Rightarrow 3(y - 3) = x - 4$$

$$\Rightarrow x - 3y + 9 - 4 = 0$$

$$\Rightarrow x - 3y + 5 = 0$$



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Q14

The required straight line passes through $(0,0)$ and trisects the part of the line $3x + y = 12$ that lies between the axes of coordinates.

The line $3x + y = 12$ has $A(4,0)$ and $B(0,12)$ as x and y intercepts.

Let P and Q be the points of trisection of AB .

Since P divides AB in the ratio $1:2$, coordinates of P are:

$$P = \frac{1(0) + 2(4)}{1+2}, \frac{1(12) + 2(0)}{1+2} = \left(\frac{8}{3}, 4\right)$$

Since Q divides BA in the ratio $1:2$, coordinates of Q are:

$$Q = \frac{2(0) + 1(4)}{1+2}, \frac{1(0) + 2(12)}{1+2} = \left(\frac{4}{3}, 8\right)$$

Equation of line through $(0,0)$ and $P\left(\frac{8}{3}, 4\right)$ is:

$$y - 0 = \frac{4 - 0}{\frac{8}{3} - 0}(x - 0)$$

$$y - 0 = \frac{12}{8}x$$

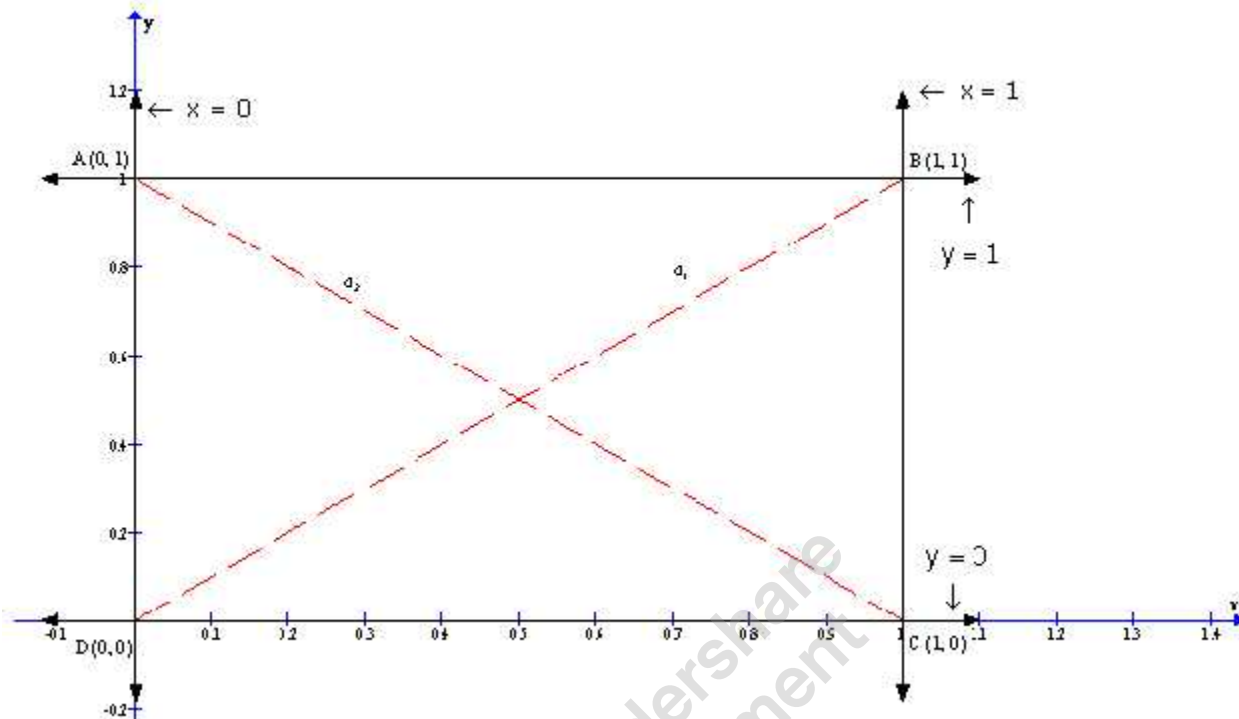
$$2y = 3x$$

Equation of line through $(0,0)$ and $Q\left(\frac{4}{3}, 8\right)$ is:

$$y - 0 = \frac{8 - 0}{\frac{4}{3} - 0}(x - 0) = 6x$$

$$y = 6x$$

Q15



When we draw all the given equations of lines on the graph we get the points of intersection A(0, 1), B(1,1), C(1,0) and D(0,0).

Let d_1 be the diagonal formed by joining the points B and D.

Let d_2 be the diagonal formed by joining the points A and C.

Equation of the diagonal d_1 is given by,

$$(y - 1) = \frac{(0 - 1)}{(0 - 1)}(x - 1)$$

$$(y - 1) = 1(x - 1)$$

$$y = x$$

Equation of the diagonal d_2 is given by,

$$(y - 1) = \frac{(0 - 1)}{(1 - 0)}(x - 0)$$

$$(y - 1) = -1(x)$$

$$y + x = 1$$

$\therefore$ The equations of the diagonals are $y = x$ and $y + x = 1$.

Ex 23.6

Q1

(i)

If $(a, 0)$ and $(0, b)$ are the intercepts of a line then the intercept form of equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Here, $a = 3, b = 2$

∴ The required equation is

$$\frac{x}{3} + \frac{y}{2} = 1$$

$$\Rightarrow 2x + 3y = 6$$

(ii) If $(a, 0)$ and $(0, b)$ are the intercepts of a line then the intercept form of equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Here, $a = -5, b = 6$

∴ The required equation is

$$\frac{x}{-5} + \frac{y}{6} = 1$$

$$\Rightarrow 6x - 5y = -30$$

Q2

The equation of straight line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{--- (1)}$$

If (1) passes through the point $(1, -2)$ and has equal intercepts ($a = b = k$), we get,

$$\frac{1}{k} + \frac{(-2)}{k} = 1$$

$$\frac{1}{k} - \frac{2}{k} = 1$$

$$1 - 2 = k$$

$$k = -1$$

$$\Rightarrow a = b = -1$$

Putting in (1)

$$\frac{x}{-1} + \frac{y}{-1} = 1$$

$$x + y = -1$$

Q3

(i) Intercepts are equal and positive

$$\Rightarrow a = b = k$$

The equation of straight line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{--- (1)}$$

Since this line passes through (5, 6) and $a = b = k$, we get:

$$\begin{aligned} \frac{5}{k} + \frac{6}{k} &= 1 \\ k &= 1 \end{aligned}$$

$$\therefore \frac{x}{11} + \frac{y}{11} = 1$$

$$\Rightarrow x + y = 11$$

(ii) Intercepts are equal but opposite in sign

$$\text{Let, } a = k, b = -k$$

Putting in (1), we get,

$$\frac{5}{k} + \frac{6}{-k} = 1$$

$$\frac{5}{k} - \frac{6}{k} = 1$$

$$\Rightarrow k = -1$$

thus from (1)

$$x - y = -1$$



Q4

The equation of the given line is,

$$ax + by + 8 = 0$$

$$\Rightarrow -\frac{x}{\frac{8}{a}} - \frac{y}{\frac{8}{b}} = 1$$

It cuts the axes at $A\left(\frac{-8}{a}, 0\right)$ and $B\left(0, \frac{-8}{b}\right)$.

The equation of the given line is,

$$2x - 3y + 6 = 0$$

$$\Rightarrow \frac{-x}{\frac{6}{2}} + \frac{y}{\frac{6}{3}} = 1$$

It cuts the axes at $C(-3, 0)$ and $D(0, 2)$.

The intercepts of both the lines are opposite in sign

$$\Rightarrow \left(\frac{-8}{a}, 0\right) = -(-3, 0) \quad \text{and} \quad \left(0, \frac{-8}{b}\right) = -(0, 2)$$

$$\Rightarrow \frac{-8}{a} = 3 \quad \text{and} \quad \frac{-8}{b} = -2$$

$$\Rightarrow a = \frac{-8}{3} \quad \text{and} \quad b = 4$$

Q5

Let the intercepts on the axes be $(a, 0)$ and $(0, a)$.

Then,

$$a \times a = 25$$

$$a^2 = 25$$

$$a = 5$$

(Ignoring negative sign because it is given that the intercepts are positive)

$$\Rightarrow a = b = 5 \quad (\text{given the intercepts are equal})$$

$\therefore$ Putting in equation of straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{5} + \frac{y}{5} = 1$$

$$x + y = 5$$



Q6

The equation of the given line is,

$$\frac{x}{a} + \frac{y}{b} = 1$$

It cuts the axes at $A(a,0)$ and $B(0,b)$.

The portion of AB intercepted between the axis is 5:3.

$$\therefore h = \frac{3 \times a + 5 \times 0}{8} \text{ and } k = \frac{3 \times 0 + 5 \times b}{8}$$

$$\Rightarrow p = \left(\frac{3a}{8}, \frac{5b}{8} \right)$$

The line is passing through the point $(-4,3)$

$$\Rightarrow \frac{3a}{8} = -4 \quad \frac{5b}{8} = 3$$

$$\Rightarrow a = \frac{-32}{3} \quad b = \frac{24}{5}$$

$\therefore$ The equation of the given line is,

$$\frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1$$

$$\frac{-3x}{32} + \frac{5y}{24} = 1$$

$$9x - 20y + 96 = 0$$

Q7

The line intercepted by the axes are $(a,0)$ and $(0,b)$, If this line segment is bisected at point (α, β)

then $\frac{a+0}{2} = \alpha, \frac{0+b}{2} = \beta$ (Using mid point formula)

$$a = 2\alpha, b = 2\beta$$

The equation of straight line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$$

Q8

Suppose $P = (3, 4)$ divides the line joining the points $A(a, 0)$ and $B(0, b)$ in the ratio $2:3$.

Then,

$$3 = \frac{2(0) + 3(a)}{2+3} \Rightarrow 3 = \frac{3a}{5} \Rightarrow a = 5$$

$$4 = \frac{2(b) + 3(0)}{2+3} \Rightarrow 4 = \frac{2b}{5} \Rightarrow b = 10$$

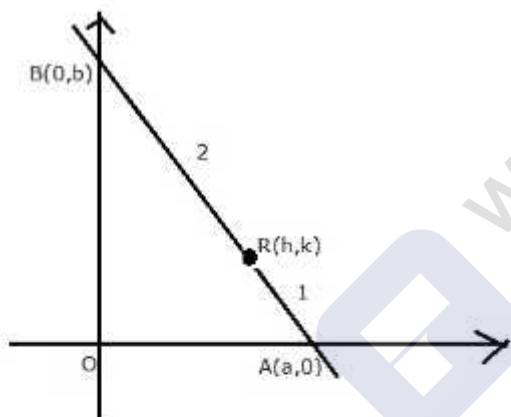
$\therefore A$ is $(5, 0)$, B is $(0, 10)$

Equation of line AB is

$$\frac{x}{5} + \frac{y}{10} = 1$$

$$2x + y = 10$$

Q9



Point (h, k) divides the line segment in the ratio $1:2$

Thus, using section point formula, we have

$$h = \frac{2 \times a + 1 \times 0}{1+2}$$

and

$$k = \frac{2 \times 0 + 1 \times b}{1+2}$$

Therefore, we have,

$$h = \frac{2a}{3} \text{ and } k = \frac{b}{3}$$

$$\Rightarrow a = \frac{3h}{2} \text{ and } b = 3k$$

Thus, the corresponding points of A and B are $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$

Thus, the equation of the line joining the points A and B is

$$\frac{y-3k}{3k-0} = \frac{x-0}{0-\frac{3h}{2}}$$

$$\Rightarrow -\frac{3h}{2}(y-3k) = x \times 3k$$

$$\Rightarrow -3hy + 9hk = 6kx$$

$$\Rightarrow 2kx + hy = 3kh$$

Q10

Let equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

then $a + b = 7$ and $a \geq 0$ and $b \geq 0$

$$\therefore \frac{x}{a} + \frac{y}{7-a} = 1 \quad \text{--- (1)}$$

The line passes through $(-3, 8)$

$$\Rightarrow \frac{-3}{a} + \frac{8}{7-a} = 1$$

$$\Rightarrow -21 + 3a + 8a = 7a - a^2$$

$$\Rightarrow -21 + 11a = 7a - a^2$$

$$\Rightarrow a^2 + 4a - 21 = 0$$

$$\Rightarrow A = 3 \text{ or } -7$$

$$a \neq -7 \text{ (as } a \geq 0)$$

$$\therefore a = 3 \text{ and } b = 4$$

$\therefore$ Equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or } 4x + 3y = 12$$

Q11

Let equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

It is given $(-4, 3)$ divides the line joining $A(a, 0)$ and $B(0, b)$ in ratio 5 : 3

$$\therefore \left(\frac{3a}{8}, \frac{5b}{8} \right) = (-4, 3)$$

$$\Rightarrow \frac{3a}{8} = -4 \quad \Rightarrow a = \frac{-32}{3}$$

And

$$\frac{5b}{8} = 3 \quad \Rightarrow b = \frac{24}{5}$$

$\therefore$ The equation of line is

$$\frac{3x}{-32} + \frac{5y}{24} = 1$$

$$\text{or } 9x - 20y + 96 = 0$$



Q12

Let equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\text{then } a = b + 5$$

$$\therefore \frac{x}{b+5} + \frac{y}{b} = 1$$

It passes through $(22, -6)$

$$\Rightarrow \frac{22}{b+5} - \frac{6}{b} = 1$$

$$\Rightarrow 22b - 6b - 30 = b^2 + 5b$$

$$\Rightarrow b^2 - 11b + 30 = 0$$

$$\Rightarrow b = 5 \text{ or } 6$$

$$\therefore a = 10 \text{ or } 11$$

$\therefore$ Equations of line are

$$\frac{x}{10} + \frac{y}{5} = 1$$

$$\text{or } x + 2y - 10 = 0$$

and

$$\frac{x}{11} + \frac{y}{6} = 1$$

$$6x + 11y = 66$$



Q13

The equation of straight line is

$$y - y_1 = m(x - x_1)$$

The line passes through (x, y) ie, $(1, -7)$ and meets the axes at A and B

$\Rightarrow A$ point is $(a, 0)$ and B is $(0, b)$

$$\frac{AP}{BP} = \frac{3}{4}$$

Using section formula $\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}$

$$l : m = 3 : 4, (a, 0) \Rightarrow (x_1, y_1), (0, b) \Rightarrow (x_2, y_2)$$

$$\Rightarrow 1 = \frac{3(0) + 4(a)}{3+4}$$

$$\Rightarrow 1 = \frac{4a}{7}$$

$$\Rightarrow a = \frac{7}{4}$$

$$-7 = \frac{3(b) + 4(0)}{3+4}$$

$$\Rightarrow -7 = \frac{3b}{7}$$

$$\Rightarrow b = \frac{-49}{3}$$

then $A\left(\frac{7}{4}, 0\right), B\left(0, \frac{-49}{3}\right)$ putting in (1)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{\frac{-49}{3} - 0}{0 - \frac{7}{4}}\left(x - \frac{7}{4}\right)$$

$$y - 0 = \frac{49}{3} \times \frac{4}{7}\left(x - \frac{7}{4}\right)$$

$$y = \frac{28}{3}\left(x - \frac{7}{4}\right)$$

$$3y - 28x + 49 = 0$$

Q14

Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

and $a + b = 9$

$\therefore b = 9 - a$

$\therefore$ Equation is

$$\frac{x}{a} + \frac{y}{9-a} = 1$$

and it passes through $(2, 2)$

$$\therefore \frac{2}{a} + \frac{2}{9-a} = 1$$

$$18 - 2a + 2a = 9a - a^2$$

$$a^2 - 9a + 18 = 0$$

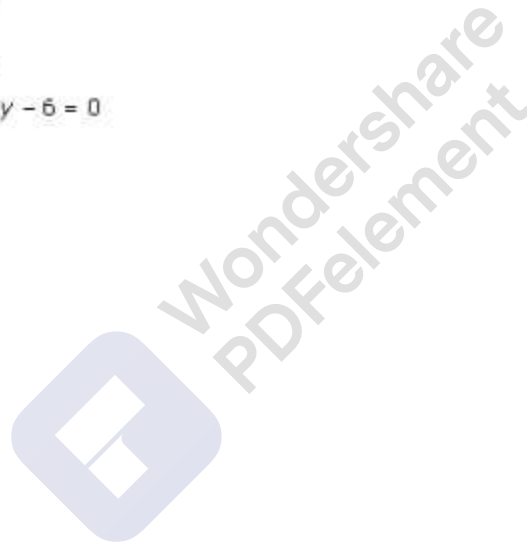
$$a = 6, 3$$

$$\therefore b = 3, 6$$

The equation of line are

$$\frac{x}{6} + \frac{y}{3} = 1 \quad \text{or} \quad \frac{x}{3} + \frac{y}{6} = 1$$

$$2x + y - 6 = 0 \quad \text{or} \quad x + 2y - 6 = 0$$



Q15

$P(2, 6)$ let A be the point on x -axis (x, y)

$$\Rightarrow A(a, 0)$$

$$(x_1, y_1)$$

B be a point on y -axis

$$\Rightarrow B(0, b)$$

$$(x_2, y_2)$$

Using section formula $x = \frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}$

$$l : m = 2 : 3$$

$$2 = \frac{2(0) + 3(a)}{2 + 3}$$

$$\Rightarrow 10 = 3a$$

$$\Rightarrow a = \frac{10}{3}$$

$$6 = \frac{2(b) + 3(0)}{2 + 3}$$

$$\Rightarrow 30 = 2b$$

$$\Rightarrow b = 15$$

$\therefore$ Point A is $\left(\frac{10}{3}, 0\right), (0, 15)$

equation of line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0 = \frac{15 - 0}{0 - \frac{10}{3}} \left(x - \frac{10}{3}\right)$$

$$y = \frac{-15 \times 3}{10} \left(x - \frac{10}{3}\right)$$

$$2y = -9x + \frac{90}{3}$$

$$9x + 2y = 30$$



Q16

The equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Here,

$$a - b = 2$$

$$\text{or } a = 2 + b$$

$$\therefore \frac{x}{b+2} + \frac{y}{b} = 1 \quad \text{--- (1)}$$

It passes through (3, 2)

$$\therefore \frac{3}{b+2} + \frac{2}{b} = 1$$

$$3b + 2b + 4 = b^2 + 2b$$

$$\Rightarrow b^2 - 3b - 4 = 0$$

$$\Rightarrow b = 4 \text{ or } -1$$

$$\Rightarrow a = 6 \text{ or } 1.$$

$\therefore$ Equations of lines are

$$\frac{x}{6} + \frac{y}{4} = 1$$

$$\Rightarrow 2x + 3y = 12$$

or

$$\frac{x}{1} - \frac{y}{1} = 1$$

$$\therefore x - y = 1$$



Q17

The line $2x + 3y = 6$ cuts coordinate axis at A (3, 0) and B (0, 2).

The portion AB intercepted between the axis is trisected by points P and Q

$$\therefore \frac{AP}{PB} = \frac{1}{2} \quad \text{and} \quad \frac{AQ}{QB} = \frac{2}{1}$$

$$\Rightarrow \text{Coordinate of } P = \left(\frac{1 \times 0 + 3 \times 2}{3}, \frac{1 \times 2 + 0}{3} \right) = \left(\frac{1}{3}, \frac{2}{3} \right)$$

$$\Rightarrow \text{Coordinate of } Q = \left(\frac{2 \times 0 + 3 \times 1}{3}, \frac{4 + 0}{3} \right) = \left(\frac{3}{3}, \frac{4}{3} \right)$$

$$\text{Equation of } OQ = y - 0 = \frac{\frac{4}{3} - 0}{\frac{3}{3} - 0} (x - 0)$$

$$3y = 4x$$

$$\text{Equation of } OP \Rightarrow y - 0 = \frac{\frac{2}{3} - 0}{\frac{1}{3} - 0} (x - 0)$$

$$x - 3y = 0$$

Q18

The equation of the given line is

$$3x - 5y = 15$$

$$\frac{x}{5} - \frac{y}{3} = 1$$

It cuts axis at (5, 0) and (-3, 0).

The position AB intercepted between the axis is 1:1

$$\therefore P = \left(\frac{5}{2}, \frac{-3}{2} \right)$$

The equation of the line passing through point (2, 1)

$$y - 1 = \frac{1 + \frac{3}{2}}{2 - \frac{5}{2}} (x - 2)$$

$$y - 1 = -5(x - 2)$$

$$5x + y = 11$$

Q19

The equation of the given line is,

$$ax + by + c = 0$$

$$ax + by = -c$$

$$\frac{x}{\frac{-c}{a}} + \frac{y}{\frac{-c}{b}} = 1$$

$$c = \left(\frac{\frac{-c}{a} + 0}{2}, \frac{0 - \frac{c}{b}}{2} \right)$$

$$c = \left(\frac{-c}{2a}, \frac{-c}{2b} \right)$$

The equation of the line is passing through the point (0,0)

$$\text{and } c = \left(\frac{-c}{2a}, \frac{-c}{2b} \right),$$

$$\left(y + \frac{c}{2b} \right) = \left(\frac{\frac{-c}{2b}}{\frac{-c}{2a}} \right) \left(x + \frac{c}{2a} \right)$$

$$\Rightarrow \frac{-c}{2a} \left(y + \frac{c}{2b} \right) = \left(\frac{-c}{2b} \right) \left(x + \frac{c}{2a} \right)$$

$$\Rightarrow \frac{-y}{a} + \frac{x}{b} = 0$$

$$\Rightarrow ax - by = 0$$



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Ex 23.7

Q1(i)

$$P = 5, \alpha = 60^\circ$$

$$x \cos \alpha + y \sin \alpha = P$$

$$\Rightarrow x \cos 60^\circ + y \sin 60^\circ = 5$$

$$\Rightarrow x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 5$$

$$\Rightarrow x + \sqrt{3}y = 10$$

Q1(ii)

$$P = 4, \alpha = 150^\circ$$

$$x \cos \alpha + y \sin \alpha = P$$

$$\Rightarrow x \cos 150^\circ + y \sin 150^\circ = 4$$

$$\Rightarrow -x \times \frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 4$$

$$\Rightarrow -\sqrt{3}x + y = 8$$

Q1(iii)

$$P = 8, \alpha = 225^\circ$$

$$x \cos \alpha + y \sin \alpha = P$$

$$\Rightarrow x \cos 225^\circ + y \sin 225^\circ = 8$$

$$\Rightarrow -x \times \frac{1}{\sqrt{2}} - y \times \frac{1}{\sqrt{2}} = 8$$

$$\Rightarrow x + y + 8\sqrt{2} = 0$$

Q1(iv)

$$P = 8, \alpha = 300^\circ$$

$$x \cos \alpha + y \sin \alpha = P$$

$$\Rightarrow x \cos 300^\circ + y \sin 300^\circ = 8$$

$$\Rightarrow x \times \frac{1}{2} - y \times \frac{\sqrt{3}}{2} = 8$$

$$\Rightarrow x - \sqrt{3}y = 16$$

Q2

Given, Inclination of perpendicular line (L) passing through origin is 30°

$$\Rightarrow \text{Slope} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Slope of perpendicular line (M) which is perpendicular to line L is $-\sqrt{3}$

So equation of line M is $y = -\sqrt{3}x + c$

Given perpendicular distance from origin to line M is 4

$$4 = \frac{c}{2} \Rightarrow c = 8$$

So equation of line M is $y = -\sqrt{3}x + 8$

Q3

Here,

$$p = 4 \text{ and } \alpha = 15^\circ$$

The equation of line is

$$x \cos \alpha + y \sin \alpha = p \quad \text{--- (1)}$$

$$x \cos 15^\circ + y \sin 15^\circ = 4$$

$$\cos 15^\circ = \cos (45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30$$

$$(\because \cos (\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{1}{2\sqrt{2}} (\sqrt{3} + 1)$$

$$\sin 15^\circ = \sin (45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{1}{2\sqrt{2}} (\sqrt{3} - 1)$$

Putting in (1)

$$x \times \frac{1}{2\sqrt{2}} (\sqrt{3} + 1) + y \times \frac{1}{2\sqrt{2}} (\sqrt{3} - 1) = 4$$

$$x (\sqrt{3} + 1) + y (\sqrt{3} - 1) = 8\sqrt{2}$$

Q4

Here $p = 3$

$$\text{and } \alpha = \tan^{-1} \left(\frac{5}{12} \right)$$

$$\Rightarrow \cos \alpha = \frac{12}{13}, \sin \alpha = \frac{5}{13}$$

Equation of straight line is:

$$x \cos \alpha + y \sin \alpha = p$$

$$x \left(\frac{12}{13} \right) + y \left(\frac{5}{13} \right) = 3$$

$$12x + 5y = 39$$



Q5

$$\text{Here } p = 2, \sin \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \frac{2\sqrt{2}}{3}$$

The equation of straight line is

$$x \cos \alpha + y \sin \alpha = p$$

$$x \left(\frac{2\sqrt{2}}{3} \right) + y \left(\frac{1}{3} \right) = 2$$

$$2\sqrt{2}x + y = 6$$

Q6

Given:

$$p = \pm 2$$

$$\tan \alpha = \frac{5}{12}$$

The equation of line is

$$x \cos \alpha + y \sin \alpha = \pm p$$

$$x \frac{12}{13} + y \frac{5}{13} = \pm 2$$

$$12x + 5y \pm 26 = 0$$



Q7

Here,

p = perpendicular distance from origin = 7

Angle made with y-axis is 150° ,

$\therefore$ Angle made with x-axis is 30°

$$\cos \alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin \alpha = \sin 30^\circ = \frac{1}{2}$$

The equation of line is

$$x \cos \alpha + y \sin \alpha = p$$

$$x \left(\frac{\sqrt{3}}{2} \right) + y \left(\frac{1}{2} \right) = 7$$

$$\sqrt{3}x + y = 14$$

Q8

We have,

$$\sqrt{3}x + y + 2 = 0$$

$$-\sqrt{3}x - y = 2$$

$$\left(\frac{-\sqrt{3}}{2} \right)x + \left(\frac{-1}{2} \right)y = 1$$

This same as $x \cos \theta + y \sin \theta = p$

Therefore, $\cos \theta = \frac{-\sqrt{3}}{2}$, $\sin \theta = -\frac{1}{2}$ and $p = 1$

$$\theta = 210^\circ \text{ and } p = 1$$

$$\theta = \frac{7\pi}{6} \text{ and } p = 1$$

Q9

Perpendicular from origin makes an angle of 30° with y-axis, thus making 60° with x-axis

Area of triangle is $= 96\sqrt{3}$

$$\frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = 96\sqrt{3}$$

$$p^2 = \frac{96\sqrt{3} \times \sqrt{3}}{2} = 48 \times 3 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$p = 12$$

$$x \cos \alpha + y \sin \alpha = p$$

$$x \cos 60^\circ + y \sin 60^\circ = 12$$

$$x \times \frac{1}{2} + y \frac{\sqrt{3}}{2} = 12$$

$$x + \sqrt{3}y = 24$$

Q10

$$\alpha = 30^\circ$$

$$\text{area of triangle} = \frac{50}{\sqrt{3}}$$

$$\text{area of triangle} = \frac{1}{2} r^2 \sin \theta = \frac{50}{\sqrt{3}}$$

$$\sin 30 = \frac{1}{2}$$

$$\frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = \frac{50}{\sqrt{3}}$$

$$p^2 = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 25$$

$$p = \pm 5$$

$$x \cos \alpha + y \sin \alpha = \pm 5$$

$$x \cos 30^\circ + y \sin 30^\circ = \pm 5$$

$$x \frac{\sqrt{3}}{2} + \frac{y}{2} = \pm 5$$

$$\sqrt{3}x + y = \pm 10$$



Ex 23.8

Q1

The equation of line through $(1, 2)$ and making an angle of 60° with the x-axis is

$$\frac{x-1}{\cos 60^\circ} = \frac{y-2}{\sin 60^\circ} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Where r is the distance of any point on the line from $A(1, 2)$.

The coordinates of P on the line are

$$\left(1 + \frac{1}{2}r, 2 + \frac{\sqrt{3}}{2}r\right)$$

and

P lies on $x + y = 6$

$$\therefore 1 + \frac{r}{2} + 2 + \frac{\sqrt{3}r}{2} = 6$$

$$\text{or } r = \frac{6}{1 + \sqrt{3}} = 3(\sqrt{3} - 1)$$

Hence length $AP = 3(\sqrt{3} - 1)$

Q2

The equation of line is

$$\frac{x-3}{\cos \frac{\pi}{6}} = \frac{y-4}{\sin \frac{\pi}{6}} = \pm r$$

$$\text{or } x = \pm \frac{\sqrt{3}}{2}r + 3 \text{ and } y = \pm \frac{1}{2}r + 4$$

$$Q\left(\pm \frac{\sqrt{3}r}{2} + 3, \pm \frac{r}{2} + 4\right) \text{ lie in } 12x + 5y + 10 = 0$$

$$\therefore 12\left(\pm \frac{\sqrt{3}r}{2} + 3\right) + 5\left(\pm \frac{r}{2} + 4\right) + 10 = 0$$

$$\pm \frac{12\sqrt{3}r}{2} + 36 \pm \frac{5r}{2} + 20 + 10 = 0$$

$$r = \frac{\pm 132}{5 + 12\sqrt{3}}$$

$$\text{Hence, length } PQ \text{ is } \frac{132}{12\sqrt{3} - 5}$$

Q3

The equation of line is

$$\frac{x-2}{\cos \alpha} = \frac{y-1}{\sin \alpha} = r$$

$$\Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$$

$$\text{or } x = \frac{1}{\sqrt{2}}r + 2, y = \frac{1}{\sqrt{2}}r + 1$$

$$B \left(\frac{r}{\sqrt{2}} + 2, \frac{r}{\sqrt{2}} + 1 \right) \text{ lie on } x + 2y + 1 = 0$$

$$\therefore \frac{r}{\sqrt{2}} + 2 + \frac{2r}{\sqrt{2}} + 2 + 1 = 0$$

$$\frac{3r}{\sqrt{2}} = \pm 5$$

$$r = \frac{5\sqrt{2}}{3}$$

The length AB is $\frac{5\sqrt{2}}{3}$ units

Q4

The required line is parallel to $3x - 4y + 1 = 0$

$$\therefore \text{Slope of the line} = \text{slope of } 3x - 4y + 1 = \frac{-3}{-4}$$

$$\tan \alpha = \frac{3}{4}$$

$$\Rightarrow \sin \alpha = \frac{3}{5} \text{ and } \cos \alpha = \frac{4}{5}$$

The equation of line is

$$\frac{x+4}{\cos \alpha} + \frac{y+1}{\sin \alpha} = r$$

$$\Rightarrow \frac{x+4}{\frac{4}{5}} + \frac{y+1}{\frac{3}{5}} = \pm 5$$

$$\Rightarrow x = 8 \text{ and } y = 2$$

or

$$x = 0 \text{ and } y = -4$$

$\therefore (8, 2)$ and $(0, -4)$ are coordinates of two points on the line which are at a distance of 5 units from $(-4, 1)$

Q5

The equation of line is

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

or

$$x = x_1 + r \cos \theta \text{ and } y = y_1 + r \sin \theta$$

$$Q(x_1 + r \cos \theta, y_1 + r \sin \theta) \text{ lie in } ax + by + c = 0$$

$$\Rightarrow a(x_1 + r \cos \theta) + b(y_1 + r \sin \theta) + c = 0$$

$$\Rightarrow r(a \cos \theta + b \sin \theta) = -a x_1 - b y_1 - c$$

$$r = \frac{|a x_1 + b y_1 + c|}{|a \cos \theta + b \sin \theta|}$$

Q6

Equation of line is

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} = r$$

$$x = \frac{r}{\sqrt{2}} + 2 \quad \text{and} \quad y = \frac{r}{\sqrt{2}} + 3$$

$$P\left(\frac{r}{\sqrt{2}} + 2, \frac{r}{\sqrt{2}} + 3\right) \text{ lie on } 2x - 3y + 9 = 0$$

$$\therefore 2\left(\frac{r+2\sqrt{2}}{\sqrt{2}}\right) - 3\left(\frac{r+3\sqrt{2}}{\sqrt{2}}\right) + 9 = 0$$

$$\Rightarrow 2r + 4\sqrt{2} - 3r - 9\sqrt{2} + 9\sqrt{2} = 0$$

$$\Rightarrow r = 4\sqrt{2}$$

$\therefore$ The point (2,3) is at a distance of $4\sqrt{2}$ from $2x - 3y + 9 = 0$

Q7

Equation of the required line is

$$\frac{x-3}{\cos \alpha} = \frac{y-5}{\sin \alpha} = r \quad \text{--- (1)}$$

$$\tan \alpha = \frac{1}{2} \Rightarrow \cos \alpha = \frac{2}{\sqrt{5}} \quad \text{and} \quad \sin \alpha = \frac{1}{\sqrt{5}}$$

$\therefore$ equation is

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{\sqrt{5}}} = r$$

$$\text{or } x = \frac{2r}{\sqrt{5}} + 3, y = \frac{r}{\sqrt{5}} + 5$$

$$P\left(\frac{2r}{\sqrt{5}} + 3, \frac{r}{\sqrt{5}} + 5\right) \text{ lie on } 2x + 3y = 14$$

$$\therefore \frac{4r}{\sqrt{5}} + 6 + \frac{3r}{\sqrt{5}} + 15 = 1 \pm 14$$

$$\frac{7r}{\sqrt{5}} = \pm 17$$

$$r = \pm\sqrt{5}$$

$$r = \sqrt{5} \quad (r \neq -\sqrt{5})$$

$\therefore$ Distance of (3,5) from $2x + 3y = 14$ is $\sqrt{5}$ units

Q8

$$\text{Slope of the line} = \tan \alpha = \frac{3}{4}$$

$$\therefore \sin \alpha = \frac{3}{5} \quad \text{and} \quad \cos \alpha = \frac{4}{5}$$

$\therefore$ Equation of line is

$$\frac{x-2}{\cos \alpha} = \frac{y-5}{\sin \alpha} = r$$

$$\Rightarrow \frac{x-2}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$$

$$\text{or } x = \frac{4r}{5} + 2 \quad \text{and} \quad y = \frac{3r}{5} + 5$$

$$\text{then } P \left(\frac{4r}{5} + 2, \frac{3r}{5} + 5 \right) \text{ lie on } 3x + y + 4 = 0$$

$$\therefore 3 \left(\frac{4r}{5} + 2 \right) + \left(\frac{3r}{5} + 5 \right) + 4 = 0$$

$$\frac{15}{5} r = \pm 15$$

$$r = \pm \frac{15 \times 5}{15}$$

$$= 5 \text{ units}$$

Q9

If m is the slope of the line $x - 2y = 1$, then

$$m = \tan \theta = \frac{-1}{-2} = \frac{1}{2}$$

$$\therefore \sin \theta = \frac{1}{\sqrt{5}} \quad \text{and} \quad \cos \theta = \frac{2}{\sqrt{5}}$$

The equation of line is

$$\frac{x - 3}{\cos \theta} = \frac{y - 5}{\sin \theta} = \pm r$$

$$\Rightarrow x = \pm \frac{2}{\sqrt{5}}r + 3 \quad \text{and} \quad y = \pm \frac{1}{\sqrt{5}}r + 5$$

$$P\left(\pm \frac{2}{\sqrt{5}}r + 3, \pm \frac{1}{\sqrt{5}}r + 5\right) \text{ lie in } 2x + 3y = 14$$

$$2\left(\pm \frac{2}{\sqrt{5}}r + 3\right) + 3\left(\pm \frac{1}{\sqrt{5}}r + 5\right) = 14$$

$$4r + 6\sqrt{5} + 3r + 15\sqrt{5} = 14\sqrt{5}$$

$$7r = -7\sqrt{5}$$

$$r = |\sqrt{5}|$$

$$r = \sqrt{5}$$

Q10

Slope of required line

$$= \text{slope of } 3x - 4y + 8 = 0 = \frac{3}{4} = \tan \theta$$

$\therefore$ Equation of required line

$$\frac{x - 2}{\frac{4}{5}} = \frac{y - 5}{\frac{3}{5}} = r$$

or

$$P \left(\frac{4}{5}r + 2, \frac{3r}{5} + 5 \right)$$

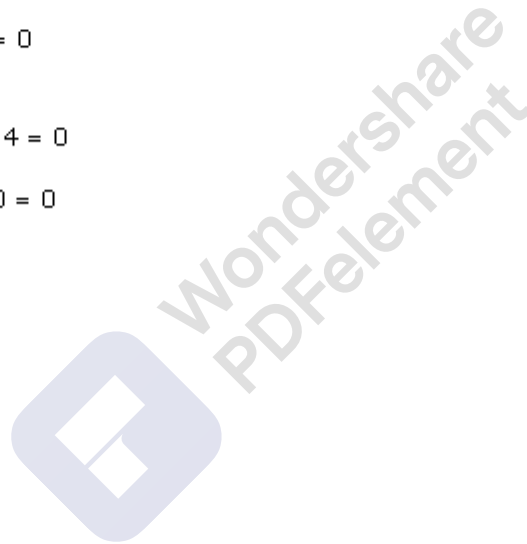
and P lies in $3x + y + 4 = 0$

$$\therefore 3 \left(\frac{4}{5}r + 2 \right) + \left(\frac{3r}{5} + 5 \right) + 4 = 0$$

$$\Rightarrow 12r + 30 + 3r + 25 + 20 = 0$$

$$\Rightarrow 15r + 75 = 0$$

$$\Rightarrow r = -5$$



Q11

The slope of the line = 1

$$\tan \theta = 1$$

$$\text{or } \theta = \frac{\pi}{4}$$

∴ Equation of line is

$$\frac{x+1}{\cos \frac{\pi}{4}} = \frac{y+3}{\sin \frac{\pi}{4}} = r$$

or

$$x = \frac{r}{\sqrt{2}} - 1 \text{ and } y = \frac{r}{\sqrt{2}} - 3$$

$$P\left(\frac{r}{\sqrt{2}} - 1, \frac{r}{\sqrt{2}} - 3\right) \text{ lie in } 2x + y = 3$$

$$\therefore 2\left(\frac{r}{\sqrt{2}} - 1\right) + \left(\frac{r}{\sqrt{2}} - 3\right) = 3$$

$$\Rightarrow \frac{3r}{\sqrt{2}} = 8$$

$$r = \frac{8\sqrt{2}}{3}$$

The distance of $2x + y = 3$ from $(-1, -3)$ is $\frac{8\sqrt{2}}{3}$ units



Q12

$$5x - y - 4 = 0 \quad \dots 1$$

$$3x + 4y - 4 = 0 \quad \dots 2$$

$$P(1, 5)$$

Let (a, b) lie on 2; (c, d) on 1

$$\text{we get } 3a + 4b = 4; \dots 3$$

$$5c - d = 4 \quad \dots 4$$

From midpoint formula, we have

$$a + c = 2 \quad \dots 5$$

$$b + d = 10 \quad \dots 6$$

$$\text{Solving 3 and 5 we get } 4b - 3c = -2 \quad \dots 7$$

$$\text{Solving 4 and 6 we get } 5c + b = 14 \quad \dots 8$$

$$\text{Solving 7 and 8 we get } c = \frac{58}{23}$$

$$\text{Substitute } c \text{ in 5 we get } a = \frac{-12}{23}$$

Substitute above values similarly in other equations we get

$$(a, b) = \left(\frac{-12}{23}, \frac{32}{23} \right)$$

$$(c, d) = \left(\frac{58}{23}, \frac{198}{23} \right)$$

$$\text{Slope of line connecting above points is } \frac{198 - 32}{58 - (-12)} = \frac{83}{35}$$

Required equation of line is

$$y - 5 = \frac{83}{35}(x - 1)$$

$$35y - 175 = 83x - 83$$

$$83x - 35y + 92 = 0$$

Q13

The equation of any line passing through $(-2, -7)$ is

$$\frac{x+2}{\cos \theta} = \frac{y+7}{\sin \theta} = r$$

B and C are at distance r and $(r+3)$

Thus, Coordinates of B and C are $(-2+r \cos \theta, -7+r \sin \theta)$ and $(-2+(r+3) \cos \theta, -7+(r+3) \sin \theta)$

B lies on $4x+3y=12$

$$\Rightarrow 4(-2+r \cos \theta) + 3(-7+r \sin \theta) = 12 \quad \text{--- (1)}$$

C lies on $4x+3y=3$

$$\Rightarrow 4(-2+(r+3) \cos \theta) + 3(-7+(r+3) \sin \theta) = 3 \quad \text{--- (2)}$$

Subtracting (1) from (2)

$$12 \cos \theta + 9 \sin \theta = -9$$

$$\Rightarrow 4 \cos \theta = -3(1 + \sin \theta)$$

$$\Rightarrow 16 \cos^2 \theta = 9(1 + \sin^2 \theta - 2 \sin \theta)$$

$$\Rightarrow 16(1 - \sin^2 \theta) = 9(1 + \sin^2 \theta - 2 \sin \theta)$$

$$\Rightarrow 16 - 16 \sin^2 \theta = 9 + 9 \sin^2 \theta - 18 \sin \theta$$

$$\Rightarrow 25 \sin^2 \theta - 18 \sin \theta - 7 = 0$$

$$\Rightarrow 25 \sin^2 \theta - 25 \sin \theta + 7 \sin \theta - 7 = 0$$

$$\Rightarrow 25 \sin \theta (\sin \theta - 1) - 7 (\sin \theta - 1) = 0$$

$$\sin \theta = 1, \sin \theta = \frac{7}{25}$$

$$\text{Now, } \sin \theta = 1 \Rightarrow \cos \theta = 0$$

$$\therefore x+2=0 \quad \text{--- (1)}$$

$$\text{and if } \sin \theta = \frac{7}{25} \text{ then } \cos \theta = \frac{24}{25}$$

$$\therefore \frac{x+2}{\frac{24}{25}} = \frac{y+7}{\frac{7}{25}}$$

$$\Rightarrow 7x + 24y + 182 = 0 \quad \text{--- (2)}$$

Ex 23.9

Q1

(i) Slope intercept form ($y = mx + c$)

$$\sqrt{3}x + y + 2 = 0$$

$$\Rightarrow y = -\sqrt{3}x - 2$$

$$\Rightarrow m = -\sqrt{3}, c = -2$$

$$y\text{-intercept} = -2, \text{ slope} = -\sqrt{3}$$

(ii) Intercept form ($\frac{x}{a} + \frac{y}{b} = 1$)

$$\sqrt{3}x + y + 2 = 0$$

$$\Rightarrow \sqrt{3}x + y = -2$$

$$\Rightarrow \frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$$

$$\Rightarrow \frac{x}{\frac{-2}{\sqrt{3}}} + \frac{y}{-2} = 1$$

$$\Rightarrow x \text{ intercept} = \frac{-2}{\sqrt{3}}, y \text{ intercept} = -2$$

(iii) Normal form ($x \cos \alpha + y \sin \alpha = p$)

$$\sqrt{3}x + y + 2 = 0$$

$$\Rightarrow -\sqrt{3}x - y = 2$$

$$\Rightarrow \left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$$

$$\Rightarrow \cos \alpha = \frac{-\sqrt{3}}{2} = \cos 210^\circ \text{ and } \sin \alpha = \frac{-1}{2} = \sin 210^\circ$$

$$\Rightarrow p = 1, \alpha = 210^\circ$$

Q2(i)

$$x + \sqrt{3}y - 4 = 0$$

Divide the equation by 2, we get

$$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 2$$

$$x \cos 60 + y \sin 60 = 2$$

$$\text{So, } p=2 \text{ and } \alpha=60$$

Q2(ii)

$$x + y + \sqrt{2} = 0$$

$$x + y = -\sqrt{2}$$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = -1$$

$$\frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

Comparing with $x \cos \alpha + y \sin \alpha = p$

$$\cos \alpha = \frac{-1}{\sqrt{2}}, \quad \sin \alpha = \frac{-1}{\sqrt{2}}, \quad p = 1$$

Both are negative

α is in III quadrant

$$\Rightarrow \alpha = \pi \frac{\pi}{4} = \frac{5\pi}{4} = 225^\circ$$

Q2(iii)

$$x - y + 2\sqrt{2} = 0$$

$$-x + y = 2\sqrt{2}$$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$\frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$$

Comparing with $x \cos \alpha + y \sin \alpha = p$

$$\cos \alpha = \frac{-1}{\sqrt{2}}, \quad \sin \alpha = \frac{1}{\sqrt{2}}, \quad p = 2$$

α is in II quadrant

$$\Rightarrow \alpha = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} = 135^\circ, \quad p = 2$$

Q2(iv)

$$x - 3 = 0$$

$$x = 3$$

Comparing with $x \cos \alpha + y \sin \alpha = p$

$$\cos \alpha = 1$$

$$= \cos 0$$

$$\Rightarrow \alpha = 0$$

$$p = 3$$

Q2(v)

$$y - 2 = 0$$

$$y = 2$$

Comparing with $x \cos \alpha + y \sin \alpha = p$

$$\sin \alpha = 1$$

$$\alpha = \frac{\pi}{2}, \quad p = 2$$

Q3

$$\frac{x}{a} + \frac{y}{b} = 1$$

The slope intercept form is

$$y = mx + c$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$bx + ay = ab$$

$$ay = -bx + ab$$

$$y = \frac{-bx}{a} + b$$

Thus y-intercept is b .

$$\text{Slope} = \frac{-b}{a}$$

Q4

The normal form is obtained by dividing each term of the equation by $\sqrt{a^2 + b^2}$,

a = coefficient of x

b = coefficient of y

$$3x - 4y + 4 = 0 \quad \text{--- (1)}$$

$$3x - 4y = -4$$

$$-3x + 4y = 4$$

Dividing each term by $\sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$$\frac{-3}{5}x + \frac{4}{5}y = \frac{4}{5}$$

$$\Rightarrow p = \frac{4}{5} \quad \text{for equation (1)}$$

Also

$$2x + 4y - 5 = 0 \quad \text{--- (2)}$$

$$2x + 4y = 5$$

Dividing each term by $\sqrt{(2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20}$

$$\frac{2x}{\sqrt{20}} + \frac{4y}{\sqrt{20}} = \frac{5}{\sqrt{20}}$$

$$p = \frac{5}{\sqrt{20}} = \frac{5}{4.4} \quad \text{for equation (2)}$$

Comparing p of (1) and (2)

We conclude that $3x - 4y + 4 = 0$ is nearest to origin

Q5

Reduce $4x + 3y + 10 = 0$ to perpendicular form

$$4x + 3y = -10$$

$$-4x - 3y = 10$$

Dividing each term by $\sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$$\frac{-4}{5}x - \frac{3}{5}y = \frac{10}{5} = 2$$

$$\Rightarrow p_1 = 2 \quad \text{--- (1)}$$

$$5x - 12y + 26 = 0$$

$$5x - 12y = -26$$

Dividing each term by $\sqrt{(-5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$$\frac{-5}{13}x + \frac{12}{13}y = \frac{-26}{13} = -2$$

$$\Rightarrow p_2 = 2 \quad \text{--- (2)}$$

$$7x + 24y = 50$$

Dividing each term by $\sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

$$\frac{7}{25}x + \frac{24}{25}y = \frac{50}{25} = 2$$

$$\Rightarrow p_3 = 2 \quad \text{--- (3)}$$

Hence, origin is equidistant from all three lines.

Q6

$$\sqrt{3}x + y + 2 = 0$$

$$\sqrt{3}x + y = -2$$

$$-\sqrt{3}x - y = 2 \text{ ----- (1)}$$

So,

$$\cos \theta = -\frac{\sqrt{3}}{2}, \sin \theta = -\frac{1}{2}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \left(\pi + \frac{\pi}{6} \right)$$

$$= 180^\circ + 30^\circ$$

$$\theta = 210^\circ$$

$$p = 2 \quad [\text{From equation (1)}]$$

Q7

The intercept form of equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$3x - 2y + 6 = 0$$

$$3x - 2y = -6$$

$$\frac{-3x}{-6} - \frac{2y}{-6} = 1$$

$$\frac{x}{-2} + \frac{y}{-3} = 1$$

$$\frac{x}{-2} + \frac{y}{-3} = 1$$

$$\Rightarrow x\text{-intercept} = a = -2$$

$$y\text{-intercept} = b = -3$$

Q8

Perpendicular distance from the origin to the line is 5, so

$$x \cos \alpha + y \sin \alpha = 5$$

$$y \sin \alpha = -x \cos \alpha + 5$$

$$y = -\frac{\cos \alpha}{\sin \alpha} x + 5$$

$$y = -\cot \alpha x + 5$$

Comparing with $y = mx + c$

$$m = -\cot \alpha$$

$$-1 = -\cot \alpha$$

$$\cot \alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

So, equation of line is

$$x \cos \frac{\pi}{4} + y \sin \frac{\pi}{4} = 5$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 5$$

$$x + y = 5\sqrt{2}$$



Ex 23.10

Q1(i)

$$2x - y + 3 = 0 \Rightarrow y = 2x + 3$$

Putting this value in the second equation, we get

$$x + y - 5 = 0$$

$$x + (2x + 3) - 5 = 0$$

$$x + 2x + 3 - 5 = 0$$

$$3x - 2 = 0$$

$$x = \frac{2}{3}$$

Putting this value in the first equation, we get

$$\Rightarrow y = 2x + 3 = \frac{2 \times 2}{3} + 3 = \frac{4}{3} + 3 = \frac{13}{3}$$

$$\therefore \text{Point of intersection is } \left(\frac{2}{3}, \frac{13}{3} \right)$$

Q1(ii)

$$bx + ay = ab \Rightarrow x = \frac{ab - ay}{b}$$

Putting this value in the second equation, we get

$$ax + by = ab$$

$$a \left(\frac{ab - ay}{b} \right) + by = ab$$

$$a^2b - a^2y + b^2y = ab^2$$

$$y(b^2 - a^2) = ab(b - a)$$

$$y = \frac{ab(b - a)}{b^2 - a^2} = \frac{ab}{b + a}$$

Putting this value in the first equation, we get

$$\Rightarrow x = \frac{ab - \frac{a(ab)}{a+b}}{b} = \frac{ab}{a+b}$$

$$\therefore \text{Point is } \left(\frac{ab}{a+b}, \frac{ab}{a+b} \right)$$

Q1(iii)

$$y = m_1x + \frac{a}{m_1} \text{ and } y = m_2x + \frac{a}{m_2}$$

Putting value of y from one equation to another

$$m_1x + \frac{a}{m_1} = m_2x + \frac{a}{m_2}$$

$$x(m_1 - m_2) = \frac{a}{m_2} - \frac{a}{m_1} = a \left(\frac{m_1 - m_2}{m_1 m_2} \right)$$

$$\Rightarrow x = \frac{a}{m_1 m_2}$$

$$\Rightarrow y = m_1x + \frac{a}{m_1}$$

$$= m_1 \left(\frac{a}{m_1 m_2} \right) + \frac{a}{m_1}$$

$$= \frac{a}{m_2} + \frac{a}{m_1}$$

$$= a \left(\frac{m_1 + m_2}{m_1 m_2} \right)$$



Q2(i)

The point of intersection of two sides will give the vertex

$$x + y - 4 = 0 \quad (1)$$

$$2x - y + 3 = 0 \quad (2)$$

$$x - 3y + 2 = 0 \quad (3)$$

Solving (1) and (2)

$$x + y = 4$$

$$y = 4 - x$$

Putting y in (2)

$$2x - (4 - x) + 3 = 0$$

$$2x - 4 + x + 3 = 0$$

$$3x - 1 = 0$$

$$x = \frac{1}{3}$$

Putting x in (1)

$$\frac{1}{3} + y - 4 = 0$$

$$y = 4 - \frac{1}{3} = \frac{11}{3}$$

$$\therefore \text{One vertex is } \left(\frac{1}{3}, \frac{11}{3}\right)$$

Solving (2) and (3), we get

$$y = 2x + 3 \text{ and putting in (3)}$$

$$x - 3y + 2 = 0$$

$$x - 3(2x + 3) + 2 = 0$$

$$x - 6x - 9 + 2 = 0$$

$$-5x = +7$$

$$x = \frac{-7}{5}$$

Q2(ii)

$$y(t_1 + t_2) = 2x + 2at_1t_2, \quad 1$$

$$y(t_2 + t_3) = 2x + 2at_2t_3 \text{ and,} \quad 2$$

$$y(t_3 + t_1) = 2x + 2at_1t_3 \quad 3$$

Solving 1 and 2 gives $(x_1, y_1) = (at_2^2, 2at_2)$

Solving 2 and 3 gives $(x_2, y_2) = (at_3^2, 2at_3)$

Solving 1 and 3 gives $(x_3, y_3) = (at_1^2, 2at_1)$

Above points are the vertices of the triangle

Q3(i)

$$y = m_1x + c_1 \quad 1$$

$$y = m_2x + c_2 \quad 2$$

$$x = 0 \quad 3$$

Solving 1 and 2 gives $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right)$

Solving 2 and 3 gives $(0, c_2)$

Solving 1 and 3 gives $(0, c_1)$

Area of triangle formed by above vertices is

$$= \frac{1}{2} \left[\left(\frac{c_2 - c_1}{m_1 - m_2} \times c_1 \right) - \left(\frac{c_2 - c_1}{m_1 - m_2} \times c_2 \right) \right]$$

$$= \frac{(c_2 - c_1)^2}{2(m_1 - m_2)}$$

Q3(ii)

$$y = 0, y = 2, x + 2y = 3$$

$$y = 0 \quad \text{--- (1)}$$

$$y = 2 \quad \text{--- (2)}$$

$$x + 2y = 3 \quad \text{--- (3)}$$

Solving (1) and (2)

$$(2, 0) \quad \text{--- (A)}$$

Solving (2) and (3)

$$2 + 2y = 3$$

$$\Rightarrow y = \frac{1}{2}$$

$$\Rightarrow y = 2$$

$$\therefore \left(2, \frac{1}{2}\right) \quad \text{--- (B)}$$

Solving (1) and (3)

$$x + 0 = 3$$

$$\Rightarrow \text{Point is } (3, 0) \quad \text{--- (C)}$$

Area of triangle is

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

and treating the points A, B, C as

$(x_1 - y_1)$, $(x_2 - y_2)$ and $(x_3 - y_3)$

$$= \frac{1}{2} \left[2 \left(\frac{1}{2} - 0 \right) + 2(0 - 0) + 3 \left(0 - \frac{1}{2} \right) \right]$$

$$= \frac{1}{2} \left[1 - \frac{3}{2} \right]$$

$$= \frac{-1}{4}$$

Q3(iii)

$$x+y-6=0, \quad 1$$

$$x-3y-2=0, \text{ and } 2$$

$$5x-3y+2=0 \quad 3$$

Solving 1 and 2 gives us $(x_1, y_1) = (5, 1)$

Solving 2 and 3 gives us $(x_2, y_2) = (-1, -1)$

Solving 3 and 1 gives us $(x_3, y_3) = (2, 4)$

So Area of triangle when three vertices are given is

$$\frac{1}{2} (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$$

$$= \frac{1}{2} [|-25 - 3 + 4|]$$

$$= 12 \text{ sq units}$$



Q4

Solving the equations $3x + 2y + 6 = 0$ and $2x - 5y + 4 = 0$
we get $x = -2$ and $y = 0$.

Solving the equations $x - 3y - 6 = 0$ and $2x - 5y + 4 = 0$
we get $x = -42$ and $y = -16$.

Solving the equations $3x + 2y + 6 = 0$ and $x - 3y - 6 = 0$
we get $x = -6/11$ and $y = -24/11$.

So let the intersection points be A, B and C i.e. the triangle be ABC.

Coordinates of A, B and C will be

A(-2, 0); B(-42, -16) and C(-6/11, -24/11).

By mid-point formula the mid-point of AB will be (-22, -8)

Equation of line passing through this mid-point and
the opposite vertex C(-6/11, -24/11) will be the equation of
the median from C. The equation will be

$$\frac{y+8}{x+22} = \frac{-8 + \frac{24}{11}}{-22 + \frac{6}{11}}$$

$$\frac{y+8}{x+22} = \frac{-88+24}{-242+6} = \frac{16}{59}$$

$$16x - 59y + 352 - 472 = 0$$

$$16x - 59y - 120 = 0 \quad \text{Median through C}$$

Similar procedure has to be used for getting other medians as well.

For getting median through B find midpoint of AC and
apply the two point form of line equation. Similarly for median through A.

Final median equations are :

$$41x - 112y - 70 = 0$$

$$25x - 53y + 50 = 0$$

$$16x - 59y - 120 = 0$$



Q5

Let the line be

$$y = \sqrt{3}x + 1 \quad \text{--- (1)}$$

$$y = 4 \quad \text{--- (2)}$$

$$y = -\sqrt{3}x + 2 \quad \text{--- (3)}$$

Solve (1) and (2)

$$4 = \sqrt{3}x + 1$$

$$x = \frac{4-1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$\therefore$ Point A is $(\sqrt{3}, 4)$

Solve (2) and (3)

$$4 = -\sqrt{3}x + 2$$

$$\sqrt{3}x = -2$$

$$x = \frac{-2}{\sqrt{3}}$$

$$= \frac{-2\sqrt{3}}{3}$$

$\therefore$ Point B is $\left(\frac{-2\sqrt{3}}{3}, 4\right)$

Solve (1) and (3)

$$\sqrt{3}x + 1 = -\sqrt{3}x + 2$$

$$2\sqrt{3}x = 1$$

$$x = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

$$y = \sqrt{3}\left(\frac{\sqrt{3}}{6}\right) + 1$$

$$= \frac{3}{2}$$



Q6(i)

$$2x + y - 1 = 0, 3x + 2y + 5 = 0$$

Writing equation in the form $y = mx + c$

$$y = -2x + 1, y = -\frac{3}{2}x - \frac{5}{2}$$

$$\Rightarrow m = -2, m' = -\frac{3}{2}$$

$$m \neq m', m_1 m_2 \neq -1$$

$\Rightarrow$ The lines are intersecting

Q6(ii)

$$x - y = 0, 3x - 3y + 5 = 0$$

$$\Rightarrow y = mx + c, 3x - 3y + 5 = 0$$

$$y = x, y = x + \frac{5}{3}$$

$$\Rightarrow m = 1, m' = 1$$

Slopes of both lines are equal

$\therefore$ Lines are parallel

Q6(iii)

$$3x + 2y - 4 = 0, 6x + 4y - 8 = 0$$

$$y = -\frac{3}{2}x + \frac{4}{2}, y = -\frac{6}{4}x + \frac{8}{4}$$

$$y = -\frac{3}{2}x + 2, y = -\frac{3}{2}x + 2$$

$\Rightarrow$ Lines are coincident

$$\text{Because } m_1 = m_2 = -\frac{3}{2}$$

Intercept = 2 in both line

Q7

The point of intersection of the lines

$$4x + y - 1 = 0 \text{ and } 7x - 3y - 35 = 0$$

$$\text{is } y = 1 - 4x$$

$$7x - 3(1 - 4x) - 35 = 0$$

$$7x - 3 + 12x - 35 = 0$$

$$19x = 38$$

$$x = 2$$

$$\Rightarrow y = 1 - 4x = 1 - 8 = -7$$

$\therefore$ Let $P(2, -7)$ and $Q(3, 5)$

The equation of line PQ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-7) = \frac{5 - (-7)}{3 - 2}(x - 2)$$

$$y + 7 = 12(x - 2)$$

$$y - 12x = -31$$

$$12x - y - 31 = 0$$

Q8

Given lines are,

$$4x - 7y = 3$$

$$2x - 3y = -1$$

Solving these two, we get the point of intersection,

$$x = -8, y = -5$$

Point of intersection of given lines is $(-8, -5)$ equation of line making equal

intercepts (a) on the coordinate axes is,

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$x + y = a$$

$$-8 - 5 = a$$

$$a = -13$$

So,

Equation of required line is

$$x + y = -13$$

Q9

$$y = m_1x, y = m_2x \text{ and } y = c$$

Vertices of triangle formed by above lines are

$$A(0,0), B\left(\frac{c}{m_1}, c\right), C\left(\frac{c}{m_2}, c\right)$$

So Area of triangle when three vertices are given is

$$\frac{1}{2} (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$$

$$= \frac{1}{2} \left[\frac{c^2}{m_1} - \frac{c^2}{m_2} \right] = \frac{c^2}{2} \left[\frac{m_2 - m_1}{m_1 m_2} \right]$$

$$\text{Given } m_1 \text{ and } m_2 \text{ are roots of } x^2 + (\sqrt{3} + 2)x + \sqrt{3} - 1 = 0$$

$$\text{Product of roots} = m_1 m_2 = \sqrt{3} - 1$$

$$|m_2 - m_1| = \sqrt{(m_2 + m_1)^2 - 4m_1 m_2} = \sqrt{(\sqrt{3} + 2)^2 - 4\sqrt{3} + 4}$$

$$|m_2 - m_1| = \sqrt{3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4} = \sqrt{11}$$

$$\text{Area} = \frac{c^2}{2} \left[\frac{\sqrt{11}}{\sqrt{3} - 1} \right]$$

$$\text{Rationalising denominator gives } \frac{c^2}{4} [\sqrt{33} + \sqrt{11}]$$

Hence Proved

Q10

If point of intersection of lines $x + y = 3$ and $2x - 3y = 1$ is

$$x = 3 - y$$

$$2(3 - y) - 3y = 1$$

$$6 - 2y - 3y = 1$$

$$-5y = -5$$

$$y = 1$$

$$\Rightarrow x = 3 - 1 = 2$$

$\therefore$ Point is $(2, 1)$

Any line parallel to $x - y - 6 = 0$

Will have the same slope $= 1$

$\therefore$ Equation of line passing through $(2, 1)$ and having slope $= 1$

$$\text{is } y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 2)$$

$$y - 1 = x - 2$$

$$y - x = -2 + 1$$

$$y - x = -1$$

$$x - y = 1$$

$$\therefore a = 1, b = -1 \left(\text{Comparing with } \frac{x}{a} + \frac{y}{b} = 1 \right)$$



Q11

$$x + y = 1, \quad AB \quad 1$$

$$2x + 3y = 6 \text{ and } BC \quad 2$$

$$4x - y + 4 \quad AC \quad 3$$

Solving 1 and 2 gives B $(-3, 4)$

Solving 1 and 3 gives A $(\frac{-3}{5}, \frac{8}{5})$

Altitude from A to BC is given by

$$y - \frac{8}{5} = \frac{3}{2} \left(x + \frac{3}{5} \right)$$

$$10y - 16 = 15x + 9$$

$$15x - 10y + 25 = 0$$

$$3x - 2y + 5 = 0 \text{-----} 4$$

Similarly Altitude from B to AC is given by

$$y - 4 = \frac{-1}{4} (x + 3)$$

$$4y - 16 = -x - 3$$

$$x + 4y - 13 = 0 \text{-----} 5$$

Solving 4 and 5 gives orthocentre

$$O(\frac{3}{7}, \frac{22}{7})$$



Q12

On solving the equation of AB , BC and CA we get

$$B = (-1, -1)$$

$$A = (2, 4)$$

$$C = (5, 1)$$

The slope of $BC = \frac{1}{3}$ then slope of $AE = -3$

slope of $AC = -1$ then slope of $BD = 1$

slope of $AB = \frac{5}{3}$ then slope of $CF = \frac{-3}{5}$

Where AD, BE, CF are altitudes of $\triangle ABC$

The equation of AD, BE and CF are

$$BD = y + 1 = 1(x + 1) \Rightarrow x - y = 0$$

$$AE = y - 4 = -3(x - 2) \Rightarrow 3x + y = 10$$

$$CF = y - 1 = \frac{-3}{5}(x - 5) \Rightarrow 3x + 5y = 20$$

Are the required equations, then equation through A is $3x + y = 10$.

Q13

$AD \perp BC, CF \perp AB, BE \perp AC$

Let G be the orthocentre of triangle

Let $G(h, k)$

Now, $AG \perp BC$

$$\therefore (\text{slope of } AG) \times (\text{slope of } BC) = -1$$

$$\left(\frac{k-3}{h+1}\right)\left(\frac{0+1}{0-2}\right) = -1$$

$$k - 3 = 2(h + 1)$$

$$k - 2h = 5 \quad \text{--- (1)}$$

And $BG \perp AC$

$$\Rightarrow (\text{slope of } BG) \times (\text{slope of } AC) = -1$$

$$\left(\frac{k+1}{h-2}\right)\left(\frac{0-3}{0+1}\right) = -1$$

$$3(k + 1) = h - 2$$

$$3k - h = -5 \quad \text{--- (2)}$$

from (1) and (2)

$$\text{Orthocentre } (h, k) = (-4, -3)$$

Q14

Let ABC be the triangle whose sides BC, CA and AB have the equations

$$y - 15 = 0, BC$$

$$3x - 4y = 0, AC$$

$$5x + 12y = 0 AB$$

Solving these equations pair wise we can obtain the coordinates of the vertices A, B, C as

A(0,0), B(-36,15), C(20,15) respectively

$$\text{Centroid} \left(\frac{-36+20+0}{3}, \frac{15+15+0}{3} \right) = \left(-\frac{16}{3}, 10 \right)$$

For incentre, We have

$$a = BC = \sqrt{56^2 + 0} = 56$$

$$b = CA = \sqrt{20^2 + 15^2} = 25$$

$$c = AB = \sqrt{36^2 + 16^2} = 39$$

Coordinates of incentre are

$$\left(\frac{56 \times 0 + 25 \times -36 + 39 \times 20}{36 + 25 + 39}, \frac{56 \times 0 + 25 \times 15 + 39 \times 15}{36 + 25 + 39} \right)$$

$$= (-1, 8)$$

Q15

Let $ABCD$ be a quadrilateral with sides AB , BC , CD , DA as $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $-\sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$ respectively.

The slope of $AB = -\sqrt{3}$ --- (1)

The slope of $BC = \frac{-1}{\sqrt{3}}$ --- (2)

The slope of $CD = -\sqrt{3}$ --- (3)

The slope of $DA = \frac{-1}{\sqrt{3}}$ --- (4)

From (1), (2), (3) and (4) we observe the slope of opposite sides of quadrilateral are equal

∴ Opposite sides are parallel.

∴ $ABCD$ is a parallelogram.

We observe that distance between $(AD$ and $BC)$ and $(DC$ and $AB)$ is equal = 1 unit

∴ Sides $AD = AB = BC = DC$

∴ The given figure $ABCD$ is a rhombus

Hence, proved

Q16

$$2x + y = 5 \text{ and } x + 3y + 8 = 0$$

Intersection point of above lines is $\left(\frac{23}{5}, \frac{-21}{5}\right)$

Required line is parallel to $3x + 4y = 7$ and passing through above point

So required line equation is

$$y + \frac{21}{5} = \frac{-3}{4} \left(x - \frac{23}{5} \right)$$

$$20y + 84 = -15x + 69$$

$$15x + 20y + 15 = 0$$

$$3x + 4y + 3 = 0$$

Q17

Solving equations $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$, we get
 $x = -1$ and $y = 1$

So, the given lines intersect at the point whose coordinates are $(-1, -1)$.

We know that, the equation of the required line is perpendicular to the line $3x - 5y + 11 = 0$.

Slope of the required line $= -\frac{5}{3}$

Equation of the required line is given by,

$$(y + 1) = -\frac{5}{3}(x + 1)$$

$$3y + 5x + 8 = 0$$



Ex 23.11

Q1(i)

If the lines are concurrent then point of intersection of any two lines satisfies the third line

$$15x - 18y + 1 = 0 \quad \text{--- (1)}$$

$$12x + 10y - 3 = 0 \quad \text{--- (2)}$$

$$6x + 66y - 11 = 0 \quad \text{--- (3)}$$

Solving (1) and (2)

$$x = \frac{18y - 1}{15}$$

$$12\left(\frac{18y - 1}{15}\right) + 10y - 3 = 0$$

$$216y - 12 + 150y - 45 = 0$$

$$366y = 57$$

$$y = \frac{57}{366} = \frac{19}{122}$$

$$\Rightarrow x = \frac{18y - 1}{15}$$

$$= \frac{18 \times \frac{19}{122} - 1}{15}$$

$$= \frac{18 \times 19 - 122}{122 \times 15}$$

$$= \frac{342 - 122}{1730}$$

$$= \frac{220}{1730}$$

$$= \frac{22}{173}$$

Putting x and y in (3)

$$6\left(\frac{22}{173}\right) + 66\left(\frac{19}{122}\right) - 11 = 0$$

$$6 \times 22 \times 122 + 66 \times 19 \times 173 - 11 \times 173 \times 122 = 0$$

$$0 = 0$$

Q1(ii)

$$3x - 5y - 11 = 0, \quad 5x + 3y - 7 = 0, \quad x + 2y = 0$$

$$3x - 5y - 11 \quad \quad \quad \text{--- (1)}$$

$$5x + 3y - 7 = 0 \quad \quad \quad \text{--- (2)}$$

$$x + 2y = 0 \quad \quad \quad \text{--- (3)}$$

Solving (1) and (2)

$$x = -2y$$

$$5(-2y) + 3y - 7 = 0$$

$$-10y + 3y - 7 = 0$$

$$-7y = 7$$

$$y = -1$$

$$\Rightarrow x = 2$$

substituting x and y in (1)

$$3(2) - 5(-1) - 11 = 0$$

$$6 + 5 - 11 = 0$$

$$0 = 0$$

Hence, the lines are concurrent

Q1(iii)

$$\frac{x}{a} + \frac{y}{b} = 1, \quad \frac{x}{b} + \frac{y}{a} = 1, \quad y = x$$

$$bx + ay = ab, \quad ax + by = ab$$

Put $y = x$

$$bx + ax = ab, \quad ax + bx = ab$$

Hence the lines are concurrent

Q2

The three lines are concurrent if they have the common point of intersection.

$$2x - 5y + 3 = 0 \quad \text{---(1)}$$

$$x - 2y + 1 = 0 \quad \text{---(2)}$$

Solving (1) and (2)

$$2x = 5y - 3$$

$$x = \frac{5y - 3}{2}$$

$$\frac{5y - 3}{2} - 2y + 1 = 0$$

$$5y - 3 - 4y + 2 = 0$$

$$y = 0$$

$$\Rightarrow x = \frac{5y - 3}{2} = \frac{5 - 3}{2} = \frac{2}{2} = 1$$

Substituting x and y in $5x - 9y + \lambda = 0$

$$5(1) - 9(0) + \lambda = 0$$

$$5 - 9 + \lambda = 0$$

$$\lambda = 4$$

Q3

The three lines are

$$y = m_1x + c_1 \quad \text{---(1)}$$

$$y = m_2x + c_2 \quad \text{---(2)}$$

$$y = m_3x + c_3 \quad \text{---(3)}$$

Collinear or they meet at a point only when they have common point of intersection

Solving (1) and (2) for x and y

$$m_1x + c_1 = m_2x + c_2$$

$$x(m_1 - m_2) = c_2 - c_1$$

$$x = \frac{c_2 - c_1}{m_1 - m_2}$$

$$\Rightarrow y = m_1x + c_1$$

$$= m_1 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_1$$

$$= m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1$$

Putting x and y in (3)

$$m_1c_2 - m_1c_1 = m_3 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_3$$

$$m_1^2c_2 - m_1m_2c_2 - m_1m_2c_1 + m_2^2c_1 = m_3c_2 - m_3c_1 + m_1c_3 - m_2c_3$$

$$\Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$



Q4

If the lines are concurrent, then the lines have common point of intersection.

The given line are

$$p_1x + q_1y = 1 \quad \text{--- (1)}$$

$$p_2x + q_2y = 1 \quad \text{--- (2)}$$

$$p_3x + q_3y = 1 \quad \text{--- (3)}$$

Solving (1) and (2)

$$x = \frac{1 - q_1y}{p_1}$$

$$p_2 \left(\frac{1 - q_1y}{p_1} \right) + q_2y = 1$$

$$p_2 = p_2q_1y + p_1q_2y = p_1$$

$$y = \frac{p_1 - p_2}{p_1q_2 - p_2q_1} \Rightarrow x = \frac{1 - q_1 \left(\frac{p_1 - p_2}{p_1q_2 - p_2q_1} \right)}{p_1}$$

Putting x, y in (3)

$$p_3 \left[(p_1q_2 - p_2q_1) - q_1p_1 - q_2p_2 \right] (p_1q_2 - p_2q_1) + q_3p_1(p_1 - p_2) = 1$$

$$(p_1p_3q_2 - p_2p_3q_1 - p_1p_3q_1 + p_2p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$(p_1p_3q_2 - p_1p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$p_1^2p_3q_2^2 - p_1p_2p_3q_1q_2 - p_1^2p_3q_1q_2 + p_1p_2p_3q_1^2 + q_3p_1^2 - q_3p_1p_2 = 1 \quad \text{--- (1)}$$

Also if $(p_1q_1)(p_2q_2)(p_3q_3)$ are collinear

Then,

$$p_1(q_2 - q_3) + p_2(q_3 - q_1) + p_3(q_1 - q_2) = 0$$

From (1)

$$p_1[p_1p_3q_2^2 - p_2p_3q_1q_2 - p_1p_3q_1q_2 + p_2p_3q_1^2 + q_3p_1 - q_3p_2] = 1$$

$$p_1[p_3q_2(p_1q_2 - p_2q_1) - p_3q_1(p_1q_2 - p_2q_1) + q_3(p_1 - p_2)] = 1$$

Hence, the points are collinear



Q5

The three lines are concurrent if they have the common point of intersection

$$(b+c)x + ay + 1 = 0$$

$$(c+a)x + by + 1 = 0$$

$$(a+b)x + cy + 1 = 0$$

Solving (1) and (2)

$$y = \frac{-1 - (b+c)x}{a}$$

Putting in (2)

$$(c+a)x + b \frac{-1 - (b+c)x}{a} + 1 = 0$$

$$acx + a^2x + b - b^2x - bcx + a = 0$$

$$x(ac + a^2 - b^2 - bc) = b - a$$

$$x(ac - bc + a^2 - b^2) = b - a$$

$$x(c(a-b) + (a-b)(a+b)) = b - a$$

$$x(c+a+b) = -1 \quad [\text{Cancelling } (a-b) \text{ both sides}]$$

$$x = \frac{-1}{a+b+c}$$

$$y = \frac{-1 + \frac{(b+c)(-1)}{a+b+c}}{a} = \frac{-a-b-c-b-c}{a(a+b+c)}$$

Putting the value of x, y in (3);

$$(a+b) \left(\frac{-1}{a+b+c} \right) + c \left(\frac{-a-2b-2c}{a(a+b+c)} \right) + 1 = 0$$

$$-a^2 - ba - ac - 2bc - 2c^2 + a^2 + ab + ac = 0$$

$$0 = 0$$

Hence, the lines are concurrent.

Q6

If the three lines are concurrent then the point of intersection of (1) and (2) should verify the (3) line, where

$$ax + a^2y + 1 = 0 \quad \text{--- (1)}$$

$$bx + b^2y + 1 = 0 \quad \text{--- (2)}$$

$$cx + c^2y + 1 = 0 \quad \text{--- (3)}$$

Solving (1) and (2)

$$x = \frac{-1 - a^2y}{a} \Rightarrow b \left(\frac{-1 - a^2y}{a} \right) + b^2y + 1 = 0$$

$$-b - a^2by + ab^2y + a = 0$$

$$y = \frac{b - a}{ab(b - a)} = \frac{1}{ab}$$

$$\Rightarrow x = \frac{1 - a^2 \times \frac{1}{ab}}{a} = \frac{1 - \frac{a}{b}}{a} = \frac{b - a}{ab}$$

Putting in (3)

$$c \left(\frac{b - a}{ab} \right) + c^2 \left(\frac{1}{ab} \right) + 1 = 0$$

$$bc - ac + c^2 + ab = 0$$

$$bc + c^2 - ac + ab = 0$$

$$c(b + c) - a(c - b) = 0$$

$$\Rightarrow \text{Either } c = b \Rightarrow 2bc = 0 \Rightarrow 2c^2 = 0 \Rightarrow c = 0$$



Q7

If a, b, c are in A.P.

$$b - a = c - b$$

$$2b = a + c \quad [\text{Common difference}]$$

To prove that the straight lines are concurrent then they have the common point of intersection.

$$ax + 2y + 1 = 0 \quad \text{--- (1)}$$

$$bx + 3y + 1 = 0 \quad \text{--- (2)}$$

$$cx + 4y + 1 = 0 \quad \text{--- (3)}$$

Solving (1) and (2)

$$x = \frac{-1 - 2y}{a}$$

Put in (2)

$$b \left(\frac{-1 - 2y}{a} \right) + 3y + 1 = 0$$

$$y = \frac{b - a}{3a - 2b} \Rightarrow x = \frac{-1 - \frac{2(b - a)}{3a - 2b}}{a} = \frac{-3a + 2b - 2b + 2a}{a(3a - 2b)}$$

$$x = \frac{-1}{3a - 2b}$$

Putting x, y in (3)

$$c \left(\frac{-1}{3a - 2b} \right) + 4 \left(\frac{b - a}{3a - 2b} \right) + 1 = 0$$

$$-c + 4b - 4a + 3a - 2b = 0$$

$$-a + 2b - c = 0$$

$$-a + a + c - c = 0$$

$$0 = 0$$

Hence, Proved

Q8

Let coordinates of $\triangle ABC$ be $A(0, 0)$, $B(a, 0)$, $C(0, b)$.

Then mid points of AB , BC and CA are $\rightarrow$.

$$D\left(\frac{a}{2}, 0\right), E\left(\frac{a}{2}, \frac{b}{2}\right) \text{ and } F\left(0, \frac{b}{2}\right)$$

Then equation of CD , AE and BF are

$$CD \Rightarrow y - b = \frac{\frac{a}{2} - 0}{\frac{a}{2} - 0} (x - 0)$$

$$\Rightarrow y - b = \frac{-2b}{a} (x)$$

$$\Rightarrow ay - ab = -2bx$$

$$\Rightarrow ay + 2bx - ab = 0 \quad \text{--- (1)}$$

$$BF \Rightarrow y - 0 = \frac{\frac{b}{2} - 0}{0 - a} (x - a)$$

$$\Rightarrow y = \frac{-b}{2a} (x - a)$$

$$\Rightarrow -2ay - bx = ba \quad \text{--- (2)}$$

$$AE \Rightarrow y - 0 = \frac{0 - \frac{b}{2}}{0 - \frac{a}{2}} (x - 0)$$

$$\Rightarrow ya = +bx \quad \text{--- (3)}$$

Adding (1), (2) and (3)

$$ay + 2bx - ab + 2b^2 - 2ay - bx - ab + ay - bx = 0$$

then,

$$\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0, \text{ where } \lambda_1 = \lambda_2 = \lambda_3 = 1.$$

Hence, lines are concurrent



Ex 23.12

Q1

Equation of line through $(2, 3)$ is

$$y - y_1 = m(x - x_1) \quad \text{--- (1)}$$

$$(2, 3) \text{ is } (x_1, y_1)$$

Since the line is parallel to $3x - 4y + 5 = 0$

$\Rightarrow$ The slope will be equal

$$\text{Slope of } 3x - 4y + 5 = 0$$

$$4y = 3x + 5$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

$$\Rightarrow m = \frac{3}{4}$$

Substituting m and (x_1, y_1) in (1)

$$y - 3 = \frac{3}{4}(x - 2)$$

$$4y - 12 = 3x - 6$$

$$3x - 4y = -12 + 6 = -6$$

$$3x - 4y + 6 = 0$$

Q2

Any equation passing through $(3, -2)$ and perpendicular to given line is

$$y - y_1 = -\frac{1}{m}(x - x_1) \quad \text{--- (1)}$$

Where (x_1, y_1) is $(3, -2)$ and m is slope of line.

$-\frac{1}{m}$ is taken as lines are perpendicular

Finding slope of line $x - 3y + 5 = 0$

$$3y = x + 5$$

$$y = \frac{x}{3} + \frac{5}{3}$$

$$\Rightarrow m = \frac{1}{3}$$

Substituting the value of m and (x_1, y_1) in (1)

$$y - (-2) = -\frac{1}{\frac{1}{3}}(x - 3)$$

$$y + 2 = -3(x - 3) = -3x + 9$$

$$3x + y = 7$$

Q3

Any line which is perpendicular bisector means line is perpendicular to the given line and one end point is the mid point of that line.

The line joining $\begin{pmatrix} 1, 3 \\ (x_1, y_1) \end{pmatrix}$ and $\begin{pmatrix} 3, 1 \\ (x_2, y_2) \end{pmatrix}$.

Has the mid-point

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow (x_1, y_1) = \left(\frac{1+3}{2}, \frac{3+1}{2} \right) = (2, 2)$$

Also slope of line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{3-1} = \frac{-2}{2} = -1$$

So, the slope of required line is 1 (negative reciprocal of slope)

Thus, the equation of perpendicular bisector is

$$y - y_1 = \frac{-1}{m} (x - x_1)$$

$$y - 2 = 1(x - 2)$$

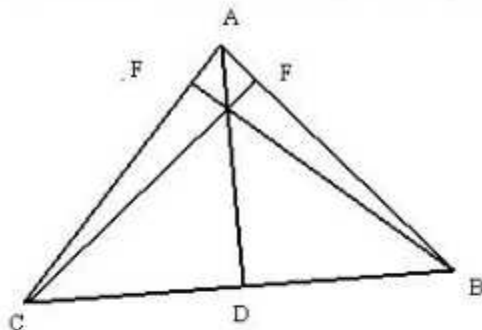
$$y - 2 = x - 2$$

$$y = x$$



Q4

Let the perpendiculars of the triangle on the side AB, BC and AC be CF, AD and FB respectively.



$$\text{Slope of the side AB} = \frac{4-2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

$$\text{Corresponding slope of CF} = -\frac{1}{1/2} = -2$$

[since $m_1 \times m_2 = -1$]

Equation of CF, $y - y_1 = m(x - x_1)$

$$y + 3 = -2(x + 5)$$

of C in place of x_1 and y_1] [Putting co-ordinates]

$$y + 3 = -2x - 10$$

$$y = -2x - 13$$

$$\text{Slope of the side BC} = \frac{2+3}{-3+5} = \frac{5}{2}$$

$$\text{Corresponding slope of AD} = -\frac{1}{5/2} = -\frac{2}{5}$$

Equation of AD,

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{5}(x - 1)$$

$$5y - 20 = -2x + 2$$

$$5y = -2x - 22$$

$$\text{Slope of the side AC} = \frac{4+3}{1+5} = \frac{7}{6}$$

Q5

Required equation of line is

$$y - y_1 = m'(x - x_1) \quad \text{--- (1)}$$

Point is $(x_1, y_1) = (0, -4)$

It is perpendicular to line $\sqrt{3}x - y + 5 = 0$

$\Rightarrow$ Slope is $y = mx + c$

$$y = \sqrt{3}x + 5$$

$$m = \sqrt{3}$$

$$m' = \frac{-1}{m} = \frac{-1}{\sqrt{3}}$$

Putting m' and (x_1, y_1) in (1)

$$y - (-4) = \frac{-1}{\sqrt{3}}(x - 0)$$

$$y + 4 = \frac{-x}{\sqrt{3}}$$

$$x + \sqrt{3}y + 4\sqrt{3} = 0$$

Q6

Here,

Let l be line mirror and B is image of A

Let m be slope of line l

So,

$$m(\text{slope of } AB) = -1$$

$$m\left(\frac{2-1}{5-2}\right) = -1$$

$$m\left(\frac{1}{3}\right) = -1$$

$$m = -3$$

M is mid point of AB

$$M = \left(\frac{2+5}{2}, \frac{2+1}{2}\right)$$

$$M = \left(\frac{7}{2}, \frac{3}{2}\right)$$

Equation line l is,

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = (-3)\left(x - \frac{7}{2}\right)$$

$$\frac{2y-3}{2} = -3x + \frac{21}{2}$$

$$2y - 3 = -6x + 21$$

$$6x + 2y = 24$$

$$3x + y = 12$$

Q7

Any line is given by equation

$$y - y_1 = m(x - x_1) \quad \text{--- (1)}$$

Where (x_1, y_1) is (α, β)

And m is negative reciprocal of slope of line $lx + my + n = 0$.

i.e; $y = \frac{-lx}{m} - \frac{n}{m}$

$$\Rightarrow \text{Slope of line} = \frac{-l}{m}$$

Putting the data in (1), we get

$$y - \beta = \frac{m}{l}(x - \alpha)$$

$$ly + mx = m\alpha + l\beta$$

$$m(x - \alpha) = l(y - \beta)$$

Q8

Let the equation of the required line be $y - y_1 = m(x - x_1)$, where 'm' denotes the slope of the line and (x_1, y_1) be the point through which the line passes.

Since the x-intercept of the line is 1 on the positive direction of the x-axis therefore the line passes through (1,0)

Also, $2x - 3y = 5$

$$3y = 2x - 5$$

$$y = \frac{2x - 5}{3}$$

Therefore, the slope of the given line is $2/3$.

$$\text{Slope of the required line} = \frac{-1}{2/3} = -\frac{3}{2}$$

Therefore the equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{2}{3}(x - 1)$$

$$y = -\frac{3}{2}(x - 1)$$

$$2y = -3x + 3$$

The equation of the required line is $3x + 2y - 3 = 0$

Q9

Slope of line through the points $(a, 2a)$, $(-2, 3)$
 (x_1, y_1) (x_2, y_2)

$$\Rightarrow m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2a}{-2 - a}$$

Also, slope of line $x - ay = 1$ in the form $y = mx + c$

$$4x + 3y + 5 = 0$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$

$$\Rightarrow m_2 = -\frac{4}{3}$$

If two lines are perpendicular then, $m_1 m_2 = -1$

$$\left(\frac{3 - 2a}{-2 - a}\right)\left(-\frac{4}{3}\right) = -1$$

$$-12 + 8a = 6 + 3a$$

$$5a = 18$$

$$a = \frac{18}{5}$$

Q10

Any line having y-intercept equal to $\frac{4}{3}$ passes through the point $(0, \frac{4}{3})$
 (x_1, y_1)

Slope of line $3x - 4y + 11 = 0$

$$y = \frac{3}{4}x + \frac{11}{4}$$

$$\Rightarrow m = \frac{3}{4}$$

The required line is perpendicular to the given line, therefore its slope is $-\frac{4}{3}$

$\Rightarrow$ Equation of required line is

$$y - y_1 = m'(x - x_1)$$

$$y - \frac{4}{3} = -\frac{4}{3}(x - 0)$$

$$4x + 3y - 4 = 0$$

**Q11**

Any line which is right bisector to another line segment passes through the mid-point of end-points and is perpendicular to it.

$\Rightarrow$ mid point of (a, b) and (a_1, b_1) is

$$(x_1, y_1) = \left(\frac{a+a_1}{2}, \frac{b+b_1}{2} \right)$$

$$\text{Slope of line } (m) = \frac{b_1 - b}{a_1 - a}$$

$$\text{Slope of required line is } m' = \frac{a - a_1}{b - b_1}$$

Equation of required line is

$$y - y_1 = m'(x - x_1)$$

$$y - \left(\frac{b+b_1}{2} \right) = \frac{a - a_1}{b - b_1} \left(x - \frac{a+a_1}{2} \right)$$

$$2x(a_1 - a) + 2y(b_1 - b) + a^2 + b^2 = a_1^2 + b_1^2$$

Q12

Let the image of the point $P(2, 1)$ in the line mirror AB be $Q(\alpha, \beta)$. Then, PQ is perpendicularly bisected at R .

The coordinates of R are

$$\left(\frac{\alpha+2}{2}, \frac{\beta+1}{2} \right)$$

And lie on the line $x + y - 5 = 0$

$$\left(\frac{\alpha+2}{2} \right) + \left(\frac{\beta+1}{2} \right) - 5 = 0$$

$$\alpha + 2 + \beta + 1 - 10 = 0$$

$$\alpha + \beta = 7 \quad \text{--- (1)}$$

Since PQ is $\perp$ to AB

$$(\text{Slope of } AB) \times (\text{Slope of } PQ) = -1$$

$$-1 \times \left(\frac{\beta - 1}{\alpha - 2} \right) = -1$$

$$\beta - 1 = \alpha - 2$$

$$\beta - \alpha = -1 \quad \text{--- (2)}$$

Solving (1) and (2), we get

$$\alpha = 5 \text{ and } \beta = 2$$

$\therefore$ Image of $(1, 2)$ in $x + y - 5 = 0$ is $(4, 3)$.

Q13

Let $Q(5, 2)$ be the mirror image of $P(2, -1)$ with respect to the line mirror $AB \times (ax + by + c = 0)$

Then,

$$(\text{Slope of } AB) \times (\text{Slope of } PQ) = -1$$

$$\frac{-a}{b} \times \left(\frac{2-1}{5-2} \right) = -1$$

$$\frac{-a}{b} \times \frac{1}{3} = -1$$

$$-a = -3b$$

$$a = 3b \quad \text{--- (1)}$$

and

(R) mid point of PQ should lie in AB , as PQ perpendicularly bisects AB .

$$\therefore \text{Coordinates of } R \text{ are } \left(\frac{5+2}{2}, \frac{2+1}{2} \right) = \left(\frac{7}{2}, \frac{3}{2} \right)$$

$$\therefore \frac{7}{2}a + \frac{3}{2}b + c = 0$$

$$7a + 3\left(\frac{a}{3}\right) + 2c = 0 \quad \left[\because b = \frac{a}{3} \text{ from (1)} \right]$$

$$8a + 2c = 0$$

$$\text{or, } -4a = c \quad \text{--- (2)}$$

$\therefore$ equation of line is $ax + by + c = 0$

$$\text{or, } ax + \frac{a}{3}y - 4a = 0$$

$$\text{or, } 3x + y - 12 = 0$$

Q14

The slope of the given line is equal to the slope of line $3x - 4y + 6 = 0$ as the two lines are parallel to each other

$$\therefore m_1 = m_2 = \frac{3}{4}$$

And the line passes through mid point of points $(2, 3)$ and $(4, -1)$

$$\text{i.e., } \left(\frac{2+4}{2}, \frac{3+(-1)}{2} \right) \quad [\text{Using mid point formula}]$$

$$\Rightarrow (3, 1)$$

$\therefore$ using one point-slope equation of line

$$(y - 1) = \frac{3}{4}(x - 3)$$

$$4y - 4 = 3x - 9$$

$$3x - 4y = 5$$

Is the required line

Q15

In a parallelogram opposite sides are parallel and parallel sides have equal slope.

Slope of line $2x - 3y + 1 = 0$

$$m_1 = \frac{2}{3} \quad \text{--- (1)}$$

Slope of line $x + y = 3$

$$m_2 = 1 \quad \text{--- (2)}$$

Slope of line $2x - 3y - 2 = 0$

$$m_3 = \frac{2}{3} \quad \text{--- (3)}$$

Slope of line $x + y = 4$

$$m_4 = -1 \quad \text{--- (4)}$$

From (1), (2), (3) and (4)

We observe that opposite sides of $ABCD$ have same slope and hence are parallel

Hence, proved, the given quadrilateral is a parallelogram

Q16

The required line is perpendicular to the given line $6x + 4y = 24$.

$$\therefore (\text{Slope of required line}) \times (\text{Slope of given line}) = -1$$

$$m_1 = \frac{-1}{\left(\frac{-6}{4}\right)} = \frac{4}{6}$$

and

The required line passes through the point (x_1, y_1) where it meets the y-axis

$$\therefore x \text{ coordinate at that point is zero. i.e; } x_1 = 0$$

$$(y - y_1) = \frac{4}{6}(x - 0)$$

$$6y - 6y_1 = 4x$$

$$2x - 3y = -3y_1 \Rightarrow y_1 = 6$$

$$2x - 3y = -18$$

$$2x - 3y + 18 = 0$$

Q17

OP is perpendicular to the given line $y = mx + c$

$$\therefore (\text{Slope of } OP) \times (\text{Slope of line}) = -1$$

$$\frac{2-0}{-1-0} \times m = -1$$

$$m = \frac{-1 \times -1}{2} = \frac{1}{2}$$

and $(-1, 2)$ lies on the line $y = \frac{1}{2}x + c$

$$\therefore 2 = \frac{1}{2}(-1) + c$$

$$c = 2 + \frac{1}{2} = \frac{5}{2}$$

$$\therefore c = \frac{5}{2} \text{ and } m = \frac{1}{2}$$

Q18

The slope of line joining $(3, 4)$ and $(-1, 2)$ is

$$\frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$$

The required line is $\perp$ to the given line

$$\therefore (\text{Slope of required line}) \times \frac{1}{2} = -1 \quad [\because m_1 \times m_2 = -1 \text{ for perpendicular lines}]$$

$$m_1 = -2$$

And the line passes through the mid point of line joining $(3, 4)$ and $(-1, 2)$

$$\text{i.e.; } \left(\frac{3-1}{2}, \frac{4+2}{2} \right) \text{ or } (1, 3)$$

$\therefore$ equation of the required line is

$$y - 3 = (-2)(x - 1)$$

$$\text{or } y - 3 = -2x + 2$$

$$\text{or } 2x + y - 5 = 0$$

Q19

If two lines intersect at right angles, then product of their slope is -1 .

Slope of $7x - 9y - 19 = 0$ is $m_1 = \frac{7}{9}$ --- (1)

Slope of line joining $(h, 3)$ and $(4, 1) = \frac{1-3}{4-h}$

or, $m_2 = \frac{2}{h-4}$ --- (2)

$$m_1 \times m_2 = -1$$

$$\frac{7}{9} \times \frac{2}{h-4} = -1$$

$$14 = -9h + 36$$

$$9h = 36 - 14$$

$$h = \frac{22}{9}$$

Q20

Let the image of $P(3, 8)$ in $x + 3y = 7$ be $Q(\alpha, \beta)$.

Then,

PQ is perpendicularly bisected at R .

Then,

$$R = \left(\frac{\alpha+3}{2}, \frac{\beta+8}{2} \right)$$

and lie on $x + 3y = 7$

$$\frac{\alpha+3}{2} + \frac{3\beta+24}{2} = 7$$

$$\alpha + 3 + 3\beta + 24 = 14$$

$$\alpha + 3\beta = -13 \quad \text{--- (1)}$$

And since PQ is perpendicular to

$$x + 3y = 7$$

$$(\text{Slope of line}) \times (\text{Slope of } PQ) = -1$$

$$\frac{-1}{3} \times \frac{\beta-8}{\alpha-3} = -1$$

$$\beta - 8 = 3\alpha - 9$$

$$\beta - 3\alpha = -1 \quad \text{--- (2)}$$

Solving (1) and (2)

$$\beta = -4, \alpha = -1$$

$\therefore Q$ is $(-1, -4)$

Q21

Let foot of perpendicular of $P(-1, 3)$ on line $3x - 4y = 16$ be $Q(\alpha, \beta)$

Then,

$$(\text{Slope of line}) \times (\text{Slope of } PQ) = -1$$

$$\frac{3}{4} \times \frac{\beta - 3}{\alpha + 1} = -1$$

$$3(\beta - 3) = -4\alpha - 4$$

$$3\beta - 9 = -4\alpha - 4$$

$$4\alpha + 3\beta = 5 \quad \text{--- (1)}$$

α and β should lie on $3x - 4y = 16$

$$\therefore 3\alpha - 4\beta = 16 \quad \text{--- (2)}$$

From (1) and (2)

$$\alpha = \left(\frac{68}{25}\right) \quad \beta = \left(\frac{-49}{25}\right)$$

$$\therefore Q \text{ is } \left(\frac{68}{25}, \frac{-49}{25}\right)$$



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Q22

Let AB be the line, $A = (-1, 2), B = (5, 4)$

Then, equation of line AB is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{4 - 2}{5 + 1}(x + 1)$$

$$y - 2 = \frac{2}{6}(x + 1)$$

$$3y - x = 7 \quad \text{--- (1)}$$

$$\text{Slope} = \frac{1}{3}.$$

Let P point $(1, 0)$ be the given point

Let $Q(x_1, y_1)$ be the projection of P

$$\text{Slope of } PQ = -3 \quad [PQ \perp AB, m_1 m_2 = -1]$$

Eq of PQ ,

$$y - 0 = -3(x - 1)$$

$$y = -3x + 3 \quad \text{--- (2)}$$

Solving (1) and (2)

$$3y - \left(\frac{y - 3}{-3}\right) = 7$$

$$-9y - y + 3 = -21$$

$$-10y = -24$$

$$y = \frac{12}{5}$$

$$\Rightarrow \frac{12}{5} = -3x + 3$$

$$-3x = +\frac{12}{5} - 3 = \frac{+12 - 15}{5} = \frac{-3}{5}$$

$$x = \frac{1}{5}$$

$$\therefore N\left(\frac{1}{5}, \frac{12}{5}\right)$$

Q23

Any line perpendicular to line $\sqrt{3}x - y + 5 = 0$

Will have the slope $\frac{-1}{m}$

Where,

$$m \Rightarrow y = mx + c$$

$$y = \sqrt{3}x + 5$$

$$m = \sqrt{3}$$

Point is $(x_1, y_1) = (3, 3)$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 3 = \frac{-1}{\sqrt{3}}(x - 3)$$

$$x + \sqrt{3}y + 6 = 0$$

Point can be $(-3, -3)$

Then, equation is

$$x + \sqrt{3}y - 6$$

$$\therefore x + \sqrt{3}y \pm 6$$



Q24

The line $2x + 3y = 12$ meets the x-axis at A and y-axis at B

$$\Rightarrow A \text{ is } 2x = 12 \Rightarrow x = 6$$

$$\therefore A \text{ is } (6, 0)$$

$$\Rightarrow B \text{ is } 3y = 12$$

$$y = 4$$

$$\therefore B \text{ is } (0, 4)$$

Line through $(5, 5)$ perpendicular to $2x + 3y = 12$ will have slope $= \frac{3}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{3}{2}(x - 5)$$

$2y - 3x = -5$ is eq of line which meets x-axis at C and the line at E

$$\therefore C \text{ is } -3x = -5$$

$$x = \frac{-5}{-3}$$

$$\therefore E \text{ is } \left(\frac{5}{3}, 0\right)$$

$E \Rightarrow$ point of intersection of two lines

$$2x + 3y = 12$$

$$2y - 3x = -5$$

The area of $OBCE$ = area of AOB - area of ACE

$$\Rightarrow \frac{1}{2} \times AO \times OB - \frac{1}{2} \times AC \times CE$$

$$\Rightarrow \frac{24}{2} - \frac{1}{2} \times \sqrt{13} \times \frac{2}{3} \sqrt{13}$$

$$\Rightarrow \frac{24}{2} - \frac{1}{2} \times \frac{2}{3} \times 13$$

$$\Rightarrow 12 - \frac{13}{3}$$

$$\Rightarrow \frac{23}{3} \text{ sq units}$$

**Q25**

The equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

Intercept on y-axis = $2a$. (given)

$\therefore$ equation is

$$\frac{x}{2a} + \frac{y}{a} = 1$$

$$ax + 2ay = 2a^2 \quad \text{--- (1)}$$

Now, perpendicular distance of (1) from origin is given unity

$$\Rightarrow \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = 1$$

$$a = a, b = 2a, c = -2a^2, x_1 = 0, y_1 = 0$$

$$= \frac{|a(0) + 2a(0) - 2a^2|}{\sqrt{(2a)^2 + (a)^2}} = 1$$

$$\Rightarrow -2a^2 = \sqrt{5}a$$

$$\Rightarrow 4a^4 = a^2 5$$

$$a^2 = \frac{5}{4} \Rightarrow a = \pm \frac{\sqrt{5}}{2}$$

$\therefore$ the intercept form of straight line is

$$\frac{x}{2a} + \frac{y}{a} = 1$$

$$\frac{x}{\pm \frac{2\sqrt{5}}{2}} + \frac{y}{\pm \frac{\sqrt{5}}{2}} = 1$$

$$x + 2y = \pm\sqrt{5}$$

$$x + 2y \pm \sqrt{5} = 0$$

Q26

Let (x_1, y_1) and (x_2, y_2) be the coordinates of B and C .

Perpendicular bisector of AB is $x - y + 5 = 0$

Its slope = 1

$$\text{Coordinates of } F = \left(\frac{x_1 + 1}{2}, \frac{y_1 - 2}{2} \right)$$

F lies on the $x - y + 5 = 0$

$$\Rightarrow \frac{x_1 + 1}{2} - \frac{y_1 - 2}{2} + 5 = 0$$

$$\Rightarrow x_1 + 1 - y_1 + 2 + 10 = 0$$

$$x_1 - y_1 + 13 = 0 \quad \text{--- (1)}$$

AB is perpendicular to HF

$$(\text{Slope of } AB)(\text{Slope of } HF) = -1$$

$$\left(\frac{y_1 + 2}{x_1 - 1} \right)(1) = -1$$

$$x_1 + y_1 + 1 = 0 \quad \text{--- (2)}$$

Solving equation (1) and (2),

$$x_1 = -7, y_1 = 6$$

Thus, B is $(-7, 6)$

Now, perpendicular bisector of AC is

$$x + 2y = 0$$

$$\text{Slope of this is } = -\frac{1}{2}$$

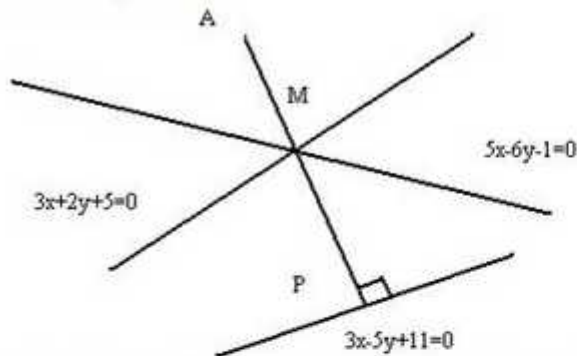
$$\text{Mid-point of } ACE = \left(\frac{x_2 + 1}{2}, \frac{y_2 - 2}{2} \right)$$

E lies on perpendicular bisector of AC

$$\Rightarrow \left(\frac{x_2 + 1}{2} \right) + 2 \left(\frac{y_2 - 2}{2} \right) = 0$$

Q27

Let M be the point of intersection of the lines $5x-6y-1=0$ and $3x+2y+5=0$.



Solving the equations $5x-6y-1=0$ and $3x+2y+5=0$, we get the point of intersection as M $(-1, -1)$.

$$3x-5y+11=0$$

$$\text{Also, } \Rightarrow 5y = 3x+11$$

$$\Rightarrow y = \frac{3}{5}x + \frac{11}{5}$$

Therefore, slope = $3/5$, Slope of AP = $-5/3$

Equation of AP, $y-y_1=m(x-x_1)$

$$y+1 = -\frac{5}{3}(x+1)$$

$$3y+3 = -5x-5$$

$$5x+3y+8=0$$

Therefore equation of the line AP, $5x+3y+8=0$

Ex 23.13

Q1(i)

Writing the equation in the form

$$y = mx + c$$

$$3x + y + 12 = 0$$

$$y = -3x - 12$$

$$\Rightarrow m_1 = -3$$

Also

$$x + 2y - 1 = 0$$

$$2y = 1 - x$$

$$y = \frac{1}{2} - \frac{x}{2}$$

$$\Rightarrow m_2 = \frac{-1}{2}$$

Angle between the lines

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{-3 - \left(\frac{-1}{2}\right)}{1 + (-3)\left(\frac{-1}{2}\right)} \right|$$

$$= \left| \frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}} \right| = \left| \frac{\frac{-6 + 1}{2}}{\frac{2 + 3}{2}} \right|$$

$$= \left| \frac{-5}{5} \right| = 1$$

$$\Rightarrow \text{angle} = \frac{\pi}{4}$$

Q1(ii)

Finding slopes of the lines by converting the equation in the form

$$\begin{aligned} y &= mx + c \\ 3x - y + 5 &= 0 \\ \Rightarrow y &= 3x + 5 \\ \Rightarrow m_1 &= 3 \end{aligned}$$

Also

$$\begin{aligned} x - 3y + 1 &= 0 \\ 3y &= x + 1 \\ y &= \frac{x}{3} + \frac{1}{3} \\ \Rightarrow m_2 &= \frac{1}{3} \end{aligned}$$

Thus angle between the lines is

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{3 - \frac{1}{3}}{1 + 3 \times \frac{1}{3}} \right| = \left| \frac{\frac{9-1}{3}}{1+1} \right| \\ &= \left| \frac{\frac{8}{3}}{2} \right| = \left| \frac{8}{6} \right| = \frac{4}{3} \\ \Rightarrow \theta &= \tan^{-1} \left(\frac{4}{3} \right) \end{aligned}$$



Q1(iii)

To find angle between the lines, convert the equations in the form

$$y = mx + c$$

$$3x + 4y - 7 = 0$$

$$\Rightarrow 4y = -3x + 7$$

$$y = \frac{-3}{4}x + \frac{7}{4}$$

$$\Rightarrow m_1 = \frac{-3}{4}$$

$$\text{Also, } 4x - 3y + 5 = 0$$

$$\Rightarrow 3y = 4x + 5$$

$$\Rightarrow y = \frac{4}{3}x + \frac{5}{3}$$

$$\Rightarrow m_2 = \frac{4}{3}$$

The angle between the lines is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$= \left| \frac{\frac{-3}{4} - \frac{4}{3}}{1 + \left(\frac{-3}{4}\right)\left(\frac{4}{3}\right)} \right| = \left| \frac{\frac{-3}{4} - \frac{4}{3}}{1 - 1} \right|$$

$$\Rightarrow \theta = \frac{\pi}{2} \text{ or } 90^\circ$$



Q1(iv)

To find angle convert the equation in the form $y = mx + c$

$$x - 4y = 3$$

$$\Rightarrow 4y = x - 3$$

$$y = \frac{x}{4} - \frac{3}{4}$$

$$\Rightarrow m_1 = \frac{1}{4}$$

Also, $6x - y = 11$

$$y = 6x - 11$$

$$\Rightarrow m_2 = 6$$

Thus, angle between the lines is

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{1}{4} - 6}{1 + \frac{1}{4} \times 6} \right|$$

$$= \left| \frac{-\frac{23}{4}}{1 + \frac{3}{2}} \right| = \left| \frac{-\frac{23}{4}}{\frac{5}{2}} \right|$$

$$\theta = \tan^{-1} \left(\frac{23}{10} \right)$$



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Q1(v)

Converting the equation in the form

$$y = mx + c$$

$$y = \frac{(mn + n^2)}{m^2 - mn}x + \frac{n^3}{(m^2 - mn)}$$

$$\Rightarrow m_1 = \frac{mn + n^2}{m^2 - mn}$$

$$\text{Also, } y = \frac{(mn - n^2)}{nm + m^2}x + \frac{m^3}{nm + m^2}$$

$$\Rightarrow m_2 = \frac{mn - n^2}{nm + m^2}$$

Thus, angle between 2 lines is $\tan \theta$

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\left(\frac{mn + n^2}{m^2 - mn} \right) - \left(\frac{mn - n^2}{nm + m^2} \right)}{1 + \left(\frac{mn + n^2}{m^2 - mn} \right) \left(\frac{mn - n^2}{nm + m^2} \right)} \right|$$

$$= \left| \frac{m^2 n^2 + m^3 n + n^3 m + n^2 m^2 - m^3 n + m^2 n^2 + n^2 m^2 - mn^3}{m^3 n + m^4 - m^2 n^2 - m^3 n + m^2 n^2 - mn^3 + mn^3 - n^4} \right|$$

$$= \left| \frac{4m^2 n^2}{m^4 - n^4} \right|$$

$$\Rightarrow \theta = \tan^{-1} \left| \frac{4m^2 n^2}{m^4 - n^4} \right|$$

Q2

Slope of line $2x - y + 3 = 0$

$$\text{is } \frac{-2}{-1} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } y)} = 2$$

$$\therefore m_1 = 2 \quad \text{---(i)}$$

Slope of line $x + y + 2 = 0$

$$\text{is } \frac{-1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$$

$$\therefore m_2 = -1 \quad \text{---(ii)}$$

Acute angle between lines

$$\begin{aligned} \theta &= \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \tan^{-1} \left| \frac{2 - (-1)}{1 - (2)(-1)} \right| \\ &= \tan^{-1} \left| \frac{3}{1 - 2} \right| = \tan^{-1} \left| \frac{3}{1} \right| = \tan^{-1} |3| \end{aligned}$$

Q3

Let $ABCD$ be a quadrilateral

$$AB = \sqrt{(0 - 2)^2 + (2 + 1)^2}$$

Using distance formula

$$\begin{aligned} &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \end{aligned}$$

$$BC = \sqrt{(2 - 0)^2 + (3 - 2)^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$CD = \sqrt{(4 - 2)^2 + (0 - 3)^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$DA = \sqrt{(4 - 2)^2 + (0 + 1)^2} = \sqrt{4 + 1} = \sqrt{5}$$

Since opposite sides (AB and CD) and (BC and DA) are equal

$\therefore$ The given quadrilateral is a parallelogram.

Q4

The equation between the points

$$\begin{matrix} (2, 0) & \text{and} & (0, 3) \\ (x_1, y_1) & & (x_2, y_2) \end{matrix}$$

$$\text{Slope of line} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{3 - 0}{0 - 2} = \frac{-3}{2}$$

Also, slope of line $x + y = 1$

Converting in the form $y = mx + c$

$$y = 1 - x$$

$$\Rightarrow m_2 = -1$$

Thus, $\tan \theta = \text{angle between the lines}$

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{-3}{2} - (-1)}{1 + \left(\frac{-3}{2}\right)(-1)} \right| = \left| \frac{\frac{-3}{2} + 1}{1 + \frac{3}{2}} \right|$$

$$= \left| \frac{\frac{-3+2}{2}}{\frac{2+3}{2}} \right| = \left| \frac{\frac{-1}{2}}{\frac{5}{2}} \right| = \frac{1}{5}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{5} \right)$$



Q5

Let l_1 be the line joining AO and

Let l_2 be the line joining BO

Then, line l_1 is $y - 0 = \left(\frac{0 - x_1}{0 - y_1} \right) (x - 0)$

$$yy_1 = x_1x = 0$$

Then, $m_1 = \frac{x_1}{y_1}$

Then line l_2 is $y - 0 = \left(\frac{0 - x_2}{0 - y_2} \right) (x - 0)$

Then, $m_2 = \frac{x_2}{y_2}$

$$\begin{aligned} \text{Then, } \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\frac{x_1}{y_1} - \frac{x_2}{y_2}}{1 + \frac{x_1 x_2}{y_1 y_2}} \right| \\ &= \left| \frac{x_1 y_2 - y_1 x_2}{y_1 y_2 + x_1 x_2} \right| \end{aligned}$$

From triangle,

$$\begin{aligned} AC &= \sqrt{(AB)^2 + (BC)^2} \\ &= \sqrt{(m_1^2 + m_2^2 - 2m_1 m_2) + (1 + m_1 m_2)^2} \\ &= \sqrt{m_1^2 + m_2^2 - 2m_1 m_2 + 1 + m_1^2 m_2^2 + 2m_1 m_2} \\ &= \sqrt{m_1^2 + m_2^2 + 1 + m_1^2 m_2^2} \end{aligned}$$

$$\cos \theta = \frac{BC}{AC} = \frac{1 + m_1 m_2}{\sqrt{m_1^2 + m_2^2 + m_1^2 m_2^2 + 1}}$$

$$\begin{aligned} &= \frac{1 + \frac{x_1 x_2}{y_1 y_2}}{\sqrt{\frac{x_1^2}{y_1^2} + \frac{x_2^2}{y_2^2} + \frac{x_1^2 x_2^2}{y_1^2 y_2^2} + 1}} \\ &= \frac{\frac{y_1 y_2 + x_1 x_2}{y_1 y_2}}{\sqrt{\frac{x_1^2 y_2^2 + x_2^2 y_1^2 + x_1^2 x_2^2 + y_1^2 y_2^2}{y_1^2 y_2^2}}} \\ &= \frac{y_1 y_2 + x_1 x_2}{\sqrt{x_1^2 (y_2^2 + x_2^2) + y_1^2 (y_2^2 + x_2^2)}} \\ &= \frac{y_1 y_2 + x_1 x_2}{\sqrt{y_1^2 + y_1^2} \sqrt{y_2^2 + x_2^2}} \end{aligned}$$

Hence proved.

Q6

$$(a+b)x + (a-b)y = 2ab \quad \text{--- (i)}$$

$$(a-b)x + (a+b)y = 2ab \quad \text{--- (ii)}$$

$$x + y = 0 \quad \text{--- (iii)}$$

Converting all the equation in the form

$$y = mx + c$$

$$y = \frac{-(a+b)x}{a-b} + \frac{2ab}{a+b}$$

$$\Rightarrow m_1 = \frac{-(a+b)}{a-b}$$

$$y = \frac{-(a-b)x}{a+b} + \frac{2ab}{a+b}$$

$$\Rightarrow m_2 = \frac{-(a-b)}{a+b}$$

$$y = -x$$

$$\Rightarrow m_3 = -1$$

Thus angle between (i) and (ii)

$$\tan \theta_1 = \frac{|m_1 - m_2|}{1 + m_1 m_2}$$

$$= \frac{\left| -\left(\frac{a+b}{a-b}\right) + \left(\frac{a-b}{a+b}\right) \right|}{1 + \left(\frac{a+b}{a-b} \times \frac{a-b}{a+b}\right)}$$

$$= \frac{2ab}{b^2 - a^2}$$

$$= \frac{2\frac{a}{b}}{1 - \left(\frac{a}{b}\right)^2}$$

$$\tan \theta_1 = \tan \left(2 \tan^{-1} \left(\frac{a}{b} \right) \right)$$

Q7

$$x = a$$

$$\Rightarrow m_1 = \frac{1}{0}$$

$$by + c = 0$$

$$y = \frac{-c}{b}$$

$$m_2 = 0$$

Comparing with $y = mx + c$

Then, putting in

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{1}{0} - 0}{1 + \frac{1}{0} \times 0} \right|$$

$$= \frac{1}{0} = \infty$$

$$\Rightarrow \theta = 90^\circ$$

Q8

$$\text{Line}_1 \text{ is } \frac{x}{3} + \frac{y}{4} = 1$$

$$\text{i.e. } 4x + 3y = 12$$

$$\text{Line}_2 \text{ is } \frac{x}{1} + \frac{y}{8} = 1$$

$$\text{i.e. } 8x + y = 8$$

Slope of line₁ and line₂ is $\frac{-4}{3}$ and $\frac{-8}{1}$ respectively.

$$\text{Thus, } \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{-4}{3} - (-8)}{1 + \left(\frac{-4}{3}\right)(-8)} \right|$$

$$= \left| \frac{\frac{-4}{3} + 8}{1 + \frac{32}{3}} \right| = \left| \frac{-4 + 24}{3 + 32} \right|$$

$$= \left| \frac{20}{35} \right| = \frac{4}{7}$$

$$\text{Thus, } \tan \theta = \frac{4}{7}$$

Q9

Slope of line through the points $(a, 2a)$, $(-2, 3)$
 (x_1, y_1) (x_2, y_2)

$$\Rightarrow m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2a}{-2 - a}$$

Also, slope of line $x - ay = 1$ in the form $y = mx + c$

$$4x + 3y + 5 = 0$$

$$y = \frac{-4}{3}x - \frac{5}{3}$$

$$\Rightarrow m_2 = \frac{-4}{3}$$

If two lines are perpendicular then, $m_1 m_2 = -1$

$$\left(\frac{3 - 2a}{-2 - a} \right) \left(\frac{-4}{3} \right) = -1$$

$$-12 + 8a = 6 + 3a$$

$$5a = 18$$

$$a = \frac{18}{5}$$

Q10

$$a^2x + ay + 1 = 0$$

$$x - ay = 1$$

Converting these two equations in the form $y = mx + c$

$$y = -\frac{a^2}{a}x - \frac{1}{a} = -ax - \frac{1}{a}$$

$$\Rightarrow m_1 = -a$$

Also, $y = \frac{x}{a} - \frac{1}{a}$

$$\Rightarrow m_2 = \frac{1}{a}$$

Thus, $m_1 m_2 = -a \times \frac{1}{a} = -1$

The two lines are perpendicular as the product of slopes is -1 .



Q11

Let the line $\frac{x}{a} + \frac{y}{b} = 1$ be AB and the line $\frac{x}{a} - \frac{y}{b} = 1$ be CD.

Equation of AB, $\frac{bx+ay}{ab} = 1$

$$\Rightarrow ay = -bx + ab$$

$$\Rightarrow y = -\frac{bx}{a} + b$$

$$\text{Therefore } m_1 = -\frac{b}{a}$$

Similarly, the equation of CD, $\frac{bx-ay}{ab} = 1$

$$\Rightarrow bx - ay = ab$$

$$\Rightarrow ay = \frac{bx}{a} - a$$

$$\text{Therefore, } m_2 = \frac{b}{a}$$

The tangent of angle between the lines AB and CD is

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)} \right| = \left| \frac{-\frac{2b}{a}}{\frac{a^2 - b^2}{a^2}} \right| = \left| \frac{-2ab}{a^2 - b^2} \right| = \frac{2ab}{a^2 - b^2}$$

$$\text{The tangent of the angle between the lines} = \frac{2ab}{a^2 - b^2}$$



Ex 23.14

Q1

Let ABC be the triangle of the equations whose sides AB , BC and CA are respectively $x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$

The coordinates of the vertices are $A(9, 3)$, $B(4, 2)$ and $C(13, 5)$.

If the point $P(\alpha, \alpha^2)$ lies in side the $\triangle ABC$, then

- (i) A and P must be on the same side of BC .
- (ii) B and P must be on the same side of AC .
- (iii) C and P must be on the same side of AB .

Now,

A and P are on the same side of BC if,

$$\begin{aligned} & \{9(1) + 3(-3) + 2\} \{\alpha^2 - 3\alpha + 2\} > 0 \\ & \{9 - 9 + 2\} \{\alpha^2 - 3\alpha + 2\} > 0 \\ & \alpha^2 - 3\alpha + 2 > 0 \\ & (\alpha - 1)(\alpha - 2) > 0 \\ & \alpha \in (-\infty, 1) \cup (2, \infty) \quad \text{---(i)} \end{aligned}$$

B and P will lie on the same side of CA if,

$$\begin{aligned} & \{13(1) + 5(-5) + 6\} \{\alpha^2 - 5\alpha + 6\} > 0 \\ \Rightarrow & (-6) \{\alpha^2 - 5\alpha + 6\} > 0 \\ \Rightarrow & \alpha^2 - 5\alpha + 6 < 0 \\ \Rightarrow & (\alpha - 2)(\alpha - 3) < 0 \\ \Rightarrow & \alpha \in (2, 3) \quad \text{---(ii)} \end{aligned}$$

C and P will lie on the same side of AB if,

$$\begin{aligned} & \{4(1) + 2(-2) - 3\} \{\alpha^2 - 2\alpha - 3\} > 0 \\ & (-3) \{\alpha^2 - 2\alpha - 3\} > 0 \\ & \alpha^2 - 2\alpha - 3 < 0 \\ & (\alpha - 3)(\alpha + 1) < 0 \\ & \alpha \in (-1, 3) \quad \text{---(iii)} \end{aligned}$$

From i, ii, iii

$$\alpha \in [2, 3]$$

Q2

Let ABC be the triangle. The coordinates of the vertices of the triangle ABC are marked in the following figure.

Point $P(a, 2)$ lie inside or on the triangle if,

- (i) A and P lie on the same side of BC .
- (ii) B and P lie on the same side of AC .
- (iii) C and P lie on the same side of AB .

A and P will lie on the same side of BC if,

$$\begin{aligned} (7(3) - 7(-3) - 6)(3a - 7(2) - 0) &> 0 \\ (21 + 21 - 6)(3a - 14 - 0) &> 0 \\ 3a - 22 &> 0 \\ a &> \frac{22}{3} \end{aligned} \quad \text{---(i)}$$

B and P will lie on the same side of AC if,

$$\begin{aligned} \left(4\left(\frac{18}{5}\right) - \left(\frac{2}{5}\right) - 31\right)(4a - 2 - 31) &> 0 \\ 4a - 33 &> 0 \\ a &> \frac{33}{4} \end{aligned} \quad \text{---(ii)}$$

C and P will lie on the same side of BC if,

$$\begin{aligned} \left(\frac{209}{25} + \frac{61}{25} - 4\right)(a + 2 - 4) &> 0 \\ a + 2 &> 0 \\ a &> -2 \end{aligned} \quad \text{---(iii)}$$

From (i), (ii), (iii)

$$a \in \left(\frac{22}{3}, \frac{33}{4}\right)$$

Q3

Let ABC be the triangle, then coordinates of the vertices are marked in the following figure.

$P(-3, 2)$ lie inside if,

- (i) A and P , B and P , C and P lie on the same side of BC , AC and BA respectively.

If A and P lie on the same side of BC then,

$$\begin{aligned} (3(7) - 7(-3) + 8)(3(-3) - 7(2) + 8) &> 0 \\ (21 + 21 + 8)(-9 - 14 + 8) &> 0 \end{aligned}$$

But, $(50)(-15)$ is not > 0

$\therefore$ The point $(-3, 2)$ is outside ABC .



Ex 23.15

Q1

Distance of a point (x_1, y_1) from $ax + by + c = 0$ is

$$= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Here, $a = 3$, $b = -5$, $c = 7$, $x_1 = 4$, $y_1 = 5$

$$\begin{aligned} \therefore \text{Distance} &= \left| \frac{3(4) - 5(5) + 7}{\sqrt{3^2 + 5^2}} \right| \\ &= \left| \frac{12 - 25 + 7}{\sqrt{9 + 25}} \right| = \left| \frac{6}{\sqrt{34}} \right| \text{ units.} \end{aligned}$$

Q2

Equation of line passing through $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is

$$y - \sin \phi = \left(\frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} \right) (x - \cos \phi)$$

$$y - \sin \phi = \left(\frac{2 \cos \frac{\theta + \phi}{2} \sin \frac{\phi - \theta}{2}}{-2 \sin \frac{\theta + \phi}{2} \sin \frac{\phi - \theta}{2}} \right) (x - \cos \phi)$$

$$y - \sin \phi = -\cot \left(\frac{\theta + \phi}{2} \right) (x - \cos \phi)$$

$$x \cot \left(\frac{\theta + \phi}{2} \right) + y - \sin \phi - \cos \phi \cot \left(\frac{\theta + \phi}{2} \right) = 0$$

Distance of this line from origin,

$$\begin{aligned} &= \left| \frac{ax_1 + by_1 + c}{a^2 + b^2} \right| \\ &= \left| \frac{0 + 0 - \sin \phi - \cos \phi \cot \left(\frac{\theta + \phi}{2} \right)}{\sqrt{\left(\cos \left(\frac{\theta + \phi}{2} \right) \right)^2 + 1}} \right| \\ &= \frac{\sin \phi + \cos \phi \cot \left(\frac{\theta + \phi}{2} \right)}{\operatorname{cosec} \left(\frac{\theta + \phi}{2} \right)} \\ &= \sin \phi \sin \left(\frac{\theta + \phi}{2} \right) + \cos \phi \cos \left(\frac{\theta + \phi}{2} \right) \\ &= \cos \left(\frac{\theta + \phi}{2} - \phi \right) \\ &= \cos \left(\frac{\theta + \phi - 2\phi}{2} \right) \\ D &= \cos \left(\frac{\theta - \phi}{2} \right) \end{aligned}$$



Q3

Line formed from joining $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$

$$\Rightarrow y - a \sin \beta = \frac{a \sin \beta - a \sin \alpha}{a \cos \beta - a \cos \alpha} \times x - a \cos \beta$$

$$\Rightarrow y - a \sin \beta = \frac{2 \sin \left(\frac{\beta - \alpha}{2} \right) \cos \left(\frac{\beta + \alpha}{2} \right)}{-2 \sin \left(\frac{\beta - \alpha}{2} \right) \sin \left(\frac{\beta + \alpha}{2} \right)} \times (x - a \cos \beta)$$

$$\Rightarrow y - a \sin \beta = -\cot \left(\frac{\beta + \alpha}{2} \right) (x - a \cos \beta)$$

$$\Rightarrow y + \cot \left(\frac{\alpha + \beta}{2} \right) x - a \cos \beta \cot \left(\frac{\beta + \alpha}{2} \right) - a \sin \beta = 0$$

Then, the length of perpendicular

$$\Rightarrow \frac{|a(y) + a - a \cos \beta \cot \left(\frac{\beta + \alpha}{2} \right) - a \sin \beta|}{\sqrt{1 + \cot^2 \left(\frac{\alpha + \beta}{2} \right)}}$$

$$\Rightarrow \frac{a \cos \beta \cot \left(\frac{\alpha + \beta}{2} \right) + a \sin \beta}{\operatorname{cosec} \left(\frac{\alpha + \beta}{2} \right)}$$

$$\Rightarrow a \cos \beta \cos \left(\frac{\alpha + \beta}{2} \right) + a \sin \beta \sin \left(\frac{\alpha + \beta}{2} \right)$$

$$\Rightarrow a \cos \left(\frac{\alpha - \beta}{2} \right) \quad \left[\text{using } \cos A \cos B + \sin A \sin B = \cos(A - B) \right]$$

Hence, proved.

Q4

Let (h, k) be the point on the line $2x + 11y - 5 = 0$

$$\Rightarrow 2h + 11k - 5 = 0 \text{ ----- (1)}$$

Let p and q be length of perpendicular from (h, k) on lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ so,

$$\begin{aligned} p &= q \\ \frac{24h + 7k - 20}{\sqrt{(24)^2 + (7)^2}} &= \frac{4h - 3k - 2}{\sqrt{(4)^2 + (-3)^2}} \\ \frac{24h + 7k - 20}{\sqrt{576 + 49}} &= \frac{4h - 3k - 2}{\sqrt{25}} \\ \frac{24h + 7k - 20}{25} &= \frac{4h - 3k - 2}{5} \\ 24h + 7k - 20 &= 20h - 15k - 10 \\ 4h &= -22k + 10 \\ 4 \left(\frac{5 - 11k}{2} \right) &= -22k + 10 \quad \left[\text{Using equation (1)} \right] \\ 10 - 22k &= -22k + 10 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

So,

Distance $24x + 7y - 20$ and $4x - 3y - 2 = 0$ from any point on the line $2x + 11y - 5 = 0$ is equal.

Q5

The point of intersection of two lines can be calculated by solving the equations

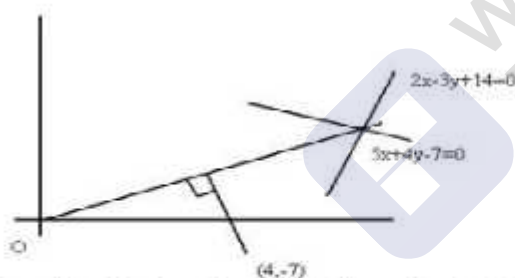
Solving $2x + 3y = 21$ and $3x - 4y + 11 = 0$, we get the point of intersection as $P(3, -5)$

Distance of P from $8x - 6y + 5 = 0$ is

Here, $a = 8$, $b = -6$, $c = 5$, $x_1 = 3$, $y_1 = -5$

$$\begin{aligned} & \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ \Rightarrow & \frac{|8(3) - 6(-5) + 5|}{\sqrt{64 + 36}} \\ \Rightarrow & \frac{|24 + 30 + 5|}{\sqrt{100}} = \frac{|59|}{10} \\ \Rightarrow & \frac{59}{10} \end{aligned}$$

Q6



The point of intersection of the lines $2x - 3y + 14 = 0$ and $5x + 4y - 7 = 0$ can be found out by solving these equations.

Solving these equations we get, $x = -\frac{35}{23}$ and $y = \frac{252}{69}$

Equation of line joining origin and the point $\left(-\frac{35}{23}, \frac{252}{69}\right)$

is $y = mx$, where $m = \frac{\frac{252}{69}}{-\frac{35}{23}} = -\frac{12}{5}$

Therefore the equation of required line is $y = -\frac{12x}{5}$

$12x + 5y = 0$

Perpendicular distance from $(4, -7)$ to $12x + 5y = 0$ is

$$p = \frac{|12(4) + 5(-7)|}{\sqrt{12^2 + (-5)^2}} = \frac{13}{13} = 1$$

Q7

Any point on x-axis is $(\pm a, 0)$
 (x_1, y_1)

Perpendicular distance from a line $bx + ay = ab$ is

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = a$$

Where,

$$a = b, \quad b = a, \quad c = -ab, \quad x_1 = \pm a, \quad y_1 = 0$$

$$= \left| \frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} \right| = a$$

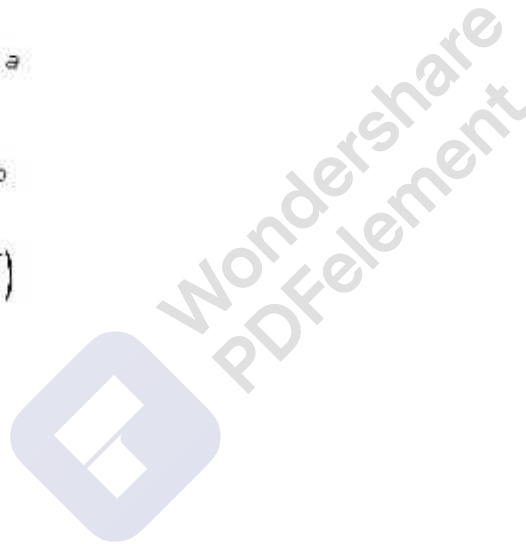
$$a = 0 \quad \text{or}$$

$$\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} = a$$

$$\frac{b}{a}x = \pm \sqrt{a^2 + b^2} + b$$

$$x = \frac{a}{b} \left(b \pm \sqrt{a^2 + b^2} \right)$$

$$x = 0$$



Q8

Perpendicular distance from $(\sqrt{a^2 - b^2}, 0)$ to $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta - 1 = 0$

$$\begin{aligned} & \left| \frac{\frac{\sqrt{a^2 - b^2}}{a} \cos \theta + \frac{0x}{b} \sin \theta - 1}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}} \right| \\ &= \frac{\frac{\sqrt{a^2 - b^2}}{a} \cos \theta - 1}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}} \quad \text{---(i)} \end{aligned}$$

Also, perpendicular distance from $(-\sqrt{a^2 - b^2}, 0)$ to $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta - 1 = 0$

$$\left| \frac{\frac{-\sqrt{a^2 - b^2}}{a} \cos \theta + 0 - 1}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}} \right| \quad \text{---(ii)}$$

(i) $\times$ (ii)

$$\frac{\left(\frac{a^2 - b^2}{a^2} \right) \cos^2 \theta - 1}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}} = b^2$$

Q9

The perpendicular of $(1, 2)$ on the straight line $x - \sqrt{3}y = -4$
Then, the equation is

$$\begin{aligned} y - y_1 &= m'(x - x_1) \\ x_1 = 1, y_1 = 2, m &= \frac{1}{\sqrt{3}}, m' = -\sqrt{3}, \\ y - 2 &= -\sqrt{3}(x - 1) \\ y + \sqrt{3}x &= (2 + \sqrt{3}) = 0 \quad \text{---(i)} \end{aligned}$$

The perpendicular distance from $(0, 0)$ to (i) is

$$\begin{aligned} & \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ & a = \sqrt{3}, b = 1, c = -(2 + \sqrt{3}) \\ & x_1 = 0, y_1 = 0 \\ &= \frac{|\sqrt{3}(0) + 1(0) + (-2 - \sqrt{3})|}{\sqrt{(\sqrt{3})^2 + (1)^2}} = \frac{2 + \sqrt{3}}{2} \end{aligned}$$

Q10

On solving $x + 2y = 5$ and $x - 3y = 7$ we get a point $A\left(\frac{29}{5}, -\frac{2}{5}\right)$

The line passing through $A\left(\frac{29}{5}, -\frac{2}{5}\right)$ and slope 5 is

$$y + \frac{2}{5} = 5\left(x - \frac{29}{5}\right)$$

$$5y + 2 = 25x - 145$$

$$25x - 5y - 147 = 0$$

The distance of $(1, 2)$ from $25x - 5y - 147 = 0$ is

$$\Rightarrow \frac{|25(1) - 5(2) - 147|}{\sqrt{25^2 + 5^2}} \quad [\text{using distance formula}]$$

$$\Rightarrow \frac{|-132|}{\sqrt{650}}$$

$$\Rightarrow \frac{132}{\sqrt{650}}$$

Q11

Let the required point be $(0, a)$

Given, distance of $(0, a)$ from line $4x + 3y - 12 = 0$ is 4 units.

$$L = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$4 = \frac{|4(0) + 3(a) - 12|}{\sqrt{4^2 + 3^2}}$$

$$4 = \frac{|3a - 12|}{5}$$

$$\Rightarrow 4 = \frac{3a - 12}{5}$$

$$\Rightarrow -3a = 20 - 12$$

$$a = -\frac{8}{3}$$

And $4 = \frac{3a - 12}{5}$

$$3a = 20 + 12$$

$$\Rightarrow a = \frac{32}{3}$$

So, Required points are

$$\left(0, \frac{32}{3}\right), \left(0, -\frac{8}{3}\right)$$

Q12

The equation of BC is

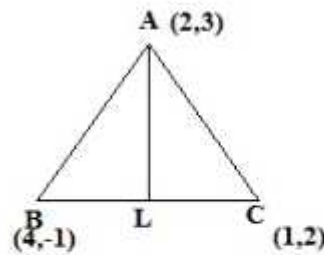
$$y+1 = \frac{2+1}{1-4}(x-4)$$

$$y+1 = -x+4$$

$$x+y-3=0$$

$$AL = \left| \frac{2+3-3}{\sqrt{1+1}} \right|$$

$$= \sqrt{2}$$



Clearly, slope of BC having equation $x+y-3=0$ is -1 .

So, slope of AL is 1. As it passes through $A(2,3)$ so, its equation is

$$y-3 = 1(x-2) \text{ or } x-y+1=0$$

Q13

Let $P(h,k)$ be a moving point such that it is equidistant from the lines $3x-2y-5=0$

and $3x+2y-5=0$, then

$$\left| \frac{3h-2k-5}{\sqrt{9+4}} \right| = \left| \frac{3h+2k-5}{\sqrt{9+4}} \right|$$

$$|3h-2k-5| = |3h+2k-5|$$

$$4k=0 \text{ or } 6h-10=0$$

$$k=0 \quad 3h=5$$

Hence, the locus of (h,k) is $y=0$ or $3x=5$, which are straight lines.

Q14

It is given that the sum of the perpendicular distances of a variable point

$P(x,y)$ from the lines $(x+y-5)=0$ and $3x-2y+7=0$ is always 10.

$$\text{Therefore, } \frac{x+y-5}{\sqrt{2}} + \frac{3x-2y+7}{\sqrt{13}} = 10$$

$$(3\sqrt{2} + \sqrt{13})x + (\sqrt{13} - 2\sqrt{2})y + (7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26}) = 0$$

Clearly, it is a straight line.

Q15

Length of perpendicular from $(1,1)$ to $ax - by + c = 0$

$$\Rightarrow \left| \frac{a(1) - b(1) + c}{\sqrt{a^2 + b^2}} \right| = 1$$

$$a - b + c = \sqrt{a^2 + b^2}$$

$$(a - b + c)^2 = a^2 + b^2$$

$$a^2 + b^2 + c^2 + 2ac - 2bc - 2ab = a^2 + b^2$$

$$c^2 + 2ac - 2bc = 2ab$$

$$c + 2a - 2b = \frac{2ab}{c}$$

$$\frac{c}{2ab} + \frac{2a}{2ab} - \frac{2b}{2ab} = \frac{1}{c}$$

$$\frac{c}{2ab} = \frac{1}{c} + \frac{1}{a} - \frac{1}{b}$$

Hence, proved.



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Ex 23.16

Q1

Determine between parallel lines

$ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

(i) $4x - 3y - 9 = 0$ and $4x - 3y - 24 = 0$

Distance between the two parallel lines is

$$\frac{|-24 - (-9)|}{\sqrt{4^2 + 3^2}} = \frac{|-24 + 9|}{5} \\ = 3 \text{ units.}$$

(ii) Distance between $8x + 15y - 34 = 0$ and $8x + 15y + 31 = 0$

$$\text{is } \frac{|-34 - 31|}{\sqrt{8^2 + 15^2}} = \frac{65}{17} \text{ units}$$

(iii) Distance between $y = mx + c$ and $y = mx + d$

$$\text{is } \frac{|c - d|}{\sqrt{m^2 + 1}}$$

(iv) Distance between $4x + 3y - 11 = 0$ and $8x + 6y = 15$

$$\text{is } \frac{|-11 - 15|}{\sqrt{4^2 + 3^2}} = \frac{7}{10} \text{ units.}$$

Q2

The two sides of square are

$$5x - 12y - 65 = 0 \text{ and } 5x - 12y + 26 = 0$$

The distance between these two parallel sides (as both have slope $\frac{5}{12}$) is

$$\frac{|-65 - 26|}{\sqrt{5^2 + 12^2}} = \frac{|-91|}{13} = 7 \text{ units.}$$

And all sides of square are equal.

∴ Area of the square is $7 \times 7 = 49$ sq units.

**Q3**

Let the required equation be $y = mx + c$ where m is slope of the line which is equal to slope of $x + 7y + 2 = 0$ (i.e. $-\frac{1}{7}$) as the two lines are parallel.

∴ The required equation is $y = -\frac{1}{7}x + c$ which is a unit distance from $(1, 1)$.

$$\therefore \left| \frac{7(1) + (1) - 7c}{\sqrt{49 + 1}} \right| = 1$$

$$8 - 7c = \sqrt{50}$$

$$64 + 49c^2 - 112c = 50$$

$$49c^2 - 112c - 14 = 0$$

$$7c^2 - 16c - 2 = 0$$

$$c = \frac{6 \pm 5\sqrt{2}}{7}$$

$$\left[\text{using } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

∴ The required equation is:

$$y = -\frac{1}{7}x + \frac{6 \pm 5\sqrt{2}}{7}$$

or $7y + x + 6 \pm 5\sqrt{2} = 0$

Q4

Since the coefficient of x and y in the equations $2x + 3y - 19 = 0$, $2x + 3y - 6 = 0$ and $2x + 3y + 7 = 0$ are same, therefore all the lines are parallel.

Distance between parallel lines is $d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$, where $ax + by + c_1 = 0$

and $ax + by + c_2 = 0$ are the lines parallel to each other.

Distance between the lines $2x + 3y - 19 = 0$ and $2x + 3y - 6 = 0$ is

$$d_1 = \left| \frac{-19 + 6}{\sqrt{2^2 + 3^2}} \right| = \left| \frac{13}{\sqrt{13}} \right| = \sqrt{13}$$

Distance between the lines $2x + 3y + 7 = 0$ and $2x + 3y - 6 = 0$ is

$$d_2 = \left| \frac{7 + 6}{\sqrt{2^2 + 3^2}} \right| = \left| \frac{13}{\sqrt{13}} \right| = \sqrt{13}$$

Since the distances of both the lines $2x + 3y + 7 = 0$ and $2x + 3y - 19 = 0$ from the line $2x + 3y - 6 = 0$ are equal, therefore the lines are equidistant.

Q5

The equation of lines are

$$3x + 2y - \frac{7}{3} = 0 \quad \text{---(i)}$$

$$3x + 2y + 6 = 0 \quad \text{---(ii)}$$

Let equation of mid way be $3x + 2y + \lambda = 0$ ---(iii)

Then, distance between (i) and (iii) and (i) and (ii) should be equal.

$$\left| \frac{\lambda + \frac{7}{3}}{\sqrt{9+4}} \right| = \left| \frac{\lambda + 6}{\sqrt{9+4}} \right|$$

$$\Rightarrow \lambda + \frac{7}{3} = -\lambda + 6$$

$$\Rightarrow \lambda = \frac{11}{6}$$

∴ The required line is $3x + 2y + \frac{11}{6} = 0$ or $18x + 12y + 11 = 0$.

Q6

Clearly, the slope of each of the given lines is same equal to $\frac{3}{4}$.
Hence, the line $3x + 4y + 2 = 0$ is parallel to each of the given lines.

Putting $y = 0$ in $3x + 4y + 2 = 0$, we get $x = -\frac{2}{3}$.

∴, the coordinates of a point on $3x + 4y + 2 = 0$ are $\left(-\frac{2}{3}, 0\right)$.

The distance d_1 between the lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is given by

$$d_1 = \frac{\left| 3\left(-\frac{2}{3}\right) - 4(0) + 5 \right|}{\sqrt{3^2 + 4^2}} = \frac{5}{5}$$

The distance d_2 between the lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is given by

$$d_2 = \frac{\left| 3\left(-\frac{2}{3}\right) + 4(0) - 5 \right|}{\sqrt{3^2 + 4^2}} = \frac{7}{5}$$

$$\frac{d_1}{d_2} = \frac{\frac{5}{5}}{\frac{7}{5}} = \frac{5}{7}$$

∴ $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ in the ratio 5:7.



Ex 23.17

Q1

Let $ABCD$ be a parallelogram the equation of whose sides AB , BC , CD and DA are $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1 = 0$ and $a_2x + b_2y + d_2 = 0$.

Let p_1 and p_2 be the distance between the pairs of parallel sides of $ABCD$

$$\sin \theta \frac{p_1}{AD} = \frac{p_2}{AB}$$

$$AD = \frac{p_1}{\sin \theta} \text{ and } AB = \frac{p_2}{\sin \theta}$$

$$\text{Area of } ABCD = AB \times p_1 = \frac{p_1 p_2}{\sin \theta}$$

$$\text{or } AD \times p_2 = \frac{p_1 p_2}{\sin \theta}$$

Now,

$$m_1 = \text{slope of } AB = -\frac{a_1}{b_1}$$

$$m_2 = \text{slope of } AD = -\frac{a_2}{b_2}$$

Since θ is angle between AB and AD ,

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{-\frac{a_2}{b_2} + \frac{a_1}{b_1}}{1 - \frac{a_1 a_2}{b_1 b_2}}$$

$$\tan \theta = \frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2} \Rightarrow \sin \theta = \frac{a_2 b_1 - a_1 b_2}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}}$$

p_1 = Distance between AB and CD

$$= \left| \frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}} \right|$$

p_2 = Distance between AD and BC .

$$= \left| \frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}} \right|$$

∴ Area of parallelogram is

$$\frac{|c_1 - d_1| |c_2 - d_2|}{|a_2 b_1 - a_1 b_2|} \quad \text{Hence, proved.}$$

(i) Rhombus is a parallelogram with all side equal.

$$\therefore p_1 = p_2$$

∴ Modifying the formula of area of parallelogram derived above.

The area of rhombus

$$\begin{aligned} &= \frac{p_1 p_2}{\sin \theta} \\ &= \frac{p_1^2}{\sin \theta} = \frac{2p_1^2}{\sin \theta} \\ &= 2 \left| \frac{(c_1 - d_1)}{a_2 b_1 - a_1 b_2} \right| \text{ or } 2 \left| \frac{(c_2 - d_2)}{a_2 b_1 - a_1 b_2} \right| \end{aligned}$$

Q2

The area of a parallelogram is

$$\begin{aligned}
 &= \frac{|c_1 - d_1| |c_2 - d_2|}{|a_2 b_1 - b_2 a_1|} \\
 &= \frac{|-a + 2a| |3a - a|}{|3(-3) - 4(-4)|} \\
 &= \frac{a \times 2a}{7} \\
 &= \frac{2}{7} a^2
 \end{aligned}$$

Hence, proved.

Q3

Let $ABCD$ be a parallelogram as shown in the following figure.

We observe that the following parallelogram is a rhombus, as distance between opposite sides (AB and CD) and (AD and BC) is equal = $(n - n')$.

And in a Rhombus, diagonals are perpendicular to each other.

$\therefore$ Angle between the two diagonals is $17/2$.





Ex 23.18

Q1

Let the required equation be $ax + by = c$ but here it passes through origin $(0, 0)$.

$$\therefore c = 0$$

$$\therefore \text{Equation is } ax + by = 0$$

$$\text{Slope of the line } (m_1) = \frac{-a}{b} \text{ and } m_2 = \frac{-\sqrt{3}}{1}$$

$$\Rightarrow \text{Angle between } \sqrt{3}x + y = 11 \text{ and } ax + by = 0 \text{ is } 45^\circ$$

$$\therefore \tan 45^\circ = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$$

$$1 = \frac{\frac{-a}{b} \pm (-\sqrt{3})}{1 \mp \frac{a}{b} \times \sqrt{3}}$$

$$1 - \frac{\sqrt{3}a}{b} = \frac{-a}{b} - \sqrt{3} \text{ and } 1 + \frac{a}{b}\sqrt{3} = \frac{-a}{b} + \sqrt{3}$$

$$b - \sqrt{3}a = -a - \sqrt{3}b \text{ and } b + a\sqrt{3} = -a + b\sqrt{3}$$

$$a(1 - \sqrt{3}) = b(-\sqrt{3} - 1) \text{ and } a(\sqrt{3} + 1) = b(\sqrt{3} - 1)$$

$$\frac{a}{b} - \frac{1 - \sqrt{3}}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{2} = 2 - \sqrt{3}$$

or

$$\frac{a}{b} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = -2 - \sqrt{3}$$

$$\therefore \text{Required lines are } \frac{y}{x} = \sqrt{3} \pm 2 \text{ or } y = (\sqrt{3} \pm 2)x$$

Q2

Let the required equation be $y = mx + c$

But, $c = 0$ as it passes through origin $(0, 0)$

$\therefore$ Equation of the lines is $y = mx$.

Slope of $x + y + \sqrt{3}y = \sqrt{3}x = a$

or $(\sqrt{3} + 1)x + (1 - \sqrt{3})y = a$ is

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}.$$

The angle between $x + y + \sqrt{3}y - \sqrt{3} = a$ and $y = mx$ is 75°

$$\tan(75^\circ) = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$$

$$\tan(30^\circ + 45^\circ) = \frac{m \pm (2 - \sqrt{3})}{1 - m(2 - \sqrt{3})}$$

$$\frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1} = \frac{m \pm 2 - \sqrt{3}}{1 - m(2 - \sqrt{3})}$$

$$2 + \sqrt{3} = \frac{m + 2 - \sqrt{3}}{1 + m(\sqrt{3} - 2)} \quad \text{and} \quad 2 + \sqrt{3} = \frac{m + \sqrt{3} - 2}{1 + m(2 - \sqrt{3})}$$

$$\therefore \frac{1}{m} = 0 \quad \text{or} \quad m = -\sqrt{3}$$

$$\therefore y = mx \quad y = -\sqrt{3}x \quad \text{and} \quad x = 0 \quad \text{are the required equations.}$$



Q3

Given equation is $6x + 5y - 8 = 0$.

Slope of given line = $m = -\frac{6}{5}$

Equations of required line is,

$$y + 1 = \frac{-\frac{6}{5} - 1}{1 + \frac{6}{5}}(x - 2)$$

$$y + 1 = \frac{-11}{11}(x - 2)$$

$$y + 1 = -x + 2$$

$$x + y - 1 = 0$$

$$y + 1 = \frac{-\frac{6}{5} + 1}{1 - \frac{6}{5}}(x - 2)$$

$$y + 1 = \frac{-1}{-1}(x - 2)$$

$$y + 1 = x - 2$$

$$x - y = 3$$



Q4

The required equation is

$$y - k = m'(x - h)$$

And this line is inclined at $\tan^{-1} m$ to straight line $y = mx + c$.

$$\text{Slope} = m = \tan \theta$$

Passing through (h, k)
 (x_1, y_1)

∴ Equation of line is

$$y - y_1 = m(x - x_1) \quad \text{---(i)}$$

$$\text{Also, } \tan \theta = \left| \frac{m - m'}{1 + mm'} \right|$$

Here, $m = m'$

$$\begin{aligned} \therefore \tan \theta &= \frac{m - m}{1 + m^2} \text{ or } \left| \frac{-m - m}{1 - m^2} \right| \\ &= 0 \text{ or } \frac{+2m}{1 - m^2} \end{aligned}$$

Substituting in (i)

$$y - k = 0$$

$$\Rightarrow y = k \quad \text{or}$$

$$y - k = \frac{+2m}{1 - m^2} (x - h)$$

$$(1 - m^2)(y - k) = +2m(x - h)$$





Q5

Here, $x_1 = 2$, $y_1 = 3$, $\alpha = 45^\circ$

$m = \text{slope of line } 3x + y - 5 = 0$

$$= \frac{-\text{coeff of } x}{\text{coeff of } y} = -3$$

The equations of the required line are

$$y - y_1 = \frac{-3 - \tan 45^\circ}{1 + (-3) \tan 45^\circ} (x - 2)$$

$$y - 3 = \frac{-3 - 1}{1 + (-3)(1)} (x - 2)$$

$$y - 3 = \frac{-4}{2} (x - 2) = 2x - 4$$

$$2x - y - 1 = 0$$

Also, $y - 3 = \frac{-3 + \tan 45^\circ}{1 - (-3) \tan 45^\circ} (x - 2)$

$$y - 3 = \frac{-3 + 1}{1 + 3} (x - 2)$$

$$y - 3 = \frac{-2}{4} (x - 2) = \frac{-x}{2} + 1$$

$$x + 2y - 8 = 0$$



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Q6

Let the isosceles right triangle be.

$$AC = 3x + 4y = 4$$

$$c(2, 2)$$

$$\text{Then, slope of } AC = \frac{-3}{4}$$

$$AB = BC$$

[$\because$ It is an isosceles right triangle]

Then, angle between $(AB \text{ and } AC)$ and $(BC \text{ and } AC)$ is 45° .

$$\tan \frac{\pi}{4} = \frac{m_1 - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-3}{4}\right)m_1} \quad [\text{when } m_1 = \text{slope of } BC]$$

$$1 = \frac{m_1 + \frac{3}{4}}{1 - \frac{3}{4}m_1}$$

$$4 - 3m_1 = 4m_1 + 3$$

$$7m_1 = 1 \quad m_1 = \frac{1}{7}$$

and, $AB \perp BC$

$$\therefore (\text{slope of } AB) \times (\text{slope of } BC) = -1$$

$$m_2 \times \frac{1}{7} = -1$$

$$m_2 = -7.$$

The equation of BC is

$$(y - 2) = \frac{1}{7}(x - 2)$$

$$7y - 14 = x - 2$$

$$x - 7y + 12 = 0$$

and

The equation of AB is

$$(y - 2) = -7(x - 2)$$

$$y - 2 = -x + 14$$

$$y + 7x - 16 = 0$$

Q7

Let $C(2 + \sqrt{3}, 5)$ be one vertex and $x = y$ be the opposite side of equilateral triangle ABC .

The other two sides makes an angle of 60° with other two sides.
slope of $x - y = 0$ is 1.

$$\therefore y - 5 = \frac{1 \pm \tan 60^\circ}{1 \mp \tan 60^\circ} (x - 2 - \sqrt{3})$$

$$y - 5 = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} (x - 2 - \sqrt{3}) \text{ and } y - 5 = \frac{1 - \sqrt{3}}{1 + \sqrt{3}} (x - 2 - \sqrt{3})$$

$$y - 5 = (\sqrt{3} - 2)(x - 2 - \sqrt{3}) \text{ and } y - 5 = (\sqrt{3} + 2)(x - 2 - \sqrt{3})$$

$$y + (2 + \sqrt{3})x = 12 + 4\sqrt{3} \text{ and } y + (2 - \sqrt{3})x = 6$$

Hence proved the 2nd side of $\triangle ABC$ is $y + (2 - \sqrt{3})x = 6$

and the 3rd side is $y + (2 + \sqrt{3})x = 12 + 4\sqrt{3}$.

Q8

Let $ABCD$ be a square whose diagonal BD is $4x + 7y = 12$

Then, slope of $BD = \frac{-4}{7}$

Let slope of $AB = m$

$$\text{Then, } \tan 45^\circ = \frac{m + \frac{-4}{7}}{1 - \frac{4}{7}m}$$

$$7 - 4m = 7m + 4$$

$$11m = 3$$

$$\therefore m = \frac{3}{11}$$

$$\therefore \text{Slope of } BC = \frac{-1}{\text{slope of } AB}$$

$$= \frac{-11}{3}$$

Equation of AB is

$$(y - 2) = \frac{3}{11}(x - 1)$$

$$11y - 22 = 3x - 3$$

$$3x - 11y + 19 = 0$$

and

Equation of BC is

$$(y - 2) = \frac{-11}{3}(x - 1)$$

$$11x + 3y - 17 = 0$$



Q9

AC and BC are inclined to $\{AB\}x + y = 0$ at an angle of 60° .

$\therefore \triangle ABC$ is equilateral triangle.

The slope of AB is -1 and let slope of AC be m_1

$$\tan 60^\circ = \frac{m_1 + 1}{1 - m_1} \quad \text{or} \quad \sqrt{3}(1 - m_1) = m_1 + 1$$

$$\sqrt{3} - 1 = m_1 + \sqrt{3}m_1$$

$$\Rightarrow m_1 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$$

and, slope of BC is m_2

$$\tan 60^\circ = \frac{m_2 - 1}{1 + m_1} = \sqrt{3}$$

$$\therefore m_2 = \sqrt{3} + 2$$

$\therefore$ Equations of AC and BC are

$$y - 2 = (2 - \sqrt{3})(x - 1) \quad \text{--- (i)}$$

$$y - 2 = (2 + \sqrt{3})(x - 1)$$

using (i) and $x + y = 0$

$$A \text{ is } \left(\frac{-1 - \sqrt{3}}{2}, \frac{1 + \sqrt{3}}{2} \right)$$

$$AC \text{ is } \sqrt{\left(\frac{2 + 1 + \sqrt{3}}{2} \right)^2 + \left(\frac{3 - \sqrt{3}}{2} \right)^2}$$

$$= \sqrt{\frac{9 + 3 + 6\sqrt{3} + 9 + 3 - 6\sqrt{3}}{4}}$$

$$AC = \sqrt{\frac{24}{4}}$$

$$= \sqrt{6}$$

The area of $\triangle ABC$

$$= \frac{\sqrt{3}}{4} (AC)^2$$

$$= \frac{\sqrt{3}}{4} \times (\sqrt{6})^2$$

$$= \frac{3}{2} \sqrt{3} \text{ sq units.}$$

Q10

Solving $7x - y + 3 = 0$ and $x + y - 3 = 0$ we get, $A(0, 3)$

The slope of $7x - y + 3 = 0$ (m_1) and $x + y - 3 = 0$ (m_2) are 7 and -1 respectively.

Any line through the point $(1, -10)$ is

$$y + 10 = m(x - 1) \quad \text{---(i)}$$

Since it make equal angle say θ with the given lines, therefore

$$\tan \theta = \frac{m - 7}{1 + 7m} = -\frac{m - (-1)}{1 + m(-1)}$$

$$\Rightarrow m = -3 \text{ or } \frac{1}{3}$$

Putting in (i)

$$y + 10 = -3(x - 1)$$

$$y + 10 = -3x + 3$$

$$3x + y + 7 = 0$$

$$y + 10 = \frac{1}{3}(x - 1) \Rightarrow \frac{x}{3} - \frac{1}{3}$$

$$3y - x + 31 = 0$$

Q11

The distance from $(3, -5)$ to the line $2x + 3y - 7 = 0$ is

$$\begin{aligned} & \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|2(3) + 3(-5) - 7|}{\sqrt{(2)^2 + (3)^2}} \\ &= \frac{|6 - 15 - 7|}{\sqrt{13}} \\ &= \frac{16}{\sqrt{13}} \quad \text{---(i)} \end{aligned}$$

Also distance of $(3, -5)$ from the second line $5x - 3y - 12 = 0$

$$\begin{aligned} & \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|5(3) + 3(-5) - 12|}{\sqrt{(5)^2 + (3)^2}} \\ &= \frac{|15 - 15 - 12|}{\sqrt{34}} \\ &= \frac{12}{\sqrt{34}} \quad \text{---(ii)} \end{aligned}$$

Now $C_1 - C_2 = 12 - 7 = 5$

Also difference between (i) and (ii) is 5

$(3, -5)$ lies between the two lines equation of line through $(3, -5)$ cutting the lines at 45° is

$$\begin{aligned} \tan \theta &= \frac{m_1 - m_2}{1 + m_1 m_2} \\ \tan 45^\circ &= \frac{m - \left(-\frac{2}{3}\right)}{1 + \frac{2}{3}m} = \pm 1 \end{aligned}$$

$$m + \frac{2}{3} = 1 - \frac{2}{3}m \quad \text{or} \quad m + \frac{2}{3} = -1 + \frac{2}{3}m$$

$$m\left(1 + \frac{2}{3}\right) = 1 - \frac{2}{3} \quad \text{or} \quad m\left(1 - \frac{2}{3}\right) = -1 - \frac{2}{3}$$

Q12

The slope of $AB = -1$

Let slope of AC be m

Then,

$$\tan 60^\circ = \frac{m+1}{1-m}$$

$$m = 2 - \sqrt{3}$$

And similarly slope of $AB = 2 + \sqrt{3}$.

Equation of AC and AB are

$$(y+1) = (2-\sqrt{3})(x-2)$$

$$\text{or, } (2-\sqrt{3})x - y - 5 + 2\sqrt{3} = 0 \quad \text{---(i)}$$

and,

$$(y-1) = (2+\sqrt{3})(x-2)$$

$$\text{or, } (2+\sqrt{3})x - y - 5 - 2\sqrt{3} = 0 \quad \text{---(ii)}$$

On solving (i) with $x+y=2$, we get

$$A \left(\frac{21-11\sqrt{3}}{6}, \frac{11\sqrt{3}-9}{6} \right)$$

$$AB = AC = BC$$

$$= \sqrt{\left(\frac{21-11\sqrt{3}-1}{6} \right)^2 + \left(\frac{11\sqrt{3}-9-1}{6} \right)^2}$$

$$= \sqrt{\frac{225 + 363 - 330\sqrt{3} + 363 + 225 - 330\sqrt{3}}{36}}$$

$$= \sqrt{\frac{2}{3}}$$

Q13

Let $A(1, 2)$, $C(5, 8)$, $B(x_1, y_1)$, $D(x_2, y_2)$

$$\text{Slope of } AC = \frac{8-2}{5-1} = \frac{6}{4} = \frac{3}{2}$$

Let m be the slope of a line making an angle 45° with AC

$$\therefore \tan 45^\circ = \left| \frac{m_1 - \frac{3}{2}}{1 + m \times \frac{3}{2}} \right|$$

$$1 = \frac{m - \frac{3}{2}}{1 + \frac{3m}{2}}$$

$$1 + \frac{3m}{2} = m - \frac{3}{2} \quad \text{or,} \quad 1 + \frac{3m}{2} = -\left(m - \frac{3}{2}\right)$$

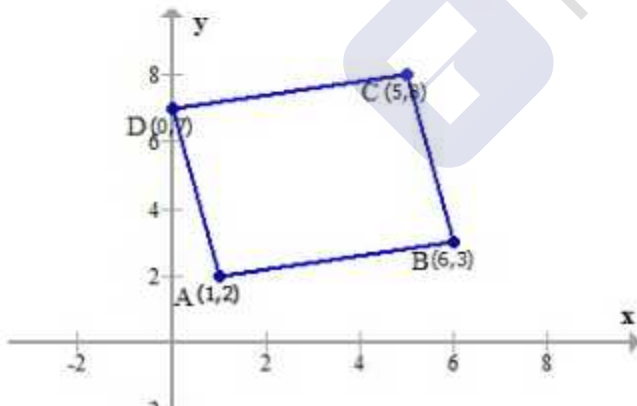
$$\frac{3m}{2} - m = -\frac{3}{2} - 1 \quad \text{or,} \quad 1 + \frac{3m}{2} = -m + \frac{3}{2}$$

$$\frac{1}{2}m = -\frac{5}{2} \quad \text{or,} \quad \frac{3m}{2} + m = \frac{3}{2} - 1$$

$$m = -5 \quad \text{or,} \quad \frac{5m}{2} = \frac{1}{2}$$

$$m = \frac{1}{5}$$

Consider the following figure:





Ex 23.19

Q1

Line through the intersection of $4x - 3y = 0$ and $2x - 5y + 3 = 0$ is

$$(4x - 3y) + \lambda(2x - 5y + 3) = 0 \quad \text{--- (i)}$$

$$\text{or, } x(4 + 2\lambda) - y(3 + 5\lambda) + 3\lambda = 0$$

And the required line is parallel to $4x + 5y + 6$

$$\therefore \text{ slope of required} = \text{slope of } 4x + 5y + 6 = \frac{-4}{3}$$

$$\therefore \frac{-(4 + 2\lambda)}{-(3 + 5\lambda)} = \frac{-4}{3}$$

$$\Rightarrow 5(4 + 2\lambda) = -4(3 + 5\lambda)$$

$$\Rightarrow 20 + 10\lambda = -12 - 20\lambda$$

$$\Rightarrow 30\lambda = -32$$

$$\Rightarrow \lambda = \frac{-16}{15}$$

Putting λ in equation (i)

$$(4x - 3y) - \frac{16}{15}(2x - 5y + 3) = 0$$

$$\Rightarrow 60x - 45y - 32x + 80y - 48 = 0$$

$$\Rightarrow 28x + 35y - 48 = 0$$

Is the required line

Q2

The equation of the required line is

$$(x + 2y + 3) + \lambda(3x - 4y + 7) = 0$$

$$\text{or, } x(1 + 3\lambda) + y(2 - 4\lambda) + 3 + 7\lambda = 0$$

$$m_1 = \text{slope of the line} = -\left(\frac{1 + 3\lambda}{2 - 4\lambda}\right)$$

The line is perpendicular to $x - y + 9 = 0$ whose slope $(m_2 = 1)$

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow -\left(\frac{1 + 3\lambda}{2 - 4\lambda}\right) \times 1 = -1$$

$$\Rightarrow 1 + 3\lambda = 2 - 4\lambda$$

$$\Rightarrow \lambda = -1$$

$\therefore$ The required line is

$$x + 2y + 3 - (3x - 4y + 7) = 0$$

$$-2x - 2y - 4 = 0$$

$$\text{or, } x + y + 2 = 0$$

Q3

The required line is

$$2x - 7y + 11 + \lambda(x + 3y - 8) = 0$$

$$\text{or, } x(2 + \lambda) + y(-7 + 3\lambda) + 11 - 8\lambda = 0$$

(i) When the line is parallel to x-axis. Its slope is 0

$$\therefore -\frac{(2 + \lambda)}{3\lambda - 7} = 0$$

$$\lambda = -2$$

$\therefore$ Equation of line is

$$2x - 7y + 11 - 2(x + 3y - 8) = 0$$

$$-13y + 27 = 0$$

(ii) When the line is parallel to y-axis then,

$$\frac{-1}{\text{slope}} = 0$$

$$\text{i.e. } \frac{3\lambda - 7}{2 + \lambda} = 0$$

$$\lambda = \frac{7}{3}$$

$\therefore$ Equation of line is

$$2x - 7y + 11 + \frac{7}{3}(x + 3y - 8) = 0$$

$$\Rightarrow \frac{6x - 21y + 33 + 7x + 21y - 56}{3} = 0$$

$$\Rightarrow 6x - 21y + 33 + 7x + 21y - 56 = 0$$

$$\Rightarrow 13x - 23 = 0$$

$$\Rightarrow 13x = 23$$

Q4

The required line is

$$(2x + 3y - 1) + \lambda(3x - 5y - 5) = 0$$

$$\text{or, } x(2 + 3\lambda) + y(3 - 5\lambda) - 1 - 5\lambda = 0$$

Since this line is equally inclined to both the axes, its slope should be 1 or -1

$$\begin{aligned} \therefore \frac{-2 - 3\lambda}{3 - 5\lambda} &= 1 & \text{or, } \frac{-2 - 3\lambda}{3 - 5\lambda} &= -1 \\ \Rightarrow 3 - 5\lambda &= -2 - 3\lambda & \text{or, } \Rightarrow -2 - 3\lambda &= -3 + 5\lambda \\ \Rightarrow 5 &= 2\lambda & \text{or, } \Rightarrow 1 &= 8\lambda \\ \Rightarrow \lambda &= \frac{5}{2} & \text{or, } \Rightarrow \lambda &= \frac{1}{8} \end{aligned}$$

$\therefore$ The required line is

$$2x + 3y + 1 + \frac{5}{2}(3x - 5y - 5) = 0$$

$$4x + 6y + 2 + 15x - 25y - 25 = 0$$

$$19x - 19y - 23 = 0$$

or

$$(2x + 3y + 1) + \frac{1}{8}(3x - 5y - 5) = 0$$

$$16x + 24y + 8 + 3x - 5y - 5 = 0$$

$$19x + 19y + 3 = 0$$

$\therefore$ The two possible equations are

$$19x - 19y - 23 = 0 \quad \text{or} \quad 19x + 19y + 3 = 0$$

Q5

The required line is

$$(x + y - 4) + \lambda(2x - 3y - 1) = 0$$

$$\text{or, } x(1 + 2\lambda) + y(1 - 3\lambda) - 4 - \lambda = 0$$

$$\text{And it is perpendicular to } \frac{x}{5} + \frac{y}{6} = 1$$

$$\therefore (\text{slope of required line}) \times (\text{slope of } \frac{x}{5} + \frac{y}{6} = 1) = -1$$

$$\Rightarrow -\left(\frac{1 + 2\lambda}{1 - 3\lambda}\right) \times \frac{-6}{5} = -1$$

$$\Rightarrow \frac{1 + 2\lambda}{1 - 3\lambda} = \frac{-5}{6}$$

$$\Rightarrow 6 + 12\lambda = -5 + 15\lambda$$

$$\Rightarrow 11 = 3\lambda \quad \text{or } \lambda = \frac{11}{3}$$

$\therefore$ The required line is

$$(x + y - 4) + \frac{11}{3}(2x - 3y - 1) = 0$$

$$3x + 3y - 12 + 22x - 33y - 11 = 0$$

$$25x - 30y - 23 = 0$$

Q6

$$x(1 + \lambda) + y(2 - \lambda) + 5 = 0$$

$$\Rightarrow x + x\lambda + 2y - \lambda y + 5 = 0$$

$$\Rightarrow \lambda(x - y) + (x + 2y + 5) = 0$$

$$\Rightarrow (x + 2y + 5) + \lambda(x - y) = 0$$

This is of the form $L_1 + \lambda L_2 = 0$

So it represents a line passing through the intersection of $x - y = 0$ and $x + 2y = -5$.

Solving the two equations, we get $\left(\frac{-5}{3}, \frac{-5}{3}\right)$ which is the fixed point through which the given family of lines passes for any value of λ .



Q7

$$(2+k)x + (1+k)y = 5 + 7k$$

$$\text{or, } (2x + y - 5) + k(x + y - 7) = 0$$

It is of the form $L_1 + kL_2 = 0$ i.e., the equation of line passing through the intersection of 2 lines L_1 and L_2 .

So, it represents a line passing through $2x + y - 5 = 0$ and $x + y - 7 = 0$.

Solving the two equation we get, $(-2, 9)$. Which is the fixed point through which the given line pass. For any value of k .

Q8

$L_1 + \lambda L_2 = 0$ is the equation of line passing through two lines. L_1 and L_2 .

$$\therefore (2x + y - 1) + \lambda(x + 3y - 2) = 0 \text{ is the required equation.} \quad \text{---(i)}$$

$$\text{or, } x(2 + \lambda) + y(1 + 3\lambda) - 1 - 2\lambda = 0$$

$$\frac{x}{\frac{1+2\lambda}{2+\lambda}} + \frac{4}{\frac{1+2\lambda}{1+3\lambda}} = 1$$

$$\text{Area of } \Delta = \frac{1}{2} \times OB \times OA$$

$$\frac{8}{3} = \frac{1}{2} \times (y \text{ intercept}) \times (x \text{ intercept})$$

$$\frac{8}{3} = \frac{1}{2} \times \left(\frac{1+2\lambda}{1+3\lambda} \right) \times \left(\frac{1+2\lambda}{2+\lambda} \right)$$

$$\frac{16}{3} = \frac{1+4\lambda^2+4\lambda}{2+3\lambda^2+7\lambda}$$

$$32 + 48\lambda^2 + 112\lambda = -3 - 12\lambda^2 - 12\lambda$$

$$60\lambda^2 + 124\lambda + 35 = 0$$

$$\lambda = \frac{-124 \pm \sqrt{(124)^2 - 4 \times 60 \times 35}}{2 \times 60}$$

$$= \frac{-124 \pm \sqrt{15376 - 8400}}{120}$$

Approximately = 1

$$\therefore \text{Substituting in (i)} \Rightarrow 3x + 4y - 3 = 0, 12x + y - 3 = 0$$

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Q9

The required line is

$$(3x - y - 5) + \lambda(x + 3y - 1) = 0$$

$$\text{or, } (3 + \lambda)x + (-1 + 3\lambda)y - 5 - \lambda = 0$$

$$\text{or, } \frac{x}{\left(\frac{5+\lambda}{3+\lambda}\right)} + \frac{y}{\frac{5+\lambda}{3\lambda-1}} = 1$$

And the line makes equal and positive intercepts with the line (given)

$$\therefore \frac{5+\lambda}{3+\lambda} = \frac{5+\lambda}{3\lambda-1}$$

$$3\lambda - 1 = 3 + \lambda$$

$$2\lambda = 4$$

$$\lambda = 2$$

$\therefore$ The required line is

$$3x - y - 5 + 2x + 6y - 2 = 0$$

$$\text{or, } 5x + 5y = 7$$

Q10

The required line is

$$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$$

$$\text{or, } (1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$$

Distance from origin of this line is

$$\left| \frac{(1+2\lambda) \cdot 0 + (-3+5\lambda) \cdot 0 + 1-9\lambda}{\sqrt{(1+2\lambda)^2 + (5\lambda-3)^2}} \right| \quad \left[\text{using } \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right]$$

$$\sqrt{5} = \left| \frac{1-9\lambda}{\sqrt{1+4\lambda^2-4\lambda+25\lambda^2+9-30\lambda}} \right|$$

$$\Rightarrow \sqrt{5} = \left| \frac{1-9\lambda}{\sqrt{26\lambda^2-26\lambda+10}} \right|$$

$$\Rightarrow \sqrt{(10+26\lambda^2-26\lambda)} = (1-9\lambda)^2$$

$$\Rightarrow 50 + 145\lambda^2 - 130\lambda = 1 - 31\lambda^2 - 18\lambda^2$$

$$\Rightarrow 54\lambda^2 - 112\lambda + 49 = 0$$

$$\Rightarrow (8\lambda - 7)^2 = 0 \quad \text{or, } \lambda = \frac{7}{8}$$

$\therefore$ Required line is

$$x - 3y + 1 + \frac{7}{8}(2x + 5y - 9) = 0$$

$$\Rightarrow 8x - 24y + 8 + 14x - 35y - 63 = 0$$

$$\Rightarrow 22x + 11y - 55 = 0$$

$$\Rightarrow 2x + y - 5 = 0$$

Q11

Solving two equations of lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ we get, intersection point (2,3).

Let equation of a line passing through (2,3) be $y = mx + c$

$$\therefore 3 = 2m + c$$

$$c = 3 - 2m$$

Equation of the line is $y = mx + 3 - 2m \dots (1)$

Perpendicular distance of above line from (3,2) = $\frac{7}{5}$

$$\left| \frac{3m - 2 + 3 - 2m}{\sqrt{m^2 + 1}} \right| = \frac{7}{5}$$

$$\left| \frac{m + 1}{\sqrt{m^2 + 1}} \right| = \frac{7}{5}$$

$$\frac{(m + 1)^2}{m^2 + 1} = \frac{49}{25}$$

$$25(m^2 + 2m + 1) = 49m^2 + 49$$

$$25m^2 + 50m + 25 = 49m^2 + 49$$

$$24m^2 - 50m + 24 = 0$$

$$12m^2 - 25m + 12 = 0$$

$$m = \frac{4}{3}, m = \frac{3}{4}$$

Substituting m in (1), we get,

$$y = \frac{4}{3}x + 3 - \frac{2 \times 4}{3}$$

$$3y = 4x + 1$$

$$4x - 3y + 1 = 0$$

$$y = \frac{3}{4}x + 3 - \frac{2 \times 3}{4}$$

$$4y - 3y + 1 = 0$$

Equations of lines are $4x - 3y + 1 = 0$ and $4y - 3x + 1 = 0$